

# Fuzzy Adaptive Gains Fault Tolerant Control Based on Feedback Linearization of the Two Tanks System

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Abstract—To keep the robustness and the advantage of the fuzzy logic control technique and to reduce the high energy level of the feedback linearization technique, we suggest a novel FTC approach based on the feedback linearization control (FLC) with adaptive gains with fuzzy logic (FTCFAGFLC) applied to the two tanks coupled system with an actuator fault. The proposed control scheme makes the hybridization between the two techniques, intelligent and guarantees the stability studied by the feedback linearization technique. An adaptation with a fuzzy technique adjusted the parameter of the FLC used to adjust the controller gains in real-time and the proposed FTCFAGFLC was compared to the FLC. The results obtained indicate the efficiency of the suggested strategy in the presence of the actuator fault effect.

#### Keywords—Fuzzy Logic Control, Feedback Linearization, Adaptive Gains, Two Tanks System.

#### I. INTRODUCTION

Due to the continuous modernization of production tools, hydraulic industrial systems with coupled reservoirs are becoming more and more complex and sophisticated. In addition, technological advances in the field of numerical control and the development of control processors and the increased demand for reliability, availability, reconfigurability and operational safety of systems now allow the application more and more complex modern control algorithms at a lower cost. Automation, which is based on the notion of a system representing a group of parts constituting a structured whole, has allowed Man to create supervisory techniques such as diagnostics and fault-tolerant control of systems [1].

To determine the industrial requirements and expectations of reliability in hydraulic systems, most of the studies that are carried out show that the sets of faults that appear in fluidic systems with coupled reservoirs are related to the actuators [2]. A fault can be defined as a nonconformity, undesirable, or degradation of the parametric properties of the system. A fault can appear in different parts of the system, failure, or total loss of the system. Three classes of faults can be defined, faults in the actuator, system components, or sensor faults [3]. A faulttolerant control (FTC) system is characterized by its aptitude to maintain or recuperate acceptable performances close to those desired at nominal speed as well as in a degraded operating mode. An FTC has the ability to automatically adapt to faults that may affect its many components. The task incumbent on fault-tolerant systems is to synthesize control laws guaranteeing the stability and dynamic performance, in both of functional mode, in the faultless case, and after the manifestation of faults [4].

FTC commands use multiple control methods to deliver fault tolerance [5]. This paper propose a back-propagation neural network to control the system with actuator faults and components (leakage) in the level control process, Himanshukumar and all [6-7] explained that after a lot of work on passive FTC of tanks systems based on artificial intelligence, using the hybrid controller (Neural Network Plus PID Controller), fuzzy controller and takagi-sugeno fuzzy logic...etc. In [8], a modified sliding mode control (SMC) with PI-D type fractional order sliding surface for level control in a coupled two-tank system is developed, the performance of the proposed method improves with an increase in the fractional value of the derivative term. The author's in [9] study the proposed method must work under a control installed with its FTC connected. The fault is injected independently for each component. then, a controller (with inserted FTC) is reconfigured when a fault has occurred in one of the components of the coupled two-tank hydraulic system.

The control based on the feedback linearization is proposed by Tahir and all and Laucas and all [10] and [11] the objective is to realize the designed controller based on feedback linearization respectively. The papers of Nail et al. present a comparison betwin feedback linearization and backstepping controllers for coupled tanks system, the authors use artificial intelligence theory such as type-1 and type-2 fuzzy controls based on the robust controller to fault-tolerant of single and two tanks system [8] [12].

The important contribution of this work is the usage of the passive FTC based on the hybrid control between the adaptive gains of the feedback linearization with the fuzzy logic (FTCFAGFLC), Since this controller with gains adaptation is the newest to determine controller gains, the proposed FTCFAGFLC method was compared with feedback linearization control without gain adaptation.

This article is presented as follows. Mathematical modeling of the two tanks system is presented in section 2. Section 3 explains the FTCFAGFLC control based on the fuzzy logic feedback linearization method. The simulation results to validate the robustness of the proposed approach is presented in Section 4. Finally, the conclusion in the present paper are driven.

### II. MATHEMATICAL MODELLING OF THE TWO TANKS SYSTEM

This hydraulic system has two coupled reservoirs connected by a flow channel, "Fig. 1" [13]. The input of the hydraulic system is fed by a variable speed pump which delivers water to tank 1.



Fig. 1. Hydraulic system with coupled tanks [13].

The goal of the control problem is to adjust the command  $q_1$  to maintain the level in tank 2,  $h_2$  close to the set point level [14]. The two flow equilibrium equations, for reservoirs, are given by:

$$\begin{cases} \frac{dh_1}{dt} = \frac{1}{C}(-q_{12} + q_1) \\ \frac{dh_2}{dt} = \frac{1}{C}(q_{12} - q_0) \end{cases}$$
(1)

where

$$q_0 = c_2 \sqrt{2gh_2} \tag{2}$$

$$q_{12} = c_{12}\sqrt{2g(h_1 - h_2)}$$
 for  $h_1 > h_2$  (3)

$$c_2 = s_2 . a_2 \tag{4}$$

$$c_{12} = s_{12}.a_{12} \tag{5}$$

and

 $h_i(t)$ : the level of the liquid in the tank *i*;

C: the section of the two tanks 1 and 2;

 $q_1$ : the inlet flow generated by the pump;

 $q_{\rm 12}$ : the flow between the two tanks;

 $q_0$ : the flow rate out of tank 2;

 $c_{12}$ : the area of the coupling orifice;

 $c_2$ : the area of the outlet orifice;

g : the gravitation constant;

 $s_{12}$ : the channel of section 1;

$$s_2$$
: the channel of section 2;

 $a_{12}$ ,  $a_2$  the discharge coefficients of valve 1 and valve 2, respectively. For the coupled tank system, the inlet fluid flow,  $q_1 \ge 0$  is always positive, because the pump is continuously pumping water into tank 1. Finally, the nonlinear differential equation of the hydraulic system is given by:

$$\left| \frac{dh_1}{dt} = \frac{1}{C} (-s_{12}.a_{12}\sqrt{2g(h_1 - h_2)} + k_p.u) \\
\frac{dh_2}{dt} = \frac{1}{C} (s_{12}.a_{12}\sqrt{2g(h_1 - h_2)} - s_2.a_0\sqrt{2gh_2})$$
(6)

$$y = h_2 \tag{7}$$

with

y: the output of the system.

where

$$u = \begin{cases} u_{\max} & if \quad u \ge u_{\max} \\ 0 & if \quad u \le 0 \end{cases}$$
(8)

For this system, we define the state model [13], with

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} h_2 \\ h_1 \end{bmatrix}$$
(9)

such as

$$x = \begin{cases} \dot{x}_{1} = \alpha_{1} \cdot \sqrt{x_{2} - x_{1}} - \alpha_{2} \cdot \sqrt{x_{1}} \\ \dot{x}_{2} = -\alpha_{1} \cdot \sqrt{x_{2} - x_{1}} + k_{c} \cdot u \end{cases}$$
(10)

$$y = x_1 \tag{11}$$

and

$$\alpha_{1} = \frac{s_{12} \cdot a_{12} \sqrt{2g}}{C}; \alpha_{2} = \frac{s_{2} \cdot a_{2} \sqrt{2g}}{C}; k_{c} = \frac{k_{p}}{C}$$

#### III. FEEDBACK LINEARIZATION CONTROL DESIGN

In the last years, the theory of control with state feedback control theory has known significant developments. This method is based on the theory of differential geometry for the control of nonlinear systems. Among the techniques developed, we can particularly cite the method of feedback linearization with input-output decoupling (input output linearization control) [15].

Consider a system (10), described by the following nonlinear state representation:

$$\begin{cases} \dot{x} = f(x) + g(x)u\\ y = h(x) \end{cases}$$
(12)

With *u* is the control vector (input), *y* is the output vector, f(x) and g(x) are the vector fields respectively and h(x) is the output. The elements of the vector fields *f*, *g* and *h* are smooth functions. If we consider the case of systems with *m* inputs and *m* outputs, we seek a static loop of the form:

$$u = \alpha(x) + \beta(x)v \tag{13}$$

Such that the input-output behavior of the system (12) after looping is linear and decoupled. Thus we obtain a set of m independent mono-output subsystems where the inputs of subsystem i do not affect the output  $y_j$  and vice versa [12].

With *v*: new command variable of the linear system,  $\beta$ : Non-singular matrix of dimension  $m \times m$  and  $\alpha$ : Vector of dimension  $m \times 1$ . Notions of differential geometry and topology are often used, such as Lie derivative, Lie bracket, diffeomorphism, distribution, and involutivity.

The new command v makes it possible to reduce the inputoutput behavior of the system, defined by equation (12) to that of a linear system, with differentiating the outputs  $y_i$  of the system until the appearance of the old commands  $u_i$  using the derived from Lie:

- choice of system output quantities,
- calculation of vector relative degree and change of coordinates,
- non-linear state feedback,
- Asymptotic stability and reference tracking.

#### A. Step 1: Relative Degree Calculation

The linearization condition for checking whether a nonlinear system admits input-output linearization is the order of the relative degree of the system.

The relative degree  $r_i$  associated with each chosen output quantity  $y_i$  is calculated, which corresponds to the number of times that this output must be derived to make one of the control quantities appear explicitly. In our case, the commands appear for the first time in the second derivatives.

$$\begin{cases} y = h(x) = x_{1} \\ y^{(1)} = h_{1}(x) = L_{f}h(x) = \dot{x}_{1} = \alpha_{1}\sqrt{x_{2} - x_{1}} - \alpha_{2}\sqrt{x_{1}} \\ y^{(1)} = \alpha_{1}.(x_{2} - x_{1})^{0.5} - \alpha_{2}.x_{1}^{0.5} \\ y^{(2)} = h_{2}(x) = L_{f}^{2}h(x) = 0.5.\alpha_{1}.(\dot{x}_{2} - \dot{x}_{1})(x_{2} - x_{1})^{-0.5} \\ -0.5.\alpha_{2}.\dot{x}_{1}x_{1}^{-0.5} \\ y^{(2)} = 0.5.\alpha_{1}.(-\alpha_{1}.(x_{2} - x_{1})^{0.5} + k_{u}u - \alpha_{1}.(x_{2} - x_{1})^{0.5} \\ -\alpha_{2}.x_{1}^{0.5})(x_{2} - x_{1})^{-0.5} - 0.5\alpha_{2}(\alpha_{1}.(x_{2} - x_{1})^{0.5} \\ -\alpha_{2}.x_{1}^{0.5})x_{1}^{-0.5} \end{cases}$$
(14)

We note, for this output, a relative degree, r=2.

#### B. Step 2: Calculate of the Diffeomorphism

Diffeomorphism is used to transform a nonlinear system into another nonlinear system with performing a change of variables of the form:

$$\begin{cases} z_{1} = \phi_{1}(x) = h(x) = x_{1} \\ z_{2} = \phi_{2}(x) = L_{f}h(x) = \dot{x}_{1} = \alpha_{1} \cdot (x_{2} - x_{1})^{0.5} - \alpha_{2} \cdot x_{1}^{0.5} \\ z_{3} = \phi_{3}(x) = L_{f}^{2}h(x) = \dot{x}_{1} = 0.5 \cdot \alpha_{1} \cdot [(-\alpha_{1}(x_{2} - x_{1})^{0.5} + k_{u} \cdot u - \alpha_{1} \cdot (x_{2} - x_{1})^{0.5} + \alpha_{2} \cdot x_{1}^{0.5})(x_{2} - x_{1})^{-0.5}] - \\ 0.5 \cdot \alpha_{2}(\alpha_{1}(x_{2} - x_{1})^{0.5} + \alpha_{2} \cdot x_{1}^{0.5})x_{1}^{-0.5} \end{cases}$$
(15)

### C. Step 3: Controller Design using Pole Placement

The control law which converts the nonlinear model of the system (15) into an exact linear representation is given by:

$$\begin{cases} \dot{z}_{1} = z_{2} = \dot{x}_{1} \\ \dot{z}_{2} = z_{3} = 0.5.\alpha_{1}.[(-\alpha_{1}(x_{2} - x_{1})^{0.5} + (x_{2} - x_{1})^{0.5} + (x_{2} - x_{1})^{0.5} + (x_{2} - x_{1})^{0.5}] \\ -0.5.\alpha_{2}(\alpha_{1}(x_{2} - x_{1})^{0.5} + (x_{2} - x_{1})^{0.5} + (x_{2} - x_{1})^{0.5}) = v \end{cases}$$

$$(16)$$

Where

$$\begin{cases} (v + \alpha_1^2 - 0.5.\alpha_1.k_u u(x_2 - x_1)^{-0.5} - 0.5.\alpha_2^2 \\ u = \frac{+0.5.\alpha_1.\alpha_2.x_1^{-0.5}(x_2 - x_1)^{0.5})}{0.5.\alpha_1.k_u.(x_2 - x_1)^{-0.5}} \\ v = -k_1.z_1 - k_2.z_2 \end{cases}$$
(17)

#### IV. FUZZY ADAPTIVE GAINS FEEDBACK LINEARIZATION CONTROLLER DESIGN

One of the main drawbacks of the feedback linearization control is the choice of the gains  $k_1$  and  $k_2$  of the linear part, because it can damage the actuators affected by faults that change the operation and the performance of the system. In order to reduce these problems, several solutions have been provided, such as the adaptation of the gain  $k_1$  and  $k_2$  of the linear control with a continuous fuzzy logic. the control is calculated by:

$$v = -k_{1FL} \cdot z_1 - k_{2FL} \cdot z_2 \tag{18}$$

The configuration of the proposed method gain-scheduled fuzzy logic feedback linearization controller structure is exposed in "Fig. 2".



Fig. 2. Gain-scheduled fuzzy logic feedback linearization controller structure.

The membership functions of the input, the error and the outputs  $k_1$  and  $k_2$  are presented by "Fig. 3".





Fig. 3. The membership functions: (a) the input error and (b) the outputs.

With NB: negative big; NM: negative medium; EZ: about zero; PM: positive medium; PB: positive big. The table of fuzzy rules which to be used or inference mechanism is given by Table 1 :

	e					
k		NG	NM	EZ	PM	PB
	NG					R5
	NM				R4	
	EZ			R3		
	РМ		R2			
	PB	R1				

TABLE 2. THE FUZZY RULES

The membership functions of the input *e* and the outputs,  $k_1$  and  $k_2$  are normalized in the interval [-1,+1].

#### V. FUZZY GAIN-ADAPTIVE FEEDBACK LINEARIZATION FAULT TOLERANT CONTROL OF THE TWO TANKS SYSTEM

The dynamics mathematic model (10) of the the two tanks system presented in the first part of this article can be rewritten in the state-space with:

$$\dot{x} = f(x,t) + g(x,t)u_f \tag{19}$$

where

$$u_{f_a} = u + f_a(t) \tag{20}$$

and

 $f_a(t)$  is the actuator fault, with  $|f_a(t)| < |u_{\text{max}}|$ .

The actuator fault is assumed to be additive and modeled by an increase of H% in the control signals (*rect*). We assume that the fault appears at time t = 300 s and disappears at time t = 310 s "Fig. 4".



Fig. 4. Type of the actuator fault added to command u.

The function can be considered as a signal:

$$f_a(t) = H.rect(\frac{t-\tau}{T}) = u(t-t_1) - u(t-t_2)$$
 (21)

such as

rect : is the rectangular function;

H: is the amplitude;

T: is the fault duration;

 $\tau$ : is the center of the rectangular function *rect*;

u: is the the step function, with  $t_2 > t_1$ .

#### VI. SIMULATION RESULTS

In this part and in order to present the results of simulations we present the parameters of our system in "Table 2".

Definition	Parameter	value			
Section of each tank	С	9350.10-6 <i>m</i> <sup>2</sup>			
Section of the variable	\$ <sub>12</sub>	78.5. 10-6 <i>m</i> <sup>2</sup>			
opening of each valve	<i>S</i> <sub>2</sub>	78.5. 10-6 <i>m</i> <sup>2</sup>			
Discharge coefficient	C <sub>12</sub>	1			
Discharge coefficient	<i>C</i> <sub>2</sub>	0.6			
Pump gain	k <sub>p</sub>	450.10-6 <i>m</i> <sup>3</sup> / <i>s</i> . <i>v</i>			
Sensor gain	ks	41 v/m			
The gravitation constant	g	9.81 $m/sec^2$			

TABLE 2. THE PARAMETERS OF THE COUPLED TANKS [13].

#### A. Simulation without actuator faults:

The simulation results of the FLC and the fuzzy adaptive gains feedback linearization (GAFL) without faults are shown in Figures 5 and 6, for a step reference. It can be seen that the proposed controller (AGFL) provides good trajectory tracking performance and minimize the response time when comparing the results with the FLC control.





Fig. 5. Simulation results of the feedback linearization (FLC) without actuator faults applied to the hydraulic system: (a) tracking error, (b) command and (c) output y.



## B. Simulation with faults:

The type of fault is actuator and modeled by an increase of  $45\% u_{\text{max}}$  of the commands, the results of the simulation in the presence of actuator faults are given in Figures 8 and 9, and the evolution of the actuator faults is given in "Fig. 7".



Fig. 7. Evolution of the actuator faults.



Fig. 6. Comparison between the proposed controller GAFL and the FLC without actuator faults: (a) tracking error, (b) command and (c) output y of FLC & GAFL.

Fig. 8. Simulation results of the FLC with actuator faults applied to the hydraulic system: (a) tracking error, (b) command and (c) output y.



Fig. 9. Comparison between the proposed controller FTCFAGFLC and the FLC with actuator faults: (a) tracking error, (b) command and (c) output y of FLC & FTCFAGFLC.

From figures 8 and 9 we notice that when the actuator fault injected into the command u, the regulation and following errors are low and that there is a good tracking of the desired trajectories and the control signals are relatively smooth. Compared with the regulator FLC, oscillations with large amplitude are observed, so the regulator FTCFAGFLC reduces the effect of faults on the control performance with rapidly varying the values of the regulator parameters to quickly bring the system outputs to their desired values (t = 300s, t=310s). When the actuator faults occur, the value of the gains decreases to avoid the overshoot which may be caused by the increase in command u.

### VII. CONCLUSION

The work presented in this work summarizes the control of a two tanks system based on the FTCFAGFLC in the presence of an actuator fault. This involves developing an adaptive gains  $k_1$  and  $k_2$  of the FLC using fuzzy logic in order to ensure tracking performance in the case of fault addition. Initially, a theoretical study and the modeling of the hydraulic system were presented, then the synthesis of the FLC and the hybridization with the fuzzy logic in the case of the presence of an actuator fault. The results obtained show that this regulation by FTCFAGFLC presents good performances.

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