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NATURAL CONVECTION IN A CYLINDRICAL AND DIVERGENT ANNULAR DUCT FITTED WITH FINS

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ABSTRACT

The paper deals with a numerical study of natural convective heat transfer flow in a three dimensional cylindrical and divergent annular duct. The inner cylinder is subjected to a volumetric heat generation and fitted with longitudinal fins. The governing equations of mass, momentum and energy equation for both the fluid and the solid are solved by the finite volume method, using the commercially available CFD software Fluent.The effect of the inclination angle ϕ of the divergent and the fins number N on the profiles and contour fields of temperature and velocity as well as the average Nusselt number were investigated for ϕ=0°, 15°, 23° and 45°, for a number of fins, N=1, 2 and 3. Simulations were carried out for the Rayleigh numbers Ra = $3.1.10^4$ and $6.2.10^4$, corresponding respectively to the volumetric heat generation, Qv = $5.10^5$and 1.10^6 W/m^3. The results reveal that the increasing of the inclination angle and the fins enhance the heat transfer.

Keywords: natural convection, fins, annular space, divergent duct, heat sink, volumetric heat generation

1. INTRODUCTION

Natural convective heat transfer in annular duct has been the object of considerable interest by many researchers because of its importance in various application areas (food, heat exchangers, power stations, turbines, and solar collectors, fluid storage, cooling of electronic device...). Numerical and experimental natural convective heat transfer with different geometrical parameters and boundary conditions have been extensively studied over the last decades. The experimental studies of Elenbaas [1-2] have reported an expression for the optimum spacing for natural convection between parallel plates and other cross sectional shapes and were followed by the first numerical study of Bodoia and Osterle [3], Churchill and Usagi [4]. Natural convection cooling with the help of the finned surfaces often offers an economical and free solution in many situations. Fins are used in a variety of engineering applications to dissipate heat to the surroundings .They also find applications in electronics equipment like a heat sink. A heat sink is a passive heat exchanger that transfers thermal

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energy from a higher temperature device to a lower temperature fluid medium. The most common configurations of naturally cooled heat sinks are vertical and horizontal plates according to their geometry and applications such as plate heat sinks or radial heat sinks. Radial heat sinks are used with high-power semiconductor devices such as power transistors and optoelectronics such as lasers and light emitting diodes (LED) which are used in general illumination and increasingly important in many applications due to their lower power consumption, longer life, smaller and more durable structure.

In a great number of related studies, circular or rectangular base with various fin configurations have been extensively considered by researchers. In the experimental investigation, Starner and Mc Manus [5] measured the average heat transfer coefficient of four heat sinks of differing dimensions oriented vertically, horizontally and inclined at 45°. Toshio [6] presented a study to obtain a reasonable method to predict the characteristics of non-isothermal fin arrays with isothermal base-plates. A good agreement between theoretical predictions and experiments was obtained. Zhang and Faghri [7] studied numerically a stationary flow in pipes with internal fins to enhance the heat transfer of PCM system. They showed that a small Reynolds number and a thermal conductivity of fins can be a very effective manner to increase the heat transfer. Ranganathan [8] obtained that the heat transfer increased for high radius ratios and fins number and decreased by decreasing the aspect ratio, the angle and the thickness of the fins. Bocu and Altak [9] investigated the effect of pin number, length, diameter and configuration to enhance the heat transferring three-dimensional rectangular enclosures with pin arrays attached to the hot wall. Byoung et al. [10], in their experimental investigation in natural convection from cylinders with vertically oriented plate fins, proposed a correlation for the purpose of estimating the Nusselt number at Ra in the range 10^5 and $6.10^5$and for limited fins number, high and cylinder diameter ratio. Seung-HwanYu et al. [11-12] investigated numerically and experimentally a radial heat sink composed of a horizontal circular base and rectangular fins in natural convection. Their parametric studies were performed in the purpose to compare the effects of three geometric parameters as well as the radiation on the total heat transfer. The experimental investigations for natural convection in Refs [13-14], were focused in the proposal of empirical correlations to estimate the Nusselt number from cylinders with plate, triangular and vertically oriented plate fins. In the numerical and experimental study of Daeseok et al. [15], they optimized the cooling performances and mass of a pin-fin radial heat sink. The aim of this work was to study the heat transfer enhancement in the annular space of thin cylinder immersed in a cylindrical or divergent tube. The divergent configuration was provided to enhance thermal performance of a heat transfer device. The inner cylinder fitted with longitudinal fins was subjected to a volumetric heat generation, while the opposite and inclined wall of the outer cylinder was assumed adiabatic. Much attention has not been given to natural convection in divergent duct. Literature concerning this geometry shows that works in this configuration are few and have not been yet explored. The objective of the present work is to investigate the effect of the inclination angle of the divergent (ϕ), the number of fins (N) and the Rayleigh number (Ra) on the structure of the fluid flow and the rate of heat transfer.

PROBLEM DESCRIPTION

A perspective view of the physical model considered in this study is shown in Figure 1. It is a flow in a three-dimensional annular space between two concentric vertical cylinders. The inner cylinder subjected to a volumetric heat generation is provided with longitudinal fins distributed on the external wall, while the outer cylinder which is inclined is supposed adiabatic. The three-dimensional configuration was defined by exploiting the symmetry of the duct. A vertical symmetry plan allows reducing the computational domain to the total half-volume. The fluid is entering by the bottom of the duct with initial velocity w_0 and at ambient temperature T_0. The top of the annular space is open to the atmospheric pressure Figure 2.

The geometric parameters are fixed throughout the parametric study. The dimensions considered here are as follows: The length of the duct L = 50mm, the radius of the inner cylinder R_{ii} = 3mm, the radius of the outer cylinder R_{oi} = 10mm.The fluid, air (the Prandtl number Pr = 0.71) is brought to initial temperature T_0 = 298 K.

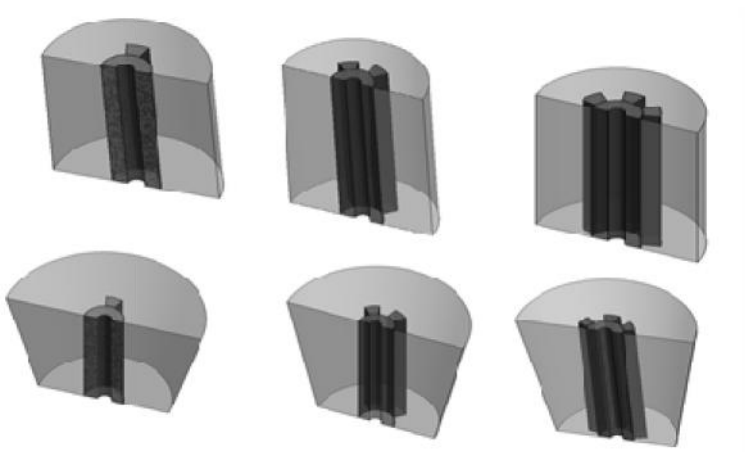


Figure 1. Studied configurations.

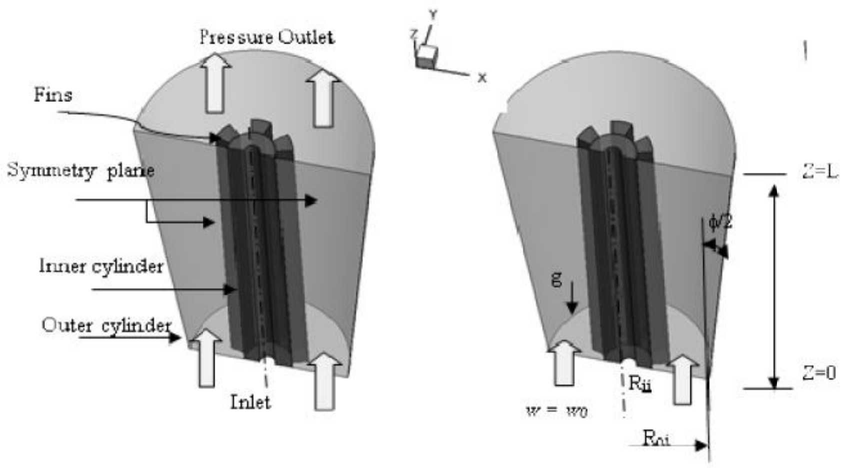


Figure 2. Schematic view of the physical problem for N=3.

3. Mathematical Formulation

The flow and temperature distributions are governed by continuity, Navier-Stokes, fluid, and solid energy equations. The flow is laminar and steady; the fluid is Newtonian and incompressible, with constant properties, except in the term of gravity where the Boussinesq assumption is adopted. The thermo physical properties of the fluid (C_p, k_f, ν, ρ and β) are constant and evaluated at inlet temperature. Radiation transfers and viscous dissipations are supposed negligible. Considering of the simplifying assumptions formulated above, the governing equations in cylindrical coordinates in directions (r, θ, z) are as follows:

3.1. Governing Equations

Continuity equation

$$\frac{1}{r}\frac{\partial(ru)}{\partial r}+\frac{1}{r}\frac{\partial v}{\partial \theta}+\frac{\partial w}{\partial z}=0 \tag{1}$$

r-momentum equation

$$\begin{aligned}\rho\left(\frac{\partial u}{\partial r}+\frac{v}{r}\frac{\partial u}{\partial \theta}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}\right)&=-\frac{\partial p}{\partial r}+\\ \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(ru)\right)+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial v}{\partial \theta}+\frac{\partial^2 u}{\partial z^2}\right]\end{aligned} \tag{2}$$

Θ-momentum equation

$$\begin{aligned}\rho\left(u\frac{\partial v}{\partial r}+\frac{v}{r}\frac{\partial v}{\partial \theta}+w\frac{\partial v}{\partial z}+\frac{uv}{r}\right)&=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\\ \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv)\right)+\frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial u}{\partial \theta}+\frac{\partial^2 v}{\partial z^2}\right]\end{aligned} \tag{3}$$

z-momentum equation

$$\begin{aligned}\rho\left(u\frac{\partial w}{\partial r}+\frac{v}{r}\frac{\partial v}{\partial \theta}+w\frac{\partial w}{\partial z}\right)&=-\frac{\partial p}{\partial z}+\\ \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}+\frac{\partial^2 w}{\partial z^2}\right]+\rho g\beta(T-T_0)\end{aligned} \tag{4}$$

energy equation

$$\begin{aligned}\frac{1}{r}\frac{\partial(ruT)}{\partial r}+\frac{1}{r}\frac{\partial(vT)}{\partial \theta}+\frac{\partial(wT)}{\partial z}&=\\ \frac{k_f}{(\rho C_p)}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right]\end{aligned}$$

Energy equation for solid wall

$$k_s\left[\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right]=Q_v \tag{6}$$

3.2. Boundary Conditions

The velocity boundary condition is (u =v=w=0) for the walls of the duct and the fins.

- For the fluid at inlet: $T=T_0$.
- For the adiabatic wall of the outer cylinder: at z:(0,L) θ:(0,π) : $\frac{\partial T(z,\theta,R)}{\partial r}=0$
- At interface solid- fluid $-\frac{\partial T}{\partial r}(R_{oi},\theta,z)\Big|_{fluid}=K\frac{\partial T}{\partial r}(R_{oi},\theta,z)\Big|_{solid}$

With $K=\frac{k_s}{k_f}$ the thermal conductivity ratio, defined by the ratio of thermal conductivities of the fluid and the solid wall. For the heated inner cylinder the flux q'' is defined versus the volumetric heat generation Q_v and the volume of the inner cylinder as:

$q''=Q_v.V$, where V is the volume of the inner cylinder

3.3. Calculation of the Nusselt Number

The Nusselt number expressing the relative ratio of the convective and conductive heat transfer is reported at the inner cylinder and at the fin. The local Nusselt number depends on the angular positions (Θ) and (z) is expressed by the following equation as follows:

$$Nu(\theta,z)=\frac{h(\theta,z)R}{k_f}=\frac{R}{k_f}\frac{1}{(T_w(R_f,\theta,z)-T_b(z))}\frac{\partial T}{\partial r}\Big|_{r=R_f} \tag{7}$$

The mean Nusselt number is expressed as:

$$\overline{Nu}=\frac{\int_0^L\int_0^\pi Nu(z,\theta)dzd\theta}{\int_0^L\int_0^\pi d\theta dz} \tag{8}$$

$T_w(R_f,\Theta,z)$ is the local temperature of the fin, R_f is the radius of the fin, Tb (z) is the bulk temperature in the section (r-Θ) and R is the radius of the divergent cylinder.

$$T_b(z) = \frac{\int_0^R \int_0^\pi T(r,\theta,z)\,dr\,d\theta}{\int_0^R \int_0^\pi r\,dr\,d\theta} \tag{9}$$

The characteristic parameter of natural convection is defined by The Rayleigh number as function of the volumetric heat generation Q_v:

$$Ra = \frac{g\beta Q_v \delta^5}{\nu\alpha\, k_f}$$

With Q_v the volumetric heat generation, δ is the gap of annulus at inlet, g, α, ν, β and k_f respectively gravitational acceleration; thermal diffusivity; kinematic viscosity; thermal expansion coefficient and thermal conductivity of the fluid.

4. NUMERICAL PROCEDURE

The continuity, momentum and energy equation are solved using commercially available software Fluent 6.0[16].The simple algorithm proposed by Patankar [17]is used for coupling between pressure and velocity with a power- law approximation scheme. A numerical solution is supposed to converge when the following test is checked.

$$\left|\frac{\left(\Phi^{k+1} - \Phi^{k}\right)}{\Phi^{k+1}}\right|_{max} \leq 10^{-6}$$

i.e., the residues for different physical quantities become more low 10^{-6}. Φ_k are the variables u, v, w, T and p. k is the number of iterations. A non-uniform mesh in axial and radial directions, are refined in regions where temperature variations are relatively important (near the walls). To examine the effect of the mesh on the numerical solution, three meshes have been considered. $(30x99x25)_{nodes}$, $(35x134x50)_{nodes}$, $(35x134x100)_{nodes}$. The grid (35 x 134 x 100) is taken as reference grid to calculate the variations expressed as a percentage. We have considered Rayleigh number Ra = $3.7.10^3$, ϕ=0° and N=3. The Table.1 gives the values of the average Nusselt number ($\overline{Nu}$) and the percentage deviation, of the front level of the fin for different meshes as well as the computing times of each one. From these results, we can see that the values obtained for these different meshes are very close, the mesh size (35 x 134 x 100) gives good results but require a higher execution time compared to the others. Our choice was directed towards the grid (35x134x50) which ensures a good compromise between the computing time and the precision of the results, Figure 3.

Table1. Comparison of $\overline{Nu}$ in axial direction for various grids

Mesh	30x99x25	35x134x50	35x134x100
Time of computing	1h	2h30min	6h30min
$\overline{Nu}$	3,94935	3,96688	3,98983
Deviation $\overline{Nu}$%	0.578	1.012	-

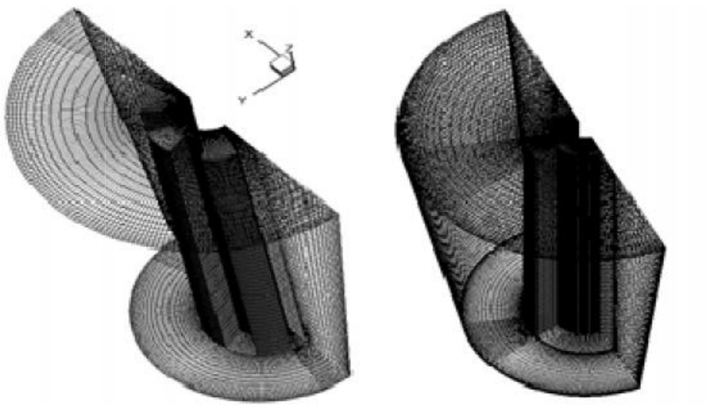


Figure 3. Grid system for ϕ=15°, N=3 and N=1.

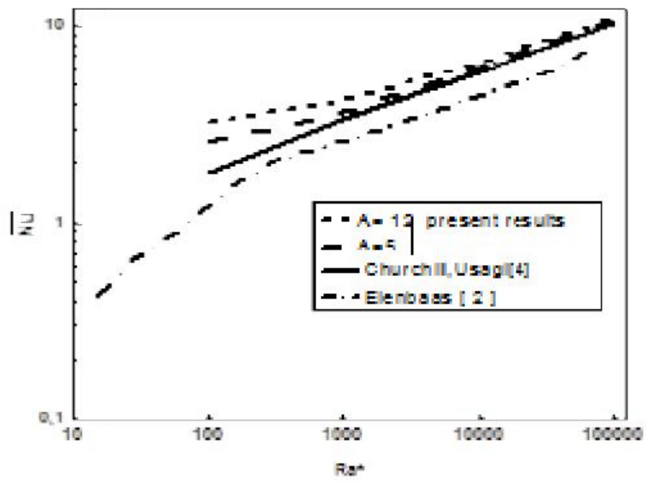


Figure 4. Comparison of the present results with experimental and correlations available in the literature for $\overline{Nu}$ of parallel plates.

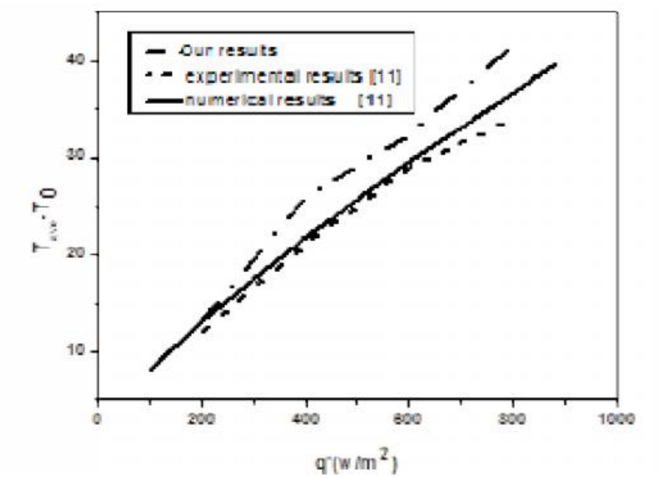


Figure 5. Comparison between the temperature difference of a radial heat sink obtained in the present work and that of obtained by [11].

In order to validate the presented numerical results, comparisons were made with those reported by Elenbaas [2],Churchill and Usagi [4],for parallel plates as first step, with different modified Rayleigh number Ra*($Ra* = \frac{g\beta(T_w - T_0)\delta_{opt}^4}{\alpha\nu L}$),δ_{opt} is the optimum thickness between the plates)and for the aspect ratio A = 5,A = 12(A = δ_{opt}/L). It is seen from Figure 4 that the average Nusselt numbers are in good agreement.

The results are also validated with the experimental and numerical results of Seung-Hwan [11] of a radial heat sink with a number of fins N = 20. Figure 5, gives the results of the temperature differences obtained from the present study for N = 6 in terms of the heat flux applied to the inner cylinder. As can be seen in this figure, the temperature increases with heat flux. The results in the present study compare well with those of Seung-Hwan [11] a difference about 8.29% to 23.79% is found between the experimental results and about 0.34% to 19% between the numerical results.

5. RESULTS AND DISCUSSIONS

This work aims to study the effect of the inclination angle of the outer cylinder as well as the number of fins on the flow and heat transfer characteristics.

The numerical simulations were carried out for wide physical and geometrical parameters like Rayleigh numbers (Ra = $3.1.10^4$ and Ra=$6.2.10^4$) which correspond to the values of the volumetric heat generation (Q_v = 5.10^5 and 1.10^6 W/m^3), the inclination angle of the divergent (ϕ=0°, 15°, 23°and ϕ=45°), the number of fins (N=1, 2, 3). The fluid, air (Pr = 0.71) is brought at temperature T_0 = 298 K.

5.1. Effect of the Inclination Angle

Figures 6.1-6.2, show the distribution of the contour fields of the temperature and the velocity in the stream-wise direction of the flow for N = 1 and for Ra = $3.1.10^4$. The effect of the inclination angle on the temperature and the velocity is visible in this figure. It can be seen that for ϕ=0°, both temperature and velocity contours are symmetric about the median plane, the temperature of the fluid increases in the direction of the main flow and reaches the maximum in the top. The fluid close to the inner cylinder and the fin absorbs heat, so its buoyancy force augments and moves towards up, the cold fluid that is near the right wall of the outer cylinder replaces it, the maximum temperature reaches is about 487K. The fluid layers are concentric circles, which the temperature decreases in moving away from the fin. The velocity for this case presents a maximum in the middle of the duct and a minimum near the walls. The velocity variation in the span-wise direction is unaffected as the flow approaches the exit; further the velocity vectors have a uniform structure in this location as shown in Figure 7. In the cell plume which is near the heated wall, the velocities vectors present a maximum. It is depicted from this Figure that, in the other side of the duct i.e., in the recirculating region; the velocity vectors are diverted and presented by a negative values. For ϕ=15°, 23° and ϕ= 45°, the temperature decreases when ϕ increases that is due to the sudden enlargement at the outlet of the duct and the maximum of the temperature for this case is about 355K for ϕ=15° about 351K for ϕ=23° and 348K for ϕ=45°.

As shown in Figures 6.1-6.2, for ϕ=15° and ϕ=23° the symmetry of the flow is destroyed by the effect of the inclination angle of the divergent for both temperature and velocity fields, in this case the plume has a large size and it rises to the top in the opposite direction of the gravitational acceleration. The effect of the inclination angle of the divergent on the temperature and velocity profiles in direction x is illustrated in Figures 8.1-8.2. Three values of the temperature and the velocity are monitored, at inlet, middle and at the outlet of the duct. As shown in these figures, the profiles of temperature and velocity are symmetrical for ϕ=0°, the maximum temperature is at the outlet, while the maximum velocity is at the inlet near the inner cylinder.

For ϕ>0° the symmetry is destroyed for both temperature and velocity due to the sudden expansion of the divergent duct and the cell plume showed in Figures 6.1-6.2, is presented by the maximum velocity and temperature in the left and right sides at outlet, the velocity profiles are not uniform in all locations.

Figure 9 depicts, the local Nusselt number along the axial direction for N=1 and the Rayleigh number and the inclination angle cited above. The Nusselt number is very large at the inlet; it decreases with increasing the axial distance approaching the fully developed value asymptotically. It is observed too, that the increasing of the inclination angle causes enhancement of the local Nusselt number.

5.2. Effect of the Fins

The effects of the number of fins N and the inclination angle ϕ on the temperature and the velocity profiles at the outlet of the duct in radial direction is depicted in Figures 10,11 and 12. For Ra=$6.2.10^4$, as it can be noticed in this figure, the maximum temperature is near the front wall of the fin and reaches the value of 668.19K for N=1 and ϕ=0°, while it is about

635.18K for N=2 and 616.52K for N=3. This temperature decreases with the increasing of ϕ and Ra for both N=1, = 2 and N=3. The velocity profiles show that the velocity is zero at the level of the fin and increases towards the center of the annular space. For a single fin the velocity increases with the increasing of ϕ also the symmetry is destroyed for ϕ=15°, 23° and ϕ=45°.

The higher velocities are seen for N=1 and N=3 at ϕ=45° so what are for N=2 at ϕ=23°. Figure 13, depicts the variation of the average Nusselt number versus the fins number for Ra=3.1.10^4. From this figure, it is observed that the mean Nusselt number is a decreasing function of the fin numbers for the inclination angles studied and the inclination angle ϕ=45°gives more heat transfer. The heat transfer results are correlated in the following form as: $\overline{Nu} = A.N^{B}$ where A and B are constants which vary with ϕ and N in the form:

For ϕ=0°, $\overline{Nu} = 5.43N^{-0.231}$, For ϕ=15°, $\overline{Nu} = 6.46N^{-0.2153}$

For ϕ=23°, $\overline{Nu} = 7.681N^{-0.1647}$, For ϕ=45°, $\overline{Nu} = 10.925N^{-0.0966}$

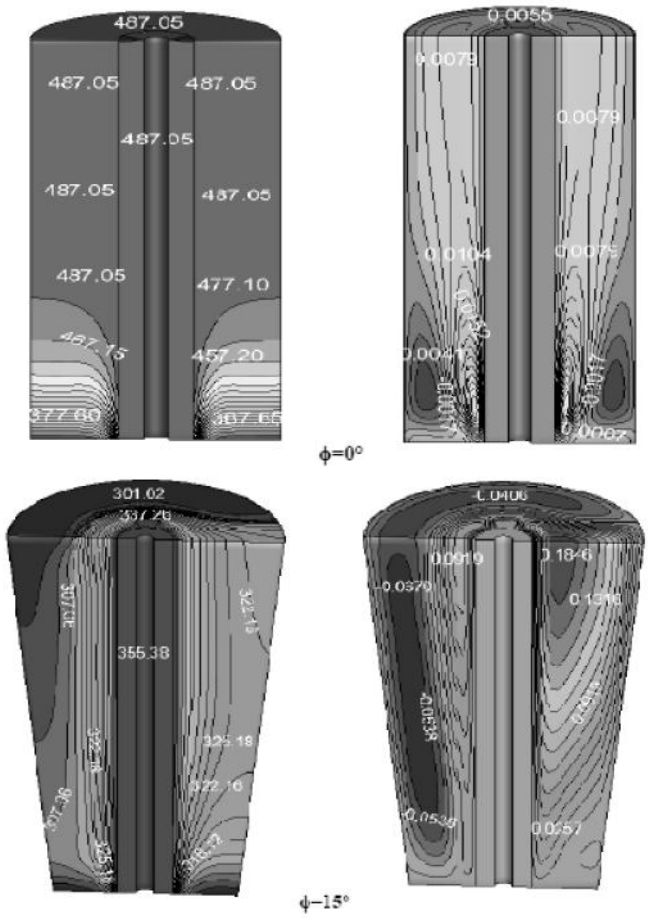


Figure 6.1. Effect of the inclination angle on the contour fields of the temperature (On the left) and the velocity (on the right) for N=1, Ra=3.1.10^4, ϕ=0° and ϕ=15°.

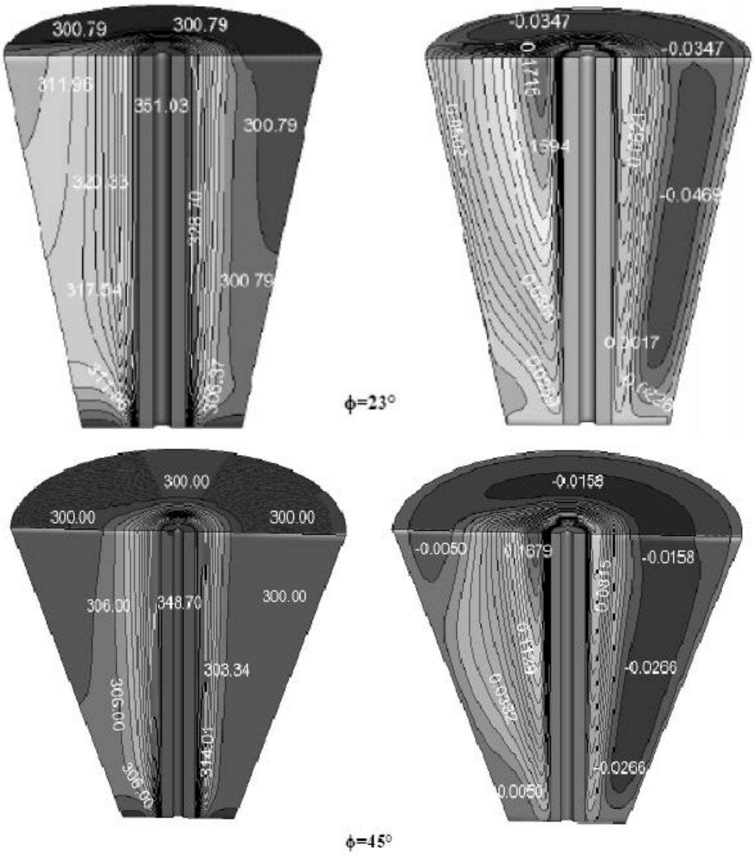


Figure 6.2. Effect of the inclination angle on the contour fields of the temperature (On the left) and the velocity (on the right) for N=1, Ra=3.1.10^4, ϕ=23° and ϕ=45°.

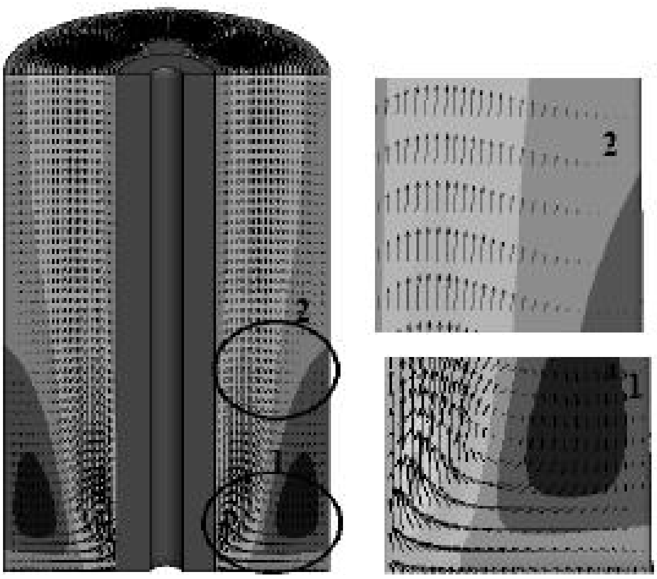


Figure7. Velocity vectors for ϕ=0°, N=1.

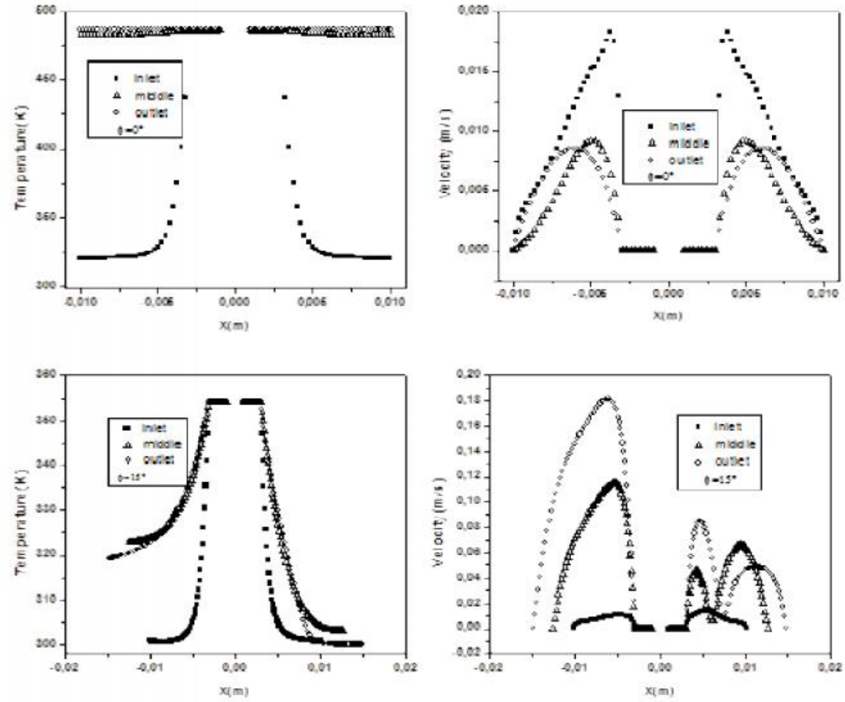


Figure8.1. Temperature and velocity profiles in different locations of the duct, for N=1, Ra=3.1.10^4, ϕ=0° and ϕ=15°.

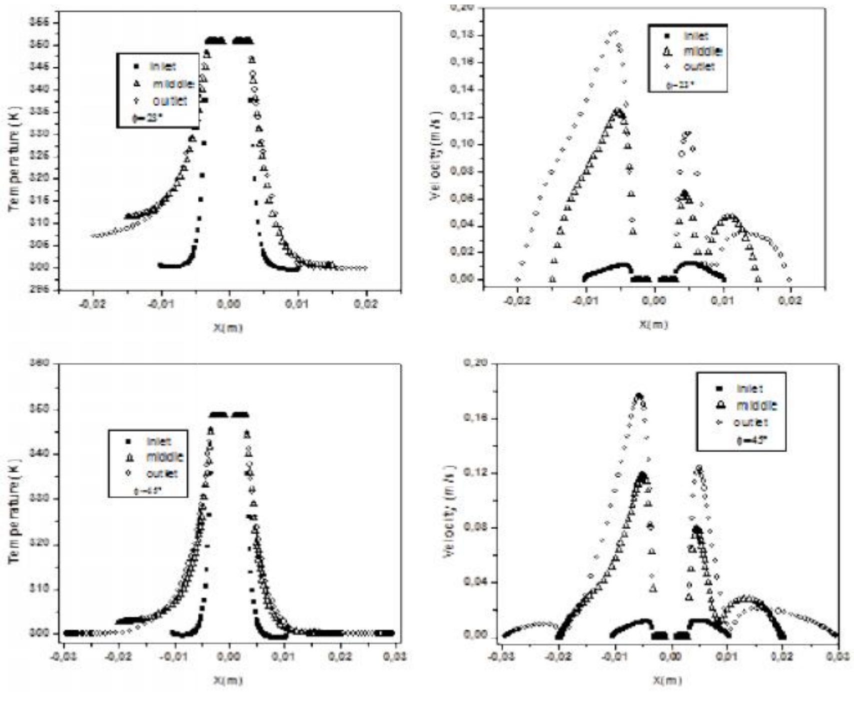


Figure 8.2. Temperature and velocity profiles in different locations of the duct, for N=1, Ra=3.1.10^4, ϕ=23° and ϕ=45°.

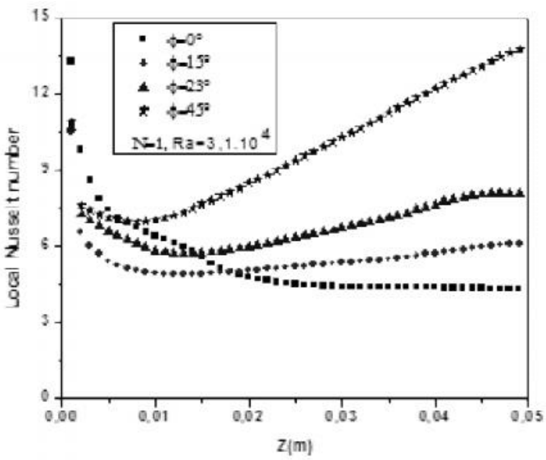
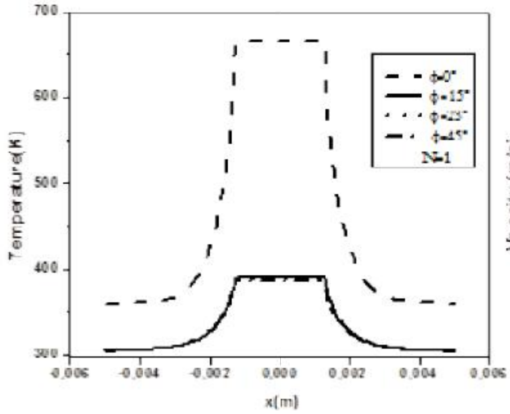


Figure 9.Variation of the local Nusselt number along the fin.



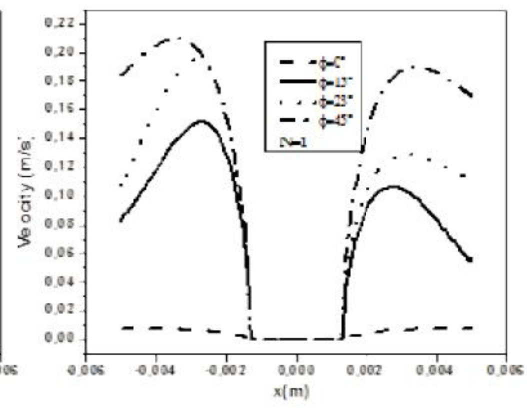
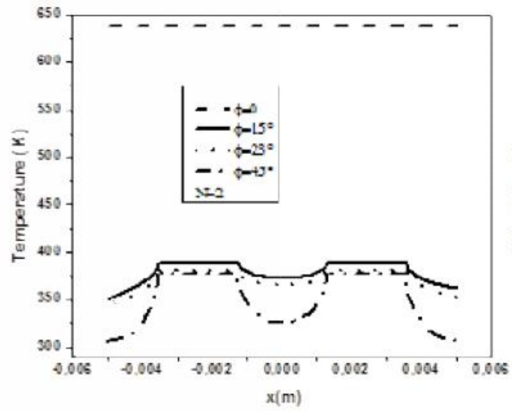


Figure 10. Temperature and velocity profiles in the radial line of coordinates (R=0.005m, $0\leq\theta\leq\pi$, Z=0.049m), N=1, Ra=$6.2.10^4$.



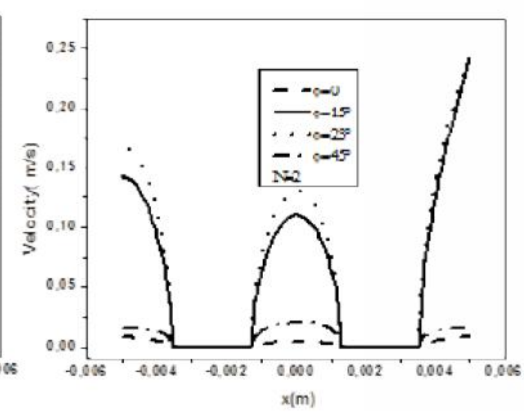


Figure 11. Temperature and velocity profiles in the radial line of coordinates (R=0.005m, $0\leq\theta\leq\pi$, Z=0.049m), N=2, Ra=$6.2.10^4$.

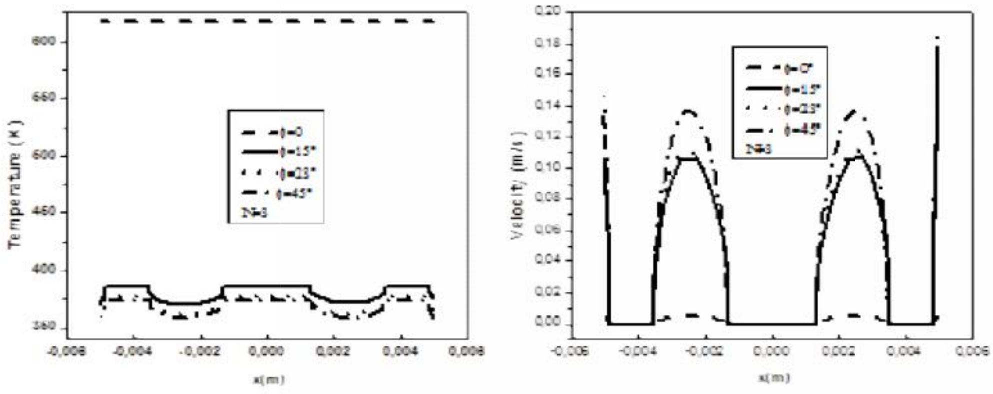


Figure 12. Temperature and velocity profiles in the radial line of coordinates (R=0.005m, $0\leq\theta\leq\pi$, Z=0.049m), N=3, Ra=6.2.10^{4}.

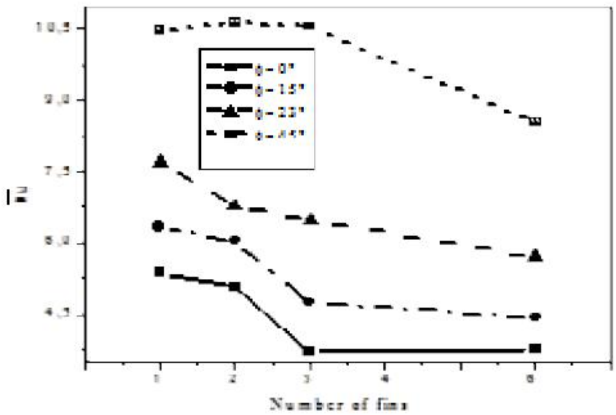


Figure 13. Variation of the average Nusselt number with fin numbers and the inclination angle for Ra=3.1.10^{4}.

CONCLUSION

The effect of the inclination angle of the divergent as well as the number of fins on natural convection in three-dimensional vertical annular ducts has been numerically studied. The inner cylinder subjected to uniform volumetric heat generation was fitted with longitudinal fins. The governing parameters are ϕ, Ra and the fin number N .The results of this study lead to the following conclusions:

- For a given Rayleigh number and a number of fins, the temperature decreases and the velocity increases with the increasing of the inclination angle of the divergent and yields a significant heat transfer
- A loss of symmetry was observed when $\phi\neq 0°$ in the velocity and the temperature contours fields as well as the profiles.
- The temperature of the flow decreases as well as the velocity with the number of fins.

- Good correlation of results obtained for average Nusselt number was reached, further there is no prior literature dealing with a generalized correlation for average Nusselt number in annular divergent duct fitted with fins.

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