



9th (Online) International Conference on
Applied Analysis and Mathematical
Modeling

ICAAMM21 June 11-13, 2021
Istanbul-Turkey

Abstracts Book

Editors
Mustafa Bayram
Aydın Seçer

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Abstracts Book

Prof. Dr. Mustafa Bayram
Prof. Dr. Aydın Seer

Participant Statistics

297 participants from 41 different countries attended the conference, 54 of them from Turkey and the others from abroad, so 82% participants are foreigners and 18% participants are Turkish.

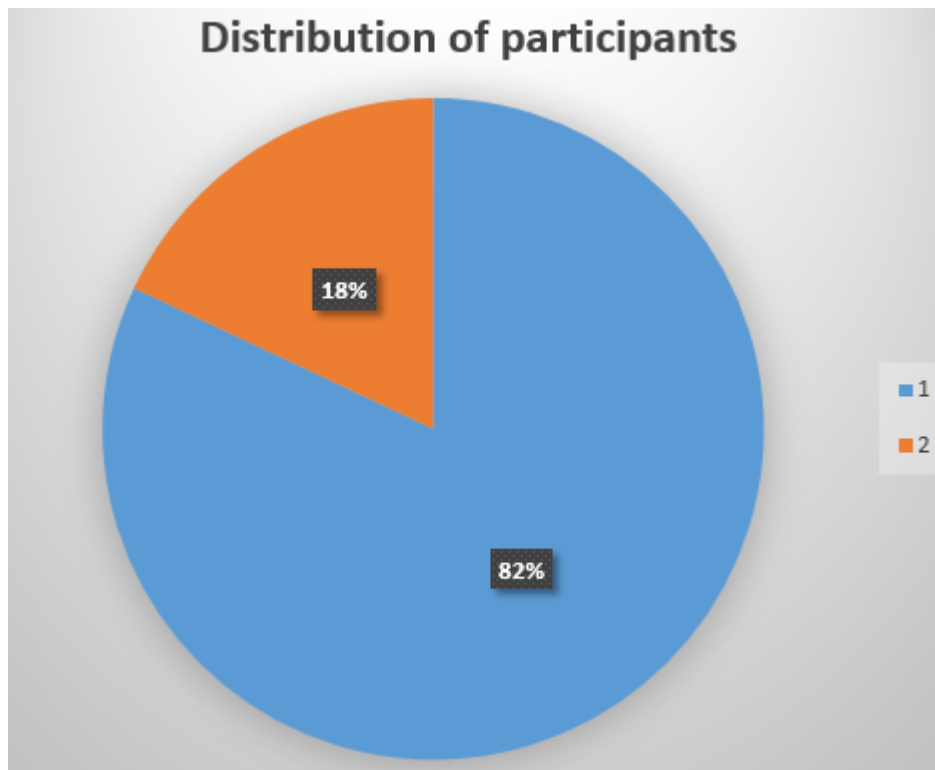


Figure 1: 1. Foreign participants, 2. Turkish participants

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MESSAGE FROM CHAIRMAN

The "9th(Online) International Conference on Applied Analysis and Mathematical Modeling, 2021" organized by Biruni University will be held on 11-13 June 2021 in Istanbul, Turkey. Due to the Covid-19 Pandemic, we could not meet face to face. For this reason, we decided to make it online by technology. The aim of this conference is to bring the Mathematics & Engineering Sciences community working in the new trends of applications of Mathematics together in a wonderful city of the world, Istanbul.



There have been quite a big number of applications from different part of the world and as you know when the number increase task of the organizing committee will increase. Thus it was a very difficult task to select and classify the abstracts for all the participants. We tried to do our best to accommodate many speakers in order to have a better and enjoyable research session which will provide more interactions, exchanges among the participants.

Besides the scientific program, we had some social activities (excursion boat trip, city tour, etc.) where we could continue some informal discussions that would serve the purpose of our meeting in such a short time. We had to cancel due to the pandemic. As we can see from the list of participants, many speeches by young researchers will also serve the purpose of this conference.

The talks will cover a wide range of mathematics and its applications such as analysis, algebra, statistics, computer mathematics, discrete mathematics, geometry, engineering, etc. as well as their use in modeling. We believe that this richness will provide the basis for interdisciplinary collaborations.

We also would very much thank to all presenters and participants for their interests in the conference and believe and hope that each of them will get the maximum benefit in terms of networking and interaction from this meeting.

We would like to thank Dumitru Baleanu, Aydin Secer, Tuğçem Partal, Neslihan Ozdemir, Melih Cinar, Handenur Esen and Ismail Onder all our colleagues who worked for the organization of the conference.

Finally, we also would to thank to chairman of the board of trustees of Biruni University and Prof. Dr. Adnan Yüksel the Rector of Biruni University which is Host University.

Further we thank to all the plenary speakers that kindly accepted our invitation and spend their precious time by sharing their ideas during the conference. We also thank to all members of organizing committee.

We apologize for any shortcomings or might not be mentioned unintentionally or may have been forgotten to be mentioned explicitly here. We really hope their kind understanding, we thank all and each individual that have put their effort to make this occasion possible.

We welcome each and every one of you again to this conference; we wish a enjoyable and productive conference and hope to meet again in future occasions.

Sincerely Yours,
Prof. Dr. Mustafa Bayram,
Conference Chair

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Md. Haider Ali Biswas, Khulna University, Bangladesh

ORAL PRESENTATIONS

Applications of the extended $\exp(-\varphi(\xi))$ -expansion method to some non-linear fractional evolution equations

Berfin Elma and Emine Mısırlı

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Abstract: In this article, the new exact solutions of some non-linear fractional evolution equations have been obtained by using extended $\exp(-\varphi(\xi))$ -expansion method. The advantages of proposed method have been discussed. All solutions have been checked with the help of Mathematica program. Additionally, 3D- graphs have been drawn to observe different type of solutions.

Keywords: Nonlinear fractional evolution equations, New analytical wave solutions, Extended $\exp(-\varphi(\xi))$ -expansion method.

Mathematics Subject Classification: 35R11, 47J35.

References

- [1] Achouri, R., "Travelling wave solutions (Doctoral dissertation, Master's thesis)", School of Manchester, 2016.
- [2] Alhakim, L. A. and Moussa, A. A., "The Improved $\exp(-\varphi(\xi))$ Fractional Expansion Method and its Application to Nonlinear Fractional Sharma-Tasso-Olver Equation", J.Appl Computat. Math, 6(360),p.2, 2017.
- [3] Pinchover, Y., Rubinstein, J., "An Introduction to Partial Differential Equations", Cambridge University Press, UK, p.366, 2005.
- [4] Torvattanabun, M., Simtrakankul, C. and Duangpithak, S., "The (3+1)-dimensional Fractional Modified Kdv-Zakharov-Kuznetsov Equation By Using The Improved Generalized Tanh-Coth Method ", Journal of Mathematical Sciences: Advances and Applications , 56,pp.53-65, 2019.
- [5] Verma, P. and Kaur, L., "Extended $\exp(-\varphi(\xi))$ -expansion method for generalized breaking soliton equation", AIP Conference Proceedings, 2214(1), p.020006, 2020.
- [6] Zheng, B., "Exact solutions for some fractional partial differential equations by the (G'/G) -method", Mathematical Problems in Engineering, 2013.
- [7] Zheng, S., "Nonlinear evolution equations", CRC Press, 2004.

Determination of Material Deformation Rate Based on Artificial Intelligence using Surface Microstructure Images

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Abstract: Making evaluations on the ever-increasing data and thus obtaining meaningful outcome becomes more and more important in the digital age. Thanks to the recent advances in artificial intelligence technologies, processing this data and making predictions is very popular. Within the scope of this study, S235 JR steel, which is used in many areas such as bridges, railways, industrial buildings, vehicle manufacturing and oil-gas exploration stations, has been deformed at different rates in the laboratory environment. Microstructure images of the deformed materials were obtained with the help of a microscope after some metallographic processes. As a result of the study, it is aimed to contribute to the literature by creating a data set containing microstructure images of S235 JR steel, which has been deformed at different rates. In addition, Convolutional Neural Network (CNN) and Multilayer Artificial Neural Network (ANN) models, which are among the deep learning methods of artificial intelligence technology, were used to classify the deformation rates. In order to minimize the error in both deep learning models, Adam Optimization Algorithm has been preferred. The results obtained with the Adam Optimization Algorithm in CNN and Multilayer ANN models were compared and the highest success in the classification process was obtained in the CNN model.

Keywords: Artificial Intelligence, CNN, Multilayer YSA, S235 JR structural steel.

Mathematics Subject Classification: 1X234.

References

- [1] Schmidhuber, J., Deep learning in neural networks: An overview. *Neural networks*, 2015. 61: p. 85-117.
- [2] LeCun, Y., et al., Gradient-based learning applied to document recognition. *Proceedings of the IEEE*, 1998. 86(11): p. 2278-2324.
- [3] Krizhevsky, A., I. Sutskever, and G.E. Hinton. Imagenet classification with deep convolutional neural networks. in *Advances in neural information processing systems*. 2012.
- [4] Szegedy, C., et al. Going deeper with convolutions. in *Proceedings of the IEEE conference on computer vision and pattern recognition*. 2015.
- [5] Simonyan, K. and A. Zisserman, Very deep convolutional networks for large-scale image recognition. *arXiv preprint arXiv:1409.1556*, 2014.
- [6] He, K., et al. Deep residual learning for image recognition. in *Proceedings of the IEEE conference on computer vision and pattern recognition*. 2016.
- [7] Yu, X., et al., Deep learning in remote sensing scene classification: a data augmentation enhanced convolutional neural network framework. *GIScience & Remote Sensing*, 2017. 54(5): p. 741-758.
- [8] Delen, D., G. Walker, and A. Kadam, Predicting breast cancer survivability: a comparison of three data mining methods. *Artificial intelligence in medicine*, 2005. 34(2): p. 113-127.

- [9] Walek, B., O. Hosek, and R. Farana. Proposal of expert system for hotel booking system. in 2016 17th International Carpathian Control Conference (ICCC). 2016. IEEE.
- [10] Ayalew, A., A review on object detection from Unmanned Aerial Vehicle using CNN. 2019.
- [11] Nabiyev, V.V., Yapay Zeka, ed. 598-599. 2016, Ankara: Seçkin Yayıncılık.
- [12] Bulgarevich, D.S., et al., Pattern recognition with machine learning on optical microscopy images of typical metallurgical microstructures. Scientific Reports, 2018. 8(1): p. 2078.
- [13] Kachanov, L., Introduction to continuum damage mechanics. Vol. 10. 1986: Springer Science & Business Media.
- [14] Chaboche, J.-L., Continuum damage mechanics: Part II—Damage growth, crack initiation, and crack growth. 1988.
- [15] Chaboche, J.-L., Continuum damage mechanics: Part I—General concepts. 1988.
- [16] Chaboche, J.-L., Continuous damage mechanics—a tool to describe phenomena before crack initiation. Nuclear Engineering and Design, 1981. 64(2): p. 233-247.
- [17] Yürük, A., B. Bozkurt, and N. Kahraman, S235JR karbon çeliği ile AISI 430 ferritik paslanmaz çeliğin MIG kaynak yöntemi ile kaynak edilebilirliğinin incelenmesi. Sakarya University Journal of Science, 2017. 21(2): p. 90-97.
- [18] Şık, A., MIG/MAG kaynak yöntemi ile birleştirilen çelik malzemelerde ilave tel türleri ve koruyucu gaz karışımlarının eğmeli yorulma ömürlerine etkilerinin araştırılması. Gazi Üniversitesi Mühendislik Mimarlık Fakültesi Dergisi, 2007. 22(4): p. 769-777.
- [19] Aydın, Ş., Yapı Çeliğinin (St52-3) Mig/Mag Kaynağında Gaz Karışımlarının Çekme Dayanımı Özelliklerine Etkisi. Trakya Univ J Sci, 2006. 7(1): p. 9-15.
- [20] Ege, Y., Ferromanyetik malzemeler üzerindeki çatlakların manyetik yöntemle belirlenmesi. 1998, Balıkesir Üniversitesi Fen Bilimleri Enstitüsü.
- [21] Kaya, Y. and N. Kahraman, Farklı özellikteki malzemelerin tozaltı ark kaynak yöntemi ile birleştirilmesi ve birleştirmelerin tahribatlı ve tahribatsız muayenesi. Sakarya Üniversitesi Fen Bilimleri Enstitüsü Dergisi, 2013. 17(1): p. 85-96.
- [22] Özkan, E., Kaynak sonrası S355J2N yapı çeliğinde oluşan gerilmeleri gidermek için uygulanan ısıtma işleminin etkilerinin tahribatlı-tahribatsız muayene yöntemleriyle belirlenmesi. 2019, Namık Kemal Üniversitesi.
- [23] Askon. S235JR ve S355J2 + N. 2021; Available from: <https://www.askondemir.com/sac-grubu/siyah-sac>.
- [24] Aizenberg, I.N., N.N. Aizenberg, and J. Vandewalle, Multiple-Valued threshold logic and multi-valued neurons, in Multi-Valued and Universal Binary Neurons. 2000, Springer. p. 25-80.
- [25] Lin, T.-H., A cross model study of corporate financial distress prediction in Taiwan: Multiple discriminant analysis, logit, probit and neural networks models. Neurocomputing, 2009. 72(16-18): p. 3507-3516.
- [26] Etheridge, H.L. and R.S. Sriram, A comparison of the relative costs of financial distress models: artificial neural networks, logit and multivariate discriminant analysis. Intelligent Systems in Accounting, Finance & Management, 1997. 6(3): p. 235-248.
- [27] Hwang, J.G. and A.A. Ding, Prediction intervals for artificial neural networks. Journal of the American Statistical Association, 1997. 92(438): p. 748-757.
- [28] Mishkin, D. and J. Matas, All you need is a good init. arXiv preprint arXiv:1511.06422, 2015.

- [29] Li, Y., et al., Deep learning for remote sensing image classification: A survey. *Wiley Interdisciplinary Reviews: Data Mining and Knowledge Discovery*, 2018. 8(6): p. e1264.
- [30] LeCun, Y. and Y. Bengio, Convolutional networks for images, speech, and time series. *The handbook of brain theory and neural networks*, 1995.

On soliton solutions of some nonlinear Schrödinger equations

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Abstract: Nonlinear Schrödinger equations (NLSEs) arise in diverse areas such as engineering, biological and physical sciences. The obtaining of the exact solutions for various models represented by NLSEs has also a principal role in fluid dynamics, plasma, nuclear physics, and nonlinear optics. Especially, soliton solutions from these solutions have received quite an attention from researchers. In this study, we implement the Riccati-Bernoulli Sub ODE method in reporting the exact solutions of two nonlinear physical models. Therefore, for the equations, some singular periodic waves, dark and singular optical solitons solutions are derived. It can be reported that all solutions produced in this study satisfy the equation by replacing it in the corresponding main equation. Utilizing suitable values of the parameters, some of the obtained solutions are illustrated by three-dimensional (3D) and two-dimensional (2D) graphs with the help of the MAPLE software in order to demonstrate the importance in the real-world of the presented equations.

Keywords: Nonlinear Schrödinger equations, Riccati-Bernoulli Sub-ODE method, soliton solutions.

Mathematics Subject Classification: 35A24, 35C08, 35Q55.

References

- [1] D.V. Kaplan, “A method for simulating chiral fermions on the lattice”, *Physics Letters B* 288 (3–4), 342–347 (1992).
- [2] A. Nishino and Y. Umeno and M. Wadati, “Chiral Nonlinear Schrödinger Equation”, *Chaos, Solitons & Fractals* Vol. 9 No 7 (1998): 1063–1069.
- [3] A. Javid and N. Raza, “Chiral solitons of the (1 + 2)-dimensional nonlinear Schrodinger’s equation.” *Modern Physics Letters B* (2019): 1950401.
- [4] N. Raza and S. Arshed, “Chiral bright and dark soliton solutions of Schrödinger’s equation in (1 + 2)-dimensions”, *Ain Shams Engineering Journal* 11 (2020): 1237–1241.
- [5] A. U. Awan and M. Tahir and K. A. Abro, “Multiple soliton solutions with chiral nonlinear Schrödinger’s equation in (2+1)-dimensions”, *European Journal of Mechanics / B Fluids* 85 (2021): 68–75.
- [6] M. S. Osman and D. Baleanu and K. U. Tariq and M. Kaplan and M. Younis and S. T. R. Rizvi, “Different Types of Progressive Wave Solutions via the 2D-Chiral Nonlinear Schrödinger Equation”, *Front. Phys.* 8 (2020):215.
- [7] K. Hosseini and M. Mirzazadeh, “Soliton and other solutions to the (1+2)-dimensional chiral nonlinear Schrödinger equation”, *Commun. Theor. Phys.* 72 (2020): 125008.
- [8] H. Ur Rehman and M. A. Imran and M. Bibi and M. Riaz and A. Akgül. “New soliton solutions of the 2D-chiral nonlinear Schrodinger equation using two integration schemes”, *Math Meth Appl Sci.* (2020): 1–20.

Application of Optimal Processes in the Nonlinear Stochastic Dynamical System

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Abstract: In this paper we firstly defined nonlinear stochastic binary dynamical system and investigated the optimal control problem given by the considered system.[1,2,3]. Then we proved a theorem concerning unique solution condition.[2,3] For simplicity, we illustrated the system characterization in the two-dimensional space by applying shift operator.[4,5, 6].

Keywords: Optimal Control Problem, Stochastic Dynamical System.

Mathematics Subject Classification: 49L20, 93E20.

References

- [1] Boltyanskii, V. G., Optimal Control of Discrete Systems, John Willey, New York, 1978.
- [2] Gaishun, I. V., Completely Solvable Multidimensional Differential Equations, Nauka and Tekhnika, Minsk, 1983.
- [3] Hacı Y., Candan M., “Optimal Control Problem for Processes Represented by Stochastic Sequential Machine.” Internatinal Journal on Cybernetics and Informatics. 3(4) pp. 21-26, 2014.
- [4] Burden, R. L., Dauglas, J. Numerical Analysis, PWS Publishing Company, Boston. 1985.
- [5] Anderson, J. A., Discrete Mathematics with Combinatorics, Prentice Hall, New Jersey, 2004.
- [6] Gabasov, R., Kirillova, F. M., Paulianok, N. S. “Optimal Control of Linear Systems on Quadratic Performance Index”, Appl. and Comput. Math., 7(1), pp. 4-20, 2008.

Generating the novel analytical solutions of the system of partial differential equations with Conformable, M-truncated and Beta derivatives

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Abstract: In recent years, plenty of real-life problems have been modeled utilizing nonlinear partial differential equations. Furthermore, some of the nonlinear partial differential equations for exploring novel traits of real-life problems have been altered. In the past several decades, several researchers have analyzed that fractional differential equations are one of the best ways of clarifying real-life problems along with different engineering fields such as sensors, actuators, and many more. In this research, we study the novel traveling wave solutions and other solutions with conformable, M-truncated, and beta fractional derivatives for the nonlinear fractional system. The exact solutions of this system are acquired utilizing Riccati-Bernoulli sub-ODE method. A comparative approach is presented between the solutions with the fractional derivatives. For the validity of the solutions, the constraints conditions are determined. The 2D and 3D graphs of the acquired solutions are successfully charted by selecting appropriate values of parameters.

Keywords: Nonlinear fractional system, traveling wave solutions, Riccati-Bernoulli Sub-ODE technique.

Mathematics Subject Classification: 35A24, 35C07, 35R11.

References

- [1] Podlubny, “I. Fractional Differential Equations”, Academic: New York, NY, USA, 1999.
- [2] Samko, S.G. and Kilbas, A.A. and Marichev, O.I, “Fractional Integrals and Derivatives”, Theory and Applications; Taylor and Francis: London, UK, 1993.
- [3] Singh, S. and Sakthivel, R. and Yusuf, A. and Murugesan, K., “Computing wave solutions and conservation laws of conformable time-fractional Gardner and Benjamin–Ono equations”, *Pramana*, 95(1), 1-13, 2021.
- [4] Alquran, M. and Yousef, F. and Alquran, F. and Sulaiman, T. A. and Yusuf, A., “Dual-wave solutions for the quadratic–cubic conformable–Caputo time-fractional Klein–Fock–Gordon equation”, *Mathematics and Computers in Simulation*, 185, 62-76, 2021.
- [5] Prakash, A. and Kumar, A. and Baskonus, H. M. and Kumar, A., “Numerical analysis of nonlinear fractional Klein–Fock–Gordon equation arising in quantum field theory via Caputo–Fabrizio fractional operator”, *Mathematical Sciences*, 1-13, 2021.
- [6] Bulut, H. and Sulaiman, T. A. and Baskonus, H. M. and Rezazadeh, H., Eslami, M. and Mirzazadeh, M., “Optical solitons and other solutions to the conformable space–time fractional Fokas–Lenells equation”, *Optik*, 172, 20-27, 2021.
- [7] Inc, M. and Parto-Haghighi, M. and Akinlar, M. A. and Chu, Y. M., “New numerical solutions of fractional-order Korteweg–de Vries equation”, *Results in Physics*, 19, 103326, 2020.

Elzaki Transform Adomian Decomposition Method to Obtain the Approximate Analytical Solutions of the Nonlinear Time-Fractional Coupled Burger's Equations

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Abstract: The Elzaki transform Adomian decomposition method is applied to obtain the approximate analytical solutions of the nonlinear time-fractional partial coupled Burger's equations. The fractional derivatives are defined in the Caputo sense. Numerical experiments are analyzed by ETADM. The graphs of the solutions of the nonlinear time-fractional partial coupled Burger's equations are plotted in the MAPLE software. The applications shows that ETADM is very effective method.

Keywords: Time-Fractional Coupled Burger's Equation, Elzaki Transform Adomian Decomposition Method, Mittag-Leffler Function.

Mathematics Subject Classification: 35F20, 35R11, 65R99.

References

- [1] Singh, J. and Kumar, D. and Swroop, R., "Numerical solution of time-and space-fractional coupled Burgers' equations via homotopy algorithm", *Alexandria Engineering Journal*, 55(2), pp. 1753-1763, 2016.
- [2] Elzaki, T. M., "Application of new transform "Elzaki transform" to partial differential equations", *Global Journal of Pure and Applied Mathematics*, 7(1), pp. 65-70, 2011.
- [3] Elzaki, T. M., and Chamekh, M., "Solving nonlinear fractional differential equations using a new decomposition method", *Univ J Appl Math Comput.*, 6, pp. 27-35, 2018.
- [4] D. Baleanu, K. Diethelm, E. Scalas and J. J. Trujillo, "Fractional calculus: models and numerical methods", World Scientific, 2012.
- [5] I. Podlubny, "Fractional differential equations", Academic Press, 1999.

Exact Traveling Wave Solutions of Two Fractional Systems in Fluid Dynamics via the Exponential Rational Function Method

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Abstract: Nonlinear fractional differential equations have many important applications in applied sciences. The investigation of exact solutions of these equations are revealed many useful and effective analytical methods. In this study, the exponential rational function method is implemented to achieve two fractional systems' exact traveling wave solutions in fluid dynamics. The time-fractional derivatives are taken in terms of the conformable sense. The accuracy of achieved results and graphics are checked with the Mathematica software. This proposed method is reliable and innovative to obtain the exact solutions of equations with fractional order derivative.

Keywords: Exponential rational function method, nonlinear fractional partial differential equation, conformable fractional derivative, traveling wave solution.

Mathematics Subject Classification: 35R11, 35C07, 35Q35.

References

- [1] Abdeljawad, T., "On conformable fractional calculus", J. Comput. Appl. Math., 279, pp.57-66, 2015.
- [2] Aksoy, E. and Kaplan, M. and Bekir, A., "Exponential rational function method for space-time fractional differential equations", Wave Random Complex, 26, pp.142-151, 2016.
- [3] Khater, M.M. and Kumar, D., "New exact solutions for the time fractional coupled Boussinesq-Burger equation and approximate long water wave equation in shallow water", J. Ocean Eng. Sci., 2, p.223, 2017.
- [4] Shi, D. and Zhang, Y. and Liu, W. and Liu, J., "Some exact solutions and conservation laws of the coupled time-fractional Boussinesq-Burgers system", Symmetry, 11, p.77, 2019.
- [5] Ekici, M. and Ünal, M., "Application of the exponential rational function method to some fractional soliton equations", In Emerging Applications of Differential Equations and Game Theory, IGI Global, pp.13-32, 2020.

Spreading or Contraction of Viscous Drops Between Plates; With and Without Rotation

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Abstract: The behaviour of a viscous drop squeezed between two horizontal planes (a contracting Hele Shaw cell) will be described by both theory and experiment. For a constant squeezing force the ultimate growth of the radius $a \sim t^{(1/8)}$ with time t . An initially elliptic drop tends to become circular as t increases and is stable to small perturbations. For a reversed force, so that the plates are drawn apart, the boundary of the drop is subject to a fingering instability on a scale determined by surface tension. The effect of a trapped air bubble at the centre of the drop will also be described. The annular evolution of the drop under constant squeezing is still found to follow a "one - eighth" power law, but this is unstable, the instability originating at the boundary of the air bubble. If the plates are drawn apart, the evolution is still subject to the fingering instability driven from the outer boundary of the annulus. Fingering can also spread from the boundary of an interior trapped air bubble, and small cavitation bubbles appear in the very low pressure region far from the point of leverage. The effect of rapid rotation of the plates about a vertical axis will be described: unstable to fingering at the interface if no upper plate present; and totally stable if there is an upper plate. All these behaviors will be described, as well as being demonstrated by real time experiments and videos.

Keywords: viscous spreading; fluid instabilities; finite-time singularities; cavitation; effects of rotation.

References

- [1] Huppert, H. E. 1982 The propagation of two-dimensional and axisymmetric viscous gravity currents over a rigid horizontal surface. *J. Fluid Mech.* 121, 43-58
- [2] Moffatt, H.K., Guest, H & Huppert, H. E. 2021 Spreading or contraction of viscous drops between plates: single, multiple or annular drops, *J. Fluid Mech.* (sub juice)
- [3] Tanveer, S. Surprises in viscous fingering. *J. Fluid Mech.* 409, 273-308

Upper and lower solution method for n th order BVPs on an infinite interval

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Abstract: This work is devoted to study a n th order ordinary differential equation on a half-line with Sturm-Liouville boundary conditions. The existence results of a solution and triple solutions are established by employing a generalized version of the upper and lower solution method, Schüuder fixed point theorem, and topological degree theory. In our problem the nonlinearity depends on derivatives, and we allow solutions to be unbounded, which is an extra interesting feature. To demonstrate the usefulness of our results we illustrate two examples.

A New Mathematical Approach for Determining Kinetic Parameters of Curing Process

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Abstract: We suggest, a new algorithm (GMN) for determining parameters of kinetic modelling which are pre-exponential and activation energy of curing process. This method includes the combine of tanh fitting for the measured conversion values via least squares minimization technique and linear fitting for the kinetic parameters. Experimentally determined differential scanning calorimetry (DSC) data sets for an epoxy resin functionalized by single wall carbon nanotubes are used for the verification of the proposed method. The results obtained from the proposed algorithm are also compared with the methods reported in the literature.

Keywords: Computational material science, Numerical methods, Numerical simulation.

Mathematics Subject Classification: 42A05, 42A10, 65L05, 74E40, 74E30.

References

- [1] F.X. Perrin, T.M.H. Nguyen, J.L. Vernet, *Macromolecular Chemistry and Physics* 208, 55-67 (2007).
- [2] J. Wan, Z.-Y. Bu, C.-J. Xu, H. Fan, B.-G. Li, *Thermochimica Acta* 525, 31-39 (2011).
- [3] K. Tsuchida, J.P. Bell, *International journal of adhesion and adhesives* 20, 449-456 (2000).
- [4] W.Y. Chen, Y.Z. Wang, F.C. Chang, *Journal of applied polymer science* 92, 892-900 (2004).
- [5] Y. Liu, C. Zhang, Z. Du, H. Li, *Journal of applied polymer science* 99, 858-868 (2006).
- [6] Q.Y. Liu, J.B. Chen, S.M. Liu, J.Q. Zhao, *Polymer International* 61, 959-965 (2012).
- [7] S. Vyazovkin, A.K. Burnham, J.M. Criado, L.A. Pérez-Maqueda, C. Popescu, N. Sbirrazzuoli, *Thermochimica Acta* 520, 1-19 (2011).
- [8] F. Boey, W. Qiang, *Polymer* 41, 2081-2094 (2000).
- [9] J. Šesták, G. Berggren, *Thermochimica Acta* 3, 1-12 (1971).
- [10] S. Vyazovkin, *International Journal of Chemical Kinetics* 28, 95-101 (1996).
- [11] M. Vafayan, M.H. Beheshty, M.H.R. Ghoreishy, H. Abedini, *Thermochimica Acta* 557, 37-43 (2013).
- [12] S. Vyazovkin, N. Sbirrazzuoli, *Macromolecular Rapid Communications* 27, 1515-1532 (2006).
- [13] H. Cai, P. Li, G. Sui, Y. Yu, G. Li, X. Yang, S. Ryu, *Thermochimica Acta* 473, 101-105 (2008).
- [14] M.E. Brown, *Introduction to thermal analysis: techniques and applications*, Springer Science & Business Media, 2001.
- [15] M.E. Brown, D. Dollimore, A.K. Galwey, *Reactions in the solid state*, Elsevier, 1980.
- [16] Q. Wang, T. He, P. Xia, T. Chen, B. Huang, *Journal of applied polymer science* 66, 799

Existence of three positive solutions for nonlinear fractional boundary value problems with m point boundary conditions

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Abstract: This talk will be devoted to the existence of multiple positive solutions for nonlinear boundary value problems with m-point boundary conditions. Green' s function is obtained and some necessary inequalities of the Green' s function are derived. In the process, we use the Leggett-Williams fixed point theorem [1] to prove the existence of positive solutions. Finally, an example is presented to show the effectiveness of the main result.

Keywords: boundary value problems, fixed point theorems; integral boundary conditions.

Mathematics Subject Classification: 34B10, 34B18, 39A.

References

- [1] RW. Leggett, RW, LR. Williams, LR: Multiple positive fixed points of nonlinear operators on ordered Banach spaces. Indiana Univ. Math. J. 28, (1979) 673-688.

Comparative Study of Different Fault-Tolerant Control Strategies for Three-Phase Induction Motor

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Abstract: In this paper, we have studied a different fault tolerant control (FTC) strategies for a three-phase induction motor (3p-IM). Further we introduce Backstepping controller (BC) and Input-output linearization controller (IOLC). To provide a direct comparison between these FTCs approaches, the performances are evaluated using the control of 3p-IM under failures, variable speed, and variable parameters. A comparison between the two control strategies is proposed to prove the most robust one. The simulation results show the robustness and good performance of the fault tolerant control with Input-output linearization controller compared to one with Backstepping controller. The FTC with IOLC is more stable and robust against failures, load torque perturbation and speed reversion.

Keywords: Three-phase induction motor (3p-IM), Fault Tolerant Control (FTC), Input-output linearization controller, Backstepping controller.

Mathematics Subject Classification: 12X34, 56Y78.

References

- [1] A. Akincilar, Mathematical Model for Transporting the Arriving Passengers from the Airport to the City Centre. *Acta Physica Polonica A* Vol. 132, 2017. DOI:10.12693/APhysPolA.132.1214.
- [2] C. Bonivento , A. Isidori ,L. Marconi, A. Paoli , Implicit fault tolerant control: Application to induction motors, *IFAC*, 35(1), pp. 299-304, 2002. <https://doi.org/10.3182/20020721-6-ES-1901.00781>
- [3] C. Bonivento , A. Isidori ,L. Marconi and A. Paoli , Implicit fault tolerant control: application to induction motors. *Automatica*, 40, pp. 355-371, 2004. DOI:10.1016/j.automatica.2003.10.003
- [4] M. Bruccoleri, M. Amico, and G. Perrone, Distributed intelligent control of exceptions in reconfigurable manufacturing systems, *Int. J. Prod. Res.*, vol. 41, no. 7, pp. 1393–1412, May 2003. <https://doi.org/10.1080/1352816031000075170>
- [5] R. Isermann, R. Schwarz, and S. Stolz, Fault-tolerant drive-by-wire systems, *IEEE Control Syst. Mag.*, vol. 22, no. 5, pp. 64–81, Oct. 2002. DOI: 10.1109/MCS.2002.1035218
- [6] M. G. Mehrabi, A. G. Ulsoy, Y. Koren, and P. Heytler, Trends and perspectives in flexible and reconfigurable manufacturing systems, *J. Intell. Manuf.*, vol. 13, no. 2, pp. 135–146, 2002.
- [7] T. Roubache, S. Chaouch, Ms. Nait-Said, Backstepping design for fault detection and FTC of an induction motor drives-based Evs, *AUTOMATIKA* 57(3): 736-748. <http://dx.doi.org/10.7305/automatika.2017.02.1733>.

The hidden role of the pre-symptomatic individuals in the transmission dynamics of COVID-19

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Abstract: : In this talk, a mathematical model with four different routes of transmission, namely, asymptomatic, pre-symptomatic, symptomatic and environmental transmissions, has been proposed and analyzed to investigate the role of pre-symptomatic individuals in the transmission dynamics of COVID-19 outbreak. Using the next generation matrix method, the basic reproduction number has been derived and then sensitivity analysis of the proposed model is presented. Existence and stability analysis of disease free and endemic equilibrium points have been discussed. Numerical simulations to demonstrate the effect of some model parameters related to pre-symptomatic transmission on the disease transmission dynamics have been carried out.

Keywords: COVID-19; Pre-symptomatic individuals; Basic Reproduction Number; Stability Analysis.

Mathematics Subject Classification: 34C23, 34D23, 92D30.

References

- [1] L. Ferretti, C. Wymant, M. Kendall, L. Zhao, A. Nurtay, L. Abeler-Dorner, M. Parker, D. Bonsall, C. Fraser, Quantifying SARS-CoV-2 transmission suggests epidemic control with digital contact tracing, *Science* 368 (2020), 1-9.
- [2] M. Gatto, E. Bertuzzo, L. Mari, S. Miccoli, L. Carraro, R. Casagrandi, A. Rinaldo, Spread and dynamics of the COVID-19 epidemic in Italy: Effects of emergency containment measures, *P. NATL. ACAD. SCI.* 117(2020), 10484-10491.
- [3] X. He, E.H.Y. Lau, P. Wu, Peng, X. Deng, J. Wang, X. Hao, Y.C. Lau, Yiu, J. Y. Wong, Y. Guan, X. Tan and others, Temporal dynamics in viral shedding and transmissibility of COVID-19, *Nat. Med.* 26(2020), 672-675.
- [4] W. E. Wei, Z. Li, C. J. Chiew, Calvin, S. E. Yong, M. P. Toh, V. J. Lee, Presymptomatic Transmission of SARS-CoV-2 in Singapore, January 23–March 16, 2020, *Morbidity and Mortality Weekly Report* 69(2020), 411-415.

Bell-Based Genocchi Polynomials

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Abstract: In this study, we introduce Bell-based Genocchi polynomials and then derive multifarious correlations and formulas including some implicit summation formulas and derivative properties.

Keywords: Genocchi polynomials, Bell polynomials, Mixed-type polynomials, Stirling numbers of the second kind.

Mathematics Subject Classification: 11B83, 11S80, 05A19.

References

- [1] E.T. Bell, Exponential polynomials. *Ann. Math.* 35 (1934) 258-277.
- [2] U. Duran, S. Araci, M. Acikgoz, Bell-Based Bernoulli Polynomials with Applications. *Axioms* 2021, 10, 29.
- [3] D.S. Kim, T. Kim, Some identities of Bell polynomials, *Sci. China Math.* 58 (2015) 2095-2104.
- [4] L. Carlitz, Some remarks on the Bell numbers. *Fibonacci Quart.* 18 (1980) 66-73.
- [5] T. Kim, D. S. Kim, L.-C. Jang, H. Lee, H.-Y. Kim, Complete and incomplete Bell polynomials associated with Lah-Bell numbers and polynomials. *Adv Differ Equ* 2021, 101 (2021).
- [6] U. Duran, M. Acikgoz, On degenerate truncated special polynomials, *Mathematics*, 2020, 8(1), 144
- [7] N. Khan, S. Husain, Analysis of Bell based Euler polynomials and thier application, arXiv:2104.09129v1 [math.NT].
- [8] A.F. Horadam, Genocchi Polynomials, *The Fibonacci Quarterly*, 30(1)(1992), 21-34.

On Degenerate Truncated Frobenius-Euler Polynomials

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Abstract: In this study, we consider the truncated degenerate Frobenius-Euler polynomials. Then we examine diverse properties and formulas covering addition formulas, correlations and derivation property. Then, we derive some interesting implicit summation formulas.

Keywords: Degenerate exponential function, truncated exponential function, Frobenius-Euler polynomials, exponential generating function.

Mathematics Subject Classification: 11B73, 11B68, 33B10.

References

- [1] L. Carlitz, Degenerate Stirling, Bernoulli and Eulerian numbers. *Utilitas Math.* 15 (1975) 51-88.
- [2] G. Dattoli, C. Ceserano, D. Sacchetti, A note on truncated polynomials. *Appl. Math. Comput.* 134 (2003) 595-605.
- [3] F.T. Howard, Explicit formulas for degenerate Bernoulli numbers. *Discrete Math.* 162 (1996) 175-185.
- [4] B. Y. Yasar, M. A. Ozarslan, Frobenius-Euler and Frobenius-Genocchi polynomials and their differential equations, *New Trends in Mathematical Sciences*, 3(2) (2015) 172-180.
- [5] W. Kumam, H.M. Srivastava, S.A. Wani, S. Araci, P. Kumam, Truncated-exponential-based Frobenius–Euler polynomials. *Adv. Differ. Equ.* 2019, 530 (2019)
- [6] U. Duran, M. Acikgoz, On degenerate truncated special polynomials, *Mathematics*, 2020, 8(1), 144.

Study of time fractional Black Scholes Schrodinger equation for two stocks in the light of Islamic vision

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Abstract: This paper represents the numerical and analytical results to get real and Imaginary option prices of two stocks in form of analytic infinite series by solving Schrödinger Black Scholes time fractional ordered PDE consisting two different stocks. For that reason, the appropriate numerical and analytical methods are discovered for the models that can be expressed as partial differential equations of integer and fractional orders, subjected to preliminary or boundary situations. The discussion have been presented after getting the solution for the time fractional Black-Scholes Schrödinger equation for two stocks in the light of Islamic vision.

Keywords: Black-Scholes equation, Islamic option pricing, time fractional

Mathematics Subject Classification:

References

- [1] Rejeb, Aymen&Arfaoui, Mongi. (2016). Conventional and Islamic stock markets: what about financial performance.
- [2] IMF, World Bank acknowledge Islamic finance as powerful development tool for members (gulf-times.com)February 28 2017 09:28 PM
- [3] Islamic Finance (worldbank.org)March 31, 2015
- [4] <https://www.globenewswire.com/news-release/2019/03/20/1758003/0/en/Global-Islamic-Finance-Markets-Report-2019-Islamic-Banking-is-the-Largest-Sector-Contributing-to-71-or-USD-1-72-Trillion.html>
- [5] “World Bank; Islamic Development Bank Group. 2017. Global Report on Islamic Finance : Islamic Finance - A Catalyst for Shared Prosperity?. Washington, DC: World Bank. Islamic Development Bank Group. <https://openknowledge.worldbank.org/handle/10986/25738> License: CC BY 3.0 IGO.”
- [6] Khaliq, A., &Thaker, H. M. T. (2015). Revisiting of an Islamic Options Permissibility from Shariah Perspectives. *Global Review of Islamic Economics and Business*, 1(3), 175-184.
- [7] Aboulaich, R., &Dchieche, A. (2015). Pricing of WaadBilMourabaha. *Asian journal of applied sciences*, 3(3), ISSN: 2321 – 0893
- [8] Omrana, S., &Aboulaich, R. (2016). Islamic Financial Engineering Al Arboun Sale. *International Journal of Applied Engineering Research*, 11(8), 5584-5590.
- [9] Nurazizah, S., & Bon, A. T. Review Methods to Solve Fractional Black-Scholes. *Proceedings of the International Conference on Industrial Engineering and Operations Management Pilsen, Czech Republic*, July 23-26, 2019
- [10] Prathumwan, D., &Trachoo, K. (2020). On the solution of two-dimensional fractional Black–Scholes equation for European put option. *Advances in Difference Equations*, 2020(1), 1-9.
- [11] Podlubny, I.: *Fractional Differential Equations*. Mathematics in Science and Engineering. Academic Press, San Diego(1999)

- [12] Hadamard, J.: Implied volatility in Black–Scholes model with GARCH volatility. *J. Math. Pures Appl.* 4(8), 101–186(1892)
- [13] Abdon, A., Dumitru, B.: New fractional derivatives with nonlocal and non-singular kernel: theory and application to heat transfer model. *Therm. Sci.* 20(2), 763–769 (2016). <https://doi.org/10.2298/TSCI160111018A>
- [14] Caputo, M.: Linear models of dissipation whose q is almost frequency independent—II. *J. Funct. Spaces* 13(5), 529–539 (1967). <https://doi.org/10.1111/j.1365-246X.1967.tb02303.x>
- [15] Bayin, S.S.: Definition of the Riesz derivative and its application to space fractional quantum mechanics. *J. Math. Phys.*

Application of Extended Rational Sin-Cos and Sinh-Cosh Method for the System of Nonlinear Partial Differential Equations

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Abstract: In this research, system of the nonlinear partial differential equations with fractional order are studied. The system including both conformable and M-truncated fractional derivatives are solved. Trigonometric, dark and soliton solutions of the considered system are analytically obtained by the extended rational sin-cos and sinh-cosh method. We make a comparison between the solutions of the system with the conformable derivative and M truncated fractional derivative. The results are demonstrated in the 2D and 3D graphics.

Keywords: Extended rational sin-cos and sinh-cosh method , fractional conformable derivative, fractional M-truncated derivative.

Mathematics Subject Classification: 34K37, 35G50

References

- [1] R. Khalil, M. Al Horani, A. Yousef, M. Sababheh, A new definition of fractional derivative, *Journal of Computational and Applied Mathematics* 264 (2014) 65–70. doi:10.1016/j.cam.2014.01.002. URL <http://dx.doi.org/10.1016/j.cam.2014.01.002>
- [2] J. V. D. C. Sousa, E. C. De Oliveira, A new truncated m-fractional derivative type unifying some fractional derivative types with classical properties, *arXiv* 16 (1) (2017) 83–96. arXiv:1704.08187, doi:10.28924/2291-8639-16-2018-83
- [3] D. Lu, A. R. Seadawy, M. M. Khater, Structure of solitary wave solutions of the nonlinear complex fractional generalized Zakharov dynamical system, *Advances in Difference Equations* 2018 (1) (2018). doi:10.1186/s13662-018-1734-4. URL <http://dx.doi.org/10.1186/s13662-018-1734-4>
- [4] N. Mahak, G. Akram, Extension of rational sine-cosine and rational sinh-cosh techniques to extract solutions for the perturbed NLSE with Kerr law nonlinearity doi:10.1140/epjp/i2019-12545-x.

Numerical scheme for solving fractional integro-differential equations with Mittag-Leffler Kernel

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Abstract: In this work, we discussed a general class of fractional integro-differential equations that involve the Atangana-Baleanu derivative. The numerical scheme is developed for solving the aforementioned equations based on the shifted Vieta-Lucas polynomials. We utilize the operational matrices based on these polynomials to obtain a numerical solution of the considered equations. By approximating the unknown function and its derivative in terms of Vieta-Lucas polynomials and substituting these approximations into the original equation, the studied equation is transformed into a system of nonlinear algebraic equations. At the end of the study, some examples are included to show the accuracy and validity of the proposed method.

Keywords: Fractional integro-differential equations, Atangana-Baleanu derivative, Vieta-Lucas polynomials, Operational matrices.

Mathematics Subject Classification: 42B25, 31C15.

References

- [1] K. Kumar, R. K. Pandey*, S. Sharma, Comparative study of three numerical schemes for fractional integro-differential equations, *Journal of Computational and Applied Mathematics* 315 (2017) 287–302.
- [2] P. Agarwal, A.A. El-Sayed, Vieta–Lucas polynomials for solving a fractional-order mathematical physics model, *Advances in Difference Equations*, 2020: 624, (2020).
- [3] A. Atangana, D. Baleanu, New fractional derivatives with non-local and non-singular kernel: theory and application to heat transfer model. *Therm Sci.* 20 (2) 763-769, 2016.

Numerical schemes based on Legendre wavelets for Solving Nonlinear Partial Differential Equations

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Abstract: This study focuses on the solution of some partial differential equations by various methods. This purpose is accomplished by using the three-step wavelet Galerkin method and the three-step wavelet Collocation method. These methods are based on Legendre wavelets. These methods are the valuable and robust numerical methods that can be easily applied to linear and nonlinear problems. At the end of the study, the comparison between numerical and exact solutions proves the presented methods' efficiency and accuracy.

Keywords: Legendre Wavelets, Galerkin method, Collocation method, three-step Taylor method.

Mathematics Subject Classification: 65T60, 35Q53, 65M60.

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References

- [1] BV Rathish, K., Mani, M. (2005), "Time-accurate solutions of Korteweg–de Vries equation using wavelet Galerkin method", *Applied mathematics and computation*, 162(1), pp. 447-460.
- [2] Kumar, R. B., Mehra, M. (2006), "A three-step wavelet Galerkin method for parabolic and hyperbolic partial differential equations", *International Journal of Computer Mathematics*, 83(1), pp. 143-157.
- [3] Wazwaz, A.M. 2002, "Partial Differential Equations Methods and Applications", Saint Xavier University, Chicago, Illinois, USA.
- [4] Kumar, B. R., Mehra, M. (2005), "Time-accurate solutions of Korteweg–de Vries equation using wavelet Galerkin method", *Applied mathematics and computation*, 162(1), pp. 447-460.
- [5] Usman, M., Hamid, M., Haq, R. U., Wang, W. (2018), "An efficient algorithm based on Gegenbauer wavelets for the solutions of variable-order fractional differential equations", *The European Physical Journal Plus*, 133(8), pp. 327.

Numerical Solution of Fractional Order Partial Differential Equation via Modified Laplace Method

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Abstract: In this study, numerical solutions of the fractional-order partial differential equations are investigated. The nonlinear partial equation has Caputo type fractional order. Numerical solutions are obtained by the modified Laplace decomposition method. Due to the ease of use of the mentioned method, significant and efficient results are obtained successfully. The solutions that have fractional order are compared with the behavior of the solutions that have integer order. 2D and 3D graphs express the results.

Keywords: Modified Laplace Decomposition Method, fractional nonlinear partial differential equations.

Mathematics Subject Classification: 35R11, 44A10.

References

- [1] Atangana, Abdon, and Ali Akgül. "Can Transfer Function and Bode Diagram Be Obtained from Sumudu Transform." *Alexandria Engineering Journal*, Elsevier, 3 Jan. 2020.
- [2] Hussain, Mazhar & Khan, Majid. (2010). Modified Laplace decomposition method. *Applied Mathematical Sciences* (Ruse). 4.
- [3] Prakasha, D. G., Malagi, N. S., Veerasha, P., & Prasannakumara, B. C. (2020, October 31). An efficient computational technique for time– fractional KaupKupershmidt equation. *Wiley Online Library*.

Mathematical Modeling for Optimal Control of COVID-19 with Combined Measures of Vaccination and Non-clinical Interventions

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Abstract: More than a year has been passed since the first cases of COVID-19 were reported in Wuhan, China and in that time, the virus has caused high mortality and tremendous interruptions to socio-economic activity. It has become the most important public health challenge humans have faced since the 1918 Spanish flu pandemic. Within weeks of emergence, the highly transmissible and deadly COVID-19 pandemic spread to every part of the world, so far accounting for over 169,094,393 confirmed cases and 3,512,509 deaths (as of May 27, 2021), in addition to incurring severe economic burden, social disruptions and other human stresses, globally. Although there is finally a little hope due to unprecedented progress in SARS-CoV-2 vaccine research and development, the current approved vaccines can only provide 50-95% protection in preventing symptomatic COVID-19 diseases which is much lower when it comes to reducing SARS-CoV-2 infection. At the same time, the emergence of new coronavirus strains may affect the current vaccine efficacy. So, until an effective preventative measure (such as 100% effective vaccine) has been developed, it is of great concern to develop disease management strategies, a better understanding of SARS-CoV-2 pathogenesis and population susceptibility to infection by applying non-pharmaceutical control strategy in terms of mathematical modeling which can provide a robust strategy to reduce the infection of COVID-19. In this talk, I will discuss our work on the mathematical modeling and analysis of the spread and control of COVID-19, with emphasis on the non-pharmaceutical preventive strategy in parallel to the currently-available and large scale implemented anti-COVID vaccines. Specifically, we will explore the importance of gaining self-immunity for the elimination of the pandemic taking immune boosting foods and drugs along with other non-pharmaceutical interventions, such as face masks usage, hand sanitizing and social-distancing.

Keywords: SARS-CoV-2, mathematical model, vaccination, non-pharmaceutical interventions, optimal control, maximum principle.

Mathematics Subject Classification:

References

- [1] M. H. A. Biswas, M.A.Islam, S. Akter, S. Mondal, M. S. Khatun, S.A.Samad, A. K. Paul and M. R. Khatun, Modelling the Effect of Self-Immunity and the Impacts of Asymptomatic and Symptomatic Individuals on COVID-19 Outbreak, CMES-Computer Modeling in Engineering & Sciences, 125(3), 1033–1060, 2020. DOI:10.32604/cmescs.2020.012792.
- [2] M. H. Kabir, M. O. Gani, S. Mandal and M. H. A. Biswas, Modeling the dispersal effect to reduce the infection of COVID-19 in Bangladesh, Sensors International, Vol. 1, 2020, 100043. DOI: <https://doi.org/10.1016/j.sintl.2020.100043>.
- [3] S. Mandal, M. S. Islam and M. H. A. Biswas, Modeling the Potential Impact of Climate Change on Living Beings near Coastal Area, Modeling Earth Systems and Environment, 2020. DOI: <https://doi.org/10.1007/s40808-020-00897-5>.
- [4] S. S. Shanta and M. H. A. Biswas, The Impact of Media Awareness in Controlling the Spread of Infectious Diseases in Terms of SIR Model, Mathematical Modelling of Engineering Problems, 7(3): 368-376 (2020). DOI: <https://doi.org/10.18280/mmep.070306>.

- [5] M. S. Khatun and M. H. A. Biswas, Mathematical Analysis and Optimal Control Applied to the Treatment of Leukemia, *J. Appl. Math. Comput.* 64, 331–353 (2020). DOI: <https://doi.org/10.1007/s12190-020-01357-0>
- [6] M. H. A. Biswas, M. S. Khatun, A. K. Paul, M. R. Khatun, M. A. Islam, S. A. Samad, U. Ghosh, Modeling the Effective Control Strategy for Transmission Dynamics of Global Pandemic COVID-19, Preprint in medRxiv, 2020. DOI: <https://doi.org/10.1101/2020.04.22.20076158>.
- [7] M. N. Hasan, M. S. Uddin and M. H. A. Biswas, Interactive Effects of Disease Transmission on Predator-Prey Model, *Journal of Applied Nonlinear Dynamics*, 9(3) (2020) 401–413. DOI: 10.5890/JAND.2020.09.005.
- [8] M. S. Khatun and M. H. A. Biswas, Optimal Control Strategies for Preventing Hepatitis B Infection and Reducing Chronic Liver Cirrhosis Incidence, *Infectious Disease Modeling*, 5: 91-110 (2020). DOI: <https://doi.org/10.1016/j.idm.2019.12.006>.
- [9] M. S. Khatun and M. H. A. Biswas, Modeling the Effect of Adoptive T cell Therapy for the Treatment of Leukemia, *Comp and Math Methods*, 2(2), 2020; e1069. DOI: <https://doi.org/10.1002/cmm4.1069>.
- [10] J. K. Ghosh, U. Ghosh, M. H. A. Biswas and S. Sarkar, Qualitative Analysis and Optimal Control Strategy of an SIR Model with Saturated Incidence and Treatment, *Differential Equations and Dynamical Systems*, 12(38): 1-15, 2019. DOI: <https://doi.org/10.1007/s12591-019-00486-8>.

Statistical Approximation for Kantorovich Type q -Balázs-Szabados Operators

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Abstract: Balázs and Szabados studied approximation properties of the Bernstein type rational functions called the Balázs-Szabados operators. Different q -analogues of Balázs-Szabados operators are recently studied by several authors. New Kantorovich type q -analogue of the Balázs-Szabados operators are defined by Hamal and Sabancıgil as follows:

$$R_{n,q}^*(f, x) = \sum_{k=0}^n r_{n,k}(q, x) \int_0^1 f\left(\frac{[k]_q + q^k t}{b_n}\right) d_q t, \text{ where } f : [0, \infty) \rightarrow \mathbb{R}, q \in (0, 1), a_n = [n]_q^{\beta-1},$$

$$b_n = [n]_q^\beta, \quad 0 < \beta \leq \frac{2}{3}, \quad n \in \mathbb{N}, \quad x \geq 0, \quad r_{n,k}(q, x) = \frac{1}{(1 + a_n x)^n} \begin{bmatrix} n \\ k \end{bmatrix}_q (a_n x)^k \prod_{s=0}^{n-k-1} (1 + (1 - q) [s]_q a_n x).$$

The following notions is due to Fast [2] and Fridy [3].

. A sequence $x = (x_n)$ is statistically convergent to the number L if for every $\varepsilon > 0$, $\delta \{k \in \mathbb{N} : |x_k - L| \geq \varepsilon\} = 0$ and we write $st_A - \lim_{n \rightarrow \infty} x_n = L$.

Let $C_B[a, b]$ be the space of all continuous functions at each point in $[a, b]$ and bounded on the entire positive real line that means $|f(x)| \leq M_f, \forall x \in (0, \infty)$, where M_f is a constant depending on f .

Bohman –Korovkin type statistical approximation theorem was proved by Gadjiev and Orhan.

Now we present the main result for statistical convergence of the operators $R_{n,q}^*(f, x)$ to f .

Theorem 2. Let $q = (q_n)_{n \in \mathbb{N}}, q_n \in (0, 1)$, be a sequence $st_A - \lim_n q_n = 1$. Then for each compact interval $[0, b] \subset [0, \infty)$, we have $st_A - \lim_n \|R_{n,q}^*(f, x) - f(x)\| = 0, \quad \forall f \in C([0, b])$.

Keywords: q -Balázs-Szabados operators, Korovkin theorem, statistical approximation theorem

Mathematics Subject Classification: 47B65

References

- [1] Hamal. H, Sabancıgil, P, “Some Approximation Properties of new Kantorovich type analogue of Balázs-Szabados Operators”, Journal of Ineq &Appl. 159, 2020
- [2] Fast. H, Sur la “convergence statistique”, Colloq. Math. 2, pp. 241-244, 1951.
- [3] Fridy. J. A, “On statistical convergence”, Analysis. 5, pp. 301-313, 1985.

Sequential feature selection with machine learning techniques for heart disease diagnosing

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Abstract: In healthcare domain, the medical information treatment is very crucial for data acquisition, archiving, presentation and decision support services. For exploring these data, several techniques based on machine learning are utilized to predict a decision by building models. In this paper, we aim to develop an effective medical decision system based on machine learning techniques for heart disease detection. In this context, we used three different classification algorithms such as Decision Tree (DT), Support Vector Machines (SVM) and Linear Discriminant Analysis (LDA). In addition, we can seriously reduce the time, materials, and labor to get the final decision while increasing the prediction performance by using Sequential feature selection technique (SFS). Our experiments are conducted on real heart diseases dataset that has been collected to assess and analyze the risk factors. The obtained results show the effectiveness of the SFS technique with each classifier. Furthermore, our best system outperforms the well-known heart disease prediction methods in literature.

Keywords: Heart disease, prediction, Decision Tree, Support Vector Machines, Linear Discriminant Analysis, Sequential feature selection.

Mathematics Subject Classification: 1X234

References

- [1] M. Kumar, S. Shambhu, and A. Sharma, "Classification of heart diseases patients using data mining techniques." *International journal of research in electronics and computer engineering*, vol. 6, no. 3, pp. 1495-1499, 2018.
- [2] J. Soni, U. Ansari, D. Sharma, and S. Soni, "Predictive data mining for medical diagnosis : An overview of heart disease prediction," *International Journal of Computer Applications*, vol. 17, no. 8, pp. 43-48, 2011.
- [3] M. Ramaraj and T. A. Selvadoss, "A comparative study of cn2 rule and svm algorithm and prediction of heart disease datasets using clustering algorithms," *Network and Complex Systems*, vol. 3, no. 10, pp. 1-6, 2013.
- [4] K. Sudhakar and D. M. Manimekalai, "Study of heart disease prediction using data mining," *International journal of advanced research in computer science and software engineering*, vol. 4, no. 1, pp. 1157-1160, 2014.
- [5] S. Gupta, D. Kumar, and A. Sharma, "Performance analysis of various data mining classification techniques on healthcare data," *International journal of computer science & Information Technology (IJCSIT)*, vol. 3, no. 4, pp. 155-169, 2011.
- [6] R. Detrano, A. Janosi, W. Steinbrunn, M. Pfisterer, J.-J. Schmid, S. Sandhu, K. H. Guppy, S. Lee, and V. Froelicher, "International application of a new probability algorithm for the diagnosis of coronary artery disease," *The American journal of cardiology*, vol. 64, no. 5, pp. 304-310, 1989.
- [7] C. Ordonez, "Association rule discovery with the train and test approach for heart disease prediction," *IEEE Transactions on Information Technology in Biomedicine*, vol. 10, no. 2, pp. 334-343, 2006.

The Covid 19 Analysis With Taylor Matrix and Collocation Method

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Abstract: Early analysis of infectious diseases is very important in the spread of the disease [1]. The main aim of this study is to make important predictions and inferences for the current epidemic disease Covid 19, with mathematical modeling and numerical solution methods. We deal with the logistic growth model [2-6]. We obtain carrying capacity and growth rate with Turkey epidemic data. The obtained growth rate and carrying capacity is used in the Taylor collocation method. With this method, we estimate and make predictions close to reality with Maple. We also show the estimates with the help of graphics and tables.

Keywords: Logistic model, Covid-19, Taylor polynomials and series, Collocation points, Turkey.

Mathematics Subject Classification: 65H05.

References

- [1] N. M. Last, “Mathematical prediction in infection”, *Medicine*, Volue 37, Issue 10, (2009), 507-509.
- [2] K. Roosa, Y. Lee, R. Luo, A. Kirpich, R. Rothenberg, J. M. Hayman, P. Yan and G. Chowell, “Real-time forecasts of the Covid 19 epidemic in China from February 5th to February”, 24th, *Infectious Disease Modelling*, 5(2020), 256-263.
- [3] E. Pelinovsky, A. Kurkin, O. Kurkina, M. Kokoulina, “Logistic equation and Covid 19”, *Chaos Solitons Fractals* 140, (2020).
- [4] M. Batista, “Estimation of the final size of the coronavirus epidemic by the logistic model”, <https://doi.org/10.1101/2020.03.11.20024901>, (2020).
- [5] M. Jain, P. K. Bhati, P. Kataria, R. Kumar, “Modelling logistic growth model for Covid 19 pandemic in India”, *Proceedings of the Fifth International Conference on Communication and Electronics Systems (ICCES 2020)*, (2020).
- [6] K. Wu, D. Darcet, Q. Wang and D. Sornette, “Generalized logistic growth modeling of the Covid 19 outbreak in 29 provinces in China and in the rest of the World”, *Nonlinear Dyn* (2020) 101:1561.1581.

A review of clustering algorithms and application

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Abstract: There are many different tasks where it is important to find hidden groups in the data that are much easier to interpret than individual observations. This can be done using data clustering. According to Jain [1], the main goal of clustering is to perform accurate data grouping using observations, points, or objects. As well data clustering can be described as a machine learning method (unsupervised learning) that allows objects to be formed in groups (called clusters) while objects in different clusters are different. To assess the similarity of different observations, different distance measures are used. Distance between the two objects is smaller at which the observations are more similar to each other and vice versa. There are a particularly large number of clustering algorithms, so all these methods are divided into separate groups. According to Fraley and Raftery [2], data clustering can be distinguished into two main groups: hierarchical clustering and divisional clustering. Han and Kamber [3] propose to classify data clustering in a different way and distinguish the following groups: density-based methods, model-based methods, and grid-based methods. One of the most commonly used clustering algorithms is the k-means algorithm [4, 5], but this method does not always work properly. This method is usually suitable for distinguishing only observations that are spherical in shape. There are also many other clustering algorithms that are used in scientific research: UNIC [6], k-Medoids (PAM) [7], Gaussian Mixture [7], TCLUS [7], Trimmed k-means [8], Spectral Clustering, Density-Based Spatial Clustering, MULIC, DENCLUE, SOMs (NeuralNet), SVM, HIERDENC, Deep embedded clustering [9] and etc. However, all of these methods are often applied to specific tasks and are not universal, for this reason the aim of this paper is to introduce different clustering algorithms and present how these clustering algorithms work.

Keywords: clustering, machine learning, data science, k-means, DBSCAN

Mathematics Subject Classification:

References

- [1] K. Jain. Data Clustering: 50 years beyond k-means. *Pattern Recognition Letters*, 31 (8):651-666, 2010.
- [2] C. Fraley, A. E. Raftery. How Many Clusters? Which Clustering Method? Answers Via Model-Based Cluster Analysis. Technical Report No. 329, Department of Statistics University of Washington, 1998.
- [3] J. Han, M. Kamber, and J. Pei. *Data Mining: Concepts and Techniques*. Morgan Kaufmann Publishers, 2011.
- [4] W.Guogiu. Robust self-tuning spectral clustering. *Neurocomputing*, 1 (391):243-248, 2020.
- [5] K. Sinaga, M. Yang. Unsupervised K-means clustering algorithm. *IEEE Access*, 8 (8):80716-80727, 2020.
- [6] N.Leopold, O.Rose. A fast nonparametric clustering. *Pattern Recognition*, 100 (100):107-117, 2020.
- [7] A. E. Attar, R. Khatoun, B. Birregah and M. Lemercier. Article in proceedings. In: *Robust clustering methods for detecting smartphone's abnormal behavior*, 2014 IEEE Wireless Communications and Networking Conference (WCNC), 2014, 2552-2557. (Istanbul, Turkey)
- [8] C. Albertos, J. Antonio et.al.. Trimmed k-means: An attempt to robustify quantizers. *Annals of Statistics*, 25 (2):553-576, 1997.
- [9] R.Yazhou et. al. Semi-supervised deep embedded clustering. *Neurocomputing*, 325 (325):121-130, 2019

Quantization: Crossroads and Crossovers

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Abstract: Quantization is the mathematical procedure turning classical observables (functions on phase space) into quantum ones (often as self-adjoint operators). There exist various quantization techniques but they can be broadly classed into two: ones that are necessarily based on Hilbert spaces, and ones that do not. Of the former, there is geometric quantization, and the latter, deformation quantization and their various ramifications. We will discuss some of the pros and cons of these techniques and their interrelationships, with the focus on systems in external fields and noncommutative quantum mechanics.

Keywords: quantization, symplectic geometry, deformation of algebras, fibre bundles, gauge fields and noncommutative quantum mechanics.

Mathematics Subject Classification: 47B32, 46E22, 34A12, 74S30.

References

- [1] S. Twareque Ali & Mirosława Engliš, “Quantization Methods: A Guide for Physicists and Analysts”, *Reviews in Mathematical Physics* 17 (2005) 391-490.
- [2] C.J. Isham, “Topological and Global Aspects of Quantum Theory” in *Relativity, Groups and Topology II*, (eds.) Bryce S. DeWitt and Raymond Stora, (North-Holland, 1984) 1059-1290.
- [3] N.M.J. Woodhouse, *Geometric Quantization*, (Clarendon Press, 1997)
- [4] F. Bayen, M. Flato, C. Fronsdal, A. Lichnerowicz and D. Sternheimer, “Deformation Theory and Quantization I: Deformations of Symplectic Structures”, *Annals of Physics* 111 (1978) 61-110; “Deformation Theory and Quantization II: Physical Applications”, *Annals of Physics* 111 (1978) 111-151.
- [5] H. Zainuddin, “Group-Theoretic Quantization of a Particle on a Torus in a Constant Magnetic Field”, *Physical Review D* 40 (1989) 636-641.
- [6] Mohd Faudzi Umar, Nurisya Mohd Shah & Hishamuddin Zainuddin, “Two-Dimensional Plane, Modified Symplectic Structure and Quantization”, *Jurnal Fizik Malaysia* 39 (2018) 30022-30026.
- [7] S. Hasibul Hassan Chowdhury & Hishamuddin Zainuddin, “Wigner Functions for Gauge Equivalence Classes of Unitary Irreducible Representations of Noncommutative Quantum Mechanics”, *European Journal of Physics: Special Topics* 226 (2017) 2359-2374.

Piecewise differential and integral operators: An approach to capture real world problems with crossover behaviors

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Abstract: Mankind relies on modeling to predict future behaviors of some problems occurring in nature. Due to the complexities of nature, several approaches have been introduced, for example deterministic, stochastics, probabilistic and fuzzy. Although the listed approaches have been applied with some success, the problem of crossover still remains a great challenge. Very recently the concept of piecewise differentiation and integration was introduced, our talk will therefore be devoted to the theory, methods and applications of this approach.

Robust Profile Monitoring for Phase II Profile Monitoring via Residuals

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Abstract: Many studies were conducted for fitting models using parametric and non-parametric techniques, in fact, their fits may be biased and have inflated the estimated variances when the model is misspecified, respectively. Thus, semi-parametric techniques are used for fitting models as they combine the advantages of parametric and non-parametric fits. In this study, we introduce model robust regression technique-2 (MRR2) for Phase II profile monitoring, namely the semi-parametric approach, where it is a combination of the parametric fit with a portion of a non-parametric residuals fit. Multivariate CUSUM (MCUSUM) chart unitized for monitoring the slope of linear mixed models in Phase II based on the random effects. A comprehensive simulation study performed to evaluate the proposed approach for correlated and uncorrelated profiles assuming different profile sizes, sample sizes and several model misspecification levels. Average Run Length (ARL) and Average time to signal (ATS) criteria were used for comparing the performances of the parametric, non-parametric and semi-parametric MCUSUM charts. The results showed that the semi-parametric chart had the best performance in detecting different shifts. Also, a real data application was conducted, where it showed that the semi-parametric chart had the highest sensitivity for the out-of-control scenarios.

Keywords: Linear Mixed Models; Penalized Spline; Model Misspecification; Profile Monitoring; MCUSUM; ARL; Average Time to Signal.

Mathematics Subject Classification:

References

- [1] Kang, L., & Albin, S. L. 2000. On-Line Monitoring When the Process Yields a Linear Profile. *Journal of Quality Technology*, 32(4), 418–426. doi:10.1080/00224065.2000.11980027
- [2] Keunpyo Kim, Mahmoud A. Mahmoud, William H. Woodall. 2003. On the Monitoring of Linear Profiles, *Journal of Quality Technology*. doi: 10.1080/00224065.2003.11980225
- [3] Mahmoud, M. A., & Woodall, W. H. 2004. Phase I analysis of linear profiles with calibration applications. *Technometrics*, 46(4), 380-391.
- [4] Ding, Y., Zeng, L., & Zhou, S. 2006. Phase I Analysis for Monitoring Nonlinear Profiles in Manufacturing Processes. *Journal of Quality Technology*, 38(3), 199–216. doi:10.1080/00224065.2006.11918610
- [5] William H. Woodall, Dan J. Spitzner, Douglas C. Montgomery & Shilpa Gupta. 2004. Using Control Charts to Monitor Process and Product Quality Profiles, *Journal of Quality Technology*, 36)3(, 309-320, doi: 10.1080/00224065.2004.11980276
- [6] Jensen, W. A., Birch, J. B., & Woodall, W. H. 2008. Monitoring Correlation Within Linear Profiles Using Mixed Models. *Journal of Quality Technology*, 40(2), 167–183. doi:10.1080/00224065.2008.11917723
- [7] Abdel-Salam, A.S. G. 2009. Profile Monitoring with Fixed and Random Effects using Non-parametric and Semi-parametric Methods. Ph.D. thesis, Department of Statistics, Virginia Polytechnic Institute & State University, Blacksburg, VA.
- [8] Saghaei A, Amiri A, Mehrjoo M. 2009. Performance evaluation of control schemes under drift in simple linear profiles. *Proceedings of the International Conference of Manufacturing Engineering and Engineering Management*, London, England.

- [9] Noorossana, R., Eyvazian, M., & Vaghefi, A. 2010. Phase II monitoring of multivariate simple linear profiles. *Computers & Industrial Engineering*, 58(4), 563–570.doi:10.1016/j.cie.2009.12.003
- [10] Abdel-Salam, A.-S. G., Birch, J. B., & Jensen, W. A. 2012. A Semi-parametric Mixed Model Approach to Phase I Profile Monitoring. *Quality and Reliability Engineering International*, 29(4), 555–569.doi:10.1002/qre.1405
- [11] Chou, S.-H., Chang, S. I., & Tsai, T.-R. 2013. On monitoring of multiple non-linear profiles. *International Journal of Production Research*, 52(11), 3209–3224.doi:10.1080/00207543.2013.867088
- [12] Noorossana, R., Fatemi, A., & Zerehsaz, Y. 2015. Phase-II monitoring of linear profiles with random explanatory variable. *International Journal of Advance Manufacturing Technology*, 76(5), 779–787.
- [13] Mahmood, T., Abbasi, S. A., Riaz, M., & Abbas, N. 2018. An Efficient Phase I Analysis of Linear Profiles with Application in Photo-Voltaic System. *Arabian Journal for Science and Engineering*.doi:10.1007/s13369-018-3426-5
- [14] Abbas, T., Rafique, F., Mahmood, T., & Riaz, M. 2019. Efficient Phase II Monitoring Methods for Linear Profiles Under the Random Effect Model. *IEEE Access*, 7, 148278–148296.doi:10.1109/access.2019.2946211
- [15] Siddiqui, Z., & Abdel-Salam, A.-S. G. 2018. A semi-parametric profile monitoring via residuals. *Quality and Reliability Engineering International*.doi:10.1002/qre.2439
- [16] Demidenko, E. 2004. *Mixed Models: Theory and Applications*. John Wiley and Sons, New York, NY.
- [17] Mays, J. E., Birch, J. B., and Einsporn, R. L. 2000. An overview of model-robust regression. *Journal of Statistical Computation and Simulation*.
- [18] Schabenberger O, Pierce FJ. 2002. *Contemporary statistical models for the plant and soil sciences*. Boca Raton, FL: CRC Press.
- [19] Waterman, M. J., Birch, J. B., and Schabenberger, O. 2007. *Linear mixed model robust regression*. Technical Report, Department of Statistics, Virginia Polytechnic Institute & State University, Blacksburg, VA.
- [20] Waterman, M. J. T. 2002. *Linear mixed model robust regression*. Ph.D. thesis, Department of Statistics, Virginia Polytechnic Institute & State University, Blacksburg, VA.

The Fear of COVID-19 Scale (FCV-19S): Multidimensionality Structure in Turkey Sample

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Abstract: The global pandemic, COVID-19, started on the last day of 2020 has brought new challenges. Researchers have started a great effort not only to try and find to this unexpected pandemic, but to also find its psychological impacts. Therefore, this study aimed to provide more validity and reliability evidence for the Fear of COVID-19 (FCV-19S) Scale. Using a large representative sample from Turkey, the results supported two-factor solution of the FCV-19S. This scale will allow researchers and health professionals to understand fear of COVID-19 using this multidimensional scale. The aim of the study was to evaluate the construct validity based on the internal structure, the relationship with other variables, and the internal consistency among items of the Fear of COVID-19 Scale (FCV-19S) in a sample of Turkish peoples who are age of 18 and older. Future studies should be conducted in different population for more support.

Keywords: COVID-19, Fear, Confirmatory Factor Analysis, Measurement Invariance

References

- [1] Ahorsu, D., Lin, C., Imani, V., Saffari, M., Griffiths, M., & Pakpour, A. (2020). The Fear of COVID-19 Scale: Development and initial validation. *International Journal of Mental Health and Addiction*. doi: 10.1007/s11469-020-00270-8.
- [2] Alyami, M., Henning, M., Krägeloh, C., & Alyami, H. (2020). Psychometric evaluation of the Arabic version of the Fear of COVID-19 Scale. *International Journal of Mental Health and Addiction*. doi: 10.1007/s11469-020-00316-x
- [3] Arpacı, I., Karataş, K., & Baloğlu, M. (2020). The development and initial tests for the psychometric properties of the COVID-19 phobia scale (C-19P-S). *Personality and Individual Differences*, 164(1), 110108. <https://doi.org/10.1016/j.paid.2020.110108>.
- [4] Bakioğlu, F., Korkmaz, O., & Ercan, H. (2020). Fear of COVID-19 and positivity: Mediating role of intolerance of uncertainty, depression, anxiety, and stress. *International Journal of Mental Health and Addiction*. doi: 10.1007/s11469-020-00331-y
- [5] Doshi, D., Karunakar, P., Sukhabogi, J. R., Prasanna, J. S., & Mahajan, S. V. (2020). Assessing Coronavirus Fear in Indian Population Using the Fear of COVID-19 Scale. *International Journal of Mental Health and Addiction*, 1.
- [6] Xiang, Y., Yang, Y., Li, W., Zhang, L., Zhang, Q., Cheung, T., & Ng, C. (2020). Timely mental health care for the 2019 novel coronavirus outbreak is urgently needed. *The Lancet Psychiatry*, 7(3), 228-229. doi: 10.1016/s2215-0366(20)30046-8
- [7] Ye, Z. W., Yuan, S., Yuen, K. S., Fung, S. Y., Chan, C. P., & Jin, D. Y. (2020). Zoonotic origins of human coronaviruses. *International journal of Biological Sciences*, 16(10), 1686-1697.

Test of divisibility by prime numbers via generalization of the famous criterion of divisibility by 3

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Abstract: Everywhere in nature and in life, prime numbers are very often used. For many years, prime numbers attract the attention of many mathematicians around the world, see for example the story of the theory of numbers in [1,5]. There are a number of well-known open questions regarding prime numbers. The creation of a real rule, trick, method or test of divisibility by a prime number is a very difficult elementary problem, there are currently various algorithms used for the test of divisibility, we can for example refer to [3,4,6] and [7,8,10]. In this article, especially using decimal system base, we study a new general recursive divisibility test by a prime number. Finally, as an example of applications, we discuss that the some specific known cases remain valid.

Keywords: Divisibility, congruence, prime, test.

Mathematics Subject Classification: 11A07; 11A41

References

- [1] A. ELMASRY, C. JENSEN, J. KATAJAINEN, *The Magic of a Number system*. Springer-Verlag,(2010).
- [2] B. FINE, G. ROSENBERGER, *Number theory: An Introduction via the distribution of primes*. Birkhauser Boston,2007.
- [3] C. POMERANCE, *Primality Testing:Variation on a theme of Lucas*. 13th Meeting of the Fibonacci Association in Patras, Greec, July 2008.
- [4] C.R. BEMBLEKAR, *A test of divisibility by a Number of the form $10k+9$* . Bulletin of the Marthwada Mathematical society, Aurangarabad, Vol 9. N01, June 2008, p.1-4.
- [5] D. FENSTER, *History of the theory of Numbers: an historical Study with mathimatical implications*. Mathematics History Review, 1999, p.159–179.
- [6] M. HASSANI, *Tests for divisibility by prime numbers*. The Mathematical Gazette, Nov 2019, 494–495.
- [7] M. OLSEN, G. K. GOFF, *A divisibility test for any prime*. School Science and Mathematics.Vol.86(7). Nov (1986), 578–581.
- [8] G.S. KAWAL, *Test of divisibility by prime Numbers* . Bulletin of the Marthwada Mathematical society, Aurangarabad, Vol 9. N01, June 2008,p.55-61.
- [9] R. GANDALL, C. POMERANCE, *Prime Numbers*, Springer, (2005).
- [10] S. M. CHAUTHAIWALE, *A General divisibility test for all positive divisors*, Bulletin of the Marathwada Mathematical society, Vol.13, No.1, June (2012), 1–8.
- [11] W. STEIN, *Elementary Number theory: Prime, Congruences, and secret*. Springer, (2009).

Dual Toeplitz operators on the orthogonal complement of the Hardy space

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Abstract: In this paper we investigate products of dual Toeplitz operators acting on the orthogonal complement of the Hardy space of the polydisk. In particular, we establish a Brown-Halmos type theorem and derive characterizations of the zero divisors among dual Toeplitz operators as well as symbols giving rise to isometric, idempotent and unitary dual Toeplitz operators.

Keywords: Brown-Halmos, dual Toeplitz operator, polydisk, Hardy space.

Mathematics Subject Classification: 47B35, 47B47.

References

- [1] L. Benaissa and H. Guediri, "Properties of dual Toeplitz operators with applications to Haplitz products on the Hardy space of the polydisk", *Taiwanese J. of Math.*, **19** (1), 31–49, 2015.
- [2] X.H. Ding, "Products of Toeplitz operators on the polydisk", *Integr. Equ. Oper. Theory*, **45**, 389–403, 2003.
- [3] H. Guediri, "Products of Toeplitz and Hankel Operators on the Hardy Space of the Unit Sphere", *Operator Theory: Advances and Applications*, Vol. **236**, 243–256, 2014.
- [4] K. Stroethoff and D. Zheng, "Algebraic and spectral properties of dual Toeplitz operators", *Trans. Amer. Math. Soc.*, **354** (6), 2495–2520, 2002.

Positive Solutions for ϕ -Laplacian BVPs on bounded interval

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Abstract: Using an adapted version of the Krasnosels'kii fixed point theorem, we present new existence results for a ϕ -Laplacian boundary value problem of the type

$$\begin{cases} -(\phi(u'))'(x) = f(x, u(x), u'(x)), & 0 < x < 1, \\ u(0) - au'(0) = 0, & u'(1) = 0, \end{cases}$$

where $a > 0$, $f: [0, 1] \times \mathbb{R}^+ \times \mathbb{R} \rightarrow \mathbb{R}^+$ is a Carathéodory function and $\phi: \mathbb{R} \rightarrow \mathbb{R}$ is an increasing homeomorphism such that $\phi(0) = 0$. Some examples illustrate the results obtained.

Keywords: Positive solution, ϕ -Laplacian, BVPs, Fixed point theorems.

Mathematics Subject Classification: 47B35, 47B47

References

- [1] K. Bachouche, S. Djebali and T. Moussaoui, One-Dimensional Dirichlet ϕ -Laplacian BVPs *Advances in Dynamical Systems and Applications*, **6(2)**, 159–175, 2011.
- [2] K. Deimling, "Nonlinear Functional Analysis", Springer-Verlag, Berlin, Heidelberg, 1985.
- [3] D. R. Herlea, "Positive solutions for second-order boundary-value problems with ϕ -Laplacian", *Elect. J. Differ. Eq.*, **51**, 1–8, 2016.
- [4] D. R. Herlea, R. Precup, "Existence, localisation and multiolicity of positive solutions to ϕ -Laplacians and systems", *Taiwanese J. Math.*, Vol. **20**, 77–89, 2016.
- [5] G.L. Karakostas, "Triple positive solutions for the Φ -Laplacian when Φ is a sup-multiplicative-like function", *Elect. J. Differ. Eq.*, **69**, 1–12, 2004.
- [6] B. Yan, "Multiple positive solutions for singular boundary-value problems with derivative dependence on finite and infinite intervals", *Elect. J. Differ. Eq.*, **74**, 1–25, 2006.

Solving a coupled time -fractional partial differential equations by using generalized Gegenbauer -Humbert wavelets

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Abstract: In this paper, wavelets technique based on generalized Gegenbauer -Humbert polynomial are used for solving a coupled system of time-fractional Whitham-Broer-Kaup (WBK) and coupled Korteweg-de Vries (KdV) equations. An efficiency and accuracy of the present method are established by testing some numerical examples.

Keywords: Whitham-Broer-Kaup, Korteweg-de Vries.

Mathematics Subject Classification:

References

- [1] A. Ali, K. Shah and R. A. Khan , *Numerical treatment for traveling wave solutions of fractional Whitham-Broer-Kaup equations*, *Alexandria Engineering Journal* , **57**(2018),1991-1998.
- [2] K. Shah, H. Khalil and R. A. Khan, *A generalized scheme based on shifted Jacobi polynomials for numerical simulation of coupled systems of multi-term fractional-order partial differential equations*, *LMS J. Comput. Math.* , **20**(1)(2017),11-29.
- [3] F. Bulut, Ö. Oruç and A. Esen. , *Numerical Solutions of Fractional System of Partial Differential Equations By Haar Wavelets*, *CMES*, **108**(4)(2015),263-284.
- [4] B. Albuohimad, H. Adibi and S. Kazem. , *A numerical solution of time-fractional coupled Korteweg-de Vries equation by using spectral collection method*, *Ain Shams Engineering Journal*, **9**(2018),1897-1905.
- [5] L. Wang and X. Chen. , *Approximate Analytical Solutions of Time Fractional Whitham-Broer-Kaup Equations by a Residual Power Series Method*, *Entropy*, **17**(2015),6519-6533.
- [6] N. Ozdemir, A. Secer and M. Bayram. , *The Gegenbauer Wavelets-Based Computational Methods for the Coupled System of Burgers' Equations with Time-Fractional Derivative*, *Mathematics*, **7**(486)(2019).
- [7] M. Al-Smadi, O. Abu Arqub and S. Hadid ., *An attractive analytical technique for coupled system of fractional partial differential equations in shallow water waves with conformable derivative*, *Commun. Theor. Phys.*, **72**(2020).
- [8] A. K. Gupta., *Wavelet Methods for the Solutions of Partial and Fractional Differential Equations Arising in Physical Problems*, *National Institute of Technology Rourkela*, (2016).

Improved MPPT Based on Robust Backstepping Control via the Fractional-Order Step Size Incremental Conductance Algorithm

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Abstract: The increasing energy demands rely heavily on fossil fuels such as coal and natural gas, which have arisen the need for alternate clean energy. In this regard, solar energy becomes the trend in alternate energy sources. However, it keeps varying with solar irradiation and environmental temperature, so that it causes power low at the output of photovoltaic (PV) cells and a rapid wearing of electronic modules. As the solar cell is a sort of semiconductor, the interaction between the diffusion current and drift current of the semiconductor and the ambient temperature can be reflected in fractional order (FO) behavior. Correspondingly, to increase the efficiency of a PV power system, a robust backstepping controller based on Mittag-Leffler and Lyapunov stability is proposed to enhance the maximum power point tracking (MPPT) of the PV system. The designed controller is used to track the generated reference voltage for PV array which is determined under a fixed-fractional order step size incremental conductance algorithm, so that adjust the duty cycle of the boost converter. In this paper, MATLAB/Simulink is used to validate the desired result of the proposed controller. Consequently, the obtained results confirm the effectiveness of the designed controller and the significant performance of MPPT in transient and steady states under weather conditions.

Keywords: Fractional calculus, Photovoltaic system (PVs), MPPT, Robust Backstepping, Mittag-Leffler Stability, Fractional Lyapunov theory.

Mathematics Subject Classification: 26A33, 34A08, 90C32, 93B09, 33E12, 37B25

References

- [1] M. Arsalan, R. Iftikhar, I. Ahmad, A. Hasan, K. Sabahat and A. Javeria, MPPT for Photovoltaic System using Nonlinear Backstepping Controller with Integral Action, *Sci. Dir. Solar Energy* 170(1), pp. 192-200, 2018.
- [2] K.-N. Yo, C.-k. Liao and H.-T. Yau. A New Fractional-Order Based Intelligent Maximum Power Point Tracking Control Algorithm for Photovoltaic Power System, *Int. J. Photoenergy*. pp. 1-8, 2015.
- [3] N. Benbaha, F. Zidani, M.S. Nait Said, S. Boukebbous and H. Ammar, Robust backstepping MPPT for Photovoltaic System, *Int. Sci. Pre.* 9(1), pp. 179-188, 2016.
- [4] Y. Djourni, K. Khettab, “Improved Maximum Power Point Tracking Based on Dynamic Error Detector via Fractional Order Backstepping Control”. The 1st National Conference on Energy Transition in Algeria, March 2020.
- [5] A. Idir, K. Khettab, Y. Bensafia, “A comparative study of MPPT controllers for PV systems: NN-PID and NN-MPC approaches”. The 1st National Conference on Energy Transition in Algeria, March 2020.
- [6] K. Khettab, Y. Djourni, A. Idir, Y. Bensafia, “A New Fractional Variable Step Size Incremental Conductance for MPPT Based on Fractional Adaptive Nonlinear Controller”. First online conference on modern fractional calculus and its applications (OCMFCA'20). Tstanbul, Tutkey, Decembre 2020.

Numerical Solutions of Nonhomogeneous Rosenau Type Equations by Quintic B-Spline Collocation Method

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Abstract: In this study, a numerical scheme based on a collocation finite element method using quintic B-spline functions for getting approximate solutions of nonhomogeneous Rosenau type equations prescribed by initial and boundary conditions is proposed. The numerical scheme is tested on four model problems with known exact solutions. To show how accurate the results the proposed scheme produces, the error norms defined by L_2 and L_∞ are calculated. Additionally, the stability analysis of the scheme is carrier out by means of the von Neuman method.

Keywords: Rosenau, Rosenau-Burger, Rosenau-RLW, Rosenau-KdV-RLW, quintic B-Spline functions, collocation method.

Mathematics Subject Classification: 65N35.

References

- [1] R. Abazari, R. Abazari Numerical solution of the Rosenau equation using quintic collocation B-spline method, Iranian J. Sci. Technol(Sciences). vol. 39(A3) (2015) pp. 281-288.
- [2] N. Atouani, Y. Ouali, K. Omrani, Mixed finite element methods for the Rosenau equation, J. Appl. Math. Comput. vol. 57 (2018) pp.393–420.
- [3] J. Janwised, B. Wongsaijai, T. Mouktonglang, K. Poochinapan, A modified three-level average linear implicit finite difference method for the Rosenau-Burgers equation, Adv. Math. Phys., vol. 2014(2014) pp. 1-11.
- [4] Z. Jun, Numerical methods for a shallow water Rosenau-Burgers equation, IOP Conf. Ser.: Earth Environ. Sci., vol. 252(5) (2019) pp. 052123. doi:10.1088/1755-1315/252/5/052123
- [5] B. Wongsaijai, T. Mouktonglang, N. Sukantamala, K. Poochinapan, Compact structure-preserving approach to solitary wave in shallow water modeled by the Rosenau-RLW equation, Appl. Math. Comput., vol. 340 (2019) pp. 84–100.
- [6] N.M. Yagmurlu, B. Karaagac, S. Kutluay, Numerical solutions of Rosenau-RLW equation using Galerkin cubic B-spline finite element method, Am. J. Comput. Appl. Math., vol. 7(1) (2017) pp. 1-10.
- [7] X. Wang, W. Dai, A three-level linear implicit conservative scheme for the Rosenau-KdV-RLW equation, J. Comput. Appl. Math., vol. 330 (2018) pp. 295–306.
- [8] M. Foroutan, A. Ebadian, Chebyshev rational approximations for the Rosenau-KdV-RLW equation on the whole line, Int. J. Anal. Appl., vol. 16(1) (2018) pp. 1-15.
- [9] S. Özer, Numerical solution of the Rosenau–KdV–RLW equation by operator splitting techniques based on B-spline collocation method, Numer. Methods Partial Differential Eq., vol. 35 (2019) pp. 1928–1943.
- [10] P.M. Prenter, Splines and variational methods. John Wiley, New York, NY, 1975.

Interior point methods for convex quadratic programming based on a new kind of kernel functions

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Abstract: In this work, we propose a new type of kernel function for convex quadratic programming problems. This kind was proposed for the first time by the authors themselves for semi-definite programming problems in [7]. By simple analysis tools, we compute the worst-case iterations complexity of the algorithm based on our proposed kernel function. The obtained iteration bound for large-update methods, namely, $\mathcal{O}\left(n^{\frac{2}{3}} \log \frac{n}{\epsilon}\right)$, improves the classical iteration complexity [2] as well as first trigonometric kernel function [3], with a factor $n^{\frac{1}{3}}$. For small-update methods, we derive the iteration bound $\mathcal{O}\left(\sqrt{n} \log \frac{n}{\epsilon}\right)$, which matches the currently best known iteration bound for small-update methods.

Keywords: Convex quadratic programming, Primal-dual interior point methods, kernel functions, Complexity analysis, Large- and small-update methods

Mathematics Subject Classification: 90C20, 90C31, 90C51, C60.

References

- [1] M. Achache, *A new primal-dual path-following method for convex quadratic programming*, Computational & Applied Mathematics. 25 (1) (2006) 97-110.
- [2] Y.Q. Bai, M. EL. Ghami, C. Roos, *A comparative study of kernel functions for primal-dual interior-point algorithms in linear optimization*, SIAM J. Optim. 15 (2004) 101-128.
- [3] El Ghami M, Guennoun ZA, Bouali S, Steihaug T, *Interior point methods for linear optimization based on a kernel function with a trigonometric barrier term*. J. Comput. Appl. Math. 236 (2012) 3613-3623.
- [4] C. Roos, T. Terlaky, J.Ph. Vial, *Theory and algorithms for linear optimization*, in: *An interior point Approach*, John Wiley & Sons, Chichester, UK (1997).
- [5] I. Touil, D. Benterki, A. Yassine, *A feasible primal-dual interior point method for linear semidefinite programming*, J. Comput. Appl. Math. 312 (2017) 216-230.
- [6] I. Touil, D. Benterki, *A primal-dual interior-point method for the semidefinite programming problem based on a new kernel function*, J. Nonlinear Funct. Anal. 2019 (2019), Article ID 25.
- [7] Touil. I, Chikouche. W, *Primal-dual interior point methods for semidefinite programming based on a new type of kernel functions*, Filomat. Vol 34, No 12 (2020).

Certain fractional integral and differential formulas involving the extended incomplete generalized hypergeometric functions

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Abstract: The fractional integral and differential operators involving the family of special functions have found significant importance and applications in various fields of mathematics and engineering. The goal of this chapter is to find the fractional integral and differential formulas (also known as composition formulas) of the extended incomplete generalized hypergeometric functions by using the generalized fractional calculus operators (the Marchichev-Saigo-Maeda operators). After that, we established their image formulas by using the integral transforms like: Beta transform, Laplace transform and Whittaker transform, respectively. Moreover, the reduction formulas are also considered as special cases of our main findings associated with the well known Saigo fractional integral and differential operators, Erdélyi-Kober fractional integral and differential operators, Riemann-Liouville fractional integral and differential operators and the Weyl fractional calculus operators, respectively.

Keywords: Fractional integral operators, fractional differential operators, Saigo fractional integral and differential operators, Erdélyi-Kober fractional integral and differential operators, Riemann-Liouville fractional integral and differential operators, Weyl fractional integral operator and differential, incomplete gamma Function, extended incomplete generalized hypergeometric function, Pochhammer symbol, Gamma function.

Mathematics Subject Classification: Primary 33B20, 44A20, 65R10; Secondary 26A33, 33C20

The bi-periodic high order numbers

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Abstract: We define a new class of the bi-periodic high order the r -Fibonacci sequence. Then, we introduce a new family of the companion of these sequences, named bi-periodic r -Lucas sequence of type s , which extend the classical Fibonacci and Lucas sequences. Afterwards, we establish the link between the bi-periodic r -Fibonacci sequence and its companion sequence. Furthermore, we give their basic properties linear recurrence relations, generating functions, Binet formulas and explicit formulas.

Keywords: Bi-periodic r -Fibonacci sequence, bi-periodic r -Lucas sequence, recurrence relation, generating function, explicit formula, Binet formula

Introduction Yazlik et al. [9] introduced generalization of the bi-periodic Fibonacci r -numbers (f_n) , for r a positive integer and a, b a positive real numbers by, for $n \geq r + 1$

$$f_n = \begin{cases} af_{n-1} + f_{n-r-1}, & \text{for } n \equiv 0 \pmod{2}, \\ bf_{n-1} + f_{n-r-1}, & \text{for } n \equiv 1 \pmod{2}, \end{cases}$$

and the bi-periodic Lucas r -numbers (l_n) by, for $n \geq r + 1$

$$l_n = \begin{cases} bl_{n-1} + l_{n-r-1}, & \text{for } n \equiv 0 \pmod{2}, \\ al_{n-1} + l_{n-r-1}, & \text{for } n \equiv 1 \pmod{2}, \end{cases}$$

with the initial conditions $f_0 = 0, f_1 = 1, f_2 = a, \dots, f_r = a^{\lfloor r/2 \rfloor} b^{\lfloor (r-1)/2 \rfloor}$ and $l_0 = r + 1, l_1 = a, l_2 = ab, \dots, l_r = a^{\lfloor (r+1)/2 \rfloor} b^{\lfloor r/2 \rfloor}$, respectively.

We define a new class of the bi-periodic r -Fibonacci sequence $(U_n^{(r)})_n$ and we give its linear recurrence relation. We introduce a new family of companion sequences associated to the bi-periodic r -Fibonacci sequence indexed by the parameter s ; with $1 \leq s \leq r$; named the bi-periodic r -Lucas sequence of type s , $(V_n^{(r,s)})_n$. After that, we express $V_n^{(r,s)}$ in terms of $U_n^{(r)}$ and s . Then we give some algebraic properties.

The bi-periodic r -Fibonacci sequence In our work [1], first we define the bi-periodic r -Fibonacci sequence $(U_n^{(r)})_n$ and give its linear recurrence relation.

Definition 0.1. For a, b, c, d nonzero real numbers and $r \in \mathbb{N}$, the bi-periodic r -Fibonacci sequence $(U_n^{(r)})_n$ is defined by, for $n \geq r + 1$

$$U_n^{(r)} = \begin{cases} aU_{n-1}^{(r)} + cU_{n-r-1}^{(r)}, & \text{for } n \equiv 0 \pmod{2}, \\ bU_{n-1}^{(r)} + dU_{n-r-1}^{(r)}, & \text{for } n \equiv 1 \pmod{2}, \end{cases} \quad (0.1)$$

with the initial conditions $U_0^{(r)} = 0, U_1^{(r)} = 1, U_2^{(r)} = a, \dots, U_r^{(r)} = a^{\lfloor r/2 \rfloor} b^{\lfloor (r-1)/2 \rfloor}$.

The bi-periodic r -Fibonacci sequence can be expressed by linear recurrence relation.

Theorem 0.1. For a, b, c, d nonzero real numbers and $r \in \mathbb{N}$, the bi-periodic r -Fibonacci sequence satisfies the following linear recurrence, for $n \geq 2r + 2$

$$U_n^{(r)} = abU_{n-2}^{(r)} + (a^{\xi(r+1)}d + b^{\xi(r+1)}c)U_{n-r-1-\xi(r+1)}^{(r)} - (-1)^{r+1}cdU_{n-2r-2}^{(r)}. \quad (0.2)$$

The bi-periodic r -Lucas sequence of type s Secondly, we introduce a new family of companion sequences related to the bi-periodic r -Fibonacci sequence, called the bi-periodic r -Lucas sequence of type s , $(V_n^{(r,s)})_n$.

Definition 0.2. For any nonzero real numbers a, b, c, d and integers s, r such that $1 \leq s \leq r$, we define for $n \geq r + 1$

$$V_n^{(r,s)} = \begin{cases} bV_{n-1}^{(r,s)} + dV_{n-r-1}^{(r,s)}, & \text{for } n \equiv 0 \pmod{2}, \\ aV_{n-1}^{(r,s)} + cV_{n-r-1}^{(r,s)}, & \text{for } n \equiv 1 \pmod{2}, \end{cases}$$

with the initial conditions $V_0^{(r,s)} = s + 1, V_1^{(r,s)} = a, V_2^{(r,s)} = ab, \dots, V_r^{(r,s)} = a^{\lfloor (r+1)/2 \rfloor} b^{\lfloor r/2 \rfloor}$.

The bi-periodic r -Fibonacci sequence $(U_n^{(r)})_n$ and the bi-periodic r -Lucas sequence of type $s, (V_n^{(r,s)})_n$ can be seen as a generalization of the Fibonacci and Lucas sequences, we will list some particular cases. The bi-periodic r -Lucas sequence of type $s, 1 \leq s \leq r$ satisfy the following linear recurrence relation.

Theorem 0.2. For a nonzero real numbers a, b, c, d and s, r such that $1 \leq s \leq r$, the family of the bi-periodic r -Lucas sequence of type s satisfy, for $n \geq 2r + 2$

$$V_n^{(r,s)} = abV_{n-2}^{(r,s)} + (a^{\xi(r+1)}d + b^{\xi(r+1)}c)V_{n-r-1-\xi(r+1)}^{(r,s)} - (-1)^{r+1}cdV_{n-2r-2}^{(r,s)}. \quad (0.3)$$

After that, we express the bi-periodic r -Lucas sequence of type $s, V_n^{(r,s)}$ in terms of $U_n^{(r)}$.

Theorem 0.3. Let r and s be nonnegative integers such that $1 \leq s \leq r$, the bi-periodic r -Fibonacci sequence and the bi-periodic r -Lucas sequence of type s satisfy the following relationship

$$V_n^{(r,s)} = \begin{cases} U_{n+1}^{(r)} + sdU_{n-r}^{(r)}, & n \geq r, \quad \text{for } r \text{ odd}, \\ U_{n+1}^{(r)} + scbU_{n-r-1}^{(r)} + scdU_{n-2r-1}^{(r)}, & n \geq 2r + 1, \quad \text{for } r \text{ even}. \end{cases} \quad (0.4)$$

Main results We also give the generating functions of the bi-periodic r -Fibonacci sequence and the bi-periodic r -Lucas sequence of type s . Then, we express an explicit formulas of $(U_n^{(r)})_n$ and $(V_n^{(r,s)})_n$. Finally, we give their Binet Formulas.

References

- [1] N. R. Ait-Amrane, H. Belbachir, *Extension of the bi-periodic r -Fibonacci sequence and the bi-periodic r -Lucas sequence of type s* , Submitted.
- [2] H. Belbachir, F. Bencherif, *Linear recurrent sequences and powers of a square matrix*, Integers 6 (2006), A12, 17pp.
- [3] G. Bilgici, *Two generalizations of Lucas sequence*, Appl. Math. Comput. 245 (2014), 526-538.
- [4] L. Cerlienco, M. Mignotte, F. Piras, *Suites récurrentes linéaires, propriétés algébriques et arithmétiques*, Enseign. Math. 33 (1987), 67-108
- [5] M. Edson, O. Yayenie, *A new generalization of Fibonacci sequences and extended Binet's Formula*, Integers 9 (2009), 639-654.
- [6] M. Sahin, *The Gelin-Cesàro identity in some conditional sequences*, Hacettepe Journal of Mathematics and statistics, vol 40 (6), (2011), 855-861.
- [7] E. Tan, A.B. Ekin, *Bi-periodic Incomplete Lucas Sequences*, Ars Combinatoria, (123), 371-380, 2015.
- [8] O. Yayenie. *A note on generalized Fibonacci sequence*. Appl. Math. Comput, (2011), 217, pp 5603-5611.
- [9] Y. Yazlik, C. Kome, V. Madhusudanan, *A new generalization of Fibonacci and Lucas p -numbers*, Journal of computational analysis and applications, (2018), vol. 25, NO. 4.

Oscillatory behaviour of linear delay differential equation with nonmonotone arguments

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Abstract: We study a first order linear delay differential equation

$$x'(t) + \sum_{i=1}^m p_i(t)x(\tau_i(t)) = 0, \quad t \geq t_0, \quad (E)$$

where $p_i(t)$ and $\tau_i(t)$ are the functions of nonnegative of real numbers and $\tau_i(t)$ are not necessarily monotone for $1 \leq i \leq m$. Then, we obtain new sufficient condition for the oscillatory solutions of (E). Finally, we give an example to demonstrate our result.

Keywords: Delay equation, nonmonotone arguments, oscillatory solution, nonoscillatory solution

Mathematics Subject Classification: 34K11, 34K06

References

- [1] Akca, H., Chatzarakis, G.E. and Stavroulakis, I.P., "An oscillation criterion for delay differential equations with several non-monotone arguments," *Applied Mathematics Letters*, 59, 101-108, 2016.
- [2] Chao, J., "On the oscillation of linear differential equations with deviating arguments," *Math. in Practice and Theory*, 1, pp. 32-40, 1991.
- [3] Chatzarakis, G.E. and Péics, H., "Differential equations with several non-monotone arguments: An oscillation result," *Applied Mathematics Letters*, 68, pp. 20-26, 2017.
- [4] Elbert, A. and Stavroulakis I.P., "Oscillations of first order differential equations with deviating arguments," *University of Ioannina, T. R. N0 172 1990, Recent trends in differential equations 163-178, World Sci. Ser. Appl. Anal.*, 1, World Sci. Publishing Co., 1992.
- [5] Erbe, L.H. and Zhang, B.G., "Oscillation of first order linear differential equations with deviating arguments," *Differ. Integral Equ.*, 1, 305-314, 1988.
- [6] Erbe, L.H., Kong, Q. and Zhang, B.G., *Oscillation Theory for Functional Differential Equations*, Marcel Dekker, New York, 1995.
- [7] Fukagai, N., Kusano, T., "Oscillation theory of first order functional differential equations with deviating arguments," *Ann. Mat. Pura Appl.*, 136, pp. 95-117, 1984.
- [8] Grammatikopoulos M.K., Koplatadze R.G. and Stavroulakis I.P., "On the oscillation of solutions of first order differential equations with retarded arguments," *Georgian Math. J*, 10, pp. 63-76, 2003.
- [9] Gyóri, I. and Ladas, G., *Oscillation Theory of Delay Differential Equations with Applications*, Clarendon Press, Oxford, 1991.
- [10] Hunt, B.R. and Yorke, J.A., "When all solutions of $x' = \sum q_i(t)x(t - T_i(t))$ oscillate," *J. Differential Equations*, 53, pp. 139-145, 1984.

- [11] Jaroš, J. and Stavroulakis, I.P., "Oscillation tests for delay equations," *Rocky Mountain J. Math.*, 29, pp. 139–145, 1999.
- [12] Kon, M., Sficas, Y.G. and Stavroulakis, I.P., "Oscillation criteria for delay equations," *Proc. Amer. Math. Soc.*, 128, pp. 2989–2997, 2000.
- [13] Koplatadze, R.G. and Chanturija, T.A., "Oscillating and monotone solutions of first-order differential equations with deviating arguments," (Russian), *Differentsial'nye Uravneniya*, 8, pp. 1463-1465, 1982.
- [14] Koplatadze, R. and Kvinikadze, G., "On the oscillation of solutions of first order delay differential inequalities and equations," *Georgian Mathematical Journal*, 1(6), pp. 675-685, 1994.
- [15] Kwong, M.K., "Oscillation of first-order delay equations," *J. Math. Anal. Appl.*, 156, pp. 274-286, 1991.
- [16] Ladde, G.S., Lakshmikantham, V. and Zhang, B.G., *Oscillation Theory of Differential Equations with Deviating Arguments*, Monographs and Textbooks in Pure and Applied Mathematics, vol. 110, Marcel Dekker, Inc., New York, 1987.
- [17] Li, B., "Oscillations of first order delay differential equations," *Proc. Amer. Math. Soc.*, 124, pp. 3729-3737, 1996.
- [18] Philos, Ch.G. and Sficas Y.G., "An oscillation criterion for first order linear delay differential equations," *Canad. Math. Bull.*, 41, pp. 207-213, 1998.
- [19] Tang, X.H., "Oscillation of first order delay differential equations with distributed delay," *J. Math. Anal. Appl.*, 289, pp. 367-378, 2004.
- [20] Yu, J.S. and Wang, Z.C., "Some further results on oscillation of neutral differential equations," *Bull. Aust. Math. Soc.*, 46, pp. 149–157, 1992.
- [21] Yu, J.S., Wang, Z.C., Zhang, B.G. and Qian, X.Z., "Oscillations of differential equations with deviating arguments," *PanAmerican Math. J.*, 2, pp. 59–78, 1992.

Fractional derivatives of generalized functions and hypergeometric functions

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Abstract: In this present study we review the fractional derivatives for some elementary and certain special functions and we generalize the idea of fractional operations to linear functional in the generalized functions spaces, [1] and [8]-[11]. We provide several examples in order to illustrate the geometric representation of fractional derivatives for Dirac delta type functions, [2]-[5]. Further we also give many counterexamples where the fractional derivatives of distributions need not necessarily to be elementary functions rather the hypergeometric functions, [6,7].

Keywords: Fractional derivatives; linear functional; Dirac delta function; generalized functions; hypergeometric functions.

Mathematics Subject Classification:

References

- [1] A Kılıçman, ZAA Al Zhour, *Kronecker operational matrices for fractional calculus and some applications*, Applied Mathematics and Computation **187** (1)(2006), 250–265.
- [2] F. Shokrollahi and A. Kılıçman, *Actuarial approach in a mixed fractional Brownian motion with jumps environment for pricing currency option*, Advances in Difference Equations (2015) **2015**:257.
- [3] F. Shokrollahi, F. Kılıçman, A. *Pricing currency option in a mixed fractional Brownian motion with jumps environment*, Math. Probl. Eng. 2014, Article ID 858210 (2014).
- [4] F. Shokrollahi, F. Kılıçman, A. *Delta-hedging strategy and mixed fractional Brownian motion for pricing currency option*. Math. Probl. Eng. 2014, Article ID 718768 (2014).
- [5] A. Kılıçman and Omran, *Note on fractional Mellin transform and applications*, SpringerPlus (2016) **5**:100.
- [6] Restrepo, J., Kılıçman, A., Agarwal, P. *Weighted hypergeometric functions and fractional derivative*, Advances in Difference Equations 2017, 105 (2017).
- [7] A. Kılıçman and Z. E. Abdalnaby, *Some applications for generalized fractional operators in Analytic Functions Spaces*, Korean J. Math. **27**(3) (2019), pp. 581-594.
- [8] Aljedhi, R.A.;Kılıçman, A. *Fractional Partial Differential Equations Associated with Levy Stable Process*, Mathematics 2020, **8**, 508.
- [9] Jeng, S.W.; Kılıçman, A. *Fractional Riccati Equation and Its Applications to Rough Heston Model Using Numerical Methods*, Symmetry 2020, **12**, 959.
- [10] Kılıçman, A. : *A Note on Mellin Transform and Distributions*, Journal of Mathematical and Computational Applications, **9**(1)(2004), 65–72.
- [11] Kılıçman, A. and Hassan, A. M.: *A Note on the Differential Equations with Distributional Coefficients*, Math. Balkanica, **18**(3-4)(2004), 355–363.

A comparison of analytical solutions of a nonlinear PDE with conformable and M - truncated derivatives

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Abstract: This paper aims to find exact solutions of a nonlinear PDE concerning various differential operators and compare the obtained solutions. The exact solutions of the PDE with the conformable and M - truncated differential derivatives are obtained via extended rational *sine – cosine* and *sinh – cosh* methods. The results are compared in the $2D$ and $3D$ graphics. All computations and graphics are obtained by using Wolfram Mathematica 12.

Keywords: Analytical methods, nonlinear PDEs, conformable derivative, M - truncated derivative

References

- [1] N. Mahak, G. Akram, Exact solitary wave solutions by extended rational sine-cosine and extended rational sinh-cosh techniques, *Physica Scripta* 94 (11) (2019) 115212.
- [2] M. Inc, A. I. Aliyu, A. Yusuf, D. Baleanu, Optical solitons for Biswas-Milovic Model in nonlinear optics by Sine-Gordon equation method, *Optik* 157 (2018) 267-274.
- [3] N. Mahak, G. Akram, Extension of rational sine-cosine and rational sinh-cosh techniques to extract solutions for the perturbed NLSE with Kerr law nonlinearity.
- [4] A. M. Wazwaz, The tanh-coth and the sine-cosine methods for kinks, solitons, and periodic solutions for the Pochhammer-Chree equations, *Applied Mathematics and Computation* 195 (1) (2008) 24-33.
- [5] M. T. Darvishi, M. Naja, A. M. Wazwaz, New extended rational trigonometric methods and applications, *Waves in Random and Complex Media* 30 (1) (2020) 5-26.
- [6] N. Mahak, G. Akram, Extension of rational sine-cosine and rational sinh-cosh techniques to extract solutions for the perturbed NLSE with Kerr law nonlinearity

Hybridization of Neural Networks and Sine Cosine Algorithm for Better Classification

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Abstract: In this work, neural networks and sine cosine algorithm are used in a hybrid approach in order to increase the quality of classification problems. The proposed method allows us to design the optimal neural network architecture, while avoiding overfitting. In order to evaluate the efficiency of the proposed algorithm, experimental results in two fields are presented. The hybridization technique produces two desirable effects, a better result and a fairly low margin of error.

Keywords: Artificial Intelligence, hybridization, classification, Neural Networks, Sine Cosine Algorithm, Deep Learning.

Mathematics Subject Classification: 68T05, 68T20, 65K05

References

- [1] Nour Eddine Alaa, Maryem Hourri , "The Red Blood Cells dataset , Mendeley Data, V2", doi:10.17632/rg7f6bwvb7.2 , 2021.
- [2] K. Deepshikha, "Supervised and Unsupervised Document Classification-A survey", International Journal of Computer Science and Information Technologies, pp. 1971–1974, 2015.
- [3] P.O. Duda and P.E. Hart, "Pattern Classification and Science Analysis ", Wiley, New York, 1973.
- [4] Shubham Gupta, Kusum Deep, Seyedali Mirjalili, Joong Hoon Kim, "A modified Sine Cosine Algorithm with novel transition parameter and mutation operator for global optimization ", Expert Systems With Applications, Volume 154, 15 September 2020, 113395
- [5] G.P. Zhang, " Neural Networks for Classification: A Survey" , IEEE Transactions On Systems, Man, And Cybernetics-part C: Applications And Reviews, vol. 30, N. 4, 1958.

Estimation of the Wada Property Based on the Weighted and Truncated Shannon Entropy and the Box-counting Technique

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Abstract: Over several decades, numerous investigations have been proposed in the field of basin structures of nonlinear dynamical systems and their boundary analysis [7, 2, 3]. When the single boundary separates three or more basins of attraction, it is said that such basins of attraction have a Wada property [2, 4]. In this case, a sufficiently small perturbation in any initial condition can make the system converge to another attractor – the behavior of the system becomes unpredictable [2, 3]. In the presentation, we will introduce a Wada index which enables to determine the number of different colors and their distribution in the phase space of the initial conditions and helps to estimate the Wada property. Wada index is based on the weighted and truncated Shannon entropy and the standard box-counting technique, what results in the unconditional stability of computations [5]. Such an index is easily applied not only for the identification of Wada boundaries in the phase space of dynamical systems, but also to evaluate the complexity of digital images of different nature. We will demonstrate the advantages of the proposed Wada index via series of numerical experiments with Newton fractal [6], nonlinear pendulum [3], completely invertible logistic map [8], and Beddington-DeAngel-type predator-prey models [4].

Keywords: Wada property, Shannon entropy, Nonlinear dynamical systems

Mathematics Subject Classification: 37F10, 37C70, 94A17

References

- [1] H. Nusse and J. Yorke, *Ergod. Theor. Dyn. Syst.* 17, 463–481, 1997.
- [2] A. Daza, A. Wagemakers, B. Georgeot, D. Guéry-Odelin, and M.A.F. Sanjuán, *Sci. Rep.* 6, 31416, 2016.
- [3] A. Saha and U. Feudel, *Chaos* 28, 033610, 2018.
- [4] K. Yoneyama, *Tohoku Math. J. First Ser.* 12, 43–158, 1917.
- [5] L. Saunoriene, M. Ragulskis, J. Cao, and M.A.F. Sanjuán, *Nonlinear. Dyn.* 104, 739–751, 2021.
- [6] W.J. Gilbert, *Fractals*, 09, 251–262, 2001.
- [7] G. Baker and J. Blackburn, *The Pendulum: A Case Study in Physics*. Oxford University Press, New York, 2005.
- [8] G. Lu, M. Landauskas, and M. Ragulskis, *Int. J. Bifurcat. Chaos* 28, 1850129, 2018.
- [9] W. Wang, Y. Lin, L. Zhang, F. Rao, and Y. Tan, *Commun. Nonlinear Sci.* 16, 2006–2015, 2011.

Numerical simulation of a mathematical model of covid19 transmissibility in Morocco during vaccination period

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Abstract: The aim of this work is to develop a new Reservoir People(RP) transmission network model to simulate the potential transmission of the COVID-19 virus in the population of Morocco during the vaccination period. The proposed model is original since it contains parameters that depend on the period of vaccination that Morocco has adopted so far. After developing the mathematical model COVID- 19-Morocco, we define a cost function to minimize with respect to the parameters. Then, we use Neural Network Algorithm to optimize this functional. The obtained numerical simulation confirms that our model is robust and can predict the evolution of the virus in Morocco.

Keywords: Covid-19, mathematical model, basic reproduction number, metaheuristic optimization, Neural Network Algorithm, transmissibility, parameters identification.

Mathematics Subject Classification: 12X34,56Y78,92D30,92D25,35Q92,93A30

References

- [1] T. M. Chen, J. Rui, Q. P. Wang, Z. Y. Zhao, J. A. Cui & L. Yin, "A mathematical model for simulating the phase-based transmissibility of a novel coronavirus", *Infectious Diseases of Poverty* **9**, no. 24, 2020.
- [2] T. Chen, J. Rui, Q. Wang, Z. Zhao, J.-A. Cui, L. Yin, "A mathematical model for simulating the transmission of Wuhan novel Coronavirus", *bioRxiv*. 2020:2020.2001.2019.911669, 2020.
- [3] J.-A. Cui, S. Zhao, S. Guo, Y. Bai, X. Wang, T. Chen, "Global dynamics of an epidemiological model with acute and chronic", *Applied Mathematics Letters* **103**, 106203, 2020.
- [4] A. Sadollah, H. Sayyaadi, A. Yadav, "A dynamic metaheuristic optimization model inspired by biological nervous systems: Neural network algorithm", *Applied Soft Computing*, 71, pp 747–782, 2018.

A class of Fubini polynomials

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Abstract: The main object of this talk is to investigate a new class of generalized Fubini polynomials. Some relationships between generalized Fubini polynomials and more polynomials as Fubini, Bell, Eulerian and Frobenius-Euler polynomials are given. Also, we derive the probabilistic representation.

Keywords: Fubini polynomials, Stirling numbers, Bell polynomials, Eulerian polynomials, Frobenius-Euler polynomials, explicit formulas.

Mathematics Subject Classification: 05A19, 11B83, 33C45.

References

- [1] K.N. Boyadzhiev and A. Dil, Geometric polynomials: properties and applications to series with zeta values, *Analysis Mathematica* 42 (2016), no. 3, 203–224.
- [2] G. Guettai, D. Laissaoui, M. Rahmani, and M. Sebaoui, On poly-Bell numbers and polynomials. *Quaestiones Mathematicae*, 1–21 (2020).
- [3] H.M. Srivastava, M.A. Boutiche and M. Rahmani, A class of Frobenius-type Eulerian polynomials, *Rocky mountain journal of mathematics*. 48 (2018), Number 3, 1003–1013.
- [4] H.M. Srivastava, M.A. Boutiche and M. Rahmani, Some Explicit Formulas for the Frobenius-Euler Polynomials of Higher Order, *Appl. Math. Inf. Sci.* 11(2) (2017), 621–626.

Some Optical Soliton Solutions for Generalized Fokas- Lenells Equations

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Abstract: Soliton propagation studies in optical fibers have been flourishing because of the rich features of models describing these phenomena over the past few decades. In the field of optics, by careful selection of higher order nonlinear and linear effects, Fokas-Lenells equation (FLE) emerges as a model governing femtosecond pulse propagation through single mode silica optical fiber. Accordingly it is a useful model to understand the physical insight of ultra short pulses in media. It is one of the models that govern pulse transmissions technology first appeared about a decade ago. The existence of solitons in the nonlinear dispersive media has been studied within the framework of the FLE. Here, the model is considered in the presence of perturbation terms that provide a picture much closer to reality. The FLE is studied with full nonlinearity in the generalized form using Jacobi elliptic functions in this work. It is obtained Jacobi elliptic function solutions and bright, dark and singular optical soliton solutions are also attained with the help of modulus limit of Jacobi elliptic functions for FLE. These solitons appear with constraint conditions on their parameters and they are also presented.

Keywords: Fokas Lenells equation, Optical soliton solutions, Jacobi elliptic functions

Mathematics Subject Classification: 35Q35, 37K45

References

- [1] Biswas, A., Yıldırım, Y., Yasar, E., Zhou Q., Moshokoa S.P., Belic M (2018). Optical soliton solutions to Fokas-Lenells equation using some different methods, *Optik-International Journal for Light and Electron Optics*, 173, 21–31.
- [2] Arshed, S., Raza, N. (2020). Optical solitons perturbations of Fokas-Lenells equation with full nonlinearity and dual dispersion, *Chinese Journal of Physics*, 63, 314-324.
- [3] Ates, E., Inc, M. (2017). Travelling wave solutions of generalized Klein-Gordon equations using Jacobi elliptic functions. *Nonlinear Dyn*, 88, 2281-2290.
- [4] Inc, M., Ates, E., Tchier F. (2016). Optical solitons of the coupled nonlinear Schrödinger's equation with spatiotemporal dispersion, *Nonlinear Dyn*, 84 (2), 1-11.
- [5] Kudryashov, N.A., (2019). First integrals and general solution of the Fokas-Lenells equation, *Optik-International Journal for Light and Electron Optics*, 195, 163135.
- [6] Biswas, A., Ekici, M., Sonmezoglu, A., Alqahtani, R.T., (2018). Optical soliton perturbation with full nonlinearity for Fokas-Lenells equation, *Optik* 165, 29-34.
- [7] Triki, H., Wazwaz, A.M., (2017). New types of chirped soliton solutions for the Fokas-Lenells equation, *Int. J. Numer. Methods Heat Fluid Flow* 27 (7), 1596-1601.

Some New Banach Spaces of Double Sequences Derived by Jordan Totient Function

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Abstract: We define new Banach spaces of double sequences derived by Jordan totient function in this work. Moreover, we prove some inclusion relations, examine some topological and algebraic properties, compute the α -, β -, (ϑ) - and γ - duals of these spaces and finally, we characterize some new 4-dimensional $(4 - d)$ matrix classes.

Keywords: Jordan totient function, RH-regular matrix, Matrix domain, Double sequence space.

Mathematics Subject Classification: 40C05, 46A45.

References

- [1] C.R. Adams, "On non-factorable transformations of double sequences", Proc. Natl. Acad. Sci. USA, 19(5) (1933), 564-567.
- [2] M. İlkan, N. Şimşek and E.E. Kara, "A new regular infinite matrix defined by Jordan totient function and its matrix domain in l_p ", Math. Methods Appl. Sci. (2020), 1-12.
- [3] F. Moricz and B.E. Rhoades, "Almost convergence of double sequences and strong regularity of summability matrices", Math. Proc. Cambridge Philos. Soc., 104(1988), 283-294.
- [4] M. Mursaleen and F. Başar, "Sequence Spaces: Topics in Modern Summability Theory", CRC Press, Taylor and Francis Group, Series: Mathematics and Its Applications, Boca Raton & London & New York, 2020.
- [5] O. Tuğ, "Four-dimensional generalized difference matrix and some double sequence spaces", J. Inequal. Appl., 2017(1), 149 (2017).
- [6] M. Yeşilkayagil and F. Başar, "Domain of Riesz mean in some spaces of double sequences", Indag. Math. (N.S) 29 (2018) No:3, 1009-1029.
- [7] M. Zeltser, "On conservative matrix methods for double sequence spaces", Acta Math. Hungar., 95(3) (2002), 225-242.

A Note on the Binomial Double Sequence Spaces

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Abstract: In the current study, the most apparent aspect is to submit new sequence spaces as the domain of 4-dimensional $(4 - d)$ binomial matrix. We investigate their topological properties and inclusion relations. In addition, α -, $\beta(bp)$ - and γ - duals were determined and finally, some matrix transformations were characterized.

Keywords: $4 - d$ binomial matrix, RH-regular matrix, Matrix domain, Double sequence space, Duals, Matrix transformations.

Mathematics Subject Classification: 40C05, 46A45.

References

- [1] C.R. Adams, "On non-factorable transformations of double sequences", Proc. Natl. Acad. Sci. USA, 19(5) (1933), 564-567.
- [2] M.C. Bişgin, "The Binomial Almost Convergent and Null Sequence Spaces", Commun.Fac.Sci.Univ.Ank.Series A1, 67(1) (2018), 211-224.
- [3] F. Moricz and B.E. Rhoades, "Almost convergence of double sequences and strong regularity of summability matrices", Math. Proc. Cambridge Philos. Soc., 104(1988), 283-294.
- [4] M. Mursaleen and F. Başar, "Sequence Spaces: Topics in Modern Summability Theory", CRC Press, Taylor and Francis Group, Series: Mathematics and Its Applications, Boca Raton & London & New York, 2020.
- [5] O. Tuğ, "Four-dimensional generalized difference matrix and some double sequence spaces", J. Inequal. Appl., 2017(1), 149 (2017).
- [6] M.Yeşilkayağil and F. Başar, "Domain of Riesz mean in some spaces of double sequences", Indag. Math. (N.S) 29 (2018) No:3, 1009-1029.
- [7] M. Zeltser, "On conservative matrix methods for double sequence spaces", Acta Math. Hungar., 95(3) (2002), 225-242.

Free transverse vibration of nonhomogeneous and elastically restrained monoclinic rectangular plate by Rayleigh-Ritz method

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Abstract: This paper focuses on a boundary value problem which deals with free transverse vibration of a thin elastic simply-supported nonhomogeneous monoclinic rectangular plate. The plate is elastically restrained against rotation and thickness of the plate is varying in two directions. The Rayleigh-Ritz method with orthogonal polynomials is used in the analysis. The orthogonal polynomials are generated using Gram-Schmidt process. Due to orthogonal polynomials, standard eigenvalue problem is obtained which is solved numerically to obtain the natural frequencies of the plate. The effects of nonhomogeneity parameters, thickness parameters, aspect ratio and flexibility parameters have been studied on the frequencies of the plate. Three dimensional mode shapes have been plotted.

Keywords: Rayleigh-Ritz, restrained, nonhomogeneous, monoclinic.

Mathematics Subject Classification: 70J10, 70J30, 74K20, 74S30, 74-10

References

- [1] R. Szilard, "Theory and Analysis of Plates", Prentice-Hall Inc., NJ, 1974.
- [2] V.S. Haussuhl, "Elastische und Thermoelastische Eigenschaften CaSO₄.2H₂O (Gips). – Zeitschrift für Kristallographie", Bd. 122, pp. 311-314, 1965.
- [3] Y. Kumar and S.K. Tomar, "Free transverse vibrations of monoclinic rectangular plates with continuously varying thickness and density", Int. J. Appl. Mech. Eng. 11(4), pp. 881-900, 2006.

A secure communication Scheme based on adaptive modified projective combination synchronization of fractional-order hyper-chaotic systems

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Abstract: In this work, a novel technique called "adaptive modified projective combination synchronization (AM-PCS)" for synchronizing non-identical fractional-order hyper-chaotic systems with unknown parameters has been introduced. The purpose of the suggested technique is to ensure synchronization between two non-identical master systems and one slave system by employing a diagonal matrix, Lyapunov stability theory, adaptive control, adaptive law of parameter, and some techniques of fractional calculus. An application of synchronization in secure communication has been performed. The important feature of the suggested (AMPCS) technique is to create high security in secure communication.

Keywords: Adaptive control, Hyper-chaotic systems, Combination synchronization, Secure communication

Mathematics Subject Classification: 34D06, 37B25, 34A08, 34A34

References

- [1] L. X. Yang, J. Jun, "Adaptive synchronization of drive response fractional-order complex dynamical networks with uncertain parameters", *Communications in Nonlinear Science and Numerical Simulation*, Vol.19, No.5, 1496-1506, 2014
- [2] S. Bhalekar and V. Daftardar-Gejji, "Synchronization of different fractional order chaotic systems using active control", *Nonlinear Sci Numer Simul*, 15, 3536-46, 2010.
- [3] Labid M. and Hamri N., "Chaos synchronization and antisynchronization of two fractional-order systems via global synchronization and active control", *Non Dyn Syst Theory*, 19(3), 416-426, 2019.
- [4] He S. B., Sun K.H., Wang H.H., Mei X.Y. and Sun Y.F., "Generalized synchronization of fractional-order hyperchaotic systems and its DSP implementation", *Nonlinear Dyn*, 92, 85-96, 2018.
- [5] He J, Chen F, Lei T, Fractional matrix and inverse matrix projective synchronization methods for synchronizing the disturbed fractional-order hyperchaotic system, *Math. Methods Appl. Sci*, 41(16), 6907-6920, 2018.
- [6] Zerimeche H., Houmor T. and Berkane A., "Combination synchronization of different dimensions fractional-order non-autonomous chaotic systems using scaling matrix", *Int. J. Dynam. Control*, <https://doi.org/10.1007/s40435-020-00660-9>, 2020.
- [7] He, J and Cai, J. " Finite-time combination-combination synchronization of hyperchaotic systems and its application in secure communication".*Phys. Sci. Int. J.* 4(10), 1326, 2014.
- [8] Pan L., Zhou W., Zhou L. and Sun K., "Chaos synchronization between two different fractional-order hyper-chaotic systems", *Commun. Nonlinear Sci. Numer. Simulat*, 16 (6), 2628-2640, 2011.

CSG: Towards a Comprehensive Model of Growth

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Abstract: The main purpose of this study is to propose a better model than Sloboda and Gompertz models. So, a comprehensive growth model (CSG) is defined by combining Sloboda and Gompertz models, which are well known in literature and properties of the CSG model are given. In order to demonstrate the performance of the proposed model in modeling growth and to compare the performance of the new model with Gompertz and Sloboda models, an application was made on the real data set and the results were presented in table and graphics.

Keywords: Curve Fitting, Gompertz Growth Model, Sigmoid Functions, Sloboda Growth Model.

Mathematics Subject Classification: 91B99, 00A71, 62P99, 91C99

On variational-iterative method for solving of a static beam problem

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Abstract:In the paper, the boundary value problem is solved by the variation-iterative method, which describes the stationary state of a beam. The equation is an equation of the Kirchhoff type [1]. Some computational aspects of this equation and its various modifications are investigated in many works [2-6]. The accuracy of the method is estimated, and its effectiveness is checked by an example.

Keywords:static beam, Galerkin method, iteration method, Newton method

Mathematics Subject Classification: 65H10, 65L10, 65L60, 74K10

References

- [1] G. Kirchhoff, Vorlesungen über mathematische Physik. I. Mechanik, Teubner, Leipzig, 1876.
- [2] G.Berikelashvili, A.Papukashvili, J.Peradze, "Iterative solution of a nonlinear static beam equation", Ukr. Math. J., 72, pp. 1185-1196, 2021.
- [3] Q. A.Dang, V.T. Luan, "Iterative method for solving a nonlinear fourth order boundary value problem", Comput. Math. Appl., 60(1), pp. 112-121, 2010.
- [4] Q. A. Dang, T. H. Nguyen, "Existence results and iterative method for solving a nonlinear biharmonic equation of Kirchhoff type". Comput. Math. Appl., 76(1), 2018.
- [5] N. Kachakhidze, N. Khomeriki, J. Peradze, Z. Tsiklauri, "Chipot's method for a one-dimensional Kirchhoff static equation", Numerical Algorithms, 73(4), pp. 1091-1106, 2016.
- [6] Q.Ren , H.Tian, "Numerical solution of the static beam problem by Bernoulli collocation method", Appl. Math. Model., 40(21-22), pp. 8886-8897, 2016.

U-net based MRI brain tumor segmentation

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Abstract: In recent years, the implementation of medical information technologies and e-health care framework allows healthcare specialists to provide the patient with quality health care. Brain tumor is one of the primary causes of an increase in human mortality. Segmentation is used to detect contaminated tumor tissues from medical imaging modalities. The U-net is one of the most well-known convolutional neural network (CNN) architectures used for biomedical image segmentation because of the cascade connection in the up-sampling process. It was designed specifically for medical image processing. In this paper, we proposed a U-net based method for brain tumor segmentation. We have applied our approach on 3064 T1-weighted contrast-enhanced images from 233 patients. Data were analyzed in terms of accuracy and IOU-metric. Analysis of the data obtained shows that in accuracy results, on the one hand, training accuracy is greater than validation accuracy. The former is 99.30%, whereas the latter is 99.26%. Similarly, in IOU-metric, on the other hand, validation data (61.52%) are less than training data (64.39%).

Keywords: Image processing, MRI Brain tumor, segmentation, U-net.

Stability analysis of conformable fractional-order nonlinear systems

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Abstract: In this paper, we study the uniformly fractional exponential stability for some class of systems like class of perturbed systems and class of nonlinear fractional-order equations with control using the Lyapunov-like function.

Keywords: Conformable fractional derivative, fractional practical exponential stability.

References

- [1] Abdeljawad, T., On conformable fractional calculus," J. Comput. Appl. Math., vol.279, pp 57-66, 2015.
- [2] Batar, H., Losada, J., Nieto, J J. and Shammakh, W. Three-point boundary Value problems for conformable fractional differential equations," J.Funct. Spaces, pp. Art. ID 706383, 6, 2015.
- [3] Ben Hamed, B.,Haj Salem, Z., Hammami, M.A.(2013)."Stability of nonlinear time- varying perturbed differential equations," Nonlinear Dynamics.,vol. 73, pp 1353-1365 2013.
- [4] Ben Makhlof, A. and Hammami, M. A. The convergence relation between ordinary and delay-integro-differential equations," Int. J.Dyn. Syst. Differ. Equ., vol. 5, pp 236-247, 2015
- [5] BenAbdallah, A., Dlala, M., Hammami, M A A new Lyapunov function for stability of time-varying nonlinear perturbed systems," Systems Control Lett., vol. 56, pp 179-187, 2007.
- [6] Benaoumeur B.and Delm F.M.T., Existence of solution to a local fractional Nonlinear differential equation," J.Comput. Appl. Math., vol. 312, pp 127-133, 2016
- [7] Chung, W. S., Fractional Newton mechanics with conformable fractional derivative," J. Comput. Appl. Math., vol. 290 pp 150-158, 2015.

Agent Based Modeling and Simulation for Geographic Routing Protocol in the Wireless Sensor Networks

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Abstract: Agent Based Modeling and Simulation (ABMS) is an approach aiming to model autonomous systems and interacting agents. This method is required more and more for its efficiency and its simplicity; it constitutes an interesting issue in the field of the modeling of complex systems. Indeed, ABMS offers, unlike other types of simulations, the possibility of directly representing the simulated entities, their behaviors and their interactions without having to resort to mathematical equations. The obtained models are composed of autonomous interacting agents. Simple rules or processes that are more complicated can describe agent behaviors and interactions. The interactions between the agents society influence the individual and the general behavior of the system. This work is a contribution in this way; the goal is to propose an agent-based model to simulate interacting wireless sensor nodes that manipulate sensed data in order to send it to remote base stations. The proposed approach is validated by a case study of the geographic protocol with avoidance of the minima problem (void problem).

Keywords: Multi-Agent System (MAS), Wireless Sensor Networks, Geographic Routing Protocol, Modeling and Simulation, Void Problem.

Mathematics Subject Classification: General theory of simulation.

References

- [1] Fahmy, H. M. A. Wireless Sensor Networks. Signals and Communication Technology: 3-39, 2020. doi:10.1007/978-3-030-29700-8
- [2] Obaidat, M.S., Misra, S. Principles of wireless sensor networks. Cambridge, UK: Cambridge University Press; 2014. <https://doi.org/10.1017/CBO9781139030960>
- [3] Mohamed, R.E., Saleh, A.I., Abdelrazzak, M. et al. Survey on Wireless Sensor Network Applications and Energy Efficient Routing Protocols. Wireless Pers Commun 101, 1019–1055, 2018. <https://doi.org/10.1007/s11277-018-5747-9>
- [4] Dhobley, A., Ghodichor, N.A., Golait, S.S.. An overview of wireless sensor networks for health monitoring in hospitals via mobile. Int J Adv Res Comput Commun Eng;4(1):169–71, 2015.
- [5] Belghith, A., and Obaidat, M. S. Wireless sensor networks applications to smart homes and cities. Smart Cities and Homes 17-40, 2016. doi:10.1016/b978-0-12-803454-5.00002-x
- [6] Tomar, P., Kaur, G., and Singh, P. A Prototype of IoT-Based Real Time Smart Street Parking System for Smart Cities. Internet of Things and Big Data Analytics Toward Next-Generation Intelligence, 243–263, 2017. doi:10.1007/978-3-319-60435-0_10
- [7] Hilmani, A., Maizate, A., and Hassouni, L. Designing and managing a smart parking system using wireless sensor networks. Journal of Sensor and Actuator Networks, 7(2), 24, 2018.
- [8] Faheem, M., Butt, R. A., Raza, B., Ashraf, M. W., Begum, S., Ngadi, M. A., and Gungor, V. C. Bio-inspired routing protocol for WSN-based smart grid applications in the context of Industry 4.0. Transactions on Emerging Telecommunications Technologies, 30(8), e3503, 2019.

- [9] Rida, N., Ouadoud, M., and Hasbi, A. Traffic Signal Control for a Single Intersection-Based Intelligent Transportation System. In *Digital Transformation and Innovative Services for Business and Learning* (pp. 159-180). IGI Global, 2020.
- [10] Derakhshan, F., and Yousefi, S. A review on the applications of multiagent systems in wireless sensor networks. *International Journal of Distributed Sensor Networks*, 15(5), 2019. 1550147719850767.
- [11] Dong, M., Ota, K., Yang, L. T., Chang, S., Zhu, H., and Zhou, Z. Mobile agent-based energy-aware and user-centric data collection in wireless sensor networks. *Computer networks*, 74, 58-70, 2014.
- [12] Bennur, V. S., Shirabur, S. S., and Sutagundar, A. V. Multiagent based multipath routing in wireless sensor networks. *International Journal of Wireless and Mobile Networks*, 6(2), 67, 2014.
- [13] Taboun, M. S., and Brennan, R. W. An embedded agent-based intelligent industrial wireless sensor network. In *International Conference on Industrial Applications of Holonic and Multi-Agent Systems* (pp. 227-239). Springer, Cham, (2017, August).
- [14] Dhobley, A., Ghodichor, N.A., Golait, S.S.. "An overview of wireless sensor networks for health monitoring in hospitals via mobile". *Int J Adv Res Comput Commun Eng*; 4(1):169–71, 2015.
- [15] Belghith, A., and Obaidat, M. S. Wireless sensor networks applications to smart homes and cities. *Smart Cities and Homes*, 17–40, 2016. doi:10.1016/b978-0-12-803454-5.00002-x
- [16] Zhu, X., Ding, B., Li, W. et al. On development of security monitoring system via wireless sensing network. *J Wireless Com Network* 2018, 221, 2018. <https://doi.org/10.1186/s13638-018-1235-x>
- [17] Haseeb, K., Ud Din, I., Almogren, A., and Islam, N. An energy efficient and secure IoT-based WSN framework: An application to smart agriculture. *Sensors*, 20(7), 2081, 2020.
- [18] Kand ris, D., Nakas, C., Vomvas, D., and Koulouras, G. Applications of wireless sensor networks: an up-to-date survey. *Applied System Innovation*, 3(1), 14, 2020.
- [19] Singh, J., Kaur, R., and Singh, D. A Survey and Taxonomy on Energy Management Schemes in Wireless Sensor Networks. *Journal of Systems Architecture*, 101782, 2020. doi:10.1016/j.sysarc.2020.101782
- [20] Kumar, A., Shwe, H. Y., Wong, K. J., and Chong, P. H. Location-based routing protocols for wireless sensor networks: A survey. *Wireless Sensor Network*, 9(1), 25-72, 2017.
- [21] Singh, M., and Kumar, S. A survey: Ad-hoc on demand distance vector (AODV) protocol. *International Journal of Computer Applications*, 161(1), 38-44, 2017.
- [22] J. Mulert, I. Welch, and W. K.G. Seah. Security threats and solutions in Manets: A case study using aodv and saodv. *Journal of Network and Computer Applications*, 35(4):1249-1259, 2012.
- [23] S. A. Ade, P. A. Tijare. Performance comparison of aodv, dsdv, olsr and dsr routing protocols in mobile ad hoc networks. *International Journal of Information Technology and Knowledge Management*, 2(2):545-548, 2010.
- [24] Mehic, M., Fazio, P., and Voznak, M. Usability of Destination-Sequenced Distance Vector Routing Protocol Routes. 2019 11th International Congress on Ultra Modern Telecommunications and Control Systems and Workshops (ICUMT), 2019. doi:10.1109/icumt48472.2019.8970752
- [25] Slimani, A., Redjimi, M., and Slimani, D. Weighted Density Center (WDC-LEACH-C) Clustering Protocol for Wireless Sensor Networks (WSN). *Informatica*, 42(2), 2017.
- [26] Slimani, A., Redjimi, M., and Slimani, D. An agent-based cluster head management strategy for hierarchical wireless sensor network. *International Journal of Systems, Control and Communications*, 11(4), 321-333, 2020.

- [27] S. Tyagi, N. Kumar, A systematic review on clustering and routing techniques based upon LEACH protocol for wireless sensor networks, *Journal of Network and Computer Applications*, 36(2):623-645, 2013.
- [28] B. Karp and H. Kung, "GPSR: Greedy perimeter stateless routing for wireless networks. In Proceedings of the 6th Annual International Conference on Mobile Computing and Networking. ACM Press, pages 243–254, 2000
- [29] R.H. Milocco, H. Costantini, S. Boumerdassi. Improved geographic routing in sensor networks subjected to localization errors. *Ad Hoc Networks*, 13: 476–486, 2014.
- [30] B. Peng, A.H. Kemp. "Energy-efficient geographic routing in the presence of localization errors". *Computer Networks*, 55(3): 856–872, 2011.
- [31] S. Lee, B. Bhattacharjee, S. Banerjee, B. Han. A general framework for efficient geographic routing in wireless networks. *Computer Networks*, 54(5): 844–861, 2010.
- [32] M. Boulaiche, L. Bouallouche-Medjkoune. EGGR : Energy-aware and delivery Guarantee Geographic Routing protocol. *Wireless Networks*, 21(6):1765-1774, 2015.
- [33] S. Tao, A.L. An, and a, Mun Choon Chan, Greedy face routing with face identification support in wireless networks, *Computer Networks*, 54 (2010): 3431–3448, 2010.
- [34] A. Kleerekoper, N.P. Filer, Perfect link routing for energy efficient forwarding in geographic routing, *Ad hoc Networks*, 30: 46–62, 2015.
- [35] M. Al-shugran, O. Ghazali, S. Hassan, K. Nisar , A.Suki, M.Arif, A qualitative comparison evaluation of the greedy forwarding strategies in Mobile Ad Hoc Network, *Journal of Network and Computer Applications*, 36:887–897, 2013.
- [36] Hasan, M. Z., Al-Turjman, F., and Al-Rizzo, H. Analysis of cross-layer design of quality-of-service forward geographic wireless sensor network routing strategies in green internet of things. *IEEE Access*, 6, 20371-20389, 2018.
- [37] Lyu, C., Zhang, X., Liu, Z., and Chi, C. H. Selective authentication based geographic opportunistic routing in wireless sensor networks for Internet of Things against DoS attacks. *IEEE Access*, 7, 31068-31082, 2019.
- [38] Hameed, A. R., Islam, S. ul, Raza, M., and Khattak, H. A. Towards energy and performance aware geographic routing for IoT enabled sensor networks. *Computers and Electrical Engineering*, 85, 106643, 2020. doi:10.1016/j.compeleceng.2020.106643
- [39] Boussoufa-Lahlah, S., Semchedine, F., and Bouallouche-Medjkoune, L. Geographic routing protocols for Vehicular Ad hoc NETWORKS (VANETs): A survey. *Vehicular Communications*, 11, 20-31, 2018. <https://doi.org/10.1016/j.vehcom.2018.01.006>
- [40] M. Al-shugran, O. Ghazali, S. Hassan, K. Nisar , A.Suki, M.Arif, "A qualitative comparison evaluation of the greedy forwarding strategies in Mobile Ad Hoc Network", *Journal of Network and Computer Applications*, 36:887–897, 2013.
- [41] Cao, N., Wang, Y., Ding, J., Zhou, C., Li, Y., Zhang, Y., . . . Li, H. The Comparisons of Different Location-Based Routing Protocols in Wireless Sensor Networks. 2017 IEEE International Conference on Computational Science and Engineering (CSE) and IEEE International Conference on Embedded and Ubiquitous Computing (EUC). doi:10.1109/cse-euc.2017.246, 2017.
- [42] A. Boukerche, B. Turgut, N. Aydin, M. Z. Ahmad, L. Boloni, D. Turgut, "Routing protocols in ad hoc networks: A survey", *Computer Networks*, 55: 3032–3080, 2011.
- [43] Cao, N., Liu, P., Li, G., Zhang, C., Cao, S., Cao, G., ... and Gupta, B. B. Evaluation models for the nearest closer routing protocol in wireless sensor networks. *IEEE Access*, 6, 77043-77054, 2018.

- [44] Gharat, C., and Krishnan, S. Effects of Duplicate Packet Transmission in Timer based Co-ordination Opportunistic Routing Scheme. In 2019 International Conference on Smart Systems and Inventive Technology (ICSSIT) (pp. 401-405). IEEE, (2019, November).
- [45] Coutinho, R. W. L., Boukerche, A., and Loureiro, A. A. F. A novel opportunistic power controlled routing protocol for internet of underwater things. *Computer Communications*, 2019. doi:10.1016/j.comcom.2019.10.020
- [46] D. Chen, P. K. Varshney, A Survey of Void Hand ling Techniques for Geographic Routing in Wireless Networks, *IEEE Communications Surveys and Tutorials*, 9(1):50-67, 2007.
- [47] Lima, M. M., Oliveira, H. A. B. F., Guidoni, D. L., and Loureiro, A. A. F. Geographic routing and hole bypass using long range sinks for wireless sensor networks. *Ad Hoc Networks*, 67, 1–10, 2017. doi:10.1016/j.adhoc.2017.08.010
- [48] Huang, H., Yin, H., Min, G., Zhang, X., Zhu, W., and Wu, Y. Coordinate-assisted routing approach to bypass routing holes in wireless sensor networks. *IEEE Communications Magazine*, 55(7), 180-185, 2017.
- [49] Nguyen, KV., Nguyen, CH., Le Nguyen, P. et al. Energy-efficient routing in the proximity of a complicated hole in wireless sensor networks. *Wireless Netw* , 2021. <https://doi.org/10.1007/s11276-021-02569-3>
- [50] J. A. Bondy and U. S. R. Murty. *Graph Theory with Applications*. The Macmillan Press Ltd North-Holland , 1976.
- [51] P. Bose, P. Morin, I. Stojmenovic, and J. Urrutia. Routing with Guaranteed Delivery in Ad Hoc Wireless Networks. *Wireless Networks*, 7(6):609–616, 2001.
- [52] Gupta, N. K., Yadav, R. S., and Nagaria, R. K. 3D geographical routing protocols in wireless ad hoc and sensor networks: an overview. *Wireless Networks*. 2019. doi:10.1007/s11276-019-01983-y
- [53] J. Kuruvila, A. Nayak, and I. Stojmenovic. Progress and Location Based Localized Power Aware Routing For Ad Hoc Sensor Wireless Networks. *International Journal of Distributed Sensor Networks*, 2:147–159, 2006.
- [54] E. Dijkstra. Solution of a problem in concurrent programming control. *Communications of the ACM*, 11(2):147–148, 1968.
- [55] I. Stojmenovic and X. Lin. Power-Aware Localized Routing in Wireless Networks. *IEEE Transactions on Parallel and Distributed Systems*. 12(11):1122-1133,2001.
- [56] V. Rodoplu and T. Meng. Minimum Energy Mobile Wireless Networks. *IEEE Journal on Selected Areas in Communications*, 17(8):1333–1347, 1999.
- [57] Ghaffari, A. An energy efficient routing protocol for wireless sensor networks using A-star algorithm. *Journal of applied research and technology*, 12(4), 815-822. 2014.
- [58] R.C. Prim, Shortest connection networks and some generalizations, *Bell System Technical Journal*, 36 (6) 1389–1401, 1957.

Hybrid Predictive Models for Water Quality Assessment Based on Water Quality Index Using ANN, LSSVM and multivariate statistical Methods

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Abstract: The study and use of water is essential for assessing the quality of surface waters. Several indices have over the years been proposed using s statistical, mathematical, and computational techniques to enhance the understanding of the phenomena that occur in these environments. For this purpose, the variables which influence the quality of the water should be known. Nowadays, indices need to be developed that can address climate change in its variables, making it even more realistic. In this study, multivariate statistical techniques such as PCA are aimed at reduce the number of variables used to recover the costs, laboratory tests and greater representativeness of indices. Searching for improvement and accuracy in indices of water quality, certain computational artificial intelligence techniques, such as LSVM and ANN, are increasingly utilized and achieve expressive research results. These Two machine learning methods have been applied in the current research to investigate and try to emulate WQI's relationship with water quality variables in Cheliff's dam in Mostaganem (Algeria). Moreover, a comprehensive analysis has been performed for the performance assessment and sensitivity analysis of the variables. With high performance accuracy in two used reduced models, the results achieved are promising. The proposed approach also provides an efficient alternative to calculate and predict the WQI by including long computing methods, transformations, the use of various subindex formulas for every value of the water quality component variables and time consumption.

Keywords: Water quality assessment, Computational artificial intelligence techniques, Water quality index, Features selection, PCA, ANN, LSSVM, Algeria.

Mathematics Subject Classification: MSC2020 database: 68Uxx

References

- [1] Song, Y., Xie, X., Wang, Y., Yang, S., Ma, W., & Wang, P. (2021). Energy consumption prediction method based on LSSVM-PSO model for autonomous underwater gliders. *Ocean Engineering*, 230, 108982. <https://doi.org/10.1016/j.oceaneng.2021.108982>
- [2] Tiyasha, Tung, T. M., & Yaseen, Z. M. (2020). A survey on river water quality modelling using artificial intelligence models : 2000–2020. *Journal of Hydrology*, 585, 124670. <https://doi.org/10.1016/j.jhydrol.2020.124670>
- [3] Hameed, M., Sharqi, S. S., Yaseen, Z. M., Afan, H. A., Hussain, A., & Elshafie, A. (2016). Application of artificial intelligence (AI) techniques in water quality index prediction : a case study in tropical region, Malaysia. *Neural Computing and Applications*, 28(S1), 893-905. <https://doi.org/10.1007/s00521-016-2404-7>
- [4] Hmoud Al-Adhaileh, M., & Waselallah Alsaade, F. (2021). Modelling and Prediction of Water Quality by Using Artificial Intelligence. *Sustainability*, 13(8), 4259. <https://doi.org/10.3390/su13084259>

Remark on Convergence of the Associated Filters

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Abstract: Let (X, τ) be a topological space. For a proper ideal on (X, τ) , the associated filter F_I was defined and investigated in [21]. In this presentation, we shall discuss the convergence of the associated filters in terms of the points which are coming from different types of operator of the ideal topological spaces.

Keywords: Filter, Associated filter, Local function, Hayashi-Samuel space.

Mathematics Subject Classification: 54A20

References

- [1] Bandhopadhyaya, C. and Modak, S., "A new topology via ψ -operator", Proc. Nat. Acad. Sci. India, Sect. A Phys. Sci., 76(A)(IV), 317–320, 2006.
- [2] Dontchev, J., "Idealization of Ganster-Reilly decomposition theorems", arXIV:math. Gn/9901017v1 [math.GN], 5 Jan 1999.
- [3] Dontchev, J., Ganster, M. and Rose, D., "Ideal resolvability", Topology Appl., 93, 1–16, 1999. [https://doi.org/10.1016/S0166-8641\(97\)00257-5](https://doi.org/10.1016/S0166-8641(97)00257-5).
- [4] Hamlett, T. R. and Jankovic', D., "Ideals in topological spaces and the set operator ψ ", Boll. Un. Mat.Ital., 4-B(7), 863–874, 1990.
- [5] Hashimoto, H., "On the $*$ -topology and its applications", Fund. Math., 91, 5–10, 1976.
- [6] Hayashi, E., "Topologies defined by local properties", Math. Ann., 156, 205–215, 1964.
- [7] Jankovic', D. and Hamlett, T. R., "New topologies from old via ideals", Amer. Math. Monthly 97, 295–310, 1990. <https://doi.org/10.1080/00029890.1990.11995593>.
- [8] K. D. Joshi, "Introduction to general topology", Wiley, 1983.
- [9] Kuratowski, K., "Topology Vol. I", New York, Academic Press, 1966.
- [10] Modak, S., "Some new topologies on ideal topological spaces", Proc. Natl. Acad. Sci., India, Sect. A Phys. Sci., 82(3), 233–243, 2012. <https://doi.org/10.1007/s40010-0120039-3>.
- [11] Modak, S. and Bandyopadhyay, C., "A note on ψ -operator", Bull. Malays. Math. Sci. Soc., 30(1), 43–48, 2007.
- [12] Modak, S. and Noiri, T., "Connectedness of ideal topological spaces", Filomat, 29(4), 661–665, 2015. <https://doi.org/10.2298/FIL1504661M>.
- [13] Modak, S. and Selim, Sk., "Set operators and associated functions", Commun. Fac. Sci. Univ. Ank. Ser. A1 Math. Stat., 70(1), 456–467, 2021. <https://doi.org/10.31801/cfsuasmas.644689>.
- [14] Modak, S., Selim, Sk. and Islam., Md. M., "Sets and functions in terms of local function", Submitted.
- [15] N. Bourbaki, "General Topology", Chapter 1-4, Springer, 1989.

- [16] Newcomb, R. L., “Topologies which are compact modulo an ideal”, Ph. D. Dissertation, Univ. of Cal. at Santa Barbara, 1967.
- [17] Natkaniec, T., “On I-continuity and I-semicontinuity points”, *Math. Slovaca*, 36(3), 297–312, 1986.
- [18] Noiri, T., Selim, Sk. and Modak, S., “Convergence of the Associated Filters via Set-Operators”, *Cankaya Univ. j. sci. eng.*, 17(2), 101–107, 2020.
- [19] Samuel, P., “A topology formed from a given topology and ideal”, *J. London Math. Soc.*, 10, 409–416, 1975.
- [20] Selim, Sk., Islam., Md. M. and Modak, S., “Characterizations of Hayashi-Samuel Spaces via Boundary Points”, *Commun. Adv. Math. Sci.*, 2(3), 219–226, 2019. <https://doi.org/10.33434/cams.546925>.
- [21] Selim, Sk., Noiri, T. and Modak, S., “Ideals and the associated filters on topological spaces”, *Euras. Bull. Math.*, 2(3), 80–85, 2019.
- [22] Selim, Sk., Noiri, T. and Modak, S., “Some set-operators on ideal topological spaces”, Submitted.
- [23] Vaidyanathswamy, R., “The localization theory in set-topology”, *Proc. Indian Acad. Sci.*, 20, 51–61, 1945. <https://doi.org/10.1007/BF03048958>.

Dynamical Modeling and Backstepping Control Applied to the 6-DOF Quadrotor UAV

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Abstract: A quadrotor helicopter control includes nonlinearities, uncertainties and external perturbations that should be taken into account in the design of the control laws. In this paper, we present a control strategy based on nonlinear dynamic modeling and backstepping control for an underactuated six-degree of freedom helicopter quadrotor (6 DOF). These quadrotors are very difficult, because in most cases these are nonlinear, multivariate, highly coupled and underactuated systems that should be taken into account when designing control laws. The nonlinear control strategy is based on the Backstepping technique for the two subsystems (translational and rotational). It was justified to select this method through its robustness in terms of the modeling errors and external perturbations. The stability of the closed loop is evaluated by the use of the Lyapunov's theory. The result of the simulation has been successful and demonstrates the efficiency in quasi-stationary flights and trajectories of the proposed Quadrotor helicopter control strategy.

Keywords: 6 - DOF Quadrotors, Dynamic Modelling, Underactuated Systems, Strongly-coupled, Backstepping control, Theory of Lyapunov

Mathematics Subject Classification: MSC2020 database 93Dxx

References

- [1] Zeghlache, S., Benslimane, T., & Bouguerra, A. (2017). Active fault tolerant control based on interval type-2 fuzzy sliding mode controller and non linear adaptive observer for 3-DOF laboratory helicopter. *ISA Transactions*, 71, 280-303. <https://doi.org/10.1016/j.isatra.2017.09.006>
- [2] Zhang, X., & Huang, W. (2021). Adaptive sliding mode fault tolerant control for interval Type-2 fuzzy singular fractional-order systems. *Journal of Vibration and Control*, 107754632098018. <https://doi.org/10.1177/1077546320980181>
- [3] Xu, X., Watanabe, K., & Nagai, I. (2021). Backstepping Control for a Tandem Rotor UAV Robot with Two 2-DOF Tilttable Coaxial Rotors. *Journal of Robotics and Control (JRC)*, 2(5). <https://doi.org/10.18196/jrc.25116>
- [4] Wang, R., & Liu, J. (2018). Trajectory tracking control of a 6-DOF quadrotor UAV with input saturation via backstepping. *Journal of the Franklin Institute*, 355(7), 3288-3309. <https://doi.org/10.1016/j.jfranklin.2018.01.039>
- [5] Lungu, M., & Lungu, R. (2012). Adaptive backstepping flight control for a mini-UAV. *International Journal of Adaptive Control and Signal Processing*, 27(8), 635-650. <https://doi.org/10.1002/acs.2330>
- [6] Ahmed, B., Pota, H. R., & Garratt, M. (2009). Flight control of a rotary wing UAV using backstepping. *International Journal of Robust and Nonlinear Control*, 20(6), 639-658. <https://doi.org/10.1002/rnc.1458>

A New AR-ANN-framework for time series Modeling and Identification enhanced using IWO and CMA-ES metaheuristics approaches: A pilot Study

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Abstract: We attempt to design artificial neural networks (ANN) that can help in the automatic identification of the Autoregressive (AR) model. Within classic time series approaches, a time series model can be studied under three groups, namely AR (autoregressive model), MA (moving averages model) and ARMA (autoregressive moving averages model). In this paper, a new AR-ANN scheme applied for times series modeling is presented. It is based on neural networks. This approach will deal with local minima problem of the neuronal networks architecture and simultaneously preserve the fitting quality. The proposed model comprises a parallel interconnection of tow sub-ANN models. The first is primary sub-ARMA-ANN model, which represents an ordinary model with a low resolution for the time series under consideration, the second is an AR-ANN sub-model called the error model, which represents uncertainty in the primary model. Identification is achieved by innovative metaheuristic optimization algorithms such as The invasive weed optimization algorithm (IWO) and covariance matrix adaptation evolution strategy (CMA-ES). The method's effectiveness is evaluated through testing on benchmark function and real signals. In addition, a detailed comparative study with several benchmark methods would make. Intensive computer experimentations confirm that the proposed method can significantly improve convergence and resolution.

Keywords: Time series fitting, ANN, AR, Metaheuristics algorithms.

Mathematics Subject Classification: MSC2010 database: 70E60, 93C85

References

- [1] Abdulhamit Subasia, Ahmet Alkan, Etem Koklukaya, M. Kemal Kiyimik, Wavelet neural network classification of EEG signals by using AR model with MLE preprocessing. *Neural Networks* 18 (2005) 985–997
- [2] Jilei Tian, Martti Juhola, Tapio Gronfors, AR parameter estimation by a feedback neural network. *Computational Statistics & Data Analysis* 25 (1997) 17–24.
- [3] I. Rojasa, O. Valenzuelab, F. Rojasa, A. Guillena, L.J. Herreraa, H. Pomaresa, L. Marquezb, M. Pasadasb. Soft-computing techniques and ARMA model for time series prediction. *Neurocomputing* 71 (2008) 519–537.
- [4] Hongjun Xiao, Daoping Huang, Yongping Pan, Yiqi Liu, Kang Song, Fault diagnosis and prognosis of wastewater processes with incomplete data by the auto-associative neural networks and ARMA model. *Chemometrics and Intelligent Laboratory Systems* 161 (2017) 96–107.
- [5] MojganMisaghi, Mahdi Yaghoobi, Improved invasive weed optimization algorithm (IWO) based on chaos theory for optimal design of PID controller, *Journal of Computational Design and Engineering*, Volume 6, Issue 3, July 2019, Pages 284-295
- [6] Yajun Liang, Xiaofei Wang, Hui Zhao, Tong Han, Zhenglei Wei, Yintong Li, A covariance matrix adaptation evolution strategy variant and its engineering application, *Applied Soft Computing Journal*, 83 (2019).

An improved ANN-framework for dynamic systems Modeling and Identification using ICA and TLO metaheuristics approaches: A Pilot Study

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Abstract: Neural Network Modeling and Identification of Dynamical Systems presents a new approach on how to obtain the adaptive neural network models for complex systems that are typically found in real-world applications. Neural networks are used in many applications such as image recognition, classification, control and system identification. In this paper, a new hybrid Artificial Neural Network Autoregressive Moving Average (ANNARMA) and Artificial Neural Network Autoregressive (ANNAR) scheme applied for dynamical systems modeling is presented. This approach will deal with local minima problem of the neuronal networks architecture and simultaneously preserve the fitting quality. The proposed model comprises a parallel interconnection of two sub-ANN models. The first sub-ANN model is the primary model, which represents an ordinary model with a low resolution for the dynamical system under consideration. To overcome resolution quality problem, and obtain a model with higher resolution, we will introduce a second ANN sub model called Error model which will represent a model for the error modelling between the primary model and the real nonlinear dynamic system. Identification is achieved by innovative metaheuristic algorithms such as Imperialistic Competitive Algorithm (ICA) and Teaching-learning-based optimization (TLO). The method's effectiveness is evaluated through testing on the three nonlinear dynamical systems described by Narendra in the literature. In addition, a detailed comparative study with several benchmark methods will be given. Intensive computer experimentations confirm that the proposed approach can significantly improve convergence and resolution.

Keywords: Dynamical systems, Artificial neural network, AR, ARMA, Metaheuristics algorithms.

Mathematics Subject Classification: MSC2010 database: 70E60, 93C85

References

- [1] Abdulhamit Subasia, Ahmet Alkan, Etem Koklukaya, M. Kemal Kiyimik, Wavelet neural network classification of EEG signals by using AR model with MLE preprocessing. *Neural Networks* 18 (2005) 985–997
- [2] I. Rojasa, O. Valenzuelab, F. Rojasa, A. Guillena, L.J. Herreraa, H. Pomaresa, L. Marquezb, M. Pasadasb. Soft-computing techniques and ARMA model for time series prediction. *Neurocomputing* 71 (2008) 519–537.
- [3] Hongjun Xiao, Daoping Huang, Yongping Pan, Yiqi Liu, Kang Song, Fault diagnosis and prognosis of wastewater processes with incomplete data by the auto-associative neural networks and ARMA model. *Chemometrics and Intelligent Laboratory Systems* 161 (2017) 96–107.
- [4] Xu Chen, Congli Mei, Bin Xu, Kunjie Yu, Xiuhui Huang, Quadratic interpolation based teaching-learning-based optimization for chemical dynamic system optimization, *Knowledge-Based Systems*, Vol. 145, 1 April 2018, Pages 250-263
- [5] Ali Reza Kashani a , Amir Hossein Gandomi b , Mehdi Mousavi, Imperialistic Competitive Algorithm: A metaheuristic algorithm for locating the critical slip surface in 2-Dimensional soil slopes, *Geoscience Frontiers* Volume 7, Issue 1, January 2016, Pages 83-89

Primitive Divisor Theorem and An Application to the Diophantine Equations

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Abstract: One of the powerful tools to solve the Diophantine equations is Primitive Divisor Theorem. In this study we discuss how we can use this tool to solve some exponential Diophantine equations and we give a concrete application of the primitive divisor theorem to a family of exponential Diophantine equations. This theorem enable us to obtain some bounds on the variable of the equation we handle and then we use computer algebra system Maple to complete the solution of equations

Keywords: Primitive Divisor Theorem, Diophantine equations

Mathematics Subject Classification: 11D61, 11Y55

References

- [1] Bilu, Y., Hanrot, G., & Voutier, P.M. (2001) Existence of primitive divisors of Lucas and Lehmer numbers (with Appendix by Mignotte), *J. Reine Angew. Math.*, 539, 75–122., (2001)
- [2] Le, M. Some exponential Diophantine equations I: the equation $D_1x^2 - D_2y^2 = k^z$, *J. Number. Theory* 55(2), 209-221, (1995)
- [3] Terai, N.: On the exponential Diophantine equation $(4m^2 + 1)^x + (5m^2 - 1)^y = (3m)^z$. *Int J Algebra*, 6, 1135-1146 (2012)
- [4] Le, M., Scott, R. & Styer, R. A Survey on the Ternary Purely Exponential Diophantine Equation $a^x + b^y = c^z$ *Surveys in Mathematics and its Applications*, 214, 109-140, (2019)

A CAD System for breast cancer diagnosis using TAR calculation

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Abstract: Breast cancer is the most feared cause of death in woman worldwide. There were more than 2.3 million new cases and 685 000 deaths across the world in 2020 according to the World Health Organization. In the last few decades mammography become the most efficient radiological technique and mostly used. To help radiologists in the process of detection and diagnosis, Computer Aided Diagnosis systems (CADx) using digital mammograms and computer vision techniques represent a powerful tool to insure double reading of image screening. In addition of segmentation and classification, specific descriptors are the key of an effective CADx systems. Many descriptors have been developed but automatic diagnosis still a very hard task due to the fact that breast masses vary in shape and size which make their description difficult. In this research we proposed a novel descriptor based on triangle-area representation (TAR) calculation. Spiculations are the most significant features used to discriminate malignant and benign masses main. TAR descriptor browses all the point of contour's mass, detect and measure the convexity/concavity of each pixel with different steps. CBIS-DDSM dataset is used to perform training and testing with Fuzzy C-Means classifier. Experimentation results shows that accuracy of correct classification was 82.80

Keywords: Breast Cancer, CADx system, mammography, CBIS-DDSM, Shape description, Triangle-area Representation

Stability of Riemann-Liouville fractional-order delay nonlinear systems

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Abstract: Various processes with anomalous dynamics in science and engineering can be formulated mathematically using fractional differential operators ([1]). When the Riemann-Liouville (RL) fractional derivative is applied in differential equations the statement of initial conditions is very important. Fractional differential equations in terms of the RL derivative require initial conditions expressed in terms of initial values of fractional derivatives of the unknown function ([2]). In the case of zero initial conditions the RL, Grunwald-Letnikov (GL) and Caputo fractional derivatives coincide ([2]). For this reason, some authors either study Caputo derivatives, or use RL derivatives but avoid the problem of initial values of fractional derivatives by treating only the case of zero initial conditions. This leads to the consideration of mathematical correct problems, but without taking into account the physical nature of the described process. Sometimes, such as in the case of impulse response, nonzero initial conditions appear (see, for example, [2]). In connection with the main idea of stability properties we will consider in this talk nonzero initial conditions for RL fractional equations and we will define in an appropriate way stability properties which are slightly different than those for Caputo fractional differential equations. More detailed, in this talk the initial value problem for nonlinear delay differential equations with the RL fractional derivative is studied. Based on the arguments in the book [2], we set up initial conditions expressed in terms of initial values of fractional derivatives of the unknown function. Any solution of the defined initial conditions with RL fractional derivatives is not continuous at zero (the initial point). We require a new definition for stability excluding a small interval around zero. We define stability in time and generalize Mittag-Leffler stability in time for RL fractional differential equations. The stability properties of the zero solution are studied by Lyapunov functions. An appropriate modification of the Razumikhin method is suggested. Two types of derivatives of Lyapunov functions are applied: the RL fractional derivative when the argument of the Lyapunov function is a solution of the studied problem and the Dini fractional derivative among the studied problem.

Keywords: Riemann-Liouville fractional derivative, time-varying delay, stability, Lyapunov functions, fractional derivatives of Lyapunov functions, Razumikhin method

Mathematics Subject Classification: 34A08, 34K37, 34K20

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References

- [1] Hilfer R., "Applications of Fractional Calculus in Physic", World Sci., 2000.
- [2] Podlubny I., "Fractional Differential Equations", Academic Press: San Diego, 1999.

Coefficients of singularities for boundary value problems governed by the Lamé (elasticity) system in a plane sector

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Abstract: This paper represents the solution to boundary value problems governed by the Lamé (elasticity) system in a plane sector. By using appropriate Betti function, we establish a biorthogonality relation between the terms of the series, which allow us to calculate the coefficients of singularities in the important case crack.

Keywords: Lamé system, Crack, Singularity, Sector.

Mathematics Subject Classification: 35B40, 35B65, 35C20

References

- [1] P. Grisvard, G. Geymonat- Singularities and constructive methods for treatment, Proceeding Oberwolfach, Springer-Verlag, 1983, p. 123-126.
- [2] B. Merouani, Solutions singulières du système de l'élasticité dans un polygone pour différentes conditions aux limites, Maghreb math, Rev, Vol 5, Nos 1 & 2, 1996, pp. 95-112
- [3] B. Merouani and R. Boufenouche, Trigonometric series adapted for the study of Dirichlet boundary-value problems of lame systems; electronic journal of differential equations, vol. 2015 (2015), no. 181, pp. 1-6. ISSN: 1072-6691.
- [4] O. Tcha-Kondor, Nouvelles séries trigonométriques adaptées à l'étude de problèmes aux limites pour l'équation biharmonique. Étude du cas de la fissure [New trigonometric series adapted to the study of boundary value problems for the biharmonic equation. Cas of the crack], C. R. Acad. Sci. Paris, t. 315, Série I, p. 541-544, 1992(French).

Generalized memory and fractional calculus: A point of view

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Abstract: Fractional calculus deals with the study of so-called fractional order integral and derivative operators over real or complex domains, and their applications. In this talk I will present some new results regarding the generalised memory and fractional calculus. A new physical meaning of fractional operators will be presented.

Keywords: fractional calculus, physical meaning, generalized memory, extended kernels.

Mathematics Subject Classification: 34A08

References

- [1] D. Baleanu, R. P. Agarwal, Fractional calculus in the sky, *Adv. Differ. Equ.* 2021(1), Article Number: 117, 2021
- [2] A. Fernandez, D. Baleanu, Classes of operators in fractional calculus: A case study *Math. Meth. Appl. Sci.*, Early access icon Early Access, 2021
- [3] R. Nigmatullin, D. Baleanu, A. Fernandez, Balance equations with generalised memory and the emerging fractional kernels, *Nonlinear Dyn.*, to appear.
- [4] H. T. Nguyen Huy, D. Baleanu, N. T. Tran Ngoc, et al., Final value problem for nonlinear time fractional reaction-diffusion equation with discrete data, *J. Comput. Appl. Math.*, 376 Article Number: 112883, 2020
- [5] D. Baleanu, S.S. Samaneh, J. H. Asad, Jihad, et al., Hyperchaotic behaviors, optimal control, and synchronization of a nonautonomous cardiac conduction system, *Adv. Differ. Equ.* , 2021(1) Article Number: 157, 2021 , 2021

Evolution Problem with prox-regular nonconvex constraint sets

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Abstract: In this work we discuss the evolution problem known as differential inclusion for a class of prox-regular non-convex sets. An existence result of the evolution problem is proved in the finite dimensional setting. Assuming that such sets depend both on the time and on the state.

Keywords: Evolution problem, differential inclusion, prox-regular sets.

Mathematics Subject Classification: 34G20. 49J52. 70H03

References

- [1] F.H.Clarke, Yu.S.Ledyayev, R.J.Stern, P.R.Wolenski, *Nonsmooth Analysis and Control Theory*, Springer-Verlang New York (1998).
- [2] G. Colombo, L. Thibault *Prox-regular sets and applications*, *HandBook of Nonconvex Analysis*, International. Somerville, (2010).
- [3] Touma Haddad and Tahar Haddad, *Delay perturbed state-dependent sweeping process*, *Applicable Analysis*, 95(2) (2016), 270-282.
- [4] R.A. Poliquin, R.T. Rockafellar and L. Thibault, *Local differentiability of distance functions*, *M Trans. Amer. Math. Soc.* Vol. 352, (2000), N. 11, 5231-5249.

A Dynamical Study of Fractional Order Obesity Model by a Combined Legendre Wavelet Method

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Abstract: We have a new compartmental fractional order model for the simulation of epidemic obesity dynamics. Using the Legendre Wavelet method combined with the decoupling and quasi-linearization technique, we demonstrate the validity and applicability of our model. We also present a new compartmental ordinary order model for the complication of obesity. We will present some fractional differential illustrative example to demonstrate the applicability and efficiency of the method via Matlab.

Keywords: Obesity, Epidemiology, Fractional derivative, Legendre wavelet method.

References

- [1] N. Ablouei-Lahmar and O. Belhamiti, Numerical study of convection-reaction-diffusion equation by the legendre wavelet finite difference method, *Adv. Nonlinear Var. Inequal.* 19 (2016), 94–112.
- [2] O. Belhamiti, A new approach to solve a set of nonlinear split boundary value problems, *Commun. Nonlinear Sci. Numer. Simulat.* 17 (2012), 555–565.
- [3] G. A. Bray, Obesity, historical development of scientific and cultural ideas, *Int. J. Obes.* 14 (1990), 909–926.
- [4] M. Caputo, Linear model of dissipation whose q is almost frequency independent - ii, *Geophys. J. R. Astron. Soc.* 13 (1967), 529–539.
- [5] L. Corscadden and al., Obésité au canada, Rapport Conjoint de l'Agence de la Santé Publique du Canada et de l'Institut Canadien d'Information sur la Santé, Public Health Agency of Canada, 2011.
- [6] K. Diethelm, A fractional calculus based model for the simulation of an outbreak of dengue fever, *Nonlinear Dyn.* 71 (2013), 613–619.
- [7] K. Ejima, K. Aihara and H. Nishiura, Modeling the obesity epidemic: social contagion and its implications for control, *Theor. Biol. Med. Model.* 10 (2013), 1–13.
- [8] G. González-Parra, R. J. Villanueva and A. J. Arenas, An age structured model for obesity prevalence dynamics in populations, *REV MVZ CORDOBA* 15 (2010), 2051–2059.
- [9] A. L. Hill, D. G. Rand, M. A. Nowak and N. A. Christakis, Infectious disease modeling of social contagion in networks, *PLoS Comput. Biol.* 6 (2010), 1–15.
- [10] F. B. Hu, *Obesity Epidemiology*, Oxford University Press, 2008.
- [11] D. M. Thomas and al., Dynamic model predicting overweight, obesity, and extreme obesity prevalence trends, *Obesity* 22 (2014), 590–597.

Crack Propagation in Hollow Battery Electrode Particles using Bond Based Peridynamic Theory

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Abstract: Battery electrode particles suffer from diffusion induced stresses causing failure and capacity fading during lithiation and delithiation. The diffusion problem in a hollow cylindrical electrode particle is modeled using 2-d finite element method (FEM) considering critical lithiation/delithiation cases. The diffusion equation is solved by FEM utilizing Crank-Nicolson scheme for time discretization [1]. Bond based peridynamic model [2] is utilized for the elasticity problem with diffusion coupling using the obtained lithium concentration distribution from FEM analysis. Fracture of brittle solids under thermal shock is modeled using peridynamic theory can be found in the literature [3,4]. In this work, linear elastic fracture behavior of hollow cylindrical/spherical cathode electrode particles will be investigated, and a comparison will be presented for the available experimental works [5].

Keywords: Peridynamics, Diffusion-Stress, Electrode Particles.

Mathematics Subject Classification: 1X234

References

- [1] Tanrıöver, H. and Sheldon, B.W., “The impact of compositionally induced residual stress on electrochemical shock in battery electrode particles”, *Journal of The Electrochemical Society*, 162(7), A1282-8, 2015.
- [2] Silling S.A., Askari E., “A meshfree method based on the peridynamic model of solid mechanics”, *Comput. Struc.* 83:1526–1535, 2005.
- [3] Kilic, B. and Madenci E., “Prediction of crack paths in a quenched glass plate by using peridynamic theory.”, *International Journal of Fracture*, 156: 165-77, 2009.
- [4] Giannakeas, I.N., Papathanasiou, T.K. and Bahai, H., “Simulation of thermal shock cracking in ceramics using bond-based peridynamics and FEM”, *Journal of the European Ceramic Society*, 38: 3037-48, 2018.
- [5] Zhu, X., et al., "The diffusion induced stress and cracking behavior of primary particle for Li-ion battery electrode", *International Journal of Mechanical Sciences*, 178: 105608, 2020.

Some Results for Kantorovich Type Bivariate Linear Positive Operators

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Abstract: In mathematics, some functions can not be used directly in many problems because of their complex forms. Therefore, they are approximated by elementary functions as much as possible. This makes it possible to investigate the problem, readily. One of the important aspects of the theory of approximation is linear positive operators. With the proof of the theorem due to Weierstrass [1] and the fundamental convergence theorem given by Korovkin [2], various new operators have been established and their approximation properties have been examined from many different perspectives by several researchers. The talk will be centered about bivariate Bernstein-Beta Kantorovich type operators constructed by a transformation formula for the Pochhammer symbol [3]. We investigate the degree of approximation of the bivariate operators via complete and partial modulus of continuity and also we obtain Voronovskaja type theorem. Furthermore, we present some graphical examples to compare the approximation process between our modified bivariate operators and some other familiar linear positive operators.

Keywords: Lipschitz Class, Modulus of Continuity, Voronovskaja Type Theorem.

Mathematics Subject Classification: 26A16, 41A10, 41A25, 41A36

References

- [1] K. Weierstrass, “Über die analytische Darstellbarkeit sogenannter willkürlicher Functionen einer reellen Veränderlichen”, *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin*, 2 633–639, 1885.
- [2] P.P. Korovkin, “Linear Operators and the Theory of Approximation (Russian)”, Fizmatgiz, Moscow, 1959.
- [3] D.J. Bhatt, V.N. Mishra, R.K. Jana, “New class of beta type operators approximating integrable function”, *Advances in Operator Theory*, 5.2, 2020.

Rational Proper Holomorphic Mappings from \mathbb{B}^n into \mathbb{B}^{3n-2}

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Abstract: This talk is based on a joint work with Shanyu Ji and Wanke Yin. We study proper rational holomorphic maps between the complex unit balls \mathbb{B}^n and \mathbb{B}^N , and denote by $\text{Rat}(\mathbb{B}^n, \mathbb{B}^N)$ the set of all such maps. By Cayley transformation, we can identify \mathbb{B}^n with the Siegel upper half space \mathbb{H}_n . In this talk, we characterized rational proper holomorphic maps from \mathbb{B}^n into \mathbb{B}^{3n-2} .

Keywords: Rational holomorphic maps between balls, proper holomorphic maps

Mathematics Subject Classification: 32A10

References

- [A77] H. Alexander, *Proper holomorphic maps in \mathbb{C}^n* , Indiana Univ. Math. J., 26, 137-146, 1977.
- [AHJY16] J. Andrew, X. Huang, S. Ji and W. Yin, *Mapping \mathbb{B}^n into \mathbb{B}^{3n-3}* , Communications in Analysis and Geometry, Vol. 24, No. 2, 270-300(2016).
- [DA93] J. P. D'Angelo, *Several complex variables and the geometry of real hypersurfaces*, Studies in Advanced Mathematics, CRC press, 1993.
- [F82] J. Faran, *Maps from the two ball to the three ball*, Invent. math., 68(1982), 441-475.
- [FHJZ10] J. Faran, X. Huang, S. Ji and Y. Zhang, *Rational and polynomial maps between balls*, *Pure and Applied Mathematics Quarterly*, vol. 6, num. 3 (2010), p.829-842.
- [GJY18] N. Gul, S. Ji and W. Yin, *Maps from n -ball into $(3n-2)$ -ball are determined by its 3-jets*, preprint, 2018.
- [G19] N. Gul, *Maps from N -Ball into $(3N - 2)$ -Ball are determined by its 3-Jets*, Thesis, University of Houston, 2019.
- [HJ01] X. Huang and S. Ji, *Mapping \mathbb{B}^n into \mathbb{B}^{2n-1}* , Invent. Math, 145(2001), 219-250.
- [HJX06] X. Huang, S. Ji and D. Xu, *A new gap phenomenon for proper holomorphic mappings from \mathbb{B}^n to \mathbb{B}^N* , Math Research Letter, 3 (2006), no. 4, 515-529.
- [HJY14] X. Huang, S. Ji and W. Yin, *The third gap for proper holomorphic maps between balls*, Math. Ann. 358, No. 1-2, 115-142 (2014).
- [JY18b] S. Ji and W. Yin, *Criterion to characterize maps of degree two from n -ball to $(4n-6)$ -ball*, preprint, 2018.
- [W79] S. Webster, *On mappings an $(n + 1)$ -ball in the complex space*, Pac. J. Math. 81(1979), 267-272.

A Note on Projective Transformation on Riemannian and Non-Riemannian Manifolds

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Abstract: The construction of the projective transformation for the Weyl manifold which is admitting a newly constructed connection is shown in a detailed manner. Also, its unique identities concerning local coordinates, are proven. In addition to all, the differences and similarities between Riemannian, Weyl, and newly constructed Weyl manifolds' projective transformations are represented to evaluate the geometric structures of manifolds.

Keywords: Riemannian Manifolds, Non-Riemannian Manifolds, Weyl Manifolds, Projective Transformation, Semi-Symmetric Metric Connection.

Mathematics Subject Classification: 53A35, 53B20, 53C25

References

- [1] Carmo, M.P.D., Riemannian Geometry, Mathematics Theory And Applications, Birkhäuser, Basel, 1992.
- [2] Eisenhart, L.P., Non-Riemannian Geometry, The American Mathematical Society Publishing, New York, 1927.
- [3] Agashe, N.S., Chafle, M.R., A Semi-symmetric Non-metric Connection on a Riemannian Manifold, Indian J. Pure Appl. Math., 23(6), 399–409, 1992.
- [4] Barman, A., On a Special Type of Riemannian Manifold Admitting a Type of Semi-symmetric Metric Connection, Novi Sad J. Math, 42(2), 81–88, 2012.
- [5] Özdemir, F., Türkoğlu, M.D., Sectional Curvatures on Weyl Manifolds with a Special Metric Connection, Turkish Journal of Mathematics, 43, 224–240, 2019.
- [6] Gribacheva, D.K., Conformal Transformations and Their Conformal Invariants on Weyl Spaces, Tensor (N.S.), 64(8), 2003.
- [7] Thomas, T.Y., On Projective and Equiprojective Geometries of Paths, Proc. Nat. Acad. Sci., 11, 198–203, 1925.
- [8] Cartan, E., Lecons sur la Theorie des Spaces a Connexion Projective, Gauthier-Villars, 1937.
- [9] Zhao, P., Some Properties of Projective Semi-symmetric Connections, International Mathematical Forum, 3(7), 341–347, 2008.
- [10] Ercan, S., Yarı-Simetrik Rekürant Metrik Konneksiyonlu Riemann Manifoldu, Master Thesis, Marmara University, 2011.

Stabilization and hybrid synchronization via the adaptive control method of 4-D financial hyper-chaotic systems

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Abstract: In this work, we have considered the stabilization and the hybrid synchronization by the adaptive control method for two non-identical 4-D financial hyper-chaotic systems with unknown parameters, based on Lyapunov's stability theory and the theory of adaptive control. We derive new control results via adaptive control method based on Lyapunov stability theory and adaptive control theory for globally synchronizing two non-identical 4-D financial hyper-chaotic systems with unknown system parameters. The results are validated by numerical simulation using Matlab.

Keywords: hyper-chaotic system, Lyapunov exponent, stabilization, synchronization.

Mathematics Subject Classification: 34C28, 34D08, 37D45, 93C40, 93D05.

References

- [1] L.M. Pecora, T.L. Carroll: Synchronization in chaotic systems. *Physical Review Letters*. 64, 821–825, (1990).
- [2] C. Grebogi, Y. C. Lai: Controlling chaotic dynamical systems. *Systems and control letters*. 31(5), (1997), 307-312.
- [3] H. Adloo, M. Roopaei: Review article on adaptive synchronization of chaotic systems with unknown parameters. *Nonlinear Dynamics*. 65(1), (2011), 141-159.
- [4] H.K. Khalil: *Nonlinear Systems*. New York, Prentice Hall, 2002.
- [5] W. Hahn: *The Stability of Motion* Springer, New York, USA, 1967.
- [6] S. Vaidyanathan, C. Vollos, V. Pham, K. Madhavan: Analysis, adaptive. control and synchronization of a novel 4-D hyperchaotic hyperjerk system and its SPICE implementation. *Archives of Control Sciences*, 25(1), (2003), 135-158.
- [7] Hu, J., Chen, S., & Chen, L. (2005). Adaptive control for anti-synchronization of Chua's chaotic system. *Physics Letters A*, 339(6), 455-460.
- [8] Hannachi, F. (2019). Analysis, dynamics and adaptive control synchronization of a novel chaotic 3-D system. *SN Applied Sciences*, 1(2), 158.

Ab-initio calculation on optoelectronic properties of BeGeP2 for tandem solar cells applications

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Abstract: Recently, II-IV-V2 ternaries have received much concentration due to their valuables properties using in potential applications in optoelectronics, nonlinear optic, and photovoltaic absorber material in solar cells. The main aspects of interest for a material to be used in optoelectronic: emission of light and photovoltaic effect. The optoelectronic properties of BeGeP2 ternary have been theoretically calculated and investigated from the ab-initio approach by using the density functional theory within the FP-LAPW method integrated into the Wien2k code. Our calculations within the TB-mBJ approach indicate optimal bandgap energy and a very high optical absorption coefficient above 105 cm⁻¹, making this compound suitable for solar cell absorbers.

Keywords: DFT, ab-initio calculation, properties modeling, Schrödinger equation.

References

- [1] P.Blaha,K. Schwartz,G. K. H. Madsen,D.Kvasnicka and J. Luitz,Wien2k, an Augmented Plane wave plus orbital's program for calculating crystal proprieties,Vienna University Of Technology,(Austria) (2001).
- [2] W. Kohn and L. J. Sham,Phys. Rev. A1133,140 (1965).
- [3] V. L. Shaposhnikov, A. V. Krivosheeva, and V. E. Borisenko, Phys. Rev. B85 (2012) 205201.
- [4] A. Gani, et al. Chinese Journal of Physics 64 (2020) 174–182.
- [5] F.Tran and P.Blaha,Phys,Rev.le H ,102, 226401(2009).
- [6] S. Fahad et al. Journal of Alloys and Compounds 646 (2015) 211e222
- [7] BENNACER Hamza et al, Optik 159 (2018) 229–244
- [8] BENNACER Hamza et al, Indian Journal of Pure and Applied Physics, Vol.53, March 2015, PP. 181-189.

Characteristic Analysis for a Fractional-order Converter in Continuous Conduction Mode

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In the present work, a comparative study between a fractional-order DC-DC boost converter and an integer one operating in continuous conduction mode is put forward. By using the fractional definition of Riemann-Liouville, the state average model of the fractional order boost converter is presented. Since there are currently no commercial fractional components such as inductor and capacitor, the constructed model of the fractional converter is derived based on Oustaloup Recursive Approximation Method (ORAM). The approximate circuit is simulated and is compared using MATLAB/ Simulink. The results show that the inductor and the capacitor orders are considered as extra parameters, so that the capacitor voltage and the inductor current as well as their ripples are all influenced by the fractional order. Consequently, the feasible simulations confirm that the model derived under fractional calculus has more freedom and elasticity for converter applications.

Keywords: DC-DC boost converter, Fractional calculus, Oustaloup Recursive Approximation.

Mathematics Subject Classification: 00A71, 26A33, 30E10.

References

- [1] A. Oustaloup, F. Levron, B. Mathieu et al., "Frequency-Band Complex Noninteger Differentiator: Characterization and Synthesis," IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications 47(1), pp. 25-39, 2000.
- [2] P. T. Krein, J. Bentsman, R. M. Bass et al., "On the Use of Averaging for the Analysis of Power Electronic Systems," IEEE Transactions on Power Electronics 5(2), pp. 182-190, 1990.
- [3] C. Yang, F. Xie, Y. Chen et al., "Modeling and Analysis of the Fractional-Order Fly back Converter in Continuous Conduction Mode by Caputo Fractional Calculus," Electronics 9(9), pp. 1544, 2020.
- [4] I. Podlubny, Fractional Differential Equations: An Introduction to Fractional Derivatives, Fractional Differential Equations, to Methods of Their Solution and Some of Their Applications: Elsevier, 1998.
- [5] Y. Djourni, K. Khettab, "Improved Maximum Power Point Tracking Based on Dynamic Error Detector via Fractional Order Backstepping Control". The 1st National Conference on Energy Transition in Algeria, March 2020.

A 3-step block hybrid backward differentiation formulae (BHBDF) for the solution of general second order ordinary differential equation

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Abstract: In this paper, the block hybrid Backward Differentiation formulae (BHBDF) for the step number $k=3$ was developed using power series as basis function for the solution of general second order ordinary differential equation. The idea of interpolation and collocation of the power series at some selected grid and off- grid points gave rise to continuous schemes which were further evaluated at those points to produce discrete schemes combined together to form block methods. Numerical problems were solved with the proposed methods and were found to perform effectively.

Keywords: Block, Hybrid, Backward differentiation formulae, Ordinary differential equation.

Mathematics Subject Classification: 65L05, 65L06

References

- [1] Abdelrahim, R. and Omar, Z. and Kuboye, J. O, "New hybrid block method with three off-step points for solving first order ordinary differential equations", Am J Appl Sci. 1(3), pp. 3–6, 2016.
- [2] Adesanya, A. O. and Anake, T. A. and Udoh, M. O, "Improved Continuous Method for Direct Solution of General Second Order Ordinary Differential Equations. Journal of Nigerian Association of Mathematical physics, 13(1), pp. 59-62, 2008.
- [3] Badmus, A. M. and Yahaya, Y. A. and Subair, A. O, "Implicit Stomer Cowell K-Step Hybrid block methods for solution of first order ordinary differential equations (ODEs)", International Journal of Applied Mathematical Research, 3 (4), pp. 464-472, 2014
- [4] J. C. Butcher, " Implicit Runge-kutta processes", Journal of Mathematics Computer. 1(8), pp 50-64, 1964.
- [5] Yahaya, Y. A. and Tijjani, A. A, "Formulation of Corrector Methods from 3-step Hybrid Adams Type Methods for The Solution Of First Order Ordinary Differential Equation" AMSE J, 1(2), pp. 10 – 27, 2015.

Non-Instantaneous Impulsive Fractional Differential Equations: A Rigorous Survey

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Abstract: Non-instantaneous impulsive fractional differential equations can be used to model dynamic processes which have a memory effect (fractional) and are subject to external influences (impulses) which occur over non-negligible periods of time (non-instantaneous). A few different methods have been applied in the literature to rewrite differential equations in the form of equivalent integral equations. However, due to some special considerations arising only in the non-instantaneous impulsive fractional setting, these methods do not always apply in this setting, although many papers in the literature have applied them anyway, without noticing these special considerations and subtleties. We survey the various methods available, checking carefully to see whether each one can truly be applied or not, and demonstrate a correct approach for each of two different possible formulations of non-instantaneous impulsive fractional differential equations.

Keywords: fractional differential equations, impulsive differential equations, non-instantaneous impulses, fractional integrals.

Mathematics Subject Classification: 34A37, 34A08, 26A33.

References

- [1] R. Agarwal, S. Hristova, D. O'Regan, "Non-Instantaneous Impulses in Differential Equations", Springer, 2017.
- [2] R. Agarwal, S. Hristova, D. O'Regan, "Non-instantaneous impulses in Caputo fractional differential equations", *Frac. Calc. Appl. Anal.* 20(3), pp. 595–622, 2017.
- [3] E. Hernandez, D. O'Regan, "On a new class of abstract impulsive differential equations", *Proc. Amer. Math. Soc.* 141(5), pp. 1641–1649, 2013.
- [4] A. Fernandez, S. Ali, A. Zada, "On non-instantaneous impulsive fractional differential equations and their equivalent integral equations", *Math. Meth. Appl. Sci.*, under review, 2021.

The SOR iterative method for new preconditioned linear algebraic systems

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Abstract: Over the years, a good number of preconditioners have been introduced to improve the convergence of iterative methods for solving linear systems. A common feature of most of these preconditioners is that the preconditioning effect is restricted to only certain entries of the coefficient matrix. In an effort to address this drawback, a new preconditioner is proposed; the effect of its application is observed on every entry of the coefficient matrix; in particular, the preconditioner eliminates the last entry on the leftmost column and scales down every other entry. Convergence and comparison theorems of the resulting preconditioned iteration technique are advanced and established. Simulated solutions of sample numerical examples via Maple 2019 Computer Algebra System are presented. It reveals that the proposed method converges faster than the SOR as well as other preconditioned iterations in literature.

Keywords: Successive overrelaxation (SOR), Convergence, Preconditioning, Spectral radius, Iterative Matrix.

Mathematics Subject Classification: 65F10

References

- [1] Evans, D. J., Martins, M. M. and Trigo, M. E. "The AOR iterative method for new preconditioned linear systems", *Journal of Computational and Applied Mathematics*, 132, pp. 461-466, 2001.
- [2] Ndanusa, A. and Adeboye, K. R. "Preconditioned SOR iterative methods for L-matrices", *American Journal of Computational and Applied Mathematics*, 2(6), pp. 300-305, 2012.
- [3] Dehghan, M. and Hajarian, M. "Improving preconditioned SOR-type iterative methods for L-matrices", *International Journal for Numerical Methods in Biomedical Engineering*, 27, pp. 774-784.
- [4] Gunawardena, A. D., Jain, S. K. and Snyder, L. "Modified iterative methods for consistent linear systems", *Linear Algebra and its Applications*, 154-156, pp. 123-143, 1991.
- [5] Kohno, T., Kotakemori, H., Niki, H. and Usui, M. "Improving modified Gauss-Seidel method for Z-matrices", *Linear Algebra and its Applications*, 267: 113-123, 1997.
- [6] Kotakemori, H., Harada, K., Morimoto, M. and Niki, H. "A comparison theorem for the iterative method with the preconditioner $(I + S_{max})$ ", *Journal of Computational and Applied Mathematics*, 145, pp. 373-378, 2002.
- [7] Kotakemori, H., Niki, H. and Okamoto, N. "Accelerated iteration method for Z-matrices", *Journal of Computational and Applied Mathematics*, 75, pp. 87-97, 1996.
- [8] Li, W. and Sun, W. "Modified Gauss-Seidel type methods and Jacobi type methods for Z-matrices", *Linear Algebra and its Applications*, 317, pp. 227-240, 2000.

- [9] Milaszewicz, J. P. "Improving Jacobi and Gauss-Seidel iterations", *Linear Algebra and its Applications*, 93, pp. 161-170, 1987.
- [10] Morimoto, M., Harada, K., Sakakihara, M. and Sawami, H. "The Gauss-Seidel iterative method with the preconditioning matrix $(I + S_{max} + S_m)$ ", *Japan J. Indust. Appl. Math.*, 21, pp. 25-34, 2004.
- [11] Niki, H., Harada, K., Morimoto, M. and Sakakihara, M. "The survey of preconditioners used for accelerating the rate of convergence in the Gauss-Seidel method", *Journal of Computational and Applied Mathematics*, 164-165, pp. 587-600, 2004.
- [12] Mayaki, Z. and Ndanusa, A. "Modified successive overrelaxation (SOR) type methods for M-matrices", *Science world Journal*, 14(4), pp. 1-5, 2019.
- [13] Faruk, A. I. and Ndanusa, A. "Improvements of successive overrelaxation (SOR) methods for L-matrices", *Savanna Journal of Basic and Applied Sciences*, 1(2), pp. 218-223, 2019.
- [14] Abdullahi, I. and Ndanusa, A. "A new modified preconditioned accelerated overrelaxation (AOR) iterative methods for L-matrix linear algebraic systems", *Science World Journal*, 15(2), pp. 45-50, 2020.
- [15] Young, D. M. "Iterative solution of large linear systems", Academic Press, 1971.
- [16] Saad, Y. "Iterative methods for sparse linear systems", 2nd ed., Society for Industrial and Applied Mathematics, 2000.
- [17] Varga, R. S. "Matrix iterative analysis", Prentice-Hall, 1962.

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On the Exact Solutions of Wick Type Stochastic Boussinesq Equation

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Abstract: In this study we consider the Wick type stochastic Boussinesq equation with variable coefficients driven by a white noise is undertaken. The exact solutions of a Wick-type Boussinesq equation have been obtained by using the Hermite transform and the sub-equation method. By taking the Hermite transform of the Wick-type stochastic evolution equation allows to turn the Wick products into an ordinary one. After then, the inverse Hermite transform have been applied to obtain the stochastic solution in the white noise space.

Keywords: *Hermite transform, Wick product, Boussinesq equation*

References

- [1] Kim, Hyunsoo, and Rathinasamy Sakthivel. "Exact solutions of Wick-type stochastic equations with variable coefficients." *Reports on Mathematical Physics* 67.3, 415-429, (2011):
- [2] Ginovart, Frédéric. "Some exact Wick type stochastic generalized Boussinesq equation solutions." *Journal of computational and applied mathematics* 220.1-2,559-565, (2008).
- [3] Ray, S. Saha, and S. Singh. "New exact solutions for the Wick-type stochastic modified Boussinesq equation for describing wave propagation in nonlinear dispersive systems." *Chinese journal of physics* 55.4,1653-1662, (2017).
- [4] Li, Yin, Yulin Zhao, and Zheng-an Yao. "Stochastic exact solutions of the Wick-type stochastic NLS equation." *Applied Mathematics and Computation* 249, 209-221, (2014).
- [5] Wang, Ben-Hai, and Yue-Yue Wang. "Fractional white noise functional soliton solutions of a wick-type stochastic fractional NLSE." *Applied Mathematics Letters* 110, 106583, (2020).

Non-integer derivatives in selected areas of physics

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Abstract: The talk will be about the non-integer (fractional) derivative, its mathematical formulation by Abel in 1823, and present day applications in modeling power-law behavior. These applications are in acoustics of complex media like tissue and sediments as well as in rheology, turbulence, and dielectrics. It will build on my book “Waves with Power-Law Attenuation”, Springer, 2019.

Keywords: fractional calculus, physics, acoustics

Denoising of degraded facial image sequences based PCA

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Abstract: One of the most stringent issues in image sequence processing lies in the high dimensionality of the image sequences. This issue is further made worse by the high correlation between the frames, on the one hand, and by the difficulty of noisy motion estimation, on the other hand. As a result, the de-noising algorithms are complex and require intense computational workload. To address this issue, we propose a novel algorithm based on the Principal Components Analysis (PCA) for dimension reduction, followed by a spatiotemporal filter for de-noising the dimensionally-reduced frames.

The de-noising process is performed in the image reduced space by applying a temporal filter for motion compensation combined with a weighted average filter, and an anisotropic diffusion filter as a spatial filter to remove the noise. The performance of the proposed method is proven using the Cohen-Canade facial expressions (CKFE) database tested against different noise levels with PSNR and UIQ as metrics for objective evaluation. The experiments show that the proposed method gives better results than state-of-the-art classical de-noising competitors.

Keywords: Image denoising, Spatiotemporal filter, anisotropic diffusion filter ,PCA.

Mathematics Subject Classification: 68U05, 68U10, 68T05, 68T10.

References

- [1] Rajni, R., and Anutam, A., "Image Denoising Techniques - An Overview", *Int. J. Computer Applications.*, 86(16), (2013).
- [2] Turk, M., and Pentland, A., "Eigenfaces for recognition", *Int. J. Cogn. Neuroscence.*, 3(1), pp. 71-86, (1991).
- [3] Meguro Mitsuhiro, Taguchi Akira, Hamada Nozomu, "Data-dependent weighted average filtering for image sequence enhancement", In: *Proc. IEEE-EURASIP Work., Nonlinear Signal Image Process, Antalya, TURKEY*, (1999).
- [4] Boyce, J. "Noise reduction of image sequences using adaptive motion compensated frame averaging", *IEEE International Conference on Acoust. Speech, Signal Process*, (1992).
- [5] Kanade Takeo, Cohn Jeffrey F, Tian Yingli, "Comprehensive database for facial expression analysis", *Proceeding Fourth IEEE International Conference on. Autom. Face Gesture Recognition, Grenoble*, (2000).
- [6] Wang, Z. and Bovik, A., "A universal image quality index", *IEEE Signal. Process. Lett.* 9 (3), pp. 81-84, (2002).

On Lah-Ribarič inequality for $(h, g; m)$ -convex functions

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Abstract: We study a new class of $(h, g; m)$ -convex functions that unifies a certain range of convexity and allows the generalizations of know results. For this class we present Lah-Ribarič type inequalities from which we obtain inequalities of Hermite-Hadamard, Fejér, Giaccardi, Popoviciu and Petrović.

Keywords: Convex function, Lah-Ribarič inequality, Giaccardi inequality.

Mathematics Subject Classification: 26A51, 26D15

References

- [1] P. Lah and M. Ribarič, “Converse of Jensen’s inequality for convex functions”, Publ. Elektroteh. Fak. Univ. Beogr., Ser. Mat. Fiz. 412–460, 201–205, 1973.
- [2] J. E. Pečarić, F. Proschan and Y. C. Tong, “Convex Functions, Partial Orderings and Statistical Applications”, Academic Press, Inc., San Diego, 1992.
- [3] A. U. Rehman, G. Farid and V. N. Mishra, “Generalized convex function and associated Petrović’s inequality”, Int. J. Anal. Appl. 17(1), 122–131, 2019.
- [4] G. H. Toader, “Some generalizations of the convexity”, Proc. Colloq. Approx. Optim, Cluj-Napoca (Romania), 329–338, 1984.
- [5] S. Varošanec, “On h -convexity”, J. Math. Anal. Appl. 326, 303–311, 2007.

Modelling and Analysis of Proteins Aggregation in Neurodegenerative Diseases

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Abstract: Protein aggregation leading to the formation of amyloid fibrils is involved in several neurodegenerative diseases such as Alzheimer’s disease, Parkinson’s disease, and prion diseases. However, these mechanisms of the fibrils aggregation are still misunderstood by the specialists, and, for the most part, hypothetical only.

To clarify how these fibrils are able to incorporate additional units, prion fibril aggregation and disaggregation kinetics were experimentally studied.

An initial model, adapted from the Becker-Döring system is considered, and compared to the experimental data. Aiming to reproduce the observed in vitro behaviour.

Our second model involves an additional compartment of fibrils unable to incorporate more prion units. This model leads to kinetic coefficients which are biologically plausible and correctly simulates the first experimental steps for prion aggregation.

Keywords: Mathematical Modelling ; Simulation ; Data Fitting ; Numerical Analysis ; Parameter Estimation ; Prion Diseases ; Prion aggregation

Mathematics Subject Classification:

References

- [1] H. W. HAFFAF, S. Prigent, "MODELLING AND ANALYSIS OF PROTEIN AGGREGATION - COMPETING PATHWAYS IN PRION (PRP) POLYMERISATION"; ESAIM: PROCEEDINGS AND SURVEYS, Volume 45, pp. 189-198, (2014).
- [2] H. Wafaâ HAFFAF, "Analyse de l’agrégation des protéines dans les maladies neurodégénératives amyloïdes : application aux maladies à prion", Thèse Université Pierre et Marie Curie – Paris VI – tel-01143554 (2014).

The Flux, Energy and Heating Calculations for Reactor Core Designed with Monte Carlo Modelling

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Abstract: In this study, a boiling water reactor (BWR) core modeling was performed using the Monte Carlo mathematical modelling (MCNPX-2.7.0 code and ENDF/B-VII library). In the modeling, an 8x8 square lattice was designed for the reactor core. In the designed square lattice; 0.65-0.7-0.75% NpO_2 and NpF_4 were used as fuel rod, Zr-2 and SiC for fuel cladding, water as coolant and B_4C as control rod. In this study, neutron flux, fission energy and heating neutronic values were calculated for the selected fuel and clad in the designed BWR.

Keywords: MCNPX-2.7.0, flux, heating

Investigation of Some Neutronic Calculations for Vanadium Carbide Cladding Material in a Boiling Water Reactor Modelling

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Abstract: In this study, a boiling water reactor was modelled and Vanadium Carbide (VC) was used as cladding material in the design. In the designed reactor core, small squares lattices were placed in square lattices. Neptunium mixed fuel rod, VC cladding material, water cooler were used in each small square lattice. NpO_2 and NpF_4 fuels were used as the Neptunium mixture in the fuel rods. In this study, the criticality (k_{eff}), neutron flux, fission energy, heating and deposition values were investigated for the VC cladding material. In the research, the reactor design was made in three dimensions using MCNPX-2.7.0 Monte Carlo code and ENDF/B-VII library.

Keywords: k_{eff} , MCNPX-2.7.0, cladding material.

Mathematics Subject Classification:

Upshot of Marangoni nanofluid thin film flow under the influence of an inclined magnetic field

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Abstract: A numerical simulation is conducted for the applied inclined magnetic field on a two dimensional electrically conducting unsteady Marangoni nanofluid thin-film flow past a horizontal extending sheet. The composite of nanofluid comprises Copper nanoparticles and kerosene oil base fluid. The envisioned flow is also affected by the non-uniform source/sink and viscous dissipation. The success of the Shooting technique amalgamated with the Fehlberg Runge-Kutta computational scheme is secured for the solution of a highly nonlinear system of equations. The significant impacts of the prominent arising parameters versus involved fields are examined through graphs. The outcomes show that the fluid velocity of the thin film nanofluid flow is curbed for a strong magnetic field nevertheless an opposing trend is witnessed in case of the liquid temperature. Furthermore, the rate of heat flux is escalated versus estimates of nanoparticle volume fraction and time-dependent parameter.

Keywords: Marangoni effect, Nanofluid thin film flow; Numerical solution; Non-uniform source/sink.

Mathematics Subject Classification: 76D05, 76Dxx

References

- [1] Sheikholeslami, M., Ganji, D. D., Gorji-Bandpy, M., & Soleimani, S. (2014). Magnetic field effect on nanofluid flow and heat transfer using KKL model. *Journal of the Taiwan Institute of Chemical Engineers*, 45(3), 795-807.
- [2] Khan, M., Malik, M. Y., Salahuddin, T., & Hussian, A. (2018). Heat and mass transfer of Williamson nanofluid flow yield by an inclined Lorentz force over a nonlinear stretching sheet. *Results in Physics*, 8, 862-868.
- [3] Sulochana, C., Samrat, S. P., & Sandeep, N. (2018). Magnetohydrodynamic radiative liquid thin film flow of kerosene based nanofluid with the aligned magnetic field. *Alexandria Engineering Journal*, 57(4), 3009-3017.
- [4] Mehmood, K., Hussain, S., & Sagheer, M. (2017). Numerical simulation of MHD mixed convection in alumina-water nanofluid filled square porous cavity using KKL model: Effects of non-linear thermal radiation and inclined magnetic field. *Journal of Molecular Liquids*, 238, 485-498.
- [5] Alshomrani, A. S., & Ramzan, M. (2019). Upshot of magnetic dipole on the flow of nanofluid along a stretched cylinder with gyrotactic microorganism in a stratified medium. *Physica Scripta*, 95(2), 025702.
- [6] Ramzan, M., & Shaheen, N. (2019). Thermally stratified Darcy-Forchheimer nanofluid flow comprising carbon nanotubes with effects of Cattaneo-Christov heat flux and homogeneous-heterogeneous reactions. *Physica Scripta*, 95(1), 015701.
- [7] Rasool, G., Shafiq, A., Khaliq, C. M., & Zhang, T. (2019). Magnetohydrodynamic Darcy-Forchheimer nanofluid flow over a nonlinear stretching sheet. *Physica Scripta*, 94(10), 105221.
- [8] Ramzan, M., Mohammad, M., & Howari, F. (2019). Magnetized suspended carbon nanotubes based nanofluid flow with bio-convection and entropy generation past a vertical cone. *Scientific reports*, 9(1), 1-15.

- [9] Farooq, U., Lu, D., Munir, S., Ramzan, M., Suleman, M., & Hussain, S. (2019). MHD flow of Maxwell fluid with nanomaterials due to an exponentially stretching surface. *Scientific reports*, 9(1), 1-11.
- [10] Abbasi, F. M., Shanakhat, I., & Shehzad, S. A. (2019). Analysis of entropy generation in peristaltic nanofluid flow with Ohmic heating and Hall current. *Physica Scripta*, 94(2), 025001.
- [11] Emslie, A.C.; Bonner, F.D.; Peck, L.G. Flow of a viscous liquid on a rotating disk. *Journal of Applied Physics*, 1958, 29, 858–862.
- [12] Giri, S. S., Das, K., & Kundu, P. K. (2019). Inclined magnetic field effects on unsteady nanofluid flow and heat transfer in a finite thin film with non-uniform heat source/sink. *Multidiscipline Modeling in Materials and Structures*.15 (1), 265-282.
- [13] Khan, N. S., Gul, T., Kumam, P., Shah, Z., Islam, S., Khan, W., ... & Sohail, A. (2019). Influence of inclined magnetic field on Carreau nanoliquid thin film flow and heat transfer with graphene nanoparticles. *Energies*, 12(8), 1459.
- [14] Thiele, U., Goyeau, B., & Velarde, M. G. (2009). Stability analysis of thin film flow along a heated porous wall. *Physics of Fluids*, 21(1), 014103.
- [15] Hua, T., Yu, M., Geng, H., Xu, W., Lu, Y., Qin, C., ... & Wu, P. (2018). Vortex channel flow effect in grain boundary of YBCO thin film under inclined magnetic field. *Physica C: Superconductivity and its Applications*, 554, 15-18.
- [16] Saeed, A., Shah, Z., Islam, S., Jawad, M., Ullah, A., Gul, T., & Kumam, P. (2019). Three-dimensional casson nanofluid thin film flow over an inclined rotating disk with the impact of heat generation/consumption and thermal radiation. *Coatings*, 9(4), 248.
- [17] Shah, Z., Ullah, A., Bonyah, E., Ayaz, M., Islam, S., & Khan, I. (2019). Hall effect on Titania nanofluids thin film flow and radiative thermal behavior with different base fluids on an inclined rotating surface. *AIP Advances*, 9(5), 055113.
- [18] Ramzan, M., Riasat, S., Kadry, S., Long, C., Nam, Y., & Lu, D. (2020). Numerical simulation of 3D condensation nanofluid film flow with carbon nanotubes on an inclined rotating disk. *Applied Sciences*, 10(1), 168.
- [19] Kumar, K. A., Sandeep, N., Sugunamma, V., & Animasaun, I. L. (2020). Effect of irregular heat source/sink on the radiative thin film flow of MHD hybrid ferrofluid. *Journal of Thermal Analysis and Calorimetry*, 139(3), 2145-2153.
- [20] Mishra, K. K., Instan, A. A., Kumari, S., Scott, J. F., & Katiyar, R. S. (2020). Lead palladium titanate: A room temperature nanoscale multiferroic thin film. *Scientific Reports*, 10(1), 1-11.
- [21] Lu, D., Ramzan, M., Mohammad, M., Howari, F., & Chung, J. D. (2019). A thin film flow of nanofluid comprising carbon nanotubes influenced by Cattaneo-Christov heat flux and entropy generation. *Coatings*, 9(5), 296.
- [22] Al-Mdallal, Q. M., Indumathi, N., Ganga, B., & Hakeem, A. A. (2020). Marangoni radiative effects of hybrid-nanofluids flow past a permeable surface with inclined magnetic field. *Case Studies in Thermal Engineering*, 17, 100571.
- [23] Hakeem, A. K., Indumathi, N., Ganga, B., & Nayak, M. K. (2020). Comparison of disparate solid volume fraction ratio of hybrid nanofluids flow over a permeable flat surface with aligned magnetic field and Marangoni convection. *Scientia Iranica*.
- [24] Akhtaruzzaman, R., Ahmed, A., Islam, M. Q., Saha, S., & Hasan, M. N. (2019, July). Numerical modeling of Marangoni convection in the presence of external magnetic field. In *AIP Conference Proceedings* (Vol. 2121, No. 1, p. 030026). AIP Publishing LLC.

- [25] Naz, R., Mabood, F., & Hayat, T. (2018). Inclined magnetic field effects on Marangoni flow of Carreau liquid. *Thermal Science*, (00), 211-211.
- [26] Sultan, E., Boudaoud, A., & Amar, M. B. (2005). Evaporation of a thin film: diffusion of the vapour and Marangoni instabilities. *Journal of Fluid Mechanics*, 543, 183-202.
- [27] Shah, Z., Dawar, A., Kumam, P., Khan, W., & Islam, S. (2019). Impact of Nonlinear Thermal Radiation on MHD Nanofluid Thin Film Flow over a Horizontally Rotating Disk. *Applied Sciences*, 9(8), 1533.
- [28] Dávalos-Orozco, L. A. (2016). Thermal Marangoni instability of a thin film flowing down a thick wall deformed in the backside. *Physics of fluids*, 28(5), 054103.
- [29] Sheikholeslami, M., & Ganji, D. D. (2017). Influence of magnetic field on $\text{CuO-H}_2\text{O}$ nanofluid flow considering Marangoni boundary layer. *International journal of Hydrogen energy*, 42(5), 2748-2755.
- [30] Mahanthesh, B., Gireesha, B. J., Prasannakumara, B. C., & Kumar, P. S. (2017). Magneto-Thermo-Marangoni convective flow of Cu-H₂O nanoliquid past an infinite disk with particle shape and exponential space based heat source effects. *Results in physics*, 7, 2990-2996.
- [31] Hayat, T., Shaheen, U., Shafiq, A., Alsaedi, A., & Asghar, S. (2015). Marangoni mixed convection flow with Joule heating and nonlinear radiation. *AIP advances*, 5(7), 077140.

Some convergence results for generalized α -nonexpansive mapping in Banach Spaces

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Abstract: In this paper, we introduce a new iterative scheme to approximate fixed point of generalized α -nonexpansive mappings. We also prove weak and strong convergence theorems for generalized α -nonexpansive mappings by using new iteration process. At the end, we give an example for generalized α -nonexpansive mappings and compare the convergence behavior of new iterative process with other iterative processes.

Keywords: Generalized α -nonexpansive mappings, iteration process, fixed point, convergence

Mathematics Subject Classification: 47H09; 47H10

References

- [1] M. Abbas, T. Nazir, A new faster iteration process applied to constrained minimization and feasibility problems. *Mat. Vesnik* 66(2), 223-234 (2014).
- [2] R.P. Agarwal, D. OffRegan, D.R. Sahu, Iterative construction of fixed points of nearly asymptotically nonexpansive mappings. *J. Nonlinear Convex Anal.* 8 (1), 61-79 (2007).
- [3] K.Aoyama, F Kohsaka, Fixed point theorem for α -nonexpansive mappings in Banach spaces, *Nonlinear Anal.* 74 (13)(2011), 4378-4391.
- [4] J. A. Clarkson, Uniformly convex spaces, *Trans. Amer. Math. Soc.* 40 (1936) 396-414.
- [5] N. Hussain, K. Ullah, M. Arshad, Fixed point approximation of Suzuki generalized nonexpansive mappings via new faster iteration process. *J. Nonlinear Convex Anal.* (2018), 19, 1383-1393.
- [6] F. Gursoy, V. Karakaya, A Picard-S hybrid type iteration method for solving a differential equation with retarded argument, eprint arXiv 1403.2546 (2014) 1-16.
- [7] I. Ishikawa, Fixed point by a new iteration method, *Proc. Am. Math. Soc.* 44 (1974), 147-150.
- [8] W.R. Mann, Mean value methods in iteration, *Proc. Am. Math. Soc.* 4 (1953), 506-510.
- [9] M.A. Noor, New approximation schemes for general variational inequalities, *Journal of Mathematical Analysis and Applications*, 251 (2000), 217-229.
- [10] Z. Opial, Weak convergence of successive approximations for nonexpansive mappings, *Bull. Ame. Math. Soc.* 73 (1967), 591-597.
- [11] R. Pant, R. Shukla, Approximating fixed points of generalized α -nonexpansive mappings in Banach spaces. *Numer. Funct. Anal. Optim.* 38(2) (2017), 248-266.
- [12] J. Schu, Weak and strong convergence of fixed points of asymptotically nonexpansive mappings, *Bull. Austral. Math. Soc.*, 43 (1991), 153-159.

- [13] H.F. Senter, W.G. Dotson Jr., Approximating fixed points of nonexpansive mappings, *Proc. Am. Math. Soc.* 44 (1974), 375-380
- [14] T. Suzuki, Fixed point theorems and convergence theorems for some generalized nonexpansive mappings, *Journal of Mathematical Analysis and Applications*, 340(2) (2008), 1088-1095.
- [15] B.S. Thakur, D. Thakur, M. Postolache, A new iteration scheme for numerical reckoning fixed points of Suzuki's generalized nonexpansive mappings, *Applied Mathematics and Computation*, 147-155, 275 (2016).
- [16] K. Ullah and M. Arshad, Numerical reckoning fixed points for Suzuki's generalized nonexpansive mappings via new iteration process, *Filomat* 32(1) (2018) 187–196.
- [17] K. Ullah, F. Ayaz and J. Ahmad , Some convergence results of M iterative process in Banach spaces, *Asian-European Journal of Mathematics* (2021) 2150017,12 pages, DOI: 10.1142/S1793557121500170.

Qualitative analyses of systems of integro –differential equations with time-delay retardation

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Abstract: A mathematical model consisting of a system of nonlinear IDDEs with constant time retardation was taken into account. We give three new results such that they are related to uniformly asymptotically stability of zero solution as well as square integrability of norm of solutions and boundedness of solutions. The approach used in the proofs depends upon the construction a more new suitable LKF and use it in the proofs. Two examples are provided to illustrate the applications of the results. Compared to qualitative results in the literature related to the IDDEs, our results do new contributions to theory of integro-differential equations and the related results.

Keywords: System of nonlinear integro-differential equations, constant time delay, stability, integrability, boundedness, Lyapunov – Krasovskiï functional

Mathematics Subject Classification: 34K05, 34K12 34K20

References

- [1] T. A. Burton, Stability and periodic solutions of ordinary and functional differential equations. Corrected version of the 1985 original. Dover Publications, Inc., Mineola, NY, 2005.
- [2] T. A. Burton, Volterra integral and differential equations. Second edition. Mathematics in Science and Engineering, 202. Elsevier B. V., Amsterdam, 2005.
- [3] T. Furumochi, S. Matsuoka, Stability and boundedness in Volterra integro-differential equations. Mem. Fac. Sci. Eng. Shimane Univ. Ser. B Math. Sci. 32 (1999), 25–40.
- [4] W. E. Mahfoud, Boundedness properties in Volterra integro-differential systems. Proc. Amer. Math. Soc. 100 (1987), no. 1, 37–45.
- [5] Y. Raffoul, Boundedness in nonlinear functional differential equations with applications to Volterra integro-differential equations. J. Integral Equations Appl. 16 (2004), no. 4, 375–388.
- [6] Y. Tian, Z. Wang, A new multiple integral inequality and its application to stability analysis of time-delay systems. Appl. Math. Lett. 105 (2020), 106325, 8 pp.
- [7] C. Tunç, Properties of solutions to Volterra integro-differential equations with delay. Appl. Math. Inf. Sci. 10 (2016), no. 5, 1775–1780.
- [8] C. Tunç, Qualitative properties in nonlinear Volterra integro-differential equations with delay. Journal of Taibah University for Science. 11 (2017), no. 2, 309–314.
- [9] C. Tunç, Asymptotic stability and boundedness criteria for nonlinear retarded Volterra integro-differential equations. J. King Saud Univ. Sci. 30(2016), no.4, 3531–3536.
- [10] C. Tunç, Stability and boundedness in Volterra-integro differential equations with delays. Dynam. Systems Appl. 26 (2017), no. 1, 121–130.
- [11] C. Tunç, A remark on the qualitative conditions of nonlinear IDEs. Int. J. Math. Comput. Sci. 15 (2020), no. 3, 905–922.

- [12] O. Tunç, On the qualitative analyses of integro-differential equations with constant time lag. *Appl. Math. Inf. Sci.* 14 (2020), no. 1, 57–63.
- [13] C. Tunç, O. Tunç, New results on the stability, integrability and boundedness in Volterra integro-differential equations. *Bull. Comput. Appl. Math.* 6 (2018), no. 1, 41–58.
- [14] C. Tunç, O. Tunç, New results on behaviors of functional Volterra integro-differential equations with multiple time-lags. *Jordan J. Math. Stat.* 11 (2018), no. 2, 107–124.
- [15] C. Tunç, O. Tunç, New qualitative criteria for solutions of Volterra integro-differential equations. *Arab Journal of Basic and Applied Sciences.* 25 (2018), no.3, 158–165.
- [16] C. Tunç, O. Tunç, A note on the qualitative analysis of Volterra integro-differential equations. *Journal of Taibah University for Science.* 13 (2019), no.1, 490–496.

Generalized solution of a nonlinear optimal control of the heel angle of a rocket

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Abstract: We use some recent developments in Dynamics Programming Method [2,3] to obtain a rigorous solution of the optimal control problem formulated in [5] as an unsolved problem, and studied partially in [1] We use a certain refinement of Cauchy's Method of characteristics for stratified Hamilton-Jacobi equations to describe a large set of admissible trajectories and to identify a domain on which the value function exists and is generated by a certain admissible control and, its optimality is justified by the use of one of the well-known verification theorems as an argument for sufficient optimality conditions.

Keywords: Optimal control, differential inclusion, Pontryagin's maximum principle, dynamic programming, Hamiltonian flow.

Mathematics Subject Classification: 49J15, 49L20, 34A60.

References

- [1] M. Aliane, N. Moussouni, M. Bentobache, "Nonlinear optimal control of the heel angle of a rocket", 6th International Conference on Control, Decision and Information Technologies (CoDIT'19), Paris, France / April 23-26, 2019.
- [2] St. Miricã, "Constructive Dynamic programming in optimal control Autonomous Problems", Editura Academiei Române, Bucharest, 2004.
- [3] St. Miricã, "User's Guide on Dynamic Programming for autonomous differential games and optimal control problems", Rev. Roumaine Math. Pures Appl. 49, No. 5-6, 501-529 (2004).
- [4] St. Miricã and T. Bouremani, "On the correct Solution of a trivial Optimal Control in Mathematical Economics", Math. Rep. 9(59), no. 1, 77-86 (2007).
- [5] E. Trélat, Contrôle optimal: théorie et applications. Mathématiques concrètes, Vuibert, Paris (2005)

Some thoughts on differential operators for certain subclass of analytic functions

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Abstract: This article discusses on differential operators for certain class of analytic functions. Many differential operators have been introduced in a very creative way. The most basic ones can be traced back in 1975 by Ruscheweyh[1] and later by Salagean [2]in 1985. Then in 2000 onwards, we can see the generalisation of Salagean by Al-Oboudi[3] and generalisation of Salagean and Ruscheweyh by Darus and Al-Shaqsi [4]. Afterwards, many similar ones can be seen in various platforms of publications. It become more robust when q -analogue was imposed on the differential operators (see eg. [5]). Here, some earlier works will be traced back and some recent results will be presented.

Keywords: Analytic functions, differential operator, q -analogue

Mathematics Subject Classification: 30C45

References

- [1] S. Ruscheweyh, *New criteria for univalent functions*, Proc. Amer. Math. Soc.,49 (1975), 109–115.
- [2] G.S. Salagean, *Subclasses of univalent functions*, Lect. Notes Math., 1013(1983), 362–272.
- [3] F. Al-Oboudi, *On univalent functions defined by a generalised Salagean operator*, Inter. Jour. Math. Math. Sci., 2004(27) (2004), 1429-1436.
- [4] M.Darus and K. Al-Shaqsi, *Differential sandwich theorems with generalised derivative operator*, Inter. Jour. Math. Comp. Phys. Elec. Comp. Engeer., 2(2) (2008), 149-152.
- [5] H. Aldweby and M. Darus, *Some subordination results on q -analogue of Ruscheweyh differential operator*, Abstr. Appl. Anal. 2014 (2014),958563.

Riemann conjecture for the Riemann zeta function

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Abstract: In this report we present two approaches for the detailed proof for the Riemann conjecture for the Riemann zeta function.

Keywords: Riemann zeta function, Riemann conjecture, critical line, nontrivial zeros.

Mathematics Subject Classification: 11M06, 11M26

References

- [1] Xiao-Jun Yang," Analysis of the Riemann zeta function", Reprint, 2021.
- [2] Xiao-Jun Yang," New infinite product formulas for the Riemann zeta-function applied to prove the Riemann conjecture", Reprint, 2021.

Energy uniform decay rates for the semi-linear wave equation with nonlinear localized damping and source terms of critical variable exponent

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Abstract: The goal of this paper is to consider a model of a semilinear wave equation with critical source term $f(u(x, t))$ and a localized exterior nonlinear monotone damping $g(u_t(x, t))$. The proposed approach allows considering, in a unified way, much more general classes of hyperbolic problems than addressed before in the literature. This generalization refers to an analysis aspect of the problem. The method leads to optimal decay rates for solutions of semilinear hyperbolic equations driven by a source of critical variable exponent and subjected to nonlinear damping localized in a small region adjacent to a portion of the boundary. The no-growth condition is imposed on the damping $g(u_t(x, t))$, and this is the distinct feature of the model include. Hence we consider a more general case of g , and the sole conditions assumed are, g has not a polynomial behavior near the origin, monotonicity, continuity, and $g(0) = 0$. In particular, no differentiability and no growth conditions are imposed on the damping both at the origin and at the infinity. The asymptotic decay rates for the energy function obtained by using Martinez's method [11], however, this paper can be regarded as a generalization of Martinez's results.

Keywords: Wave equation localized damping, source, decay rates, critical exponents, variable nonlinearity.

Mathematics Subject Classification: 35L05, 35B40, 35A01

References

- [1] F.D. Araruna and A.B. Maciel. Existence and boundary stabilization of the semilinear wave equation. *Nonlinear Analysis: Theory, Methods and Applications*, 67, 2007.
- [2] Marcelo M. Cavalcanti, Valéria N. Domingos Cavalcanti, and Irena Lasiecka. Well-posedness and optimal decay rates for the wave equation with nonlinear boundary damping source interaction. *Journal of Differential Equations*, 236, 2007.
- [3] L. Diening, P. Hästö, P. Harjulehto, and M. R. Růžicka. *Lebesgue and Sobolev Spaces with Variable Exponents*, volume 2017. in: Springer Lecture Notes in Mathematics, Springer-Verlag, Berlin, 2011.
- [4] L. Diening and M. Růžicka. Calderon Zygmund operators on generalized Lebesgue spaces $L^{p(x)}$ and problems related to fluid dynamics. Preprint Mathematische Fakultät, Albert-Ludwigs-Universität Freiburg, 120:197–220, (21/2002, 04.07.2002).
- [5] X. L. Fan, J. Shen, and D. Zhao. Sobolev embedding theorems for spaces $W^{(k,p(x))}(\Omega)$. *J. Math. Anal. Appl.*, 262:749–760, 2001.
- [6] Y. Fu. The existence of solutions for elliptic systems with nonuniform growth. *Studia Math.*, 151:227–246, 2002.
- [7] Ha Tae Gab. Blow-up for wave equation with weak boundary damping and source terms. *Applied Mathematics Letters*, 49, 2015.
- [8] H.A. Levine and L.E. Payne. Non existence theorems for the heat equation with nonlinear boundary conditions and for the porousmedium equation back-ward in time. *J. Differ. Equ.*, 16, 1974.

- [9] Tae Gab Ha. Blow-up for semilinear wave equation with boundary damping and source terms. *Journal of Mathematical Analysis and Applications*, 390, 2012.
- [10] O. Kovàcik and J. Rákosnik. On spaces $l^{(p(x))}(\omega)$ and $w^{(1,p(x))}(\omega)$. *Czechoslovak Math. J.*, 41:592–618, 1991.
- [11] Patrick Martinez. A new method to obtain decay rate estimates for dissipative systems. *ESAIM Control Optimisation and Calculus of Variations*, 4, 1999.
- [12] M.M.Cavalcanti, V.N.Domingos Cavalcanti, and J.A.Soriano. Global solvability and asymptotic stability for the wave equation with nonlinear feedback and source term on the boundary. *Adv. Math. Sci. Appl*, 16, 2006.
- [13] M.M.Cavalcanti, V.N.Domingos Cavalcanti, and P.Martinez. Existence and decay rate estimates for the wave equation with non linear boundary damping and source term. *J.Differ.Equ*, 203, 2004.
- [14] M.M.Cavalcanti, V.N.Domingos Cavalcanti, and P.Martinez. General decay rate estimates for viscoelastic dissipative systems. *Nonlinear Anal*, 68, 2008.
- [15] M.M.Cavalcanti and V.N.Domingos Cavalcanti. Existence and asymptotic stability for evolution problems on manifolds with damping and source terms. *J. Math. Anal. Appl*, 291, 2004.
- [16] Jong Yeoul Park, Tae Gab Ha, and Yong Han Kang. Energy decay rates for solutions of the wave equation with boundary damping and source term. *Zeitschrift für Angewandte Mathematik und Physik (ZAMP)*, 61, 2010.
- [17] Jong Yeoul Park, Tae Gab Ha, and Y. H. Kang. Energy decay rates for solutions of the wave equation with boundary damping and source term. *J. Math. Phys*, 61, 2010.
- [18] Jong Yeoul Park and Tae Gab Ha. Energy decay for nondissipative distributed systems with boundary damping and source term. *Nonlinear Analysis: Theory, Methods and Applications*, 70, 2009.
- [19] Jong Yeoul Park and Tae Gab Ha. Well-posedness and uniform decay rates for the klein–gordon equation with damping term and acoustic boundary conditions. *J. Math. Phys*, 50, 2009.
- [20] Abita Rahmoune. Existence and asymptotic stability for the semilinear wave equation with variable-exponent nonlinearities. *Journal of Mathematical Physics*, 60, 12 2019.
- [21] T.G.Ha. On viscoelastic wave equation with non linear boundary damping and source term. *Commun. PureAppl. Anal*, 9, 2010.
- [22] Enzo Vitillaro. Global nonexistence theorems for a class of evolution equations with dissipation. *Archive for Rational Mechanics and Analysis*, 149, 1999.
- [23] Enzo Vitillaro. Global existence for the wave equation with nonlinear boundary damping and source terms. *Journal of Differential Equations*, 186, 2002.
- [24] V.Komornik and E.Zuazua. On viscoelastic wave equation with non linear boundary damping and source term. *Commun. PureAppl. Anal*, 69, 1990.
- [25] Park Jong Yeoul and Ha Tae Gab. Existence and asymptotic stability for the semilinear wave equation with boundary damping and source term. *Journal of Mathematical Physics*, 49, 2008.
- [26] E. Zuazua. Exponential decay for the semilinear wave equation with locally distributed damping. *J. Math. Phys*, 15, 1990.

Navigation in Multi-Robot Systems Based on the Behavioural Fuzzy Controller

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Abstract: Controlling a highly dynamic multi-robot system (DMRS) is a challenge scientific and technological in constant expansion. This article focuses on navigation in DMRS. In this context, the navigation methods allow the robot to move from a start point to target point while avoiding obstacles. These methods must therefore take into account the following problems: Fixed Obstacle Avoidance Problem (FOAP); Mobile Obstacle Avoidance Problem (MOAP) and Robots Collision Avoidance Problem (RCAP). In the FOA Problem, the environment is considered static and contains a single robot. In the MOA Problem, the environment contains dynamic obstacles and several robots. In this case, each robot move in the environment without communication, and the robots are considered obstacles to each other. Finally, in the RCA Problem, robots could communicate with each other to find an arrangement to avoid collisions between them.

The proposed navigation method in DMRS allows the robot to move from a starting point to a target point while avoiding obstacles. Local interactions between robots are used to produce a form of advanced collective intelligence. Using the concept of fuzzy logic, reactive control is modeled to allow mobile robots to move in an unknown environment. The model must also guarantee the resolution of conflicts to determine the priority-of-way when two or more robots are on the same crossing point. The simulation results obtained on the pioneer P3-DX robot clearly show the efficiency of the proposed architecture.

Keywords: Mobile Robots, Navigation, Fuzzy logic, behavioral controller architecture, collective intelligence, Multi-robot systems.

Mathematics Subject Classification:

References

- [1] Buffet, O., Dutech, A., Charpillet, F: Adaptive combination of behaviors in an agent. In: 9th European Conference on Artificial Intelligence, pp. 48–52. (2010)
- [2] Liu, F., Liang, S., Xian, X: Multi-robot task allocation based on utility and distributed computing and centralized determination. In: The 27th Chinese Control and Decision Conference (2015 CCDC), pp. 3259–3264. IEEE, (2015)
- [3] Hoang, T. T., Hiep, D. T., Duong, P. M., Van, N. T. T., Duong, B. G., Vinh, T.Q.: Proposal of algorithms for navigation and obstacles avoidance of autonomous mobile robot. In: IEEE 8th Conference on Industrial Electronics and Applications (ICIEA), pp. 1308–1313, IEEE, (2013)
- [4] Apandi, N. I. A., Martin, A.: The Integration of Fuzzy Logic System for Obstacle Avoidance Behavior of Mobile Robot. International Journal of Electrical Engineering and Applied Sciences (IJEEAS) 2(1), 31–38 (2019)

Mathematical modelling of the denitrification process

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Abstract: A mathematical model of nitrate removal in woodchip denitrification bioreactor was developed. The model based on a system of two nonlinear differential equations was used for the analysis of the water treatment process. Using computer simulation, the influence of the length of the bioreactor as well the diffusion and pore-water velocity on the nitrate concentration was investigated. The computer simulation was carried out using finite difference technique.

Keywords: Mathematical modeling, Differential equations, Reaction-diffusion, Bioreactor, Denitrification

Mathematics Subject Classification: 35Q92, 65M06

References

- [1] Halaburka, B. J., LeFevre, G. H., & Luthy, R. G. (2017). Evaluation of mechanistic models for nitrate removal in woodchip bioreactors. *Environmental science & technology*, 51, 5156–5164.
- [2] Povilaitis, A., Rudzianskaite, A., Miseviciene, S., Gasiunas, V., Miseckaite, O., & Živatkauskienė, I. (2018). Efficiency of drainage practices for improving water quality in Lithuania. *Transactions of the ASABE*, 61, 179–196.
- [3] Samarskii, A. A. (2001). *The theory of difference schemes volume 240*. Marcel Dekker New York.
- [4] Van Driel, P., Robertson, W., & Merkle, L. (2006). Denitrification of agricultural drainage using wood-based reactors. *Transactions of the ASABE*, 49, 565–573.

Semilocal Convergence of a Family of Ehrlich-type Iterative Methods

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Abstract: One of the famous third-order iterative method for finding simultaneously all the zeros of a polynomial was introduced by Ehrlich [2] in 1967. In this study, we construct a new family of high-order iterative methods as a combination of Ehrlich's iteration function and an arbitrary iteration function. We call these methods *Ehrlich's methods with correction*. We present a semilocal convergence theorem (with computer verifiable initial condition) of presented iterative methods for a large class of iteration functions. The proof of our result is based on the results presented in [2] and [3]. As special cases of the main result, we study the semilocal convergence of Ehrlich's method with Newton's correction (Nourein's method [4]), Ehrlich's method with Weierstrass' correction, and Ehrlich's method with Halley's correction. The main result extends the recent results of the authors [5] on the semilocal convergence of Nourein's method. We end the talk with some numerical examples to show the applicability of our results.

Keywords: Iterative methods, Polynomial zeros, Ehrlich method, Semilocal convergence

Mathematics Subject Classification: 65H04

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References

- [1] L. W. Ehrlich, "A modified Newton method for polynomials", *Comm. ACM* 10, 107–108, 1967.
- [2] P. D. Proinov, "Relationships between different types of initial conditions for simultaneous root finding methods", *Appl. Math. Lett.* 52, pp. 102–111, 2016.
- [3] P. D. Proinov, "Two Classes of Iteration Functions and Q-Convergence of Two Iterative Methods for Polynomial Zeros", *Symmetry*. 13, Article No. 371, 29 pages, 2021.
- [4] A.W.M. Nourein, "An improvement on two iteration methods for simultaneous determination of the zeros of a polynomial", *Intern. J. Comput. Math.* 6, 241–252, 1977.
- [5] P.D. Proinov, M.T. Vasileva, "Local and semilocal convergence of Nourein's iterative method for finding all zeros of a polynomial simultaneously", *Symmetry* 12, Article No. 1801, 25 pages, 2020.

Local Convergence of a Family of Multi-Point Ehrlich-type Root-Finding Methods

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Abstract: This talk deals with multi-point iterative methods for finding all zeros of a polynomial simultaneously. In 1999, Tričković and Petković [1] constructed a two-point variant of the well-known Ehrlich's method [2]. They proved that the order of convergence of the two-point Ehrlich-type method is $1 + \sqrt{2}$. In this study, we construct an infinite sequence of multi-point Ehrlich-type iterative methods. The first member of this family of iterative methods is the two-point Ehrlich-type method introduced in [1].

The purpose of the talk is to present a local convergence theorem for the multi-point Ehrlich-type methods. The obtained result gives a convergence domain, a priori and a posteriori error estimates, and order of convergence of each method of the family. In particular, we prove that the order of convergence $r = r(N)$ of the N -th multipoint iterative method ($N = 1, 2, \dots$) is the unique positive root of the equation

$$1 + 2(t + \dots + t^N) = t^{N+1}.$$

Therefore, for the first iterative method ($N = 1$) of the family, we obtain the order of convergence $r = 1 + \sqrt{2}$ which coincides with above mentioned result of Tričković and Petković. Our local convergence theorem is obtained by an approach introduced in [3].

Keywords: Iterative methods, Polynomial zeros, Local convergence, Error estimates

Mathematics Subject Classification: 65H04

Acknowledgments: This talk was supported by the National Science Fund of the Bulgarian Ministry of Education and Science under Grand DN 12/12.

References

- [1] S.B. Tričković and M.S. Petković, "Multipoint methods for the determination of a polynomial", Novi Sad J. Math. 29, 221-233, 1999.
- [2] L. W. Ehrlich, "A modified Newton method for polynomials", Comm. ACM 10, 107-108, 1967.
- [3] P. D. Proinov, M. D. Petkova, "Local and semilocal convergence of a family of multi-point Weierstrass-type root-finding methods", Mediterr. J. Math. 17, Article ID 107, 2020.

Tensorial total variation based image and video restoration with optimized projection methods

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Abstract: The total variation regularization method was introduced by Rudin, Osher, and Fatemi as an efficient technique for regularizing grayscale images. In this work, we aimed to generalize the total variation method to regularize multi-dimensional problems such as color image and video restoration. A degradation model in a tensor format is proposed to recover blurred and noisy color images and videos.

The alternating direction method for multipliers (ADMM) and an optimized form of projection methods have been employed to solve the tensorial total variation minimization problem.

The structure of the developed approach allows the selection of optimal parameters. We use the TSVD to reduce the size of the problem and to accelerate the convergence of the algorithm. The convergence analysis of the proposed method is proved using convex optimization. Numerical tests for image and video restoration are given showing the effectiveness of the proposed approaches.

Keywords: Tensorial algebra, total variation, ADMM, Shrinkage formula, projection methods, discrepancy principle.

Mathematics Subject Classification: 15A69, 65K10.

References

- [1] Chan, Stanley H., et al. "An augmented Lagrangian method for total variation video restoration." *IEEE Transactions on Image Processing* 20.11 (2011): 3097-3111.
- [2] Kolda, Tamara G., and Brett W. Bader. "Tensor decompositions and applications." *SIAM review* 51.3 (2009): 455-500.
- [3] Rudin, Leonid I., Stanley Osher, and Emad Fatemi. "Nonlinear total variation based noise removal algorithms." *Physica D: nonlinear phenomena* 60.1-4 (1992): 259-268.
- [4] Saad, Yousef. *Iterative methods for sparse linear systems*. Society for Industrial and Applied Mathematics, 2003.

Color image and video restoration using tensor CP decomposition.

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Abstract: This paper proposes a new approach to image and video restoration. This approach is based on tensor representation to construct the degradation model, in which a color image is represented by a third-order tensor and a video comprised of color images is a fourth-order. Applying tensor CP decomposition, which is a low-rank decomposition, to our original problem leads to three subproblems. To solve those subproblems, we apply Global LSQR algorithm and a new algorithm based on Golub Kahan bidiagonalization. Some numerical tests are presented to show the effectiveness of the proposed methods.

Keywords: CP decomposition, Color image restoration, video restoration, LSQR.

Mathematics Subject Classification: 1X234

References

- [1] A. H. Bentbib, M. El Guide, K. Jbilou, E. Onunwor and L. Reichel. Solution methods for linear discrete ill-posed problems for color image restoration. *BIT Numerical Mathematics* 58.3, pp. 555-576 (2018).
- [2] Harshman, Richard A. "Foundations of the PARAFAC procedure: Models and conditions for an " explanatory"multimodal factor analysis." pp 1-84 (1970).
- [3] K. Jbilou A. Messaoudi H. Sadok Global FOM and GMRES algorithms for matrix equations, *Appl.Num.Math*, 31 pp 49-63 (1999).
- [4] Kolda, Tamara G., and Brett W. Bader. "Tensor decompositions and applications." *SIAM review* 51.3, pp 455-500 (2009).
- [5] Paige, Christopher C., and Michael A. Saunders. "LSQR: An algorithm for sparse linear equations and sparse least squares." *ACM Transactions on Mathematical Software (TOMS)* 8.1, pp. 43-71 (1982).

Performance of the Liu-type estimator in the Bell regression model

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Abstract: Poisson regression is generally used in which the response variable comes in the form of observed data. But, the major drawback of the Poisson regression model is the equidispersion assumption, where the variance of the model is restricted to be equal to its mean. In many real-life problems, this assumption does not hold. When the variance is greater than the mean, this problem is called an overdispersion problem. Alternative models can be used to model overdispersed data, such as the negative binomial regression model (NBRM). Castellares et al. (2018) proposed the Bell regression model (BRM) to cope with the overdispersion problem. The main advantage of the BRM to the NBRM is that the BRM has only one parameter. Another problem seen in data is the multicollinearity problem which occurs among the explanatory variables. Amin et al. (2021) defined the Ridge estimator in the BRM to simultaneously solve the overdispersion and multicollinearity problems. After this work, Bulut and Işıl (2021) proposed the Liu estimator in the BRM. In this study, we introduce the Liu-type estimator, defined by Liu (2003), in the BRM. Also, we give a small simulation study to illustrate the superiority of the proposed estimator over the Ridge and Liu estimators.

Keywords: Bell regression model, Liu-type estimator, overdispersion

Mathematics Subject Classification: 62J07, 62J12

References

- [1] Castellares, F., Ferrari, S.L.P. and Lemonte, A.J. “On the Bell distribution and its associated regression model for count data”, *Applied Mathematical Modelling*, 56, pp. 172-185, 2018.
- [2] Amin, M., Akram, M.N. and Majid, A. “On the estimation of Bell regression model using ridge estimator”, <https://doi.org/10.1080/03610918.2020.1870694>, 2021.
- [3] Işıl, M. and Bulut, Y. M. “Liu estimator for the Bell regression model”, (Submitted).
- [4] Liu, K. “Using Liu-type estimator to combat collinearity”, *Communications in Statistics-Theory and Methods*, 32(5), pp. 1009-1020, 2003.

The fluid limit of the processor sharing multi-class multi-queue

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Abstract: We present a tandem network of finite sequence of queues, each one has a single server with several classes of customers, arbitrary interarrival and service time distributions. Upon completing service, customers may leave, or reenter the same queue, possibly as customers of a different class or forwarding to some class of the next queues. The server is operating under the egalitarian processor sharing discipline. When this system has a critical data and under slightly more restrictive assumptions on initial state, we show the existence and uniqueness of the fluid solution. For asymptotic behavior, under mild assumptions of initial state, we describe the behavior of solutions as time approaches infinity.

Keywords: Fluid limit, Fluid model, Queueing Networks, Multiclass networks, Multi-station, Processor sharing.

Mathematics Subject Classification: 60K25

References

- [1] Ezzidani, A., Ben Tahar, A., and Hanini, M. "The Fluid Limit of the Critical Processor Sharing Tandem Queue", International Journal of Electrical and Computer Engineering, Open Science Index 169, (2021).
- [2] Puha A.L, Williams R.J. "Asymptotic behavior of a critical fluid model for a processor sharing queue via relative entropy", Stochastic Systems, Vol. 6, No. 2 pp. 251300, (2016).
- [3] Ben Tahar, A. and Jean-Marie, A. "The fluid limit of the multiclass processor sharing queue", Queueing Syst. 71(4): 347-404, (2012).
- [4] Zhang, Jiheng and Dai, JG and Zwart, Bert "Law of large number limits of limited processor-sharing queues", Mathematics of Operations Research, vol. 34, No. 4, pp. 937-970, (2009).
- [5] Puha A.L, Williams R.J, "Invariant states and rates of convergence for the fluid limit of a heavily loaded processor sharing queue", Ann. Appl. Probab., Vol.14, pp. 517-554, (2004).

An Optimal Energy Control For a Serially Connected Euler-Bernoulli Beam

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Abstract: A system of a serially connected Euler-Bernoulli beam and its optimal energy control are studied by means of semigroup of linear operators in the present paper. The system is formulated by partial differential equations with the boundary conditions. The spectral analysis and semigroup generation of the system are discussed in the appropriate Hilbert spaces. Finally, an optimal energy control is proposed, and existence and uniqueness of the optimal control are demonstrated. Eventually, an approximation theorem is proved in terms of semigroup approach and geometric method.

Keywords: Serially Connected Euler-Bernoulli Beam, Optimal Energy Control, Semigroup of Linear Operators

Mathematics Subject Classification: 35B35, 93C20

References

- [1] Aassila, M., Cavalcanti, M.M. and Soriano, J.A., asymptotic Stability and Energy Decay Rates for Solutions of the Wave Equation Memory in a Star-Shaped Domain, *SIAM Journal of Control and Optimization*, Vol.38 (2000), 1581-1602.
- [2] Guo, B. Z. and Luo, Y. H., Controllability and Stability of a Second Order Hyperbolic System with Collocated Sensor/Actuator, *System and Control Letter*, Vol.46 (2002), 45-65.
- [3] K.S. Liu, F.L. Huang and G. Chen, Exponential stability analysis of a long chain of connected vibrating strings with dissipative linkage, *SIAM J. Appl. Math.*, 49(1989), 1694-1707.
- [4] B.Z. Guo, Y. Xie and Xuezhang Hou, "On Spectrum of a General Petrowsky Type Equation and Riesz Basis of N-Connected Beam with Linear Feedback at Joints", *Journal of Dynamical and Control Systems*, Vol.10, No.2 (2004), 187-211.
- [5] Ciarletta, M., A Differential Problem for Heat Equation with a Boundary Condition with Memory. *Applied Mathematics Letter*, Vol.10 (1997), 95-191.
- [6] Zhang, Q. and Guo, B.Z. Stabilization of an Elastic Plate with Viscoelastic Boundary Conditions, *Journal of Optimization Theory and Applications*, 212(3) (2004), 669-690.
- [7] Barbu, V. and Precupana, Th. *Convexity and Optimization in Banach Space*. Ed. Acad. Rep. Soe Romania, Bucuresti, 1978.
- [8] Hou, X. and Tsui, S.K. Analysis and Control of a Two-link and Three-join Elastic Robot Arm. *Applied Mathematics and Computation*, 152(2004), 759-777.
- [9] Pazy, A. *Semigroups of Linear Operators and Applications to Partial Differential Equations*. Springer- Verlag, New York, 1983.

On Λ -Fractional Differential Geometry

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Abstract: Applying a new Fractional derivative, the Λ - Fractional Derivative, with the corresponding Λ -Fractional space, Fractional Differential Geometry is discussed. The Λ -Fractional derivative satisfies the conditions for the existence of a differential, demanded by the Differential Topology, in the Λ -fractional space, where the Λ -derivatives behave like the conventional ones. Therefore Fractional Differential Geometry is established in that Λ -space in a conventional way. The results are pulled back to the initial space. The present work concerns the geometry of fractional curves and surfaces.

Keywords: Λ -Fractional Derivative, Λ -Fractional space, Λ -Fractional Differential Geometry, Λ - Fractional Tangent, Λ -Fractional Curvature, Λ -fractional tangent space of surfaces.

Mathematics Subject Classification: 53Z05

References

- [1] Lazopoulos, K. A., Lazopoulos, A.K., On the Mathematical Formulation of Fractional Derivatives.Prog. Fract. Diff. Appl. 5(4),pp.261-267, 2019.
- [2] Lazopoulos K.A, Lazopoulos A.K, On fractional bending of beams with Λ -fractional derivative. Arch.App.Mech., 90, pp. 573-584, 2020.
- [3] Lazopoulos K.A., Lazopoulos A.K., On plane Λ -fractional linear elasticity theory, Theoretical & Applied Mechanics Letters, 10, pp.270-275 ,2020.
- [4] Lazopoulos K.A., Lazopoulos A.K., On Λ -fractional geometry, Prog. Fract. Dif. & Appl. , accepted.

Majorization Features For Analytic Functions Involving A Linear Operator

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Abstract: The first studies of majorization in univalent functions has been investigated by MacGregor in 1967. Later, several researchers studied majorization problems for univalent and multivalent functions, which are subordinate to the functions having positive real part, involving various different operators. In this paper, we define a new subclass of Ma-Minda type function class by using Carlson-Shaffer linear operator and cosine hyperbolic function. We introduce majorization properties for this function class. Moreover, some corollaries for this function class are presented.

Keywords: Analytic function, Carlson-Shaffer linear operator, Majorization, Uniformly starlike functions
Mathematics Subject Classification: 30C45, 30C50

References

- [1] A. Alotaibi, M. Arif, M.A. Alghamdi and S. Hussain, Starlikeness associated with Cosine Hyperbolic function, *Mathematics*, **8**(7) (2020), 1118.
- [2] B.C. Carlson and D.B. Shaffer, Starlike and prestarlike hypergeometric functions, *SIAM J. Math. Anal.* **15** (1984), 737–745.
- [3] P.L. Duren, *Univalent Functions*, Springer, New York, 1983.
- [4] A.W. Goodman, On uniformly convex functions, *Ann. Polon. Math.* **56** (1991), 87–92.
- [5] W. Janowski, Some extremal problems for certain families of analytic functions I, *Ann. Polon. Math.* **28** (1973), 297–326.
- [6] S. Kanas and A. Wiśniowska, Conic domains and starlike functions, *Rev. Roumaine Math. Pures Appl.* **45** (2000), 647–657.
- [7] T.H. MacGregor, Majorization by univalent function, *Duke Math. J.* **34**(1) (1967), 95-102.
- [8] W.C. Ma and D. Minda, A unified treatment of some special classes of univalent functions, In: *Proceedings of the Conference on Complex Analysis, Tianjin, (1992)*, 157–169.
- [9] Z. Nehari, *Conformal mappings*, McGraw-Hill Book Company, New York, 1952.

A Data-Driven Machine Learning Algorithm for Financial Market Prediction

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Abstract: A machine learning algorithm, that is based on Gabor transform (spectrogram) and singular value decomposition, is built to classify and predict financial market trends. The reliability of the classifier is demonstrated by randomly generated time series, and effectiveness of the trend prediction algorithm is examined by the practical implementation of the NASDAQ-100 stock markets. It is manifested that price pattern of stock markets can be recognized and trend of the considered stock markets can be predicted effectively via the developed algorithm.

Keywords: Stock market prediction, Machine learning, Pattern recognition, Data-driven methods

Mathematics Subject Classification: 68Q32, 62P05, 91G15

References

- [1] Strader, T. J., Rozycki, J. J., Root, T. H. and Huang, Y.-H. J., "Machine learning stock market prediction studies: Review and research directions", *Journal of International Technology and Information Management*, 28(4), 63-83, 2020.
- [2] J. N. Kutz, "Data-Driven Modeling and Scientific Computation", Oxford University Press, New York, 2013.
- [3] Kim, K. J., and Lee, W. B., "Stock market prediction using artificial neural networks with optimal feature transformation. *Neural computing and applications*", 13(3), 255-260, 2004.
- [4] Basak, S., Kar, S., Saha, S., Khaidem, L. and Dey, S. R., "Predicting the direction of stock market prices using tree-based classifiers", *The North American Journal of Economics and Finance*, 47, 552-567, 2019.
- [5] Yu, L., Chen, H., Wang, S., and Lai, K. K., "Evolving least squares support vector machines for stock market trend mining", *IEEE Transactions on evolutionary computation*, 13(1), 87-102, 2008.
- [6] Kim, H. J., and Shin, K. S., "A hybrid approach based on neural networks and genetic algorithms for detecting temporal patterns in stock markets", *Applied Soft Computing*, 7(2), 569-576, 2007.

Application of machine learning techniques for predicting the WQI for water quality monitoring: a case study in Algeria

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Abstract: Surveillance of surface water quality is a major environmental challenge. The Water Quality Index (WQI) describes a number of water quality variables at a certain location environment and time. WQI is usually calculated by traditional methods involving long-term calculation, a timing consumption, and accidental errors occasionally associated with subindex calculation. Thus, it is highly necessary to provide an accurate WQI prediction model. Recently, similar prediction applications were explored in artificial neural networks (ANNs) and the capability to capture the pattern of nonlinearity between forecast and prediction is remarkable. Two machine learning methods have been applied in the current research: ANFIS and SVM models to investigate and try to emulate WQI's relationship with water quality variables in Cheliff's dam in Mostaganem (Algeria). Moreover, a comprehensive analysis has been performed for the performance assessment and sensitivity analysis of the variables. With high performance accuracy in two used models, the results achieved are promising. The proposed approach also provides an efficient alternative to calculate and predict the WQI by including long computing methods, transformations, the use of various subindex formulas for every value of the water quality component variables and time consumption.

Keywords: Water quality prediction, supervised machine learning, water quality index (WQI), ANFIS, SVM, Algeria.

Mathematics Subject Classification: 68Uxx

References

- [1] Aldhyani, T. H. H., Al-Yaari, M., Alkahtani, H., & Maashi, M. (2020). Water Quality Prediction Using Artificial Intelligence Algorithms. *Applied Bionics and Biomechanics*, 2020, 1-12. <https://doi.org/10.1155/2020/6659314>.
- [2] Isiyaka, H. A., Mustapha, A., Juahir, H., & Phil-Eze, P. (2018). Water quality modelling using artificial neural network and multivariate statistical techniques. *Modeling Earth Systems and Environment*, 5(2), 583-593. <https://doi.org/10.1007/s40808-018-0551-9>.
- [3] Hameed, M., Sharqi, S. S., Yaseen, Z. M., Afan, H. A., Hussain, A., & Elshafie, A. (2016). Application of artificial intelligence (AI) techniques in water quality index prediction : a case study in tropical region, Malaysia. *Neural Computing and Applications*, 28(S1), 893-905. <https://doi.org/10.1007/s00521-016-2404-7>
- [4] Hmoud Al-Adhaileh, M., & Waselallah Alsaade, F. (2021). Modelling and Prediction of Water Quality by Using Artificial Intelligence. *Sustainability*, 13(8), 4259. <https://doi.org/10.3390/su13084259>

A new implementation of the Block QMR method for solving non-symmetric systems of linear equations with multiple right-hand sides

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Abstract: The block QMR is a block Krylov solver for solving non-symmetric systems of linear equations with multiple right-hand sides. This method is classically implemented by first applying the Lanczos iteration as a block biorthogonalization process to create two bases. The first one is of the block Krylov subspace generated by the matrix of the system from the initial residual and the second one is of the block Krylov subspace generated by the matrix transpose of the system from the initial residual. Next, the method is solving a block least-squares problem, which is equivalent to solving several least-squares problems implying the same tridiagonal matrix. These latter are usually solved by using a block updating procedure for the QR decomposition based on givens rotations or the Housholder decomposition of the tridiagonal matrix like in the BI-GMRES. In this work, we develop a new block version of the QMR method by using a simple implementation, based on a particular partitioning of the tridiagonale matrix given by the Lanczos iteration as a block biorthogonalization process.

Keywords: Linear system, Krylov subspace method, projector, pseudo-inverse.

Mathematics Subject Classification: 65F10, 65F20, 65F50, 65L20

References

- [1] E. H. Ayachour, A fast implementation for GMRES method, *J. comput. Appl. Math.*, vol. 159 (1986), pp. 269–283.
- [2] Roland W. Freund, A block QMR algorithm for non-Hermitian linear systems with multiple right-hand sides, *Linear algebra and its applications* 254 (1997), pp. 119—157.
- [3] Y. Saad, *Iterative methods for sparse linear systems*, PWS. Publishing Company (1996).

The magnetic dipole effect framework on the thixotropic nanofluid flow past the continuous curved stretched surface

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Abstract: The magnetic dipole influence of a thixotropic nanofluid with heat and mass transfer as well as the concentration of microorganisms past the curved sheet is investigated. The dynamic problem model is converted to non-linear ordinary differential equations through similarity transformations, which are resolved by an efficient and robust method, i.e. the homotopy analysis method. The solution is evaluated on the basis of all built-in parameters. The velocity is reduced by the dipole influence while it is increased by the thixotropic nanofluid parameter. The temperature increases with the dipole effect and the concentration increases with the increase of the curvature parameter and the density of the motile microorganisms decreases with the Peclet number. Streamlines show that trapping on the curved stretched surface is uniform.

Keywords: curved stretching sheet, electromagnetism, Heat convective conditions, porous medium, radiation, thixotropic ferrofluid.

Mathematics Subject Classification: 35Q35, 76D05, 80A20

References

- [1] de Deus HPA, Dupim, GS. On behavior of the thixotropic fluids. *Physics Letters A*; 377(6), 478—485, 2013.
- [2] Hayat T, Waqas M, Shehzad SA, Alsaedi A. A model of solar radiation and Joule heating in magnetohydrodynamic (MHD) convective flow of thixotropic nanofluid. *Journal of Molecular Liquids*; 215, 704—710, 2016.
- [3] Hayat T, Waqas M, Khan MI, Alsaedi A. Analysis of thixotropic nanomaterial in a doubly stratified medium considering magnetic field effects. *International Journal of Heat and Mass Transfer*; 102: 1123-9, 2016.
- [4] Hayat T, Ahmad S, Khan MI, Alsaedi A. Exploring magnetic dipole contribution on radiative flow of ferro-magnetic Williamson fluid. *Results in physics*; 8: 545-51, 2018.
- [5] Titus LR, Abraham A. Heat transfer in ferrofluid flow over a stretching sheet with radiation. *Int. J. Eng. Res. Tech.*; 3: 2198-203, 2014.
- [6] Hayat T, Rashid M, Imtiaz M, Alsaedi A. MHD convective flow due to a curved surface with thermal radiation and chemical reaction. *Journal of Molecular Liquids*; 225:482-9, 2017.
- [7] Liao SJ. An explicit, totally analytic approximate solution for Blasius' viscous flow problems. *International Journal of Non-Linear Mechanics*; 34: 759-78, 1999.

Further Properties of L-closed topological spaces

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Abstract: In this research, we focus on some properties of almost, weakly and nearly L-closed topological spaces. We proposed lots of characterizations. Relationships between types of L-closed topological spaces are explored and discussed.

Keywords: L-closed space, Almost L-closed, Nearly L-closed, Weakly L-closed.

Mathematics Subject Classification: 00A05

References

- [1] Balasubramanian, G., On some generalizations of compact spaces, *Glasnik Mat.*, 17, 37: 367-380,(1982).
- [2] Cammarota, F., Almost Homeomorphisms and Almost Topological Properties, *Revista Colombiana de Matemáticas*, XX: 51-71,(1986).
- [3] Cammaroto, F. Santoro, G., Some Counterexamples and Properties on Generalizations of Lindelöf Spaces, *International Journal of Mathematics & Mathematical Sciences*, 19, 4 :737-746,(1996).
- [4] Engelking, R., *General Topology*, PWN, Warszawa,(1979).
- [5] Frolik, Z., Generalizations of Compact and Lindelöf Spaces, *Czechoslovak Mathematical Journal*. 9, 84: 172-217,(1959).
- [6] Hdeib, H. and Pareek, C. M., On Spaces In Which Lindelöf Sets are Closed, *Questions & Answers in General Topology*, 4: 67-72,(1986).
- [7] Henriksen, M. Woods, R., Weak P-Space and L-closed Spaces, *Questions & Answers in General Topology*, 6: 201-206,(1988).
- [8] Jankovic, D. Ellen, M., On Some Generalizations of L-closed spaces, *Acta Math Univ Comeniana*, 2: 345-353,(1999).
- [9] Kelly, J.C., Bitopological Spaces, *Proceedings of the London Mathematical Society*, 13: 71-89,(1963).
- [10] Singal, M.K and ARYA, S.P., On Almost Regular Spaces, *Glasnik Matematicki Series*. III, 4 ,24: 89-99,(1969).
- [11] Willard, S. and Dissanayake, N.B., The Almost Lindelöf Degree, *Canadian Mathematical Bulletin*, 27, 4,(1984).
- [12] Dieudonne . J. A., Une generalisation des espaces compacts, *J. Math. Pures Appl.* 23, 9: 65-76,(1944).
- [13] Tamano. H., A Characterization of Paracompactness, *Fundamenta Mathematicae*. 72:180-201,(1971).
- [14] Comfort. W., Hindman.N. and Negrepointis. S., F-Spaces and their Products with P-Spaces, *Pacific Journal of Mathematics*. 28: 489-502.(1969).
- [15] Ulmer. M., Products of Weakly-R-Compact Spaces, *Transactions of the American Mathematical Society*., 170.(1972).
- [16] Cammaroto. F., Santoro. G., Some Counterexamples and Properties on Generalizations of Lindelof Spaces. *International journal of Mathematics and mathematical Sciences*, 19, 4, 737-746,(1996).

The refinement of local fractional Hölder's inequality and its applications

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Abstract: Recently, a whole variety of classical real inequalities has been extended to hold on to certain fractal spaces. A rich collection of generalizations included inequalities with more general kernels, weight functions and integration domains and extension to a multidimensional case. In this paper by using the improved Young inequality and local fractional calculus, a refinement of local fractional Hölder's inequality is given. As an application, a multidimensional version of local fractional Hilbert-type inequalities is obtained.

Keywords: Hölder's inequality, conjugate parameters, local fractional calculus, multidimensional form.

Mathematics Subject Classification: 26D15

References

- [1] Y. Basci, D. Baleanu, "Hardy-type inequalities within fractional derivatives without singular kernel", *J. Inequal. Appl.* 2018:304 (2018), 11pp.
- [2] Ts. Batbold, M. Krnić, P. Vuković, "A unified approach to fractal Hilbert-type inequalities", *J. Inequal. Appl.* 2019:117 (2019), 13pp.
- [3] Q. Liu, "A Hilbert-type fractional integral inequality with the kernel of Mittag-Leffler function and its applications", *Math. Inequal. Appl.* **21** (2018), 729–737.
- [4] X.J. Yang, "Local fractional functional analysis and its applications", Asian Academic publisher Limited, Hong Kong, 2011.
- [5] X.J. Yang, "Advanced local fractional calculus and its applications", Word Science Publisher, New York, 2012.

Iyengar-Hilfer Fractional Inequalities

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Abstract: Here we present Iyengar type integral inequalities. At the univariate level they involve psi-Hilfer left and right fractional derivatives. At the multivariate level they involve Hilfer left and right fractional derivatives, and they deal with radial and non-radial functions on the ball and spherical shell. All estimates are with respect to norms $\|\cdot\|_p$, $1 \leq p \leq \infty$. At the end we provide an application.

Keywords: fractional integral inequalities

Mathematics Subject Classification:

Mathematical Modeling of Mixed LDL-C and Blood Flow through an inclined Channel with Heat in the presence of Magnetic Field

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Abstract: We investigated mixed LDL-C and blood flow through an inclined channel with heat in the presence of magnetic field. In the investigation, mathematical models for the mixed LDL-C and blood flow and energy transfer were formulated as a coupled system of partial differential equations (PDEs), the PDEs are scaled using the dimensionless quantities to dimensionless partial differential equations, they are further reduced to ordinary differential equations (ODEs) using the oscillatory perturbation method of which the non-homogenous governing equations are solved directly using the method of undetermined coefficient. The velocity and temperature functions are obtained with some governing parameters involved, and Mathematica codes was developed to simulate the functions to study the effect of the various governing parameters on the flow profiles. It is observed that the governing parameters influenced the flow profiles; also the angle of inclination also influences the flow profile.

Keywords: Blood, Mathematica, PDS, ODE, LDL-C

Mathematics Subject Classification: 76D05, 76D10, 76A02

References

- [1] Okoro, E. O. (2015). Aspirin and Diabetes Care in Nigeria: Treatment or Exploitation?. *Journal of Clinical Research & Bioethics*, 6(4), 1.
- [2] Makris, K., Haliassos, A., Chondrogianni, M., & Tsvigoulis, G. (2018). Blood biomarkers in ischemic stroke: potential role and challenges in clinical practice and research. *Critical reviews in clinical laboratory sciences*, 55(5), 294-328.
- [3] Sankar, D. S., & Ismail, A. (2009). Two-fluid mathematical models for blood flow in stenosed arteries: a comparative study. *Boundary Value Problems*, 2009, 1-15.
- [4] Oladapo, O. O., Bolaji, M. O., & Abdul-Malik, O. (2017). Comparative Study of Artherogenic Effects of Common Nigerian Edible Oils in Male Rabbits. *Journal of Advances in Medicine and Medical Research*, 1-10.
- [5] Basarab, M. A., Konnova, N. S., Basarab, D. A., & Kravchenko, O. V. (2019, June). Numerical Simulation of Magnetohydrodynamic Blood Flow through Stenosed Arteries Using the R-functions Method. In *2019 Photonics & Electromagnetics Research Symposium-Spring (PIERS-Spring)* (pp. 355-361).

Entire sequence spaces on 2-normed space defined by a Musielak -Orlicz function

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Abstract: In this paper we introduce entire sequence spaces defined by a Musielak -Orlicz function $M = (M_k)$, $(\Delta_{(vm)}^n)$ is generalized difference operator and $r = (r_k)$ is a sequence of positive real numbers. Also we study various properties and obtain some inclusion relations involving these spaces.

Keywords: Difference operator, Musielak-Orlicz function, Sequence spaces, Entire sequence space, Analytic sequence space.

Mathematics Subject Classification: 40A05, 46A45, 40D05, 46E30

References

- [1] Aiyub. M.,(2014). Some Lacunary sequence spaces of Invariant Means defined by Musielak-Orlicz function. British Journal of Mathematics and Computer Science. Vol.4, No.(12) ,pp. 1682-1692.
- [2] Bhardwaj.V.K and Singh . N .,(1999). Some sequence spaces defined by $[[\bar{N}_p], p_n]$ summability, Demonstratio Math. Vol.32, No.(3) ,pp. 539-546.
- [3] Bhardwaj.V.K and Singh.N.,(2000). Some sequence spaces defined by $[[\bar{N}_p], p_n]$ summability and Orlicz function . Indian J. Pure Appl. Math. vol.31, No.(3), pp. 319-325.
- [4] Dutta .H.,(2009). Characterization of certain matrix classes involving generalized difference summability spaces. Appl.Sci.Apps. Vol.11,pp. 60-67.
- [5] Et. M and Colak. R.,(1985). On generalized difference sequence spaces, Soochow J. Math. Vol.21, No. (4) , pp. 147-169.
- [6] Gähler S.,(1965). Linear-2- normierte raume Math.Nach. Vol. 28, PP. 1-43.
- [7] Gunawan. H and Mashadi.M.,(2001). On finite dimensional 2-normed spaces, Soochow J. Math.Vol. 27, No.(3), pp. 321-329.
- [8] Gurdal.M and Pehlivan.S.,(2009). Statitlial convergence in 2-normed space, Southeast Asian Bull.Math. Vol.33, pp. 257-264.
- [9] kamthan P.K and Gupta.M.,(1981). Sequence spaces and series Marcel Dekker Inc., Newyork.
- [10] Kizmaz.H.,(1981). On certain sequence spaces, Canad .Math. Bull. Vol. 24 , pp. 169-176.
- [11] Lindendstrauss.J. and Tzafriri.L.,(1972). On Orlicz sequence spaces, Israel J. Math. Vol.10, pp. 379-390.
- [12] Maddox.I.J.,(1986). Sequence spaces defined by a modulus, Mat. Proc. Camb. Phil. Soc.Vol. 100, pp. 161-166.
- [13] Maligranda.L.,(1989). Orlicz spaces and interpolation, seminars in Mathematics 5, polish Academy of Science.
- [14] Musielak.J.,(1983). Orlicz spaces and modular spaces, Lecture Notes in Mathematics., 1034, springer Verlage.

- [15] Nakano.H.,(1953). Concave modulars. J. Math. Soc. Japan.Vol. 5, pp. 29-49.
- [16] Ruckle.W.H.,(1973). FK spaces in which the sequence of coordinate vector is bounded, Canad. J. Math.Vol. 25 , pp. 973-978.
- [17] Raymood. W., Frees.Y and Cho.J.,(2001). Geometry of linear 2-normed spaces.,N. Y. Nova Science Publishers, Huntington. .
- [18] Sahiner.A, Gurdal. M , Saltan.S and Gunawan .H.,(2007). Ideal convergence in 2-normed spaces. Tiwanese J. Math.Vol. 11, No.(5), pp. 1477-1484.
- [19] Tripathy. B.C and Esi.A.,(2006). A new type of difference sequence spaces. Int.J.Sci. Technol. Vol.1 No.(1) ,pp. 11-14.
- [20] Tripathy. B.C, Esi.A and Tripathy. B.K.,(2005). On a new type of generalized difference cesaro sequence spaces.Soochow J.Math. vol. 31, No.(3), pp. 333-340.

Shape Differentiability of Semilinear Equilibrium-Constrained Optimization

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Abstract: A class of semilinear optimization problems linked to variational inequalities is studied with respect to its shape differentiability. One typical example stemming from quasi-brittle fracture describes an elastic body with a Barenblatt cohesive crack under the inequality condition of non-penetration at the crack faces. The other conceptual model is described by a generalized Stokes-Brinkman-Forchheimer's equation under divergence-free and mixed boundary conditions. Based on the Lagrange multiplier approach and using suitable regularization, an analytical formula for the shape derivative is derived from the Delfour-Zolesio theorem. The explicit expression contains both primal and adjoint states and is useful for finding descent direction of a gradient algorithm to identify an optimal shape, e.g., from boundary measurement data.

Keywords: Shape Optimization, Solid with non-penetrating crack, Stokes-Brinkman-Forchheimer flow

Mathematics Subject Classification: 49Q10, 74P15, 76B75

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References

- [1] F. Cakoni, V.A. Kovtunenکو, "Topological optimality condition for the identification of the center of an inhomogeneity", *Inverse Probl.* 34, 035009, 2018.
- [2] M.C. Delfour, J.-P. Zolésio, "Shape and Geometries: Metrics, Analysis, Differential Calculus, and Optimization", SIAM, Philadelphia, 2011.
- [3] J.R. González Granada, J. Gwinner, V.A. Kovtunenکو, "On the shape differentiability of objectives: a Lagrangian approach and the Brinkman problem", *Axioms* 7, 76, 2018.
- [4] H. Itou, S. Hirano, M. Kimura, V.A. Kovtunenکو, A.M. Khludnev (Eds.), "Mathematical Analysis of Continuum Mechanics and Industrial Applications III: Proceedings of the International Conference CoMFoS18", *Mathematics for Industry* 34, Springer, Singapore, 2020.
- [5] A.M. Khludnev, V.A. Kovtunenکو, "Analysis of Cracks in Solids", WIT-Press, Southampton, 2000.
- [6] V.A. Kovtunenکو, K. Kunisch, "High precision identification of an object: Optimality-conditions-based concept of imaging", *SIAM J. Control Optim.* 52, 773-796, 2014.
- [7] V.A. Kovtunenکو, K. Ohtsuka, "Shape differentiability of Lagrangians and application to Stokes problem", *SIAM J. Control Optim.* 56, 3668-3684, 2018.

Beyond the Standard Model in Noncommutative Geometry and Dark Matter

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Abstract: Non-commutative geometry can be loosely described as the study of spaces whose algebra of functions is non-commutative. This has been an area of interest to both mathematicians and physicists throughout this century. However, compared with classical geometry (the geometry of spaces whose algebra of functions is commuting) non-commutative geometry was very underdeveloped. Recently this has begun to change with the introduction by Connes [1] and Dubois-Violette [2] [3] of (independent) generalised by Rham differential algebras on the "non-commutative manifold". Since then, non-commutative geometry has been enhanced and refined until, in its present state [4] [5] it is highly developed and contains many of the tools of classical geometry. The purpose of this paper is to seek a connection between noncommutative geometry, and certain aspects of dark matter. The former case is based on a simple mathematical argument showing that the main manifestation of dark matter in connection with flat rotation curves in galaxies and clusters of galaxies is also a consequence of noncommutative geometry. The latter case requires an examination of the local effect of noncommutative geometry and the subsequent extension to the global phenomenon of an accelerating Universe.

Keywords: non-commutative geometry, differential algebras, dark matter

Mathematics Subject Classification: Analysis and Its Applications

References

- [1] A Connes Noncommutative Differential Geometry Part / I H E S (M/82/53) 1982
- [2] M Dubois-Violette Derivations et Calcul Differential Non Commutatif C.R.Acad.Sci 307 I (1988) 403
- [3] M Dubois-Violette, R Kerner & J Madore Non-commutative Differential Geometry and New Models of Gauge Theory J.Math.Phys 31 (1990) 316
- [4] A Connes Noncommutative Geometry Academic Press (1994)
- [5] Chamseddin, A.H., Connes, A. and Mukhanov, V. (2015) "Quanta of geometry: noncommutative aspects," Physical Review Letters, 114, Article ID: 091302.

New qualitative results for integro-differential equations with a variable retardation

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Abstract: In this work, a class of nonlinear time varying retarded integro-differential equations is investigated. Three new theorems related to the uniformly asymptotic stability of the zero solution as well as integrability and boundedness of solutions are proved by the functional Lyapunov-Krasovskii method. Two examples are given with their graphs to illustrate the applications of the proposed results.

Keywords: Stability, integrability, boundedness, Lyapunov -Krasovskii functional.

Mathematics Subject Classification: 34D05, 34K20, 45J05.

References

- [1] S. Ahmad, M. Rama Mohana Rao, Stability of Volterra diffusion equations with time delays. *Appl. Math. Comput.* 90 (1998), no. 2-3, 143-154.
- [2] T. A. Burton, *Volterra integral and differential equations*. Second edition. *Mathematics in Science and Engineering*, 202. Elsevier B. V., Amsterdam, 2005.
- [3] C. Tunç, O. Tunç, New qualitative criteria for solutions of Volterra integro-differential equations. *Arab Journal of Basic and Applied Sciences* 25(2018), no. 3, 158-165.
- [4] C. Tunç, O. Tunç, On behaviors of functional Volterra integro-differential equations with multiple time-lags. *Journal of Taibah University for Science* 12 (2), (2018), 173-179.
- [5] C. Tunç, O. Tunç, A note on the qualitative analysis of Volterra integro-differential equations. *Journal of Taibah University for Science*. 13 (2019), no.1, 490-496.
- [6] M. Bohner, O. Tunç, Qualitative analysis of integro-differential equations with variable retardation. *Discrete & Continuous Dynamical Systems-B*. (2021) in press.
- [7] J. R. Graef, O. Tunç, Asymptotic behavior of solutions of Volterra integro-differential equations with and without retardation. *Journal of Integral Equations and Applications*. (2021) in press.

Nonlinear Dynamics of Quantum Entanglement

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Abstract: In this work, we will examine in a proof-of-concept experiment a new type of quantum-inspired protocol based on the idea of nonlinear dynamics of quantum entanglement. We discuss various measures of bipartite and tripartite entanglement in the context of two and three level atoms. The quantum entanglement is discussed for different systems. For the three-level systems various measures of tripartite entanglement are explored. The significant result is that a sudden death and sudden birth of entanglement can be controlled through the system parameters.

Applying ψ -Cauputo fractional q -derivative to investigate of anti-periodic fractional differential inclusions

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Abstract: In this paper, we investigate the existence of solutions for a class of fractional q -difference inclusions with anti-periodic boundary value conditions with ψ -Cauputo fractional q -derivative. By means of some standard nonlinear theorems, sufficient conditions for the existence of solutions for the fractional q -differential inclusions under ψ -Caputo q -derivatives are presented where the real function ψ is increasing. Our result generalizes the known special case if $\psi(t) = t$ and single known results to the multi-valued ones.

Keywords: Multi-Step methods, ψ -Caputo derivative

Mathematics Subject Classification: 34A08, 34B16, 39A13

References

- [1] R.P. Agarwal, *Certain fractional q -integrals and q -derivatives*, JProceedings of the Cambridge Philosophical Society, **66** (1965) 365–370. doi: 10.1017/S0305004100045060
- [2] M.H. Annaby, Z.S. Mansour, *q -Fractional Calculus and Equations*, Springer Heidelberg, Cambridge, 2012. doi: 10.1007/978-3-642-30898-7
- [3] M. Aydogan, D. Baleanu, J.F. Gómez Aguilar, S. Rezapour, M.E. Samei, *Approximate endpoint solutions for a class of fractional q -differential inclusions*, Fractals **28**(8) (2020), Article ID 2040029, 18 pages. doi: 10.1142/S0218348X20400290
- [4] R.A.C. Ferreira, *Nontrivials solutions for fractional q -difference boundary value problems*, Elect. J. Qualit. Theory Diff. Eq. **70** (2010) 1–101.
- [5] V. Hedayati, M.E. Samei, *Positive solutions of fractional differential equation with two pieces in chain interval and simultaneous Dirichlet boundary conditions*, Bound. Value Probl. (2019) 2019:141.
- [6] Ntouyas, S. K., Samei, M. E.: Existence and uniqueness of solutions for multi-term fractional q -integro-differential equations via quantum calculus, *Adv. Diff. Eq.* (2019) 2019:475.
- [7] M.E. Samei, *Existence of solutions for a system of singular sum fractional q -differential equations via quantum calculus*, *Adv. Diff. Eq.* (2019) 2019:163.
- [8] Samei, M. E., Hedayati, V., Rezapour, S.: Existence results for a fraction hybrid differential inclusion with caputo-hadamard type fractional derivative, *Adv. Diff. Eq.* (2019) 2019:163.

Mathematical Models and Comparative Analysis for Rice Irrigation crop water Requirements: A Case Study of Bida Basin Niger State, Nigeria

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Abstract: In this paper, mathematical models for crop water Requirements (C.W.R) and the size of land for irrigation (S.L.I) were formulated. We fill the gap by considering the size of the irrigation land which was not considered by the Food and Agriculture Organization (F.A. O) in CROPWAT 8.0 software. The solutions of the models for Crop Water Requirement for Rice, and the size of land for irrigation (S.L.I) of the crops were obtained. The computational Method of solutions is carried out to get the effective results. The climatic data of the study area (Bida Basin) under which our research is based includes: Rainfall, Humidity, Sunshine hours, minimum and maximum Temperature, evapotranspiration were secondary data collected from Nigeria Metrological Society (NIMET). We compared the results of CROPWAT 8.0 software developed by the Food and Agriculture Organization (F.A. O) and our computational method so that we can arrive to a new finding and better results. The results for the computational method with the size of Land for irrigation shows that there is an increase in crop water need for the crops than the results of CROPWAT 8.0 software developed by Food and Agriculture Organization (F.A. O) in which the size of the land was not considered. We therefore recommended that an integral calculus can be used to estimate irregular shape of the size of the land if the land shape is not in a regular form before solutions is given for accuracy and effective results.

Keywords: Bida Basin, crop water coefficient, evapotranspiration

Mathematics Subject Classification: Applied Mathematics

References

- [1] M.N. Nkodo, F.C. Van zyl, H. Keuris, & B. Schreine”, Proposed National Water resources Straregy 2 (NWRS) Summary Cape Town”. Department of Water Affair, South Africa 3 (2012) 9.
- [2] Food and Agriculture of the united Nation state of the Rome world’s <https://www.fao.org/011/1035e00htm>.(2010)
- [3] A.Y. Solomon, M. Girma, & M. Mengistu, “Determination of Crop Water Requirements for maize in Abshege Wreda, Gurage Zone, Eithopia”, Journal of earth science and climatic Change 9 (2018) 2157.
- [4] W.Z. Wenken, W. Dai, & Y. Zhao, A quantitative Analysis of Hydraulic Interaction Process in Stream Aquifer System”. Journal of science education and Agronomy Journal, 3 (2016) 426.
- [5] P. Rijwana, “A Study on the Crop Water Requirement for Agriculture in Typical River Basin of India”, International Journal of Water Research; 2 (2014) 67.
- [6] I. N. Abdullahi, “Estimating Aquifer Hydraulic properties in Bida Basin, Central Nigeria using Empirical Methods”, Journal of Earth Science Research, 2 (2013) 1.

Hermite collocation method for solving fractional Lane-Emden type equations with conformable derivative

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Abstract: In this study, Hermite collocation method is used to obtain an approximate solution of fractional Lane-Emden type equations. Conformable derivative is considered for the fractional derivatives. A numerical example is solved and the absolute errors are presented in tables. The obtained results revealed that the method is very efficient to obtain approximate solutions of fractional Lane-Emden type equations with conformable derivative.

Keywords: Hermite polynomials, collocation method, conformable derivative, Lane-Emden type equations.

Mathematics Subject Classification: 65D15, 65L60, 65L70

References

- [1] Kumar, N., Pandey, R. K., & Cattani, C. (2011). Solution of the Lane-Emden equation using the bernstein operational matrix of integration. *International Scholarly Research Notices*, 2011.
- [2] Mechee, M. S., & Senu, N. (2012). Numerical study of fractional differential equations of lane-Emden type by the least square method. *International Journal of Differential Equations and Applications*, 11(3).
- [3] Wei, C. F. (2019). Application of the homotopy perturbation method for solving fractional Lane-Emden type equation. *Thermal Science*, 23(4), 2237-2244.
- [4] Ma, W. X., Mousa, M. M., & Ali, M. R. (2020). Application of a new hybrid method for solving singular fractional Lane-Emden-type equations in astrophysics. *Modern Physics Letters B*, 34(03), 2050049.

Face and Emotion Recognition using Deep Learning Based on Computer Vision Methods

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Abstract: Deep learning studies are among the discipline that are rapidly increasing and developing today. Especially after the concept of big data enters our lives, deep learning methods have used to process the data. In the study, it has aimed to detect the face on the picture determined by the user and to conduct emotional analysis and gender determination with the deep learning methods of the detected face. Viola-Jones algorithm has used for face recognition. "Mini_Xception" model of Convolutional Neural Networks (CNN) has used for emotion analysis and gender detection. Estimation rates have measured with 18 different experiments performed. The most successful emotion recognition has measured as 93,11% and the most successful gender recognition has measured as 90,75. Experiments within the study have supported by visual studies.

Keywords: Deep Learning, Viola-Jones Algorithm, Facial Expression Analysis, Emotion Analysis, Gender Detection.

Mathematics Subject Classification: 68T07

References

- [1] Taner Uçkan, Deniz Dal, "Opengl Tabanlı Animasyonlarda Görüntü Kalitesinin Cuda Mimarisi İle İyileştirilmesi", Uludağ University Journal of The Faculty of Engineering, 21(1), pp. 79-96, 2016.
- [2] Omkar M. Parkhi, Andrea Vedaldi, Andrew Zisserman, "Deep face recognition", British Machine Vision Association, pp.1-12, 2015.
- [3] Yi Sun, Xiaogang Wang, Xiaoou Tang, "Deep learning face representation by joint identification-verification", arXiv preprint arXiv:1406.4773, 2014.
- [4] Samira Ebrahimi Kahou, et al., "Emonets: Multimodal deep learning approaches for emotion recognition in video", Journal on Multimodal User Interfaces 10(2), pp. 99-111, 2016.
- [5] Jos van de Wolfshaar, Mahir F. Karaaba, Marco A. Wiering, "Deep convolutional neural networks and support vector machines for gender recognition", 2015 IEEE Symposium Series on Computational Intelligence. IEEE, pp. 188-195, 2015.

Asymptotic Behavior of Solutions for a Fractional Integrodifferential Problem

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Abstract: We study the asymptotic behavior of solutions for an initial value problem with a nonlinear fractional integro-differential equation. The involved fractional derivatives are of Caputo or Riemann-Liouville types of orders between 0 and 2. Reasonable sufficient conditions, on the nonlinear kernel and source function under which the solutions behave like power-type functions at infinity, are established. For this purpose, we use and generalize some well-known integral inequalities. Our findings are supported by examples. .

Keywords: Asymptotic behavior, Riemann-Liouville fractional integral, Caputo fractional derivative, integral inequalities, nonlocal source.

Mathematics Subject Classification: 34A08, 35B40, 26D10.

References

- [1] A. Aghajani, Y. Jalilian, and J. Trujillo, On the existence of solutions of fractional integro-differential equations, *Fract. Calc. Appl. Anal.*, 15(1):44-69, 2012.
- [2] Ahmad M. Ahmad, K. M. Furati and N-E Tatar, Boundedness and power-type decay of solutions for a class of generalized fractional Langevin equations, *Arab. J. Math.*, 8(2):79-94, 2019.
- [3] Ahmad M. Ahmad , The Asymptotic Behavior of Solutions of a Fractional Integro-differential Equation, *WSEAS Trans. Syst. Control. Art.* 35, pp. 341-348, Volume 15, 2020.
- [4] Ahmad M. Ahmad, K. M. Furati and N-E Tatar, Asymptotic Behavior of Solutions for a Class of Fractional Integro-differential Equations, *Mediterr. J. Math.*, 15: 188, 2018.
- [5] Ahmad M. Ahmad, K. M. Furati and N-E Tatar, Asymptotic power type behavior of solutions to a nonlinear fractional integro-differential equation, *Electron. J. Diff. Equ.*, 2017(134):1-16, 2017.
- [6] D. Băleanu, R. P. Agarwal, O. G. Mustafa, and M. Coşulschi, Asymptotic integration of some nonlinear differential equations with fractional time derivative, *J. Phys. A Math. Theor.*, 44(5):055203, 2011.
- [7] A. A. Kilbas, H. M. Srivastava, and J. J. Trujillo, Theory and applications of fractional differential equations, volume 204. Elsevier Science Limited, 2006.
- [8] M. Medved' and M. Pospíšil, Asymptotic integration of fractional differential equations with integrodifferential right-hand side, *Math. Model. Anal.*, 20(4):471.489, 2015.
- [9] M. Pinto, Integral inequalities of Bihari-type and applications, *Funkc. Ekvacioj.*, 33(3):387.403, 1990.

Power analysis of goodness of fit tests for normality

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Abstract: In modern statistics, goodness-of-fit hypothesis testing is often used to identify the distribution of data set. The problem of correct determination whether a data set follows a normal distribution is being investigated at least since 1900 and remains relevant to the present day. Many goodness-of-fit hypothesis tests have been developed. The Chi-square [1], Kolmogorov-Smirnov [2], Anderson-Darling [3] are the first well recognized tests. The test (with its modifications) developed by Desgagné, Lafaye de Micheaux and Leblanc is an example of a more recent contribution [4]. However, it is not clear how to select the most appropriate test to check the assumption of data set normality.

To tackle this problem a comprehensive study of goodness-of-fit tests, including a newly proposed N-metric test was carried out. The developed test is based on N-metric statistic and uses the proposed kernel function $K(x)$. This test is unique in a way that the shape of its kernel function is chosen to eliminate the shift, which occurs in the evaluation of density for a given data set values. In this study, a power analysis of 40 normality tests was performed. Each test was applied to a randomly generated data sets (of 6 sizes) of 15 data distributions. Based on the results, it is recommended to apply N-metric tests for symmetric data sets of size greater than or equal to 112, for asymmetric data sets of size greater than or equal to 118, and for bell-shaped distributed data sets of size greater than or equal to 88.

Keywords: goodness-of-fit hypothesis, normality test, power analysis.

Mathematics Subject Classification: 62F03, 62E10

References

- [1] G. A. Barnard, "Introduction to Pearson (1900) on the Criterion That a Given System of Deviations from the Probable in the Case of a Correlated System of Variables is Such That it Can be Reasonably Supposed to Have Arisen from Random Sampling", Springer Series in Statistics Breakthroughs in Statistics, Springer: Cham, Switzerland, pp. 1–10, 1992.
- [2] A. Kolmogorov, "Sulla determinazione empirica di una legge di distribuzione". *Inst. Ital. Attuari Giorn*, 4, 83–91, 1933.
- [3] T. W. Anderson, D. A. Darling, "Asymptotic theory of certain "goodness-of-fit" criteria based on stochastic processes". *Ann. Math. Stat.*, 23, 193–212, 1952.
- [4] A. Desgagné, P. Lafaye de Micheaux, "A powerful and interpretable alternative to the Jarque–Bera test of normality based on 2nd-power skewness and kurtosis, using the Rao's score test on the APD family". *J. Appl. Stat.*, 45, 2307–2327, 2017.

Numerical Comparisons For Solving Fractional Partial Differential Equations

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Abstract: In this paper, multivariate Padé approximation is applied to fractional power series solutions of fractional partial differential equations. Some numerical examples are presented to show the efficiency of multivariate Padé approximation. A comparison of the proposed method is made with the exact solution, Adomain Decomposition Method (ADM), generalized differential transformation method (GDTM) and Homotopy Perturbation Method (HPM). As it is seen from comparisons, multivariate Padé approximation gives reliable solutions and numerical results.

Keywords: Multivariate Padé approximation, Burgers equation, Fractional partial differential equations.

Mathematics Subject Classification: 65N20, 35R11

References

- [1] A. Saadatmandi, M. Dehghan, A tau approach for solution of the space fractional diffusion equation, *Comput. Math. Appl.* 62(3) (2011) 1135–1142.
- [2] S. Sharma, R.K. Pandey, K. Kumar, Collocation method with convergence for generalized fractional integro-differential equations, *J. Comput. Appl. Math.* 342 (2018) 419–430.
- [3] A. Saadatmandi, M. Dehghan, A Legendrecollocation method for fractional integro-differential equations, *J. Vib. Control* 17(13) (2011) 2050–2058.
- [4] A. Prakash, M. Kumar, K. K. Sharma, Numerical method for solving fractional coupled Burgers equations, *Applied Mathematics and Computation*, 260 (2015) : 314-320.
- [5] G. A. Baker and P. Graves-Morris, *Padé Approximants*, Addison-Wesley, 1981.
- [6] Celik E, Karaduman E, Bayram M, 2003, Numerical Solutions of Chemical Differential- Algebraic Equations, *Applied Mathematics and Computation*, 139(2-3): 259-264.
- [7] Celik E, Bayram M, 2004, Numerical solution of differential–algebraic equation systems and applications, *Applied Mathematics and Computation*, 154(2): 405-413.
- [8] Turut V, Guzel N, 2012, Comparing Numerical Methods for Solving Time-Fractional Reaction-Diffusion Equations, *ISRN Mathematical Analysis*, doi:10.5402/2012/737206.

Effect of behavior of predator on prey predator interactions

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Abstract: In this paper, we study the effect of behavior of predators on prey predator interactions. All the model equilibria have been found and their stability was established. The possibility of transcritical and Hopf bifurcation was also investigated and numerical simulation were given. The effect of prey refuge and fear also are detected. The cost of them allows the model to reach double transcritical points. The effect of competition of prey population is to convert the model from the stable limit cycle to a spirally stable equilibrium point of afraid prey, with predator. When it becomes large it converts the model to a stable trivial solution.

Keywords: Prey, Prey refuge, Holling type-II functional response, bifurcation.

Numerical solutions of some fractional singular equations

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Abstract: Some problems on the diffusion of heat and its equations in the mathematical physics and fluid dynamic are modeled by singular equations. In this talk, we try to present some nonlinear techniques for numerical solutions of some singular fractional problems. In this way, we use different methods and we compare the results.

Singularly Perturbed Fuzzy Initial Value Problems

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Abstract: In this work, we have firstly introduced singularly perturbed fuzzy initial value problems (SPFIVPs) and then we have given an algorithm for the solutions of them by using the extension principle given by Zadeh. We have presented some results on the behaviour of the α -cuts of the solutions. To show the robustness of the given algorithm, we have fuzzified some examples given in literature and then we have applied the algorithm.

Keywords: Initial Value Problem, Fuzzy Differential Equation, Singularly Perturbed.

References

- [1] Akin, O., Khaniev, T., Bayeg, S., & Türksen, I. B. (2016). Solving a second order fuzzy initial value problem using the heaviside function. *Turkish Journal of Mathematics and Computer Science*, 4 , 16–25.
- [2] Akin, O., Khaniev, T., Oruç, O., & Türksen, I. B. (2013). An algorithm for the solution of second order fuzzy initial value problems. *Expert Systems with Applications*, 40 , 953–957. doi:10.1016/j.eswa.2012.05.052.
- [3] Akin, O., & Oruç, O. (2012). A prey predator model with fuzzy initial values. *Hacet. J. Math. Stat.*, 41 , 387–395.
- [4] Alquran, M., & Doğan, N. (2010). Variational iteration method for solving two parameter singularly perturbed two point boundary value problem. *Applications and Applied Mathematics: An International Journal (AAM)*, 5 , 81–95.
- [5] Barros, L., Bassanezi, R., & Tonelli, P. (2000). Fuzzy modelling in population dynamics. *Ecol. Model*, 128 , 27–33.
- [6] Bede, B. (2013). *Mathematics of Fuzzy Sets and Fuzzy Logic*. Springer.
- [7] Buckley, J. J., & Feuring, T. (2001). Fuzzy initial value problem for n-th order linear differential equations. *Fuzzy Sets and Systems*, 121 , 247–255. doi:10.1016/S0165-0114(00)00028-2.
- [8] Byatt, W. J., & Kandel, A. (1978). Fuzzy differential equations. In *Proceedings of the International Conference on Cybernetics and Society*, Japan, 1 , .
- [9] Casanovas, J., & Rossell, F. (2005). Averaging fuzzy bio polymers. *Fuzzy Sets and Systems*, 152 , 139–158. doi:10.1016/j.fss.2004.10.019.
- [10] Doğan, N., AlQuran, M., Ertürk, V., & Momani, S. (2010). Variational iteration method for solving singularly perturbed two point boundary value problems. *International Journal of Pure and Applied Mathematics*, 58 , 11–19.

- [11] Doğan, N., Ertürk, V. S., & Akin, O. (2012). Numerical treatment of singularly perturbed two-point boundary value problems by using differential transformation method. *Discrete Dynamics in Nature and Society*, 2012 .doi:10.1155/2012/579431.
- [12] Doğan, N., Ertürk, V. S., Momani, S., Akin, O., & Yıldırım, A. (2011). Differential transform method for solving singularly perturbed volterra integal equations. *Journal of King Saud University Science*, 23 , 223–228. doi:10.1016/j.jksus.2010.07.013.
- [13] Doğan, N., Ertürk, V. S., & Shaher, M. (2013). Application of the variational iteration method for solving singularly perturbed volterra integral equations. *World Applied Sciences Journal*, 22 , 1657–1661.
- [14] El Naschie, M. (2005). From experimental quantum optics to quantum gravity via a fuzzy khler manifold. *Chaos Solitons Fractals*, 25 , 969–977. doi:10.1016/j.chaos.2005.02.028.
- [15] Friedrichs, K., & Wasow, W. (1946). Singular perturbations of non-linear oscillations. *Duke Mathematical Journal*, 13 , 367–381.
- [16] Hassan, Z., Hali, V., & Akbar, H. (2012). Fuzzy modeling and control of hiv infection, article id 893474, 17 pages. *Computational and Mathematical Methods in Medicine*, 2012 . doi:10.1155/2012/893474.
- [17] Kumar, M., Singh, P., & Mishra, H. K. (2010). A recent survey on computational techniques for solving singularly perturbed boundary value problems. *International Journal of Computer Mathematics*, 84 , 1439–1463. doi:10.1080/00207160701295712.
- [18] Neyfeh, A. (1973). *Perturbation Methods*. Wiley.
- [19] Oberguggenberger, M., & Pittschmann, S. (1999). Differential equations with fuzzy parameters. *Mathematical and Computer Modelling of Dynamical Systems*, 5 , 181–202.
- [20] O'Malley, R. J. (1974). *Introduction to singular perturbations*. Academic Press.
- [21] O'Malley, R. J. (1991). *Singular perturbation methods for ordinary differential equations*. Springer verlag.
- [22] Prandtl, L. (1905). "Über flüssigkeitsbewegung bei sehr kleiner reibung. In: *Verhandlung des dritten internationalen Mathematiker-Kongresses (Leipzig; Tübnr)*, (pp. 484–491).
- [23] Salama, A., & Hamdy, E. (2005). Interval schemes for singularly perturbed initial value problems. *Reliable Comput*, 11 , 41–58. doi:10.1007/s11155-005-5942-8.
- [24] Wasow, W. (1942). *On boundary layer problems in the theory of ordinary differential equations*. PhD Thesis New York University New York.
- [25] Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, 8 , 338–353.

Positive Solutions for Fractional Boundary Value Problems under a Generalized Fractional Operator

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Abstract: The work reported here concerns with study a generalized nonlinear fractional boundary value problems involving θ -fractional derivative in the Riemann-Liouville sense. The existence and uniqueness of positive solutions to the problem at hand are proved. Our discussion relies on the properties of the Green's function, the upper and lower solutions method, and the classical fixed point theorems in a cone. Moreover, building upper and lower control functions have an effective role in the analysis. Some examples are offered to justify the validity of theoretical findings.

Keywords: fractional boundary value problems, Fractional derivative, Control functions, Green's function, Fixed point theorem.

Mathematics Subject Classification: 34A08, 34B15, 34A12, 47H10, 34B18.

References

- [1] Abdo MS, Ibrahim AG, Panchal SK. Nonlinear implicit fractional differential equation involving Ψ -Caputo fractional derivative. Proc. Jangjeon Math. Soc. 2019;22(3):387-400.
- [2] Abdo MS, Panchal SK. Fractional integro-differential equations involving Ψ -Hilfer fractional derivative. Adv. Appl. Math. Mech. 2019;11(2):338-359.
- [3] Abdo MS, Panchal SK, Saeed AM. Fractional boundary value problem with Ψ -Caputo fractional derivative. Proc. Math. Sci. 2019;129(5):65. <https://doi.org/10.1007/s12044-019-0514-8>.
- [4] Abdo MS, Wahash HA, Panchal SK. Positive solution of a fractional differential equation with integral boundary conditions. J. Appl. Math. Computational Mech. 2018;17(2):5-15.
- [5] Ardjouni A, Djoudi A. Existence and uniqueness of positive solutions for first-order nonlinear Liouville-Caputo fractional differential equations. Sao Paulo J. Math. Sci. 2020;14:381390.
- [6] Almeida R. A Caputo fractional derivative of a function with respect to another function. Commun. Nonlinear Sci. Numer. Simul. 2017;44:460-481.
- [7] Almeida R. Fractional differential equations with mixed boundary conditions. Bull. Malays. Math. Sci. Soc. 2019;42(4):1687-1697.
- [8] Bai Z, Lu H. Positive solutions for boundary value problem of nonlinear fractional differential equation. J. Math. Anal. Applic. 2005;311(2):495-505.
- [9] Chidouh A, Guezane-Lakoud A, Bebbouchi R. Positive solutions of the fractional relaxation equation using lower and upper solutions. Vietnam J. Math. 2016;44(4):739-748.
- [10] Debnath L. A brief historical introduction to fractional calculus. Int. J. Math. Education Sci. Tech. 2004;35(4):487-501.

- [11] Gambo YY, Jarad F, Baleanu D, Abdeljawad T. On Caputo modification of the Hadamard fractional derivatives. *Adv. Difference Equ.* 2014;(1):10.
- [12] Hadamard J. Essai sur l'étude des fonctions données par leur développement de Taylor. *J. Math. Pures Appl.* 1892;8:101–186.
- [13] Harikrishnan S, Shah K, Kanagarajan K. Study of a boundary value problem for fractional order θ -Hilfer fractional derivative. *Arab. J. Math.* (2019);1-8.
- [14] Hilfer R. *Applications of fractional calculus in physics*, Singapore: World scientific. 2000;35(12):87-130.
- [15] Jarad F, Abdeljawad T, Baleanu D. Caputo-type modification of the Hadamard fractional derivatives. *Adv. Difference Equ.* 2012;2012(1):142.
- [16] Jarad F, Ugurlu E, Abdeljawad T, Baleanu D. On a new class of fractional operators. *Adv. Difference Equ.* 2017;2017(1):247.
- [17] Katugampola UN. New approach to a generalized fractional integral. *Appl. Math. Comput.* 2011;218:860–865.
- [18] Kilbas AA, Shrivastava HM, Trujillo JJ. *Theory and Applications of Fractional Differential Equations*. Elsevier, Amsterdam, 2006.
- [19] Khan H, Abdeljawad T, Aslam M, Khan RA, Khan A. Existence of positive solution and HyersUlam stability for a nonlinear singular-delay-fractional differential equation. *Adv. Difference Equ.* 2019;2019(1):1-13.
- [20] Kucche KD, Mali AD. Initial time difference quasilinearization method for fractional differential equations involving generalized Hilfer fractional derivative. *Comput. Appl. Math.* 2020;39(1):31.
- [21] Mainardi F. *Fractional calculus and waves in linear viscoelasticity: An introduction to Mathematical models*. World Scientific, 2010.
- [22] Malahi MA, Abdo MS, Panchal SK. Positive solution of Hilfer fractional differential equations with integral boundary conditions. *arXiv: 1910.07887v1*, 2019.
- [23] Malahi MA, Abdo MS, Panchal SK. Existence and Ulam HyersMittag-Leffler stability results of Ψ -Hilfer nonlocal Cauchy problem. *Rend. Circ. Mat. Palermo, II. Ser.* 2020. <https://doi.org/10.1007/s12215-020-00484-8>.
- [24] Mall S, Chakraverty S. Artificial neural network approach for solving fractional order initial value problems. *arXiv preprint arXiv:1810.04992*, 2018.
- [25] Li N, Wang C. New existence results of positive solution for a class of nonlinear fractional differential equations. *Acta Mathe. Scientia.* 2013;33B:847-854.
- [26] Qu H, Liu X. A Numerical Method for Solving Fractional Differential Equations by Using Neural Network. *Adv. Math. Phy.* 2015;2015:1-12.
- [27] Rostami F, Jafarian A. A new artificial neural network structure for solving high-order linear fractional differential equations. *Int. J. Computer Math.* 2017;95(3):528-539.
- [28] Podlubny I, *Fractional Differential Equations*. Academic Press, San Diego, 1999.
- [29] Khaldi R, Guezane-Lakoud A. Upper and Lower Solutions Method for Higher Order Boundary Value Problems. *Prog. Frac. Differ. Applic.* 2017;3:53-57.
- [30] Yan R, Sun S, Lu H, Zhao Y. Existence of solutions for fractional differential equations with integral boundary conditions. *Adv. Difference Equ.* 2014;2014(1):1-13.

- [31] Sousa JVC, de Oliveira CE. On the Ψ -Hilfer fractional derivative. *Commun Nonlinear Sci. Numer. Simul.* 2018;60:72-91.
- [32] Sousa JVC, Kucche KD, de Oliveira CE. Stability of Ψ -Hilfer impulsive fractional differential equations. *Appl. Math. Lett.* 2019;88:73-80.
- [33] Seemab A, Rehman MU, Alzabut J, Hamdi A. On the existence of positive solutions for generalized fractional boundary value problems. *Bound. Value Probl.* 2019;2019(1):186.
- [34] Vivek D, Elsayed E, Kanagarajan K. Theory and analysis of Ψ -fractional differential equations with boundary conditions. *Communications Appl. Anal.* 2018;22:401-414.
- [35] Sun Y, Zhao M. Positive solutions for a class of fractional differential equations with integral boundary conditions. *Appl. Math. Lett.* 2014;34:17-21.
- [36] Wang Y, Liu L, Wu Y. Positive solutions for a nonlocal fractional differential equation. *Nonlinear Anal.* 2011;74:3599-3605.
- [37] Zhou Y. Basic theory of fractional differential equations. Singapore: World Scientific, 2014.

Superquadratic functions in information theory

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Abstract: Using Jensen's inequality and the converse Jensen's inequality for superquadratic functions we obtain new estimates for Shannon's entropy of the random variable X and derive new lower and upper bounds for the Shannon entropy in the terms of the Zipf and Zipf - Mandelbrot's law.

Keywords: Superquadratic function, Jensen's inequality, Shannon entropy, Zipf-Mandelbrot's law.

Mathematics Subject Classification: 94A15, 94A17, 26D15, 26A51

References

- [1] S. Abramovich, G. Jameson and G. Sinnamon, "Refining Jensen's Inequality", *Bull. Math. Soc. Sc. Math. Roumanie* 47 (95), (2004), 3-14.
- [2] S. Abramovich, G. Jameson, G. Sinnamon, "Inequalities for averages of convex and superquadratic functions", *J. Inequal. Pure and Appl. Math.*, 5 (4), (2004), Article 91.
- [3] V. Diodato, "Dictionary of bibliometrics", New York, Haworth Press, (1994).
- [4] S. Banić, J. Pečarić, S. Varošanec, "Superquadratic functions and refinements of some classical inequalities", *J. Korean Math. Soc.* 45(2), (2008), 513-525.
- [5] M. E. J. Newman, "Power Laws, Pareto Distributions and Zipf's law", *Contemporary Physics*, (2005) 46:323-351.
- [6] M. Matić, C. E. Pearce, J. E. Pečarić, "Shannon's and related inequalities in information theory", *Survey on Classical Inequalities*, Springer, Dordrecht, (2000), 127-164.
- [7] D. S. Mitrinović, J. Pečarić, A. M. Fink, "Classical and New Inequalities in Analysis", Kluwer Academic Publishers, Dordrecht/ Boston / London, (1993).

Generalized Laguerre transforms of sequences

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Abstract: In this talk, we investigate a new class of polynomials which we call the associated polynomials. Firstly, we drive a general identity which generalizes the Laguerre transform of a sequence. More precisely, we give a three-term recurrence formula for calculating the associated polynomials. Secondly, as an application, we define the associated Fibonacci polynomials and give their characteristic properties.

Keywords: Laguerre transform, Associated Laguerre polynomials, Fibonacci numbers, explicit formulas, generating functions.

Mathematics Subject Classification: 33C45, 05A19, 11B83.

References

- [1] M. Abramowitz and I. A. Stegun, Handbook of Mathematical functions with formulas, graphs, and mathematical tables. Dover Publications, Inc., New York, 1992.
- [2] G. E. Andrews, R. Askey, R. Roy, Special functions. Encyclopedia of Mathematics and its Applications, 71. Cambridge University Press, Cambridge, 1999.
- [3] J. Riordan. An introduction to combinatorial analysis. Dover Publications, Inc., Mineola, NY, 2002.
- [4] M. Sebaoui, D. Laissaoui, G. Guettai, and M. Rahmani. On s-Lah polynomials. Ars Combinatoria 142(2019), pp. 111-118.

Dynamics of Lump-periodic, breather and two-wave solutions with the long wave in shallow water under gravity and 2D nonlinear lattice

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Abstract: A lump solution is a rational function solution which is real analytic and decays in all directions of space variables. The equation under consideration in this study is the (2+1)-dimensional generalized fifth-order KdV equation which demonstrates long wave movements under the gravity field and in a two-dimensional nonlinear lattice in shallow water. The collisions between lump and other analytic solutions is studied in this work. Using Hirota bilinear approach, lump-periodic, breather and two-wave solutions are successfully reported. In order to shade more light on the characteristics of the acquired solutions, numerical simulations have been performed by means of the 3-dimensional and contour profiles under careful choice of the values of the parameters involved.

Keywords: fifth-order KdV; Hirota bilinear; collision phenomena.

References

- [1] Y. Kivshar and G. Agrawal, *Optical Solitons: From Fibers to Photonic Crystals* (Academic Press, London 2003)
- [2] B. Frisquet, B. Kibler, and G. Millot, *Phys. Rev. X*, 3, 041032 (2013)
- [3] V. Zakharov and A. Gelash, *Physical Review Letters*, 111, 054101 (2013)
- [4] A. Chabchoub and N. Akhmediev, *Phys. Lett. A*, 377, 2590 (2013)
- [5] J.G. Liu, *Applied Mathematics Letters*, 86 36-41 (2018)

Optical solitons with the birefringent fibers without four-wave mixing via the Lakshmanan–Porsezian–Daniel equation

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Abstract: This article investigates the optical soliton to Lakshmanan–Porsezian–Daniel (LPD) model in birefringent fibers which incorporates two vector solitons. The optical solitons in the forms of bright, dark, singular, as well as combined form bright-dark, dark-singular, complex soliton and multiple soliton solutions are extracted successfully by the mechanism of Fan-extended sub equation approach with the effect of five free parameters. Furthermore, singular periodic wave solutions are obtained, as well as the constraint requirements for the existence of soliton solutions. The results suggest that the method is simple, direct, and effective, and that it may be applied to more complex phenomena with the help of symbolic calculations.

Keywords: Optical soliton; LPD equation; Extended fan-sub equation method; Generalized elliptic equation

References

- [1] A. Biswas, J.V. Guzmán, M. Ekici, Q. Zhou, H. Triki, A.S. Alshomrani, M.R. Belic, Optical solitons and conservation laws of Kudryashov's equation using undetermined coefficients, *Optik*, 202 (2020) 163417.
- [2] M. Inc, A.I. Aliyu, A. Yusuf and D. Baleanu, Dark-bright optical solitary waves and modulation instability analysis with $(2 + 1)$ -dimensional cubic-quintic non-linear Schrodinger equation, *Waves in Random and Complex Media*, 29(3) (2019) 393-402
- [3] M. Younis, U. Younas, S.U. Rehman, M. Bilal, A. Waheed, Optical bright–dark and Gaussian soliton with third order dispersion, *Optik* 134 (2017) 233-238.
- [4] D.D. Ganji, A. Asgari, Z.Z. Ganji, Exp-Function Based Solution of Nonlinear Radhakrishnan, Kundu and Laskhmanan (RKL) Equation, *Acta Appl Math* 104 (2008) 201–209, <https://doi.org/10.1007/s10440-008-9252-0>
- [5] R.T. Alqahtani, M.M. Babatin, A. Biswas, Bright optical solitons for Lakshmanan-Porsezian-Daniel model by semi-inverse variational principle, *Optik* 154 (2018) 109–114.

On the orthogonality of new combination of two orthogonal polynomials sequences

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Abstract: In this paper, we deal with a special case of an inverse problem in the theory of the orthogonal polynomials to analyze of the orthogonality of a monic polynomials sequence $\{Q_n\}_{n \geq 0}$ defined as a linear combination of a sequence of monic orthogonal polynomials sequence $\{P_n\}_{n \geq 0}$ such as

$$Q_n(x) + r_n Q_{n-1}(x) = P_n(x) + s_n P_n(x) + t_n P_{n-1}(x) + v_n P_{n-2}(x), \quad n \geq 0$$

when $v_n t_n \neq 0$ for every $n \geq 4$. Furthermore, we show that the relation between the corresponding linear functionals is

$$k(x-c)u = (x^3 + ax^2 + bx + d)v$$

where $c, a, b, d \in \mathbb{C}$ and $k \in \mathbb{C} \setminus \{0\}$. We end by giving an illustration for a special case of the above type relation.

Keywords: Orthogonal polynomials, Linear functionals, Inverse problem, Chebyshev polynomials.

Mathematics Subject Classification: 33C45, 42C05.

References

- [1] M. Alfaro, F. Marcellàn, A. Peña, M. L. Rezola, "On linearly related orthogonal polynomials and their functionals", J. Math. Anal. Appl. 287, pp. 307-319, (2003).
- [2] M. Alfaro, F. Marcellàn, A. Peña, M. L. Rezola, "On rational transformations of linear functionals: direct problem", J. Math. Anal. Appl. 298, pp. 171-183, (2004).
- [3] M. Alfaro, F. Marcellàn, A. Peña, M. L. Rezola, "When do linear combinations of orthogonal polynomials yield new sequences of orthogonal polynomials", J. comput. Appl. Math. 233, pp. 1446-1452, (2010).
- [4] M. Alfaro, A. Peña, M. L. Rezola, F. Marcellàn, "Orthogonal polynomials associated with an inverse quadratic spectral transform", Comput. Math. Appl. 61, pp. 888-900, (2011).
- [5] M. Alfaro, A. Peña, J. Petronilho, M. L. Rezola, "Orthogonal polynomials generated by a linear structure relation: inverse problem", J. Math. Anal. Appl. 401(??), pp. 182-197, (2013).
- [6] M. I. Bueno, F. Marcellàn, "Darboux transformations and perturbations of linear functionals", Linear Algebra Appl. 384, pp. 215-242, (2004).
- [7] T. S. Chihara, "An Introduction to Orthogonal Polynomials", Gordon and Breach. New York, 1978.

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