Nonlinear Dynamical Systems Modelling and Identification Using Type-2 Fuzzy Logic: Metaheuristic Algorithms Based Approach.

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Abstract—This paper presents a novel type-2 fuzzy model for nonlinear dynamical systems. This method can deal with the curve fitting and computational time problems of type-2 fuzzy systems. It is based on interval type-2 fuzzy systems and it is comprised of a parallel interconnection of two type-2 sub fuzzy models. The first sub fuzzy model is the primary model, which represents an ordinary model with low resolution for the nonlinear dynamical system under consideration. To overcome resolution quality problem, and obtain a model with higher resolution, we will introduce a second type-2 fuzzy sub model called error model which will represent a model for the error modelling between the primary model and the real nonlinear dynamical system. As the error model represents uncertainty in the primary model, it's suitable to minimize this uncertainty by simple subtraction of the error model output from the primary model output, which will lead to a parallel interconnection between them, giving then a unique whole final model possessing higher resolution. To apply this approach successfully, the model's representation and identification are considered in this investigation using type-2 fuzzy auto regressive (T2FAR) and type-2 fuzzy auto regressive moving average (T2FARMA) models. Identification is achieved by innovative metaheuristic optimization algorithms, like as firefly and biogeography-based optimization algorithms. To evaluate the effectiveness of the proposed method, it will be tested on three types of nonlinear dynamical systems. Computer investigations indicate that the proposed model may significantly improves convergence and resolution.

Keywords-dynamical system; fuzzy logic; modelling & identification; meta-heuristic algorithm.

I. INTRODUCTION

Fuzzy modelling uses the concepts of fuzzy logic to represent a given system. Fuzzy identification and fuzzy modelling are effective tools of approximating nonlinear dynamical systems, because of their universal approximation capability. The increasing demand for intelligent systems to solve complex real-world problems has highlighted the importance of fuzzy logic and it has been the focus of attention for many science and engineering researchers. A fuzzy identification system is a computational model founded upon the concepts of fuzzy set theory, fuzzy If-Then rules and fuzzy reasoning [1]. Fuzzy identification can be categorized as linguistic fuzzy modelling, fuzzy relational modelling and Takagi-Sugeno-Kang (TSK) modelling [2].

Fuzzy identification is constituted of two key steps; structure identification and parameter identification. The first step is very

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important and has to address the issue of constructing the framework for the fuzzy model using the input-output measurement pairs. There are many fuzzy methods described in the literature; these uses fuzzy partition based on particular objective functions with the aim of establishing suitable model structures [3], [4], [5], [6], [7]. Many different approaches have been put forward to identify parameters including gradient descent, Kalman filtering and nonlinear least squares.

In this investigation, a straightforward and effective identification method is proposed. It aims to offer a type-2 fuzzy model for nonlinear dynamical systems. A primary model is initially devised using input-output training data. Then the error between the measurements and the primary model output are modelled to create the error model. This latter model represents the uncertainties within the primary model; it can readily be modified by removing the error model output from the primary model output; these two models are interconnected in a parallel configuration. Having corrected the model output, there is a consequential improvement in resolution. Latest contributions in this subject are reported in [8], [9], [10], [11], [12], [13].

In this study, metaheuristic calculations or evolutionary computations such as GA, FA, PSO and BBO, which are families of stochastic algorithms, will be used to tune the fuzzy parameters adaptively. These algorithms are well suited to solving hard optimization problems and are frequently applied to numerous fields in diverse areas, as they are typically effective. The optimal solution is acquired through parallel processing in the population. These techniques are often inspired by biological and biogeographical evolution mechanisms. A population of individuals exposed to a particular environment will exhibit a range of behaviors [1]. To determine which optimization algorithm will provide optimal parameters for the proposed type-2 fuzzy system, a short comparative study of various optimization algorithm will be performed. This study will compare BBO and FA, a comparison, which according to the literature [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], does not appear to have been conducted. Note that in literature, references [23] and [24] focus mostly on the motivation behind the algorithms and differences in performance when they were applied only to particular benchmarks functions. By applying these algorithms to problems such as nonlinear dynamical systems modelling, this study will perform comprehensive comparisons.

Type-2 fuzzy models and evolutionary algorithms are briefly reviewed in sections II of this paper. Section III outlines the

proposed technique and the experimental results of the simulation studies are presented in section IV.

II. THEORY

A. Type-2 fuzzy model

Zadeh first proposed type-n fuzzy sets in 1975 [25]. These are characterized by a membership function (MF) that ranges over fuzzy sets of type-(n-1). Note that MF of a fuzzy set of type-1 ranges over the interval [0, 1]. This was followed up in 1999, when Karnik and Mendel introduced some basic definitions, mechanisms and algorithms for type-2 fuzzy sets [26]. A type-2 fuzzy set is where the membership is itself a type-1 fuzzy set. Compared to type-1 fuzzy logic systems (T1FLSs), type-2 systems (T2FLSs) are more efficient, particularly when dealing with noisy data and ambiguity [27]. A comparative study of T2FLSs and T1FLSs in real-world applications that exhibit measurement noise and modelling uncertainty, found the former to be superior [28], [29], [30].

1) Parameter update rules: The design of a type-2 fuzzy system for modelling includes determination of the unknown parameters of the antecedent and the consequent parts of the fuzzy if-then rules. In the antecedent parts, the input space is divided into a set of fuzzy regions and in the consequent parts the system behavior in those regions is designed automatically. In Gaussian type-2 fuzzy sets, uncertainties can be associated to the mean (center) and the standard deviation (STD). In this paper, the Gaussian MFs were chosen due to their utility in universal approximation and their ability to uniformly estimate continuous functions [31]. Fig. 1 (a) and (b) show Gaussian type-2 fuzzy sets with uncertain STD and uncertain mean, respectively. The mathematical expression for the Gaussian membership function (MF) is expressed as follows:

$$\widetilde{\mu} = \left(\frac{1}{2} \frac{(x-c)^2}{\sigma^2}\right),\tag{1}$$

where *c* and σ are the center and width of the MF and *x* is the input vector. In this study, type-2 fuzzy sets are formed by only considering the uncertainty on the mean, i.e., $c = [c_1, c_2]$ with a fixed STD σ , where c_1 and c_2 are the lower and upper bounds of the uncertainty interval, respectively.

B. Méta-heuristic Algorithms

This section provides a brief overview of PSO, BBO, FA meta-heuristic algorithms.



Figure 1. Gaussian type-2 fuzzy set with (a) uncertain standard deviation, (b) uncertain mean (center).

1) Particle swarm optimization (PSO): This meta-heuristic computation technique has been stimulated by the social behavior of congregating animals, such as bird flocks, fish schools and insect colonies, in which a group can effectively achieve an objective by using the common information of every element. In response to solving optimization problems, Eberhart and Kennedy introduced the PSO algorithm in 1995 as

an alternative to population-based search approaches (like genetic algorithms) [32].

2) Biogeography-based optimization (BBO): Inspired by studies of the spatial distribution of species of plants and animals together with the causes of their distribution and extinctions, BBO is a global optimization algorithm [33]. The algorithm considers island habitats and species populations, and the habitat's ability to support the populations. When an island is unable to sustain the population of a species easily, some members migrate to new islands and undergo speciation. Each island is a potential solution to the problem.

3) Firefly algorithms (FA): The firefly algorithm is another swarm intelligence algorithm [34]. FA was inspired by the flashing of fireflies. Each firefly species has a unique pattern of flashes; whilst the full range of flash functions have yet to be determined, it is known that flashing attracts mates. In some firefly species, males are attracted to sedentary females. In other species, the female copies the signal of a different species to lure males of that species, which the female then preys on. The flashing can also be used to send information between fireflies. The FA algorithm was inspired by idea of this attraction and passing of information.

III. PROPOSED TYPE2-FUZZY MODELLING APPROACH

The proposed framework constitutes of three stages:

- Stage 1: Primary model identification.
- Stage 2: Error process identification.
- Stage 3: Final model design.

The free parameters of the type-2 fuzzy system to be identified are:

- ✓ The centers of the premise Gaussian type-2 fuzzy membership functions from the data available, using the constant standard deviation $\sigma = 0.75$, lower membership function with constant amplitude 0.8 and higher membership function with constant amplitude 1.
- ✓ The centers of the consequence intervals, having a constant width equal to 0.5.

The fitness function that will be used throughout the study will be a mean square error (*MSE*) criterion that uses the actual and estimated values as follows:

$$MSE = \frac{\sum_{k=1}^{N} (y_k - \hat{y}_k)^2}{N} = \frac{\sum_{k=1}^{N} e^2}{N},$$
 (2)

with y_k is the actual measure, \hat{y}_k its estimate and N the length of data.

1) Primary model identification: For the first stage, the data set (u_1, y_1) is used to determine a type-2 fuzzy primary model (T2FPM) \hat{f}_p for the process (the considered nonlinear dynamical system). The scheme of the T2FPM identifier is mainly an online adaptation of \hat{f}_p where the lower script p is used to denote "Primary". The optimization algorithm which can be either the BBO, PSO, FA or GA algorithm, adjusts the model's parameters \hat{f}_p such that the error e_l between the process output y_1 and the T2FPM output \hat{y}_{p1} reaches its minimum value. Using a fuzzy auto regressive moving average (ARMA) model, which is a very popular stochastic time series modelling technique, we intend to identify \hat{f}_p . The ARMA model is one of a group of prediction formulas that attempt to predict an output of a system based on the previous outputs (new input). Assuming there are NR_1 fuzzy rules of a fuzzy ARMA model, the T2FPM \hat{f}_p is described as follows:

$$\begin{aligned} R_i: if \ u_1(k) \ is \ U_i(u_1) \ and \ y(k) \ is \ Y_i(y_k) \ and \ \dots \ and \ y(k) \\ & -n+1) \ is \ Y_i(y_{k-n+1}) \ then \\ \hat{y}_p = \overline{y}_i \qquad i = 1, 2, \dots, NR_1, \end{aligned}$$

where $U_i(u_1)$ is the premise type-2 membership function that is relative to regressor $u_1(k)$, $Y_i(y_{k-j})$ is the premise type-2 membership function that is relative to regressor y(k - j) and \bar{y}_i is an interval output membership function for the i^{th} rule. The parameters of \hat{f}_p to be trained by the optimization algorithm are the Gaussian centers of the type-2 fuzzy premise membership function $U_i(u_1), Y_i(y_k), \dots, Y_i(y_{k-n+1})$ and the consequence intervals $\bar{Y}_1, \bar{Y}_2, \dots, \bar{Y}_{NR1}$.

2) Error process identification: For generalization purposes, the primary model \hat{f}_p is validated with a new input u_2 to obtain a new general output signal \hat{y}_{p2} for T2FPM. Defining the error process (E_p) by the parallel interconnection of the process and T2FPM with input u_2 and output e_2 .

$$e_2 = \hat{y}_{p2} - y_2, \tag{3}$$

where e_2 is the general error, y_2 is the new process output corresponding to the input u_2 and \hat{y}_{p2} is the new T2FPM output corresponding to input u_2 . Once the error process output e_2 is obtained (Equation. 3), a second type-2 fuzzy system can be designed to model the general error signal e_2 . This model is denoted as the type-2 fuzzy general error model (T2FGEM). As e_1 is time series, it is appropriate to design a model that uses a fuzzy AR model that attempts to predict the new output based on the previous outputs. Thus, a given time series of data e_2 , is identified by the following fuzzy AR model:

$$\hat{e}_2(k+1) = \hat{E}_p[e_2(k), e_2(k-1), ..., e_2(k-n+1)].$$
 (4)

 NR_2 fuzzy rules describe the time series e_2 as follows:

$$R_i: if \ e_2(k) \ is \ E_i(e_{2,k}) \ and \ e_2(k-1) \ is \ E_i(e_{2,k-1}) \ and, ..., \\ and \ e_2(k-n+1) \ is \ E_i(e_{2,k-n+1}) \ then \\ \hat{e}_2 = \overline{E}_i \qquad i = 1, 2, ..., NR_2,$$

where $E_i(e_{2,k})$ is the premise type-2 membership function relative to regressor $e_2(k)$ and \overline{E}_i an interval output membership function for the i^{th} rule. The task is to train the T2FGEM parameters to minimize the error e_3 . The parameters of T2FGEM to be trained by the optimization algorithms are Gaussian centers of the type-2 fuzzy premise membership functions $E(e_{2,k}), E(e_{2,k-1}), \dots, E(e_{2,k-n+1})$ and the output intervals $\overline{E}_1, \overline{E}_2, \dots, \overline{E}_{NR2}$.

3) Final model design: The third and the final step is to obtain the final model \hat{f}_F by interconnecting the type-2 fuzzy primary model \hat{f}_p and the type-2 fuzzy general error \hat{E}_p in a parallel configuration. This framework enables the error modelling obtained in the T2FPM to be decreased; therfore, a sharper model \hat{f}_F will be acquire. Thus, the output \hat{y}_2 of the final model can be given by:

$$\hat{y}_2 = \hat{y}_{p2} - \hat{e}_2. \tag{5}$$

IV. RESULTS AND DISCUSSION

In this section, three models for the representation of SISO systems are used for testing the ability of our approach to

approximate large classes of nonlinear dynamical systems and to assess the effectiveness and efficiency of the cited evolutionary algorithms for the optimization of the dynamical models [35].

Model II

$$y_p(k+1) = f[y_p(k), y_p(k-1), \dots, y_p(k-n+1)] + \sum_{i=0}^{m-1} \beta_i u(k-1).$$
(6)

$$v_{i}(k+1) = \sum_{i=1}^{n-1} \alpha_{i} v_{i}(k-1) + a[u(k-1)] + a[u(k-$$

$$y_p(k+1) = f[y_p(k), y_p(k-1), \dots, y_p(k-n+1)]$$
$$+g[u(k), u(k-1), \dots, u(k-m+1)]. (8)$$

A. Determination of the metaheuristic algorithms paramaters

Population size is an important factor in determining the optimal solution. Where the population size increases, the solution in exploration space will be improved, but it increases the computation complexity. In order to make fair comparison, the corresponding population size for each algorithm is set at 200. The setting values of algorithmic control parameters of the mentioned algorithms adopted in this paper are given as follows. In PSO, self and swarm confidences $c_1 = c_2 = 2$ and the inertia factor $\omega = 0.75$. In BBO algorithm, the mutation rate m = 0.001, immigration rate δ_k and emigration rate μ_k take like the linear migration curves (Fig. 2). In FA algorithm, light absorption coefficient $\sigma = 1$, the degree of attractiveness of the firefly at r = 0 is $\beta_0 = 2$, mutation coefficient damping ratio $\alpha = 0.98$. In GA algorithm, crossover probability and mutation probability are set to 0.8 and 0.05, respectively.



Figure 2. Linear migration curves for BBO where δ is the immigration rate and β the emigration rate.

B. Modelling and identification of system I

The process to be identified is described by the following second-order difference equation:

$$y_p(k+1) = f[y_p(k), y_p(k-1)] + u(k), \qquad (9)$$

$$f[y_p(k), y_p(k-1)] = \frac{y_p(k)y_p(k-1)[y_p(k)+2.5]}{1+y_p^2(k)+y_p^2(k-1)}.$$
 (10)

f is the part of equation. 9 to be identified in the first stage by the primary model \hat{f}_p according to the following second order fuzzy AR model:

$$R_i: if y(k) is Y_i(y_k) and y(k-1) is Y_i(y_{k-1}) then$$

 $\hat{f}_{pi} = \overline{f}^i$ $i = 1, 2, ..., NR_1,$

where y(k) and y(k-1) are the regressors and $Y_i(y_k)$, $Y_i(y_{k-1})$ are type-2 fuzzy premise membership functions and \overline{f} are the adjustable intervals of the consequences.

During the second stage, the error process \hat{E}_p is identified with a second order type-2 fuzzy AR as follows:

$$R_i$$
: if $e_2(k)$ is $E_i(e_{2,k})$ and $e_2(k-1)$ is $E_i(e_{2,k-1})$ then
 $\hat{e}_2 = \bar{E}_i$ $i = 1, 2, ..., NR_2$,

where $e_2(k)$ and $e_2(k-1)$ are the regressors and $E_i(e_{2,k})$ and $E_i(e_{2,k-1})$ are type-2 fuzzy premise membership functions and \overline{E}_i are the adjustable intervals of the consequences.

We simulated the proposed method with $NR_1 = NR_2 =$ 40 rules therefore the identifier will be made up of 80 rules (40 rules for the T2FPM model and 40 rules for the T2FGEM model) which gives a total of $80 \times 3 = 240$ adjustable parameters. For the first stage we have two regressors, and one interval output for each rule, which gives 2×40 premise parameters and 1×40 consequence parameters (120 free parameters for the first stage). Same statistics for the second stage, therefore, giving us then 240 free parameters in all.

Fig. 3 and 4 present the primary model output and the final model output respectively corresponding to PSO, BBO, FA and GA for the following input data.

$$u = u_2(k) = \sin\left(\frac{2\pi k}{25}\right) \text{ for } 1 \le k \le 50 \& 150 \le k \le 200. (11)$$
$$u = u_2(k) = 0.5 \sin\left(\frac{2\pi k}{10}\right) + 0.5 \sin\left(\frac{2\pi k}{5}\right) \text{ for } 50 \le k \le 150. (12)$$

By a visual inspection of Fig. 3 and 4, we clearly see that the final model is much better than the primary model with all discussed optimization methods. We confirm this fact by the visual comparison given in Fig. 5 where we present a superposition of error curves showing that the result obtained with BBO algorithm is the better one compared to the other techniques.



In the following, we compare quantitatively the performances of the method in terms of a fitness function using optimization algorithms PSO, BBO, FA and GA. We run our algorithm 20 times as independent trials. Statistical performance measures like Worst, Mean, Best, Standard Deviation (SD) and Convergence Rate of the fitness function (Equation. 2) are

estimated and indicated in Table. I, where, BBO algorithm was able to get the best over the 20 independent trials for the



Figure 5. Comparison between the primary and the final models in terms of errors with (*a*) PSO method (*b*) BBO method (*c*) FA method (*d*) GA method.

discussed techniques, in terms of best cost function value, mean of fitness function for 20 independent trails, and lowest value of standard deviation. System identification based on PSO optimization has the second lowest best cost function value, mean and standard deviation, but FA and GA have failed in terms of mean and standard deviation. BBO and PSO seems to produce better results after every new iteration, which leads to decreased the standard deviations and mean values contrary to the FA and GA. In terms of convergence rate, we note that FA and GA have faster convergence rates compared to PSO and BBO, but BBO has faster convergence speed compared to PSO.

TABLE I. RESULTS OF FITNESS FUNCTION AFTER 200 ITERATIONS FOR 20 RUNS INDEPENDENTLY.

Algorithm	Worst	Mean	Best	SD	Converge- nce rate
PSO	7.8820e-4	1.6196e-4	3.1008e-5	1.6196e-4	1
BBO	7.7998e-5	2.2326e-5	1.2501e-5	6.9036e-5	0.9782
FA	0.5896	0.0331	5.8206e-5	0.1319	0.7861
GA	0.1036	0.0289	0.0017	0.0387	0.95

For more statistical analysis, let's consider some statistical

measures like means and confidence intervals for the 83th parameter over 20 independent trials (Table. II).

TABLE II. MEAN VALUES AND CONFIDENCE INTERVALS OF THE $83^{\mbox{\tiny TH}} PARAMETER$ OVER 20 TRIALS.

Algorithm	Mean		Confidence interval [95%]	
-	Stage 1	Stage 2	Stage 1	Stage 2
PSO	-0.4124	-2.0840	[-1.6427 : 0.8179]	[-3.2614:-0.9062]
BBO	6.6136	0.9665	[tends to zero]	[0.4568:1.4762]
FA	0.8611	-0.1887	[-1.7125:3.4347]	[-2.8145:2.4371]
GA	0.4773	-0.2173	[-1.7996:2.7542]	[-3.4451:30105]

Depending on the above statistical measures and simulation results, we consider BBO to be the best optimization algorithm for this problem of type-2 fuzzy system identification, due to it leads to more precise reconstruction for the parameters of the type-2 fuzzy identifier.

C. Validation and generalization tests

Let's now test the effectiveness of the method on more complicated system (Equation. 7) by considering the special case governed by the following difference equation:

$$y_p(k+1) = 0.3 \times y_p(k) + 0.6 \times y_p(k-1) + f[u(k)],$$
(13)

where the unknown function f to be identified has the following form:

$$f(u) = 0.6 \times \sin(\pi u) + 0.3 \times \sin(3\pi u) + 0.1 \times \sin(5\pi u).$$
(14)

the input signal u to both plant and model is chosen to be a sinusoid:

$$u(k) = \sin\left(\frac{2\pi k}{250}\right). \tag{15}$$

We simulated the method with the same number of rules as with system I. In Fig. 6 (a), (b), (c), (d) we give the final results of the type2-fuzzy identifier using PSO, BBO, FA and GA, respectively. By inspecting the zooms, we confirm the superiority of BBO algorithm.



Figure 6. System II identification results using: (a) PSO (b) BBO (c) FA (d) GA.

We consider in what follows the identification of a more complicated plant for which both input and output are present in the nonlinearity. In this case, the plant is described by Model III (Equation. 8) for which we consider the following example:

$$y_p(k+1) = \frac{y_p(k)}{1+y_p(k)^2} + u^3(k).$$
(16)

$$y_p(k+1) = f(y_p(k), u(k)).$$
 (17)

where in this case the unknown function f to be identified has more general form due to that its independent variables are the input and output signals. The input signal u is chosen to be:

$$u(k) = \sin\left(\frac{2\pi k}{25}\right) + \sin\left(\frac{2\pi k}{10}\right). \tag{18}$$

We simulated this case with the same number of rules as with system I. In Fig. 7 (a), (b), (c) and (d) we present a superposition of the plant output and the final output of the type2-fuzzy identifier using PSO, BBO, FA and GA, respectively. Always, the superiority of BBO algorithm is confirmed (see the zooms in Fig. 7).



We summarize in Table. III the performances in terms of goodness for the used optimization techniques where we show that the BBO algorithm maintains absolutely its superiority whether for nonlinear dynamical systems identification compared to PSO, FA and GA. PSO algorithm proved its superiority for nonlinear dynamical systems, compared to FA and GA, Finally, the worst results in this investigation were obtained with GA.

TABLE III. PERFORMANCES IN TERMS OF GOODNESS FOR THE USED OPTIMIZATION TECHNIQUES.

Algorithm	Nonlinear dynamical systems				
U	Model I	Model I	Model I		
PSO	Very good	Very good	Very good		
BBO	Excellent	Excellent	Excellent		
FA	good	good	good		
GA	fair	fair	fair		

V. CONCLUSION

In this paper, we have presented a method to solve the classical problem of nonlinear dynamical systems identification. The proposed technique is based on type-2 fuzzy models using metaheuristic algorithms. This approach allows to generalize the notion of identification by adding a new identification module, called error model. The introduced type-2 fuzzy error model was used as a complement of the primary identified model in order to improve the fitting quality, which gave a more precise fitting. Optimization in type-2 fuzzy logic identification has been

assured by applying some metaheuristic algorithms (GA, PSO, FA and BBO). Experimental results and comparative studies showed the effectiveness of the proposed approach for the problem of nonlinear dynamic systems modelling, and the best optimization results was obtained with BBO method.

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