Fuzzy modeling of Multiple-Input Multiple-Output systems using Takagi-Sugeno models based on Gustafson-Kessel clustering

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Abdelhafid Benyounes
Université de M'sila

Ahmed Hafaifa
Ziane Achour University of Djelfa

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Fuzzy modeling of Multiple-Input Multiple-Output systems using Takagi-Sugeno models based on Gustafson-Kessel clustering

A. Benyounes, A. Hafaifa, A.Z. Djeddi, S. Abudura

Abstract

Fuzzy identification and modeling is one of the best approaches for the representation of complex systems. In this article we use the Takagi-Sugeno fuzzy model for some class of nonlinear system, in order to use this proposed approach in various industrial applications. The validation of the proposed model was tested by the clustering technique, based on Gustafson-Kessel algorithm, to a multivariable industrial system. .

Key words: Fuzzy modeling, Takagi-Sugeno fuzzy model, Gustafson-Kessel algorithm, complex systems, industrial applications, multivariable industrial system.

1 Introduction

The fuzzy identification and modeling of the industrial system, from real exploitation data are effective tools to approximate a multivariable industrial system. Among the models that are widely used in modeling techniques we find those of Takagi-Sugeno in several applications [2, 3, 5, 15, 17]. The Takagi-Sugeno models use the idea of linearization to fuzzy regions in the state space. Based on the fuzzy regions, a nonlinear system can be decomposed into a multi-model structure consisting of several linear models that are not necessarily independent. The fuzzy sets partitioning the premises into a number of fuzzy regions space, while the consequences functions describe the system behavior in these regions [5, 9]. One of the techniques used to make the first step is the fuzzy clustering in the Cartesian product of input-output. Since the consequences functions are generally chosen linear forms, the second step was carried out in the literature by the standard methods of least squares [8; 11; 18]. We can find many clustering

A. Benyounes, A. Hafaifa, A.Z. Djeddi:Applied Automation and Industrial Diagnostic Laboratory, Faculty of Science and Technology, University of Djelfa 17000 DZ, Algeria, ab_benyounes@yahoo.fr, hafaifa@hotmail.com, a_z_djeddi@yahoo.fr.

S. Abudura:Faculty of Science and Technology, University of Medea, 26000, Algeria, abudura.salam@univ-medea.dz.

Corresponding autho. A. Benyounes: ab_benyounes@yahoo.fr

algorithms in the literature; they are mainly based on functional optimizing fuzzy C-means [7, 13]. Among these algorithms we find those who use a standard Euclidean distance, for which the detected clusters have hyper spherical forms of clusters, for which the surfaces of constant membership are hyper spheres. Other generalizes the standard Euclidian distance for a standard adaptive distance to detect clusters of different geometrical shapes within a single data set [14, 16, 19]. In this work, we propose the use of fuzzy clustering algorithms; this clustering is a cartesian product of the input-output has been extensively used to obtain the membership functions of the premises. By applying the fuzzy clustering, multidimensional fuzzy sets are initially obtained, and can be used directly in the model, or after projection on the antecedent variables (regressors). Since it is generally difficult to interpret multidimensional fuzzy sets, one-dimensional fuzzy sets are projected generally preferred. In this paper, we study the algorithm of Gustafson-Kessel (GK) for the identification of a multi-input multi-output (MIMO) system. We proposed a fuzzy modeling method based on Gustafson-Kessel (GK) clustering algorithm to estimate the membership functions of the premises and the least squares method to estimate parameters of the impact functions.

2 Complexes systems identification based on fuzzy algorithms

This part of the work focuses on the theories that are the basis of the modeling methods and qualitative reasoning. Among the existing formalisms for reasoning about the orders of magnitude, there are models based on linguistic formalism. This section aims to provide a bridge between the absolute models of a multivariable industrial system and the proposed fuzzy models based on Gustafson-Kessel (GK) clustering algorithms, in order to make them compatible for use. The two fuzzy clustering algorithms, most commonly used for identification of the parameters of the complexes systems are the Fuzzy C-mean (FCM) standard developed by Bezdek in [10] and clustering algorithms of Gustafson-Kessel (GK) proposed in [9]. The first method, allows the detection of hyperspherical classes, while the second detects hyper-ellipsoidal classes typically better adapted to the geometry of the observations. For this approaches, we consider a system described by the following equation:

$$y_k = f_{NL}(x_k) \tag{1}$$

Where:$x_k \in \Re^*$ is the observations vector.

3 Fuzzy C-Means Algorithm (FCM)

The algorithm (FCM) firstly are developed on the work of Dunn and later improved by Bezdek [10] is a benchmark among different methods of fuzzy clustering based on the minimization of the objective function given by the flowing equation:

$$J_{FCM}(Z,U,V) = \sum_{i=1}^{c} \sum_{k=1}^{N} (\mu_{ik})^m D_{ikA}^2 \tag{2}$$

Where Z is the set of data, $U = [\mu_{ik}]$ is the matrix of fuzzy partition with $C \times M$ dimension,$V_i = [V_1, V_2, ...V_n]$ is the center vector of classes $V_i \in \Re^n$ and $1 < i < c$, this vector to be determined.

$$D_{ikA}^2 = (z_k - v_i)^T A (z_k - v_i) \tag{3}$$

with $1 \le i \le c$ and $1 \le k \le N$

Equation (3) presents the standard quadratic distance in the space in question, which defines the distance measure between observation and Z_k center within the meaning of the metric induced by A and $m \in [0, \infty]$ is a factor that indicates the fuzzy degree of the partition. In equation (2), the measurement of non-similarity expressed by the term $J_{FCM}(Z, U, V)$ is the sum of the squared distances between each vector V_i and Z_k center of the corresponding class. The effect of this distance is weighted by the degree of activation $U = [\mu_{ik}]$ corresponding to the vector Z_k data value of the cost function $J_{FCM}(Z, U, V)$ can be seen as a measure of the total variance Z_k of to the relative centers V_i. The minimization of the objective function of the equation (2) is given by the flowing equation:

$$\mu_{ik} = \frac{1}{\sum_j^c (\frac{D_{ikA}}{D_{jkA}})^{\frac{2}{m-1}}} \tag{4}$$

with $1 \le i \le c$ and $1 \le k \le N$
Also, we have :

$$v_i = \frac{\sum_{K=1}^{N} (\mu_{ik})^m Z_k}{\sum_{K=1}^{N} (\mu_{ik})^m} \tag{5}$$

To apply the algorithm, since the data set$Z$, choose the number of class $1 \le c \le N$, the exponent $m > 1$ with the stopping tolerance ϵ and the standard matrix A, randomly initialize the partition matrix U, after we repeat the operation for $l = 0, 1, 2, ...$ To calculate the cluster centers we be used the flowing equation:

$$v_i = \frac{\sum_{K=1}^{N} (\mu_{ik}^{(1)})^m Z_k}{\sum_{K=1}^{N} (\mu_{ik}^{(1)})^m} \quad 1 \le i \le c \tag{6}$$

And to calculate the distances for the cluster centers we used the equation (7):

$$D_{ikA}^2 = (z_k - v_i^1)^T A (z_k - v_i^1) \tag{7}$$

with $1 \le i \le c$ and $1 \le k \le N$
Finely we will be update the partition matrix if $D_{jkA} > 0$ for $1 \le i \le c$ and $1 \le k \le N$, using the equation (8):

$$\mu_{ik}^{(1)} = \frac{1}{\sum_j^c (\frac{D_{ikA}}{D_{jkA}})^{\frac{2}{m-1}}} \tag{8}$$

So we run the program of the proposed algorithm :

$$\mu_{ik}^{(1)} = 0 \ \ if \ \ D_{ikA} < 0 \ \ and \ \ \mu_{ik}^{(1)} \in [0, 1] \tag{9}$$

with $\sum_i^c \mu_{ik}^{(1)} = 1$; up to $|| \ U^l - U^{l-1} \ ||< \epsilon$
The form class is determined by the choice of the matrix A in the equation of the objective function. The particular choice $A = I$, induces the standard Euclidean norm given by the flowing equation:

$$D_{ikA}^2 = (z_k - v_i^1)^T A (z_k - v_i^1) \tag{10}$$

this case of the classes detected the form determined is in spherical shapes, as shown in figure 1.

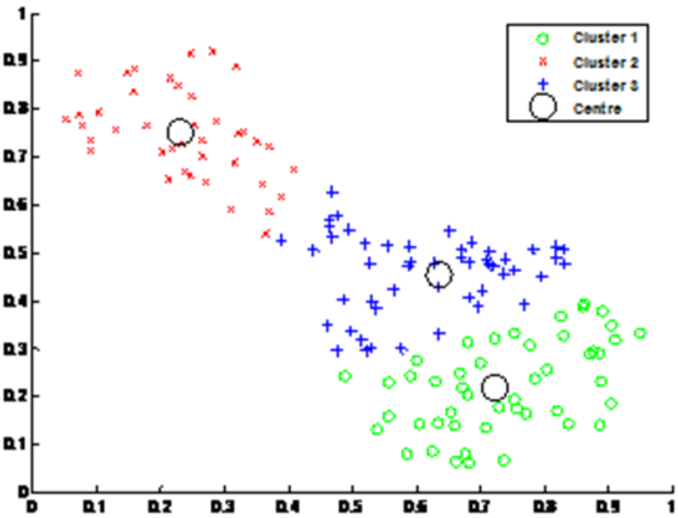

Figure 1: Form of the classes detected in spherical shapes.

4 Gustafson-Kessel Algorithm (GK)

In 1989 Gustafson and Kessel in his work have standardized the Fuzzy C-Means algorithm FCM using a standard adaptive distance in order to detect different classes of geometric shapes in a data set. In this case, each class has its own matrix norm, which leads to:

$$D^2_{ikA} = \| z_k - v_i \|^2_A = (z_k - v_i)^T A (z_k - v_i) \tag{11}$$

Assume that the matrixA_i satisfied the flowing hypothesis:

$$| A_i | = \rho_i \;\; with \;\; \rho_i > 0 \tag{12}$$

Where ρ_i is set for each class, In this case, the optimization of (2) gives the following expression for A_i :

$$| A_i | = [\rho_i det(F_i)]^{\frac{1}{n}} F_i^{-1} \tag{13}$$

Where F_i is the fuzzy covariance matrix of the i^{th} class given by:

$$F_i = \frac{\sum_{K=1}^{N} (\mu_{ik})^m (z_k - v_i)(z_k - v_i)^T}{\sum_{K=1}^{N} (\mu_{ik})^m} \tag{14}$$

The use of the fuzzy covariance matrix gives a good results in the using of the algorithm (GK), given the data set Z , choose the number of class $1 \leq c \leq N$, with the exponent $m > 1$ and the stopping tolerance is ϵ with A the norm matrix, randomly initialize the partition matrix U, after we repeat the operation for $l = 0, 1, 2, ...$ To calculate the cluster centers we will be used the flowing equation:

$$v_i^1 = \frac{\sum_{K=1}^{N} (\mu_{ik}^{(1)})^m Z_k}{\sum_{K=1}^{N} (\mu_{ik}^{(1)})^m} \;\; 1 \leq i \leq c \tag{15}$$

And to calculate the covariance matrix of this class we used the following equation:

$$F_i = \frac{\sum_{K=1}^{N} (\mu_{ik}^{l-1})^m (z_k - v_i^l)(z_k - v_i^l)^T}{\sum_{K=1}^{N} (\mu_{ik})^m} \tag{16}$$

And to calculate the distances for the cluster centers we used equation (17):

$$D_{ikA}^2 = (z_k - v_i^1)^T [\rho_i det(F_i)]^{\frac{1}{n}} F_i^{-1} (z_k - v_i^1) \tag{17}$$

with $1 \leq i \leq c$ and $1 \leq k \leq N$
Finely we will be update the partition matrix if $D_{ikA}^2 > 0$ for $1 \leq i \leq c$ and $1 \leq k \leq N$, using equation (18):

$$\mu_{ik}^1 = \frac{1}{\sum_j^c (\frac{D_{ikA}}{D_{jkA}})^{\frac{2}{m-1}}} \tag{18}$$

So we run the program of the proposed Gustafson-Kessel algorithm :

$$\mu_{ik}^1 = 0 \ \ if \ \ D_{ikA} > 0 \ \ and \ \ \mu_{ik}^1 \in [0, 1] \tag{19}$$

with $\sum_i^c \mu_{ik}^1 = 1$ up to $\| U^l - U^{j*1} \| < \epsilon$
The application results of the proposed Gustafson-Kessel algorithm detected the form of the clustering classes for random data, as shown in figure 2.

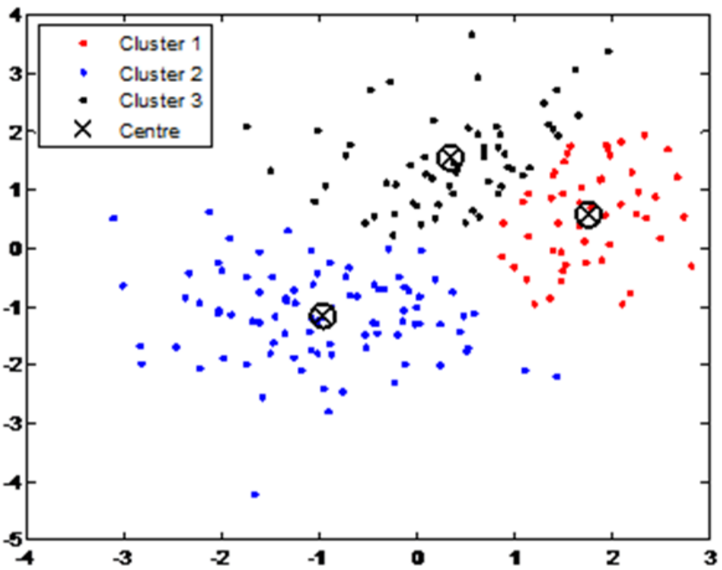

Figure 2: Form of the classes clustering for random data.

For a multivariable industrial systems using fuzzy Takagi-Sugeno models, based on Gustafson-Kessel clustering, we consider a system described by equation (1), our goal is to approximate this nonlinear function given by the equation (1) by the model of Takagi-Sugeno (TS):

$$\begin{array}{c} R_i : \ \ if \ \ x_1 \ \ is \ \ A_{i1} \ \ and \ \ X_2 \ \ is \ \ A_{i2} \ \ ....x_p \ \ is \ \ A_{ip} \\ Then \ \ y_i = a_i x + d_i \quad i = 1..r \\ a_i^T = [a_i 1.a_i 2......a_i n] \end{array} \tag{20}$$

With:R_i represent the i^{th} rule,$x_i = [x_1.x_2......x_n]$; observation vector and $A_{i1}, A_{i2}, A_{i3}...A_{in}$ represents the fuzzy sets. The Takagi-Sugeno TS modeling, obtaining the output of the model is made from any combination of the inferencing and defuzzification. The final output is calculated as the average of the corresponding outputs of the rules R_i, weighted by the degree to which standardized given by Takagi and Sugeno in [13]:

$$y = \frac{\sum_{i=1}^{r} \omega_i(x) y_i}{\sum_{i=1}^{r} \omega_i(x)} \tag{21}$$

Where:$\omega_i = \mu_{Ai}(x)$, μ_{Ai} is the fuzzy membership function.
To identify the parameters involved in the model , the regression matrix X and the vector of the output Y are constructed from measurements from the system, are $x_i = [x_1.x_2......x_n]^T$ and $y_i = [y_1.y_2......y_n]^T$. The determination of f_{NL} in two steps; firstly, by applying any clustering algorithm C-means for calculating the fuzzy partition matrix U. Subsequently, we then estimate the parameters a_i and d_i. Indeed, the defuzzification method used in the model of Takagi-Segeno is linear with respect to parameters a_i and d_i. However, these parameters can be estimated using least square techniques. The $\theta_i = [a_i, b]$ is the vector of parameters and an extension of i_{th} $X_e = [X, 1]$ rule of the X matrix. Note that $\Gamma_i \in \Re^{N \times N}$ is a diagonal matrix that contains the degrees activations $\mu_i(x_i) > 0$ with $1 \leq k \leq N$. Using the least squares method balanced, the solution $y = X_e\theta + \epsilon$ is given by the following expression:

$$\theta_i = [X_e^T \Gamma_i X_e]^{-1} X_e^T \Gamma_i Y \tag{22}$$

5 Application results

For our application, we consider a multivariable systems with two inputs and tow outputs. The identification results of this system with a signal input step, as shown in figure 3. The validation of fuzzy Takagi-Sugeno model can be obtained using the VAF (Variance For Accounting) Criteria used to calculate the standard deviation as a percentage of the variance between the measured output and the estimated output, the value of VAF is of the order of 100

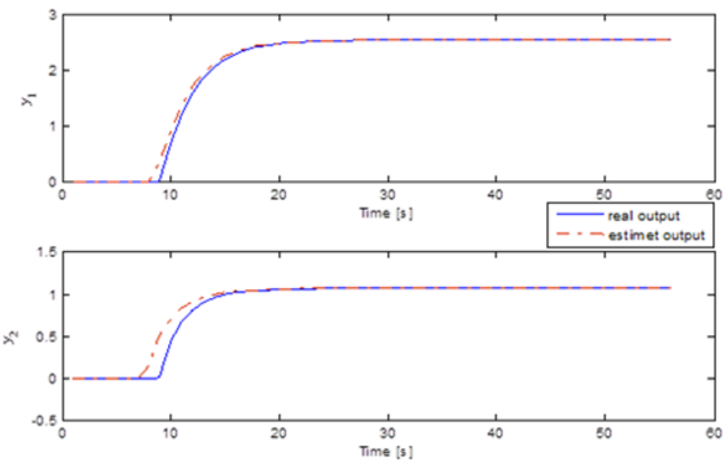

Figure 3: Identification results of this system with a signal input step.

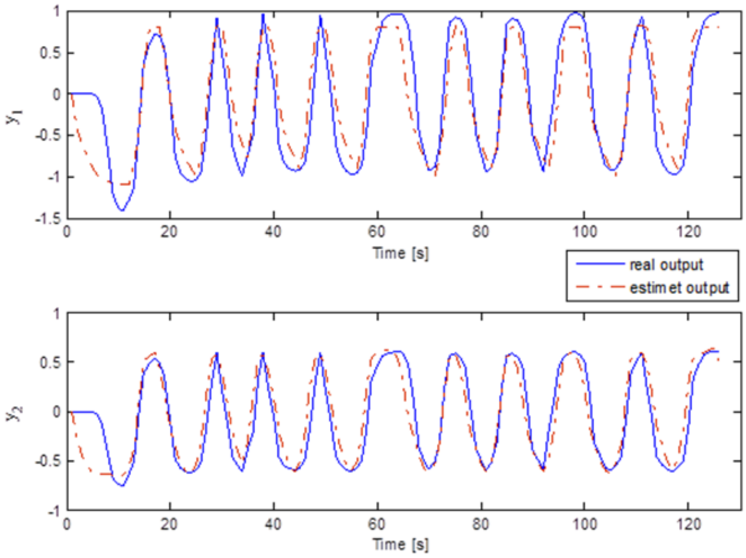

Figure 4: Identification results of this system with a signal input square.

The figure 4 show the identification result for a square input signal, the Variance For Accounting VAF of this validation test is VAF = [81.3664 ; 83.0964].

6 Conclusion

In this paper, we have studed the fuzzy identification using Takagi-Sugeno model TS algorithm based on the technique of clustering algorithm by Gustafson Kessel (GK) that gives a best results on modeling of multivariable industrial system. The validation of the developed model was tested by the clustering technique, based on Gustafson-Kessel algorithm.

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