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doi:10.15199/48.2023.12.16

# A Comparative Study between Fractionalized and Fractional Order PID Controllers for Control of a Stable System Based on Particle Swarm Optimization Algorithm

***Abstract.*** *Most industrial applications use integer-order proportional integral derivative (IOPID) controllers due to well-known characteristics such as simplicity and ease of implementation. However, because of their nonlinear nature and the underlying iso-damping feature of fractional-order operators, fractional-order PID (FOPID) and fractionalized-order PID (FrOPID) controllers outperform the IOPID controllers. In this study, three different controllers based on particle swarm optimization are used to regulate a stable system. While a FrOPID controller only has to optimize four parameters and a normal PID controller only needs to optimize three parameters, a FOPID controller requires the optimization of five parameters. Set-point tracking, and better disturbance rejection are obtained with the fractional PID controller, whereas fractionalized PID outperforms the other controllers in terms of noise attenuation.*

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***Streszczenie.*** *Większość aplikacji przemysłowych wykorzystuje regulatory IOPID rzędu liczb całkowitych ze względu na dobrze znane cechy, takie jak prostota i łatwość implementacji. Jednak ze względu na ich nieliniowy charakter i leżącą u ich podstaw funkcję izo-tłumienia operatorów ułamkowego rzędu, regulatory PID ułamkowego rzędu (FOPID) i PID ułamkowego rzędu (FrOPID) przewyższają regulatory IOPID. W tym badaniu trzy różne kontrolery oparte na optymalizacji roju cząstek są używane do regulacji stabilnego systemu. Podczas gdy regulator FrOPID musi zoptymalizować tylko cztery parametry, a normalny regulator PID tylko trzy parametry, regulator FOPID wymaga optymalizacji pięciu parametrów. Śledzenie wartości zadanej i lepsze tłumienie zakłóceń uzyskuje się za pomocą ułamkowego regulatora PID, podczas gdy ułamkowy PID przewyższa inne regulatory pod względem tłumienia szumów.. (**Badanie porównawcze między sterownikami PID o ułamkowym i ułamkowym porządku do sterowania stabilnym systemem w oparciu o algorytm optymalizacji roju cząstek**)*

**Keywords:** PID controller, Fractional/Fractionalized PID controller, transient response, robustness, comparative study.
**Słowa kluczowe:** Regulator PID, ułamkowy/frakcyjny regulator PID, odpowiedź przejściowa, solidność, badanie porównawcze.

## Introduction

Although the first PID (Proportional-Integral-Derivative)controllers, based on vacuum tube technology, date back to 1951 and earlier mechanical versions were reported in 1939, the first paper on this type of controller was published in 1922 by a Russian American engineer, Nicolas Minorsky,working for the automatic steering ship of the US Navy [1]–[3]. This was still the most common controller for decades until 1974 when K. Oldham and J. Spanier introduced the fractional calculus theory applied to controllers [4]. This has various appealing properties, including ease of design and dependable performance [4], [5]. The PID controller has been used in a variety of applications, most notably in industrial automation [6], islanded microgrid [7] and robotics [8]. As a result, it is worthwhile to take care to increase their quality and robustness. One way of improving integer order PID controllers is to use a fractional or fractionalized order controller. Fractional calculus has been widely used in the modeling and control of various types of physical systems in recent years, as documented in several control theory or application literature [9]-[21].

The FOPID controller contains two additional parameters in addition to the proportional ($K_p$), integral ($K_i$), and derivative ($K_d$) parameters that form the integer order PID: an integrator order ($\lambda$) and a differentiator order ($\mu$). In the feedback control loop, this FrOPID employs fractional order filters to approximate integer order transfers. By introducing fractional order integrators ($\alpha$) into the classical feedback control loop while leaving the overall equivalent closed loop transfer function unchanged. So, FrPID contains four parameters (one more than classical PID).

The goal of this study is to compare three distinct controllers: IOPID, FrOPID, and FOPID, analyzing their benefits and drawbacks for good reference tracking, noise attenuation, and better disturbance rejection.

The following summarizes the contribution of this paper:

- The principal contribution is that this is the first time that a comparative study between integer order PID, fractionalized order PID, and fractional order PID controllers will be done.
- Examine the proposed controllers' robustness.
- Examine the proposed controller design's degree of tracking, disturbance rejection, and noise attenuation in comparison to IOPID controllers optimized using particle swarm optimization techniques.

## Fractional Order Systems

Fractional calculus is a branch of calculus that uses non-integer order to generalize functions, allowing fractional approximation techniques to be used in fractional order systems.

The Oustaloup method is based on the function approximation from as [10,21];

(1) $$\mathrm{G_f(s) = S^{\alpha}} \qquad ,\alpha \in \mathrm{R^+}$$

By taking into account the rational function:

(2) $$\mathrm{G_f(s) = K\prod_{k=1}^{N}\frac{s+w_k'}{s+w_k}}$$

However, the poles, zeros, and gain can beevaluated as; $\mathrm{w_k^{'} = w_b.\,w_u^{(2k\text{-}1\text{-}\gamma)/N}}$ , $\mathrm{w_k = w_b.\,w_u^{(2k\text{-}1+\gamma)/N}}$ , $\mathrm{K = w_h^{\gamma}}$

Where $w_u$ represents the unity gain in frequency and the central frequency in a geometrically distributed frequency band. Let $w_u = \sqrt{w_h w_b}$, where, $w_h$ and $w_b$ represent the upper and lower frequencies, respectively. γ and N are the orders of derivative and filter, respectively.

## Particle swarm optimization algorithm

PSO is a modern heuristic search method inspired by the social‑ behavior of bird and fish schooling. PSO optimization consists of designing the optimization goal, i.e.

the fitness function and then encoding the parameters to be searched [5]. PSO exploits a swarm of particles probing promising regions of the D-dimension search space with adaptable velocity. It runs until the stop condition is satisfied. The best particle's position givesthe optimized parameters for the controller.

The flowchart of a typical PSO algorithm is shown in fig.1.

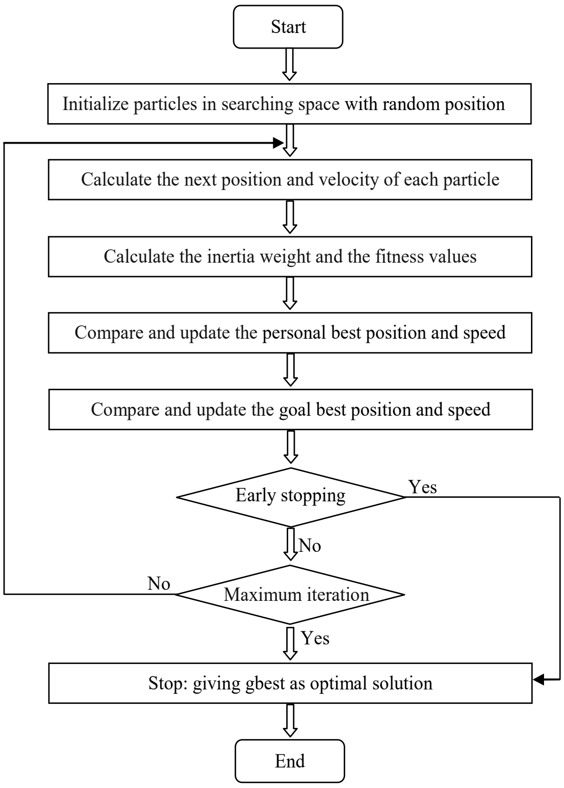

Fig.1. Flowchart of PSO algorithm procedure

The update formula of velocity and position is stated by (3) and (4):

(3) $$v_i^{k+1} = w_i v_i^k + C_1 a\left(P_i - x_i^k\right) + C_2 b\left(P_g - x_i^k\right)$$

(4) $$x_i^{k+1} = x_i^k + v_i^{k+1}$$

Where:

$v_i^k, x_i^k$: Velocity and positioning vectors of particle $i$ at iteration $k$ respectively.

$v_i^{k+1}, x_i^{k+1}$: Modified velocity and position of particle $i$ at the next iteration $k+1$ respectively.

$a, b$ : Random number between 0 and 1

$C_1, C_2$ : Positive constants

$P_i, P_g$ : Best positions found by particle $i$ and $g$ respectively

$w_i$ : Weight function for velocity of particle $i$.

## Controller's design

Three controllers are suggested in this paper: integer order PID, fractionalized order PID, and fractional order PID, with general designs given in Fig. 2.

There are numerous methods for determining tuning constant values [5],[10], [24],[25]. A method presented in [26] are used in this paper. In order to make a fair comparison, the three controllers are tested in the open-loop 3rd-order stable system given in [24] and defined as follow:

(5) $$G(s) = \frac{1}{s^3+5s^2+8s+6}$$

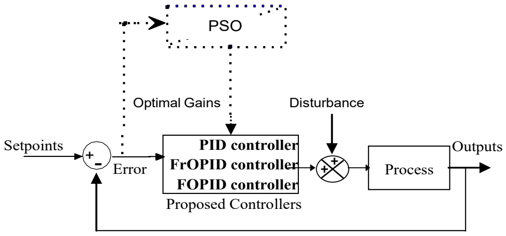

Fig. 2. General structure of the proposed optimization methods. PSO, particle swarm optimization; IOPID, integer order proportional integral derivative; FrOPID, Fractionalized Order PID; FOPID, Fractional Order PID.

## Integer Order PID Controller

The IOPID controller's mathematical expression is as follows:

(6) $$G_c(s) = K_p + K_i s^{-1} + K_d s$$

Where $K_p$, $K_i$ and $K_d$, are the proportional, integral, and derivative gains, respectively.

Then,

(7) $$G_{IOPID}(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

Where

$T_i$ : is the integral time constant

$T_d$: is the derivative time constant.

## Fractionalized Order PID Controller

As stated in Eq. 9, the rule of PID controller is enhanced by fractionalizing a control system part and the integral operator:

(8) $$\frac{1}{s} = \frac{1}{s^\alpha}\cdot\frac{1}{s^{1-\alpha}}$$

With $\alpha \in ]0 \quad , \quad 1[$ .

The proposed IOPID controller will look like this:

(9) $$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

The IOPID fractionalization controller to be generated is given as [7],[11]:

$$G_{FrOPID}(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right) = \frac{1}{s}\left(\frac{K_pT_ds^2+K_pT_is+K_p}{T_i}\right)$$

Then,

(10) $$G_{FrOPID}(s) = \frac{1}{s^\alpha}\frac{1}{s^{(1-\alpha)}}\left(\frac{K_pT_ds^2+K_pT_is+K_p}{T_i}\right)$$

Where $K_p$, $T_i$ and $T_d$, refer for proportional gain, integral time constant, and derivative time constant, respectively. $\alpha$ is the integral fractional order.

## Fractional Order PID Controller

The FOPID controller's mathematical expression is as follows:

(11) $$G_c(s) = K_p + K_i s^{-\lambda} + K_d s^{\mu}$$

The unknown five parameters that define the FOPID equation are $K_p$, $K_i$ and $K_d$; the two additional parameters $\lambda$ and $\mu$ denotes the fractional integral and fractional derivative order respectively; $\lambda, \mu$ are positive real numbers.

## Results and Discussion

To achieve good transient and steady-state performance of the system given in eq.5, the PSO algorithm was used to minimize the integral square error (ISE) criterion and find the optimal parameters of the controllers. The ISE value is calculated using Eq. 12.

(12) $$J = \int_0^{t_{sim}} (e(t))^2 dt$$

Where $J$ is the performance criteria.

The integer order PID controller optimized using particle swarm optimization of the system provided in Eq. 5 has the following parameters: $K_p = 30.70, T_i = 0.3070\ and\ T_d = 0.3257$ .

The PSO algorithm is used to design the integer order PID controller. The PSO/IOPID controller is given by:

$$G_{IOPID}(s) = 30.70\ (\ 1 + 0.3070s^{-1} + 0.3257s)$$

The fractionalized order PID controller parameters optimized by PSO are as follows: $K_p = 30.70$, $T_i = 0.3070\ and\ T_d = 0.3257$ and for $\alpha = 0.1$ the PSO/FrOPID controller can be written as:

$$G_{FrOPID}(s)\ = \frac{1}{s^{0.1}}\frac{1}{s^{0.9}}\left(\frac{(0.9989s^2+9.4249s+30.70)}{0.3070}\right)$$

For the same system model $G_2(s)$ given in Eq. (18), the parameters of the fractional order PID optimized by PSO are: $K_p = 100\ , K_i = 100\ ,\ K_d = 100\ ,\ \ \lambda = 0.335$ and $\mu = 1.557$.

The PSO/FPID controller is given by:

$$G_c(s) = 100\ + 100s^{-0.335} + 100s^{1.557}$$

A step input reference is used to compare the actions of the several proposed controllers (PSO/IOPID, PSO/FrOPID, and PSO/FOPID).

Fig.3 depicts the characteristics of the time response of the integer order PID, fractionalized order PID, and fractional order PID controllers without noise, as well as the output error signals.

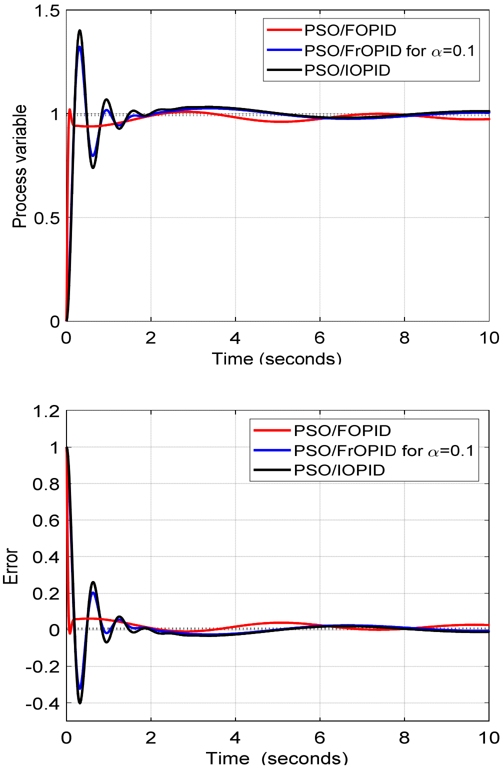

Fig. 3. Set-point unit step response for example 1. (Top) closed-loop step response comparison of IOPID, FrOPID, and FOPID controllers. (Bottom) Controller's steady state error comparison

The unit step response of the system controlled by integer order PID and fractionalized order PID controllers is extremely close, as can be seen in figure 6. However, the FrOPID controller outperforms the IOPID controller in terms of percent overshoots.

In comparison to the PSO/FrOPID and PSO/IOPID controllers with the ISE objective function provided in Table 1.

Table 1. Transient Response Stability Parameters

| Controller type | Maximum overshoot $M_p[\%]$ | Settling time $T_s[s]$ | Rise time $T_r[s]$ |
|---|---|---|---|
| PSO/FOPID [Proposed] | $\mathbf{2.0453}$ | 5.6771 | $\mathbf{0.0401}$ |
| PSO/FrOPID [Proposed] | | | |
| $\alpha = 0.1$ | 38.7462 | 4.3036 | 0.1271 |
| $\alpha = 0.2$ | 38.4238 | 1.4241 | 0.1262 |
| $\alpha = 0.3$ | 38.1085 | 1.4237 | 0.1260 |
| $\alpha = 0.4$ | 37.8652 | $\mathbf{1.4235}$ | 0.1257 |
| $\alpha = 0.5$ | 37.8080 | $\mathbf{1.4235}$ | 0.1257 |
| PSO/IOPID | 40.1425 | 4.2656 | 0.1268 |

As we can stated, the suggested PSO/FOPID controller with the examined objective function achieves superior results with two performance indices, $M_p[\%]$ and $T_r[s]$, and only slightly worse with $T_s[s]$. The proposed PSO/FrOPID controller, on the other hand, had the best performance in terms of settling time (for $\alpha = 0.4$ and $\alpha = 0.5$).

On the input side of the process to be managed, additive noise is taken into account. The temporal response characteristics of integer order PID, fractionalized order PID (for $\alpha = 0.1$), and fractional order PID controllers with random output noise of 2% and 20% of the reference signal amplitude are shown in Fig.4.

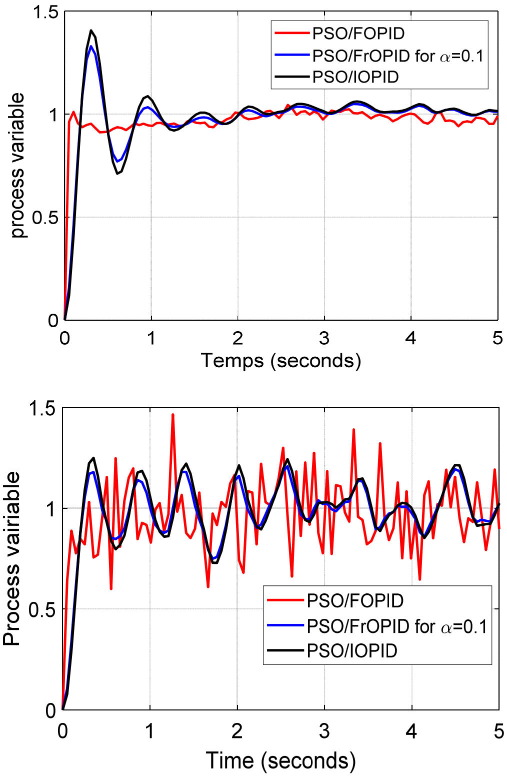

Fig. 4. Controller's comparison with random output noise; (Top) for 2% of the amplitude of the reference signal (α=0.1). (Bottom) for 20% of the amplitude of the reference signal (α=0.1).

As can be seen in Figure 7, the overshoot obtained with the integer order PID (IOPID) and fractionalized order PID (FrOPID) controllers are very close.

Due to noise, the performance of the FOPID controller has obviously degraded, with increased overshoot.

Fig.4 also shows, that the PSO-FrOPID controller performs better in terms of noise attenuation than PSO-IOPID and PSO-FOPID controllers.

## Conclusion

This study investigated numerous PID controllers and their conversion from one form to another. These forms were used to determine the control actions of FOPID and FrOPID controllers. As a proof of concept, a comparison study on a stable system with all of the controllers described was performed. The study has taken set-point tracking and noise attenuation into account. In terms of maximum overshoot, rising time, and settling time, the suggested PSO-FOPID controller is faster and has better disturbance rejection than the PSO-IOPID and PSO-FrOPID. These comparison findings show that the proposed PSO-FOPID controller has better time response characteristics than the others. The PSO-FrOPID controller, on the other hand, performs better in terms of noise attenuation than the PSO-IOPID and PSO-FOPID controllers.

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