

An Improved Recursive Least Square Algorithm For Adapting Fuzzy Channel Equalizer

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ABSTRACT

Adaptive filters have been thoroughly investigated in digital communication. They are especially exploited as equalizers, to compensate for channel distortions, although equalizers based on linear filters perform poorly in nonlinear distortion. In this paper, a nonlinear equalizer based on a fuzzy filter is proposed and a new algorithm for the adaptation parameters is presented. The followed approach is based on a regularization of the Recursive Least Square (RLS) algorithm and an incorporation of fuzzy rules in the adaptation process. The proposed approach, named Improved Fuzzy Recursive Least Square (IFRLS), enhances significantly the fuzzy equalizer performance through the acquisition of more convergence properties and lower steady-state Mean Square Error (MSE). The efficiency of the IFRLS algorithm is confirmed through extensive simulations in a nonlinear environment, beside the conventional RLS, in terms of convergence abilities, through MSE, and the equalized signal behavior. The IFRLS algorithm recovers the transmitted signal efficiently and leads to lower steady-state MSE. An improvement in convergence abilities is noticed, besides the RLS.

Keywords-channel equalization; digital communication; nonlinear channels; adaptive fuzzy filtering

I. INTRODUCTION

Channel equalization has attracted a great attention owing its success in achieving high-speed data transmissions. Equalization technique can cancel linear and nonlinear distortions and therefore recover the transmitted data [1, 2]. Linear distortion can be caused by inter-symbol, co-channel and adjacent channel interferences, in the presence of Additive White Gaussian Noise (AWGN). Burst noise, amplifiers, converters, and the modulation process can produce nonlinear distortions. Different signal processing techniques have been implemented to perform adaptive channel equalization [3-11]. Conventional adaptive filtering techniques [2, 4] perform well on linear channels, nevertheless, they perform poorly on severe and nonlinear channels [3, 6-11]. A diversity of nature and human intelligence-inspired techniques for channel equalization have been proposed [3, 5-14], including artificial neural networks [5-11], particle swarm optimization [12],

artificial bee colony [13], firefly [14], and fuzzy logic [15-19]. Among them, fuzzy logic is considered as a natural manner to represent and process uncertain data [20]. The revealed benefits of fuzzy logic are the ability to reduce the system complexity and procuring more robustness [15, 17, 19]. Fuzzy logic incorporates a simple "If-Then" rule-based approach to solve a problem rather than modeling the system mathematically [15, 20-22].

Several researches have reported that incorporating fuzzy logic in adaptive filter structures improves their performance [15, 16]. Adaptive fuzzy filters have been widely used in nonlinear channel equalization [15, 17, 19, 23-26]. If-then fuzzy rules using input-output data pairs of the channel allow implementing adaptive equalizers to eliminate severe channel distortions [16, 17]. Adaptive algorithms are applied to change parameters of the adaptive nonlinear fuzzy equalizers through membership functions such as least square and RLS algorithms [15, 16, 19, 23-26].

In this context, a new approach to adapt the nonlinear fuzzy equalizer is proposed. The new adaptation algorithm uses fuzzy logic reasoning through nonlinear membership functions in addition to the improvement procured in the RLS. The proposed approach, named Improved Fuzzy Recursive Least Square (IFRLS) algorithm, consists of a particular regularization of the RLS algorithm that improves significantly the adaptive fuzzy equalizer and guarantees a great improvement in performance by acquiring more convergence abilities and lower steady-state MSE.

II. FUZZY LOGIC FUNDAMENTALS

Fuzzy systems perform nonlinear relationships between the input and the output. They do not make assumptions concerning the process structure, or invoke any kind of probabilistic distribution model [15-18]. The basic structure of a general fuzzy logic system is illustrated in Figure 1.

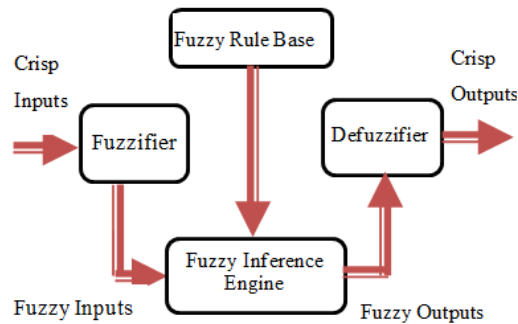


Fig. 1. Structure of a fuzzy logic system.

The principal form of a fuzzy logic system includes the fuzzy rule base, fuzzification process, inference engine, and the defuzzification process [15, 23, 26].

A. Fuzzy Rule Base

The fuzzy rule base is the main module of a fuzzy system and consists of expert knowledge formed of a set of linguistic rules such as "IF a set of condition are satisfied, THEN, a set of consequence are inferred". The fuzzy rules are combined in the inference engine to produce a fuzzy output.

B. Inference Engine

The inference engine provides the relationship between the fuzzy input in terms of membership functions and the fuzzy output.

C. Fuzzification Phase

Through the fuzzification step, the crisp input samples are converted into the fuzzy input described by the membership function, thus offering the degree to which the input scalar belongs to the fuzzy set, and are then mapped into the fuzzy output by means of the inference fuzzy rules.

D. Defuzzification Phase

The defuzzification stage is intended for mapping the fuzzy sets yielded by the inference engine and evaluates the fuzzy output to the crisp output in a weighted average way.

III. ADAPTIVE FUZZY EQUALIZER

The schematic of the adaptive fuzzy equalizer is depicted in Figure 2.

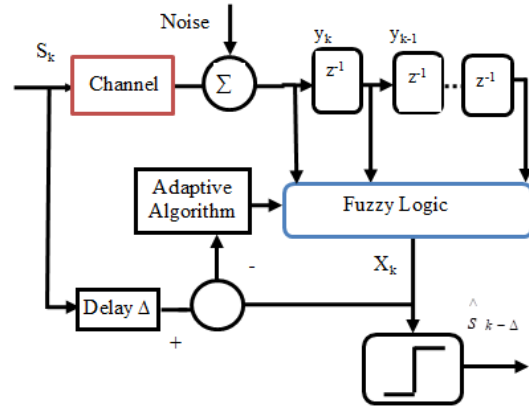


Fig. 2. The fuzzy logic-based equalizer.

A. Design of the Adaptive Fuzzy Equalizer

The equalization problematic is to recover the transmitted unobserved data sequence S_k with the highest reliability, through exploiting the information given by the noisy observation Y_k [1, 2]. The samples Y_k are connected to the fuzzy equalizer inputs, then the fuzzy sets are defined over the input space, and the membership function is incorporated into the equalizer. An adaptive algorithm performs the adaptation process on the free parameters of the fuzzy equalizer [15, 16, 24, 26]. The design of the adaptive fuzzy equalizer starts through expressing the membership functions denoted as: $\mu_{F_i^{j_i}}(\cdot): U \equiv [c_i^-, c_i^+] \rightarrow R$.

The intervals $[c_i^-, c_i^+]$ are labeled as $F_i^{j_i}$, where $i = \overline{1, n}$, $j_i = \overline{1, m_i}$, and U and R are the input and the output spaces, respectively. For each $Y_i \in [c_i^-, c_i^+]$, there is at least one $\mu_{F_i^{j_i}}(Y_i) \neq 0$. The most used membership function is of Gaussian type [15, 24]:

$$\mu_{F_i^{j_i}}(Y_i) = \exp\left(-\frac{1}{2}\left(\frac{Y_i - \bar{Y}_i^{j_i}}{\sigma_i^{j_i}}\right)^2\right) \quad (1)$$

where $\bar{Y}_i^{j_i}$ and $\sigma_i^{j_i}$ are the center and spread parameter, respectively. Afterward the fuzzy IF ... THEN ... rules (\mathfrak{R}) are defined as following:

$$\mathfrak{R}^{(j^1, \dots, j^n)}: \text{IF } y_1 \text{ is } F_1^{j^1} \text{ and } \dots \text{ and } y_n \text{ is } F_n^{j^n}, \text{ THEN } d \text{ is } g^{j^1, \dots, j^n} \quad (2)$$

where g^{j^1, \dots, j^n} are the fuzzy sets labels in the output space, they correspond to the linguistic terms of the rules (1), d is the desired output of the adaptive fuzzy equalizer $d=S_{k-\Delta}$, and k is the time index.

B. The Proposed Fuzzy Recursive Least Square Algorithm

The equalizer output X_k is performed by (3). Let $Y(k) = [y(k), y(k-1), \dots, y(k-n)]$ and $\theta^{(j^1, \dots, j^n)}$ be the points at which the g^{j^1, \dots, j^n} of the THEN parts have maximum membership values. $\theta^{(j^1, \dots, j^n)}$ are the free parameters of the

$$X_k(Y) = \frac{\sum_{j^1=1}^{m_1} \sum_{j^n=1}^{m_n} \theta^{(j^1, \dots, j^n)} (\mu_{F_1^{j^1}}(Y_1) \dots \mu_{F_n^{j^n}}(Y_n))}{\sum_{j^1=1}^{m_1} \sum_{j^n=1}^{m_n} (\mu_{F_1^{j^1}}(Y_1) \dots \mu_{F_n^{j^n}}(Y_n))} \tag{3}$$

$$\Theta = \left[\theta^{(1,1, \dots, 1)}, \dots, \theta^{(m_1, 1, \dots, 1)}; \theta^{(1, 2, 1, \dots, 1)}, \dots, \theta^{(m_1, 2, 1, \dots, 1)}; \dots; \theta^{(1, m_2, 1, \dots, 1)}, \dots, \theta^{(m_1, m_2, 1, \dots, 1)}; \dots; \theta^{(1, m_2, \dots, m_n)}, \dots, \theta^{(m_1, m_2, \dots, m_n)} \right]^T \tag{4}$$

$$Q(y) = \frac{\mu_{f_1^{j^1}}(Y_1) \dots \mu_{f_n^{j^n}}(Y_n)}{\sum_{j^1=1}^{m_1} \dots \sum_{j^n=1}^{m_n} (\mu_{f_1^{j^1}}(Y_1) \dots \mu_{f_n^{j^n}}(Y_n))} \tag{5}$$

$$Q(Y) = \left(q^{(1,1, \dots, 1)}(Y), \dots, q^{(m_1, 1, \dots, 1)}(Y); q^{(1, 2, 1, \dots, 1)}(Y), \dots, q^{(m_1, 2, 1, \dots, 1)}(Y); \dots; q^{(1, m_2, 1, \dots, 1)}(Y), \dots, q^{(m_1, m_2, 1, \dots, 1)}(Y); \dots; q^{(1, m_2, \dots, m_n)}(Y), \dots, q^{(m_1, m_2, \dots, m_n)}(Y) \right)^T \tag{6}$$

Equation (3) can be rewritten as:

$$X_k(Y) = Q^T(Y) \cdot \Theta \tag{7}$$

The adaptive process accomplishes the minimization of the cost function between the adaptive fuzzy equalizer output $X_k(Y)$ and the desired output $d_k = s_{k-\Delta}$ as:

$$J_k = \sum_{i=0}^k \lambda^{k-i} (d(i) - X_k(Y(i)))^2 + \gamma f(\Theta_k) \tag{8}$$

where λ is the forgetting parameter, γ the Lagrange multiplier ensuring regularization, and $f: R^N \rightarrow R$ a general convex function.

We developed a new approach to adapt the Θ parameters. The proposed approach consists of an improvement of the RLS algorithm [27, 28] conjointly to the incorporation of the fuzzy logic membership functions. The proposed adaptive IFRLS algorithm is partially inspired from [27, 28] but our approach incorporates fuzzy logic reasoning through membership functions thus procuring more suppleness and effectiveness.

The parameters adaptation process is performed explicitly as follows:

$$\varphi(k) = Q(Y(k)) \tag{9}$$

$$P(k) = \frac{1}{\lambda} [P(k-1) - K(k)\varphi^T(k)P(k-1)] \tag{10}$$

$$K(k) = P(k-1)\varphi(k)[\lambda + \varphi^T(k)P(k-1)\varphi(k)]^{-1} \tag{11}$$

adaptive fuzzy equalizer that are collected as in (4). Θ is of $\prod_{i=1}^n m_i$ dimension. At this phase, the fuzzy basis function is expressed as in (5). The vector $Q(y)$ is of the dimension of the vector Θ , and can be explicitly written as in (6).

$$\Theta(k) = \Theta(k-1) + K(k)\varepsilon(k) - \gamma(1-\lambda)P(k)\text{sgn} \Theta(k-1) \tag{12}$$

$$\varepsilon(k) = d(k) - \Theta(k-1)\varphi(k) \tag{13}$$

where $P(-1) = \delta.I$, δ is a small positive number, Q is defined by (5), $\text{sgn}(\cdot)$ is the sign function, and γ is the regularizing parameter set as in [27]. Note that the third term in (12) materializes the regularization term performing the improvement of the RLS algorithm. The updating principle of the above equations is similar to those obtained through the RLS algorithm. Nevertheless, the regularization parameter in addition to the fuzzy logic nonlinear membership functions procures more suppleness and nonlinear processing to the proposal.

IV. SIMULATION RESULTS AND DISCUSSION

We used MATLAB to carry out all simulations. The digital message applied to the channel is made of uniformly distributed bipolar random numbers $\{-1, 1\}$. The channel output is corrupted by zero mean AWGN with a signal to noise ratio (SNR) of 20 dB. The forgetting parameter is set to $\lambda = 0.995$ for all the algorithms. The regularization parameter for the IFRLS is set to $\gamma = 1.5$ as in [27].

A. Nonlinear Channel Model

A nonlinear channel model composed of a discrete-time equivalent modelled as a Finite Impulse Response (FIR) filter, followed by polonium nonlinearity as depicted in Figure 3 was considered.

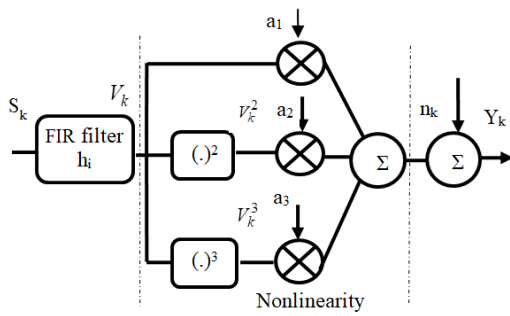


Fig. 3. Nonlinear channel model.

Channel nonlinearities can occur in data transmission from the saturation phenomena of the amplifiers when they operate in their high-gain limits [1, 17, 29]. Channel nonlinearity spreads the signal spectrum and leads to nonlinear amplitude and phase distortions. We selected two challenging nonlinear channel equalization problems, designated by CN1 and CN2. The first nonlinear channel model output is expressed as [17, 26]:

$$Y(k) = a_1(V(k)) + a_3(V(k))^3 + n(k) \tag{14}$$

$$V(k) = h_1S(k) + h_2S(k-1) + h_3S(k-2) \tag{15}$$

where $S(k)$, $Y(k)$ are the channel input and output, respectively, $[a_1 \ a_2 \ a_3]=[1 \ 0 \ -0.9]$ are coefficients that control the nonlinearity degree, $n(k)$ is the Gaussian noise, and $[h_1 \ h_2 \ h_3]=[1 \ 0.5 \ 0]$ are the FIR channel coefficients [17].

The second nonlinear channel model is more challenging, as in [17]. The nonlinear part is described by (14), whereas the FIR channel is chosen to be of non-minimum phase with coefficients set to $[h_1 \ h_2 \ h_3] = [0.3482 \ 0.8704 \ 0.3482]$. Such FIR channels are widely used in the literature to assess the equalization task [10, 12].

B. Equalizer Output Signals

Figures 4 and 5 show the equalized signals features for the linear equalizer and the nonlinear fuzzy equalizer trained respectively by the RLS and the IFRLS. The transmitted signal (Trans-Sig) is represented by a discontinued blue line, the received signal (Receiv-Sig) is symbolized by a solid black line, and the equalized signal (Eq-Sig) is represented by a red solid line for both RLS and IFRLS. For more convenience and visibility, the equalized signal is scaled down.

As we can perceive, the received signal is distorted by ISI and nonlinearity, while the additive noise some parts are amplified and some others are attenuated. The magnitude of the received signal is occasionally greater than the transmitted signal and sometimes-near zero, thus can cause error detection. Channel attenuation increases when considering the nonlinear channel CN2. Despite channel alterations, the IFRLS exhibits the best performance. The IFRLS-equalized signal is smoothed and typically close to the transmitted signal, therefore, the channel impairments are successfully cancelled. The conventional RLS algorithm performs poorly. Some residual distortion persists which is noticeable on its behavior for both nonlinear channels CN1 and CN2.

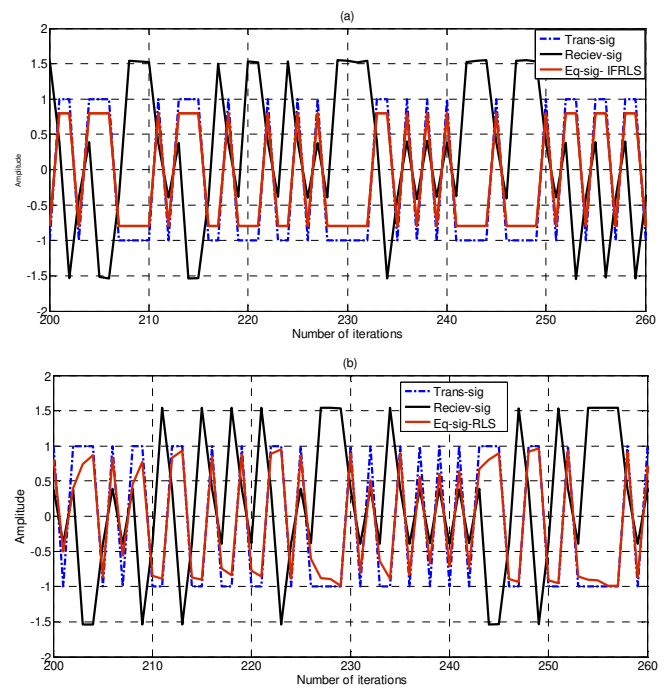


Fig. 4. Equalized signals of nonlinear channel CN1: (a) Fuzzy equalizer-IFRLS, (b) linear equalizer-RLS.

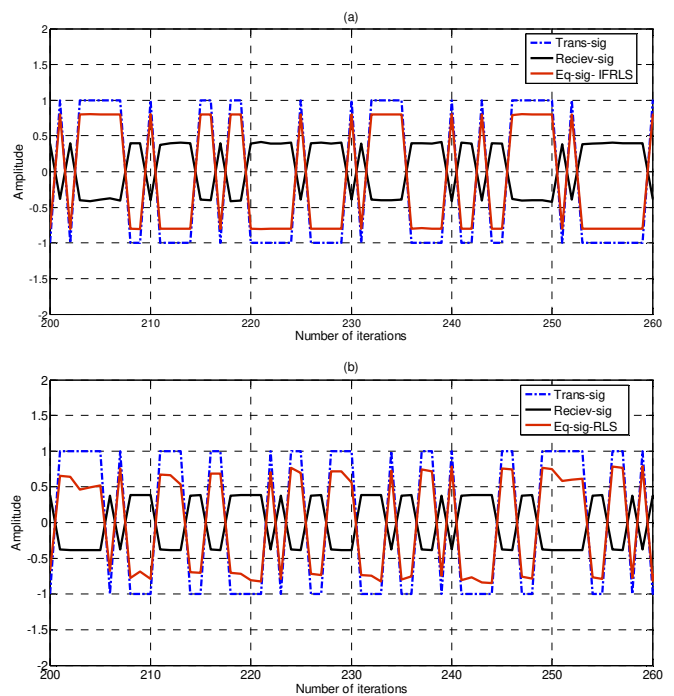


Fig. 5. Equalized signals nonlinear channel CN2: (a) Fuzzy equalizer-IFRLS, (b) linear equalizer-RLS.

C. Equalizers' Mean Square Error Convergence

The Mean Square Error (MSE) convergence is shown in Figures 6 and 7, respectively for the nonlinear channels CN1 and CN2.

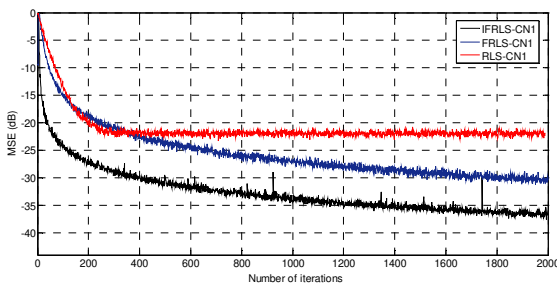


Fig. 6. Equalizers convergence, nonlinear channel CN1.

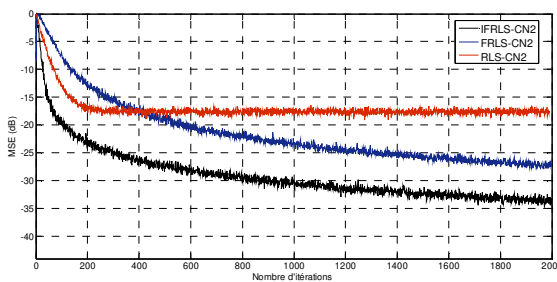


Fig. 7. Equalizers convergence, nonlinear channel CN2.

To further highlight the improvement of our approach, the convergence MSE of the IFRLS is compared with the ones of the Fuzzy Recursive Least Square (FRLS) and the conventional RLS. The IFRLS shows a significant improvement in convergence and steady-state MSE, when compared to the FRLS and the RLS algorithms. When considering convergence, we can perceive that the IFRLS reaches the MSE level of -20 dB at about 35 iterations for CN1 and 108 iterations for CN2. The FRLS attains the same MSE level at about 255 iterations for CN1 and 525 iterations for CN2. Moreover, the RLS attains the 20 dB MSE level at about 198 iterations for CN1 and presents a MSE behavior above 20 dB for CN2. Thus, the IFRLS procures a gain in convergence of 220 and 163 iterations in comparison with FRLS and RLS for CN1, and about 417 over the FRLS for CN2.

When considering the steady-state MSE, the IFRLS reaches a steady-state of about 37 dB and 34 dB for CN1 and CN2, respectively, whereas the FRLS attains 31 dB and 27 dB for CN1 and CN2. The steady-state for the RLS is -22 dB and -18 dB for CN1 and CN2, respectively. Therefore, the IFRLS exhibits an improvement in steady-state MSE of about 6 dB and 15 dB against FRLS and RLS for the nonlinear channel CN1. When considering the nonlinear channel CN2, a gain of 7 dB is realized over the FRLS and 16 dB over the RLS. These improvements are summarized in Tables I and II.

For more reliability, we situate our approach relatively to the results of [27]. The improvement obtained in the convergence through the proposed approach over the RLS is about 163 iterations in comparison with the 155 iterations obtained in [27], whereas the gain in steady-state reached by our approach over RLS is about 15 dB in comparison with the 10 dB obtained in [27]. That confirms the effectiveness of our approach in improving the convergence and steady-state MSE.

TABLE I. MSE PERFORMANCE

Algorithms	Convergence (iterations)		Steady-state (dB)	
	CN1	CN2	CN1	CN2
IFRLS	35	108	-37	-34
FRLS	255	525	-31	-27
RLS	198	-	-22	-18

TABLE II. MSE IMPROVEMENT OF THE IFRLS AGAINST FRLS AND RLS

Channels	CN1	CN2
Gain in convergence: IFRLS over FRLS (iterations)	220	417
Gain in convergence: IFRLS over RLS (iterations)	163	-
Gain in steady-state: IFRLS over FRLS (dB)	6	7
Gain in steady-state: IFRLS over RLS (dB)	15	16

V. CONCLUSION

In this paper, a new approach for the channel equalization problem was presented, that is an improvement of the recursive least square algorithm. The proposed improvement involves conjointly a regularization for the RLS and an incorporation of a fuzzy logic reasoning through membership functions. The performance of the Improved Fuzzy RLS was compared with the ones of the Fuzzy RLS and the conventional RLS. Extensive simulations revealed that the IFRLS surpasses both FRLS and RLS. The IFRLS improves considerably the convergence abilities of the adaptive fuzzy equalizer through MSE by procuring lower steady-states never reached through the FRLS and the conventional RLS. The proposed approach was also compared with [27]. The proposed IFRLS realizes an improvement in convergence and steady-state over RLS that surpasses the improvement realized in [27], confirming its effectiveness in improving convergence and steady-state MSE. Due to the performance improvement obtained with the proposed approach, the assessment of the IFRLS in a variable environment, such as a mobile channel, is a proposal for future work.

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