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Analysis Study of Radar Probability of Detection for Fluctuating and Non-fluctuating Targets

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Abstract: The radar analyst can develop and use mathematical and statistical techniques that lead to accurate prediction or adapting models for estimating the target detection performance. In radar detection theory, detection probability, false alarm probability, number of samples non-coherently integrated for a detection test, and signal-to-noise ratio (SNR) are closely interrelated. The present paper is intended to provide an overview of the calculations of radar probability of detection and its related parameters. The main methods and procedures for predicting the detection performance of either non-fluctuating or fluctuating targets are described. Performance's analysis of the studied models is included, along with some graphical simulation examples.

Keywords: Radar signal processing, radar detection, probability of detection, probability of false alarm, Swerling target model.

1. INTRODUCTION

The first applications of radio were telecommunications and radio-navigation, but by the early 20th century, precursors considered the ability to detect the presence of metal objects through the use of electromagnetic waves. The word RADAR itself, which is now universally adopted to designate a material satisfying these requirements for detecting, locating, and identifying reflecting objects over long distances. The term RADAR was coined in 1941 as an acronym for Radio Detection and Ranging. This acronym of American origin replaced the previously used British abbreviation RDF (Radio Direction Finding). However, due to its wide use, the word has become a standard noun in English, and almost all people have had an experience with radar [1-2].

In general, radar systems use modulated waveforms and directive antennas to transmit electromagnetic energy into a specific volume in space to search for targets. Objects (targets) within a search volume will reflect portions of this energy (radar returns or echoes) back to the radar. These echoes are then processed by the radar receiver to extract target information such as range, velocity, angular position, and other target identifying characteristics [3].

Basic Radar functions can be classified as detection, tracking, or imaging and the most fundamental problem in radar is detection of an object or physical phenomenon. Nevertheless, one can't never determine the range or estimate the speed without detecting the target, and it is necessary to distinguish the signal reflected from the target, from the signal containing only noise. The target detection problem in a radar system is naturally a statistical problem that random fluctuations, due to noise located at the receiver output, corrupt the target signal and influence the detector performance. The receiver detection of a target refers then to a decision made by the radar detector concerning presence or absence of the useful signal in the presence of additive noise.

The first research in detection theory that has been used in radar signal was made by Marcum [4-5]. He considered the detection of a completely known signal in white Gaussian noise using multiple received samples. This research was continued by Swerling [6-8], and since then, radar detection has been consistently developing. The fundamental theory behind detection and classification was developed in mathematical statistics and decision theory, and signal detection is a special case of hypothesis testing theory in statistical inference [9-10].

The organization of this paper is as follows: section 2 describes briefly the radar system and its operational blocs and characteristics. Section 3 formulates the problem under consideration and presents the detection techniques. Section 4 deals with the numerical results and brief discussion analysis, while section 5 contains our conclusions.

2. RADAR SYSTEM

Radar is an electromagnetic system that detects, locates, and recognizes target objects. It transmits electromagnetic signal and then receives echoes from target objects to get their location or other information. The received signal is frequently accompanied by noise and clutter. The disturbances may cause serious performance issues with radar systems by concluding these signals as targets. The basic parts of a radar system are illustrated in the simple block diagram of fig.1. Radar equipment consists of a transmitter, an antenna, a receiver, and a signal processor. Radar transmitters and receivers are usually located in the same place. Each block of radar system has a specific operation:

- The waveform generator is a unit which purpose is to create and control the waveform to be modulated and transmitted by the transmitter.
- The transmitter, for its side, produces powerful pulses and/or waveforms of electromagnetic energy at precise time intervals and sends them to the antenna system.
- The main role of the antenna is to provide a transducer between the free-space propagation and the guided-wave propagation. The antenna system includes a transmitting function and a receiving function. During transmission is to concentrate the radiated energy into a shaped directive beam which illuminates the targets in a desired direction. During reception the antenna collects the energy contained in the reflected target echo signals and delivers it to the receiver.
- The target reflected energy is received by the receiver from the antenna system. Then, the receiver performs amplification, filtering, and demodulation on the received signal.
- The computer/signal processor performs complex mathematical computations on the demodulated signal to extract target velocity and/or range information.
- The timing-and-control block affords timing information to synchronize various signals and to control the operation of other radar components.

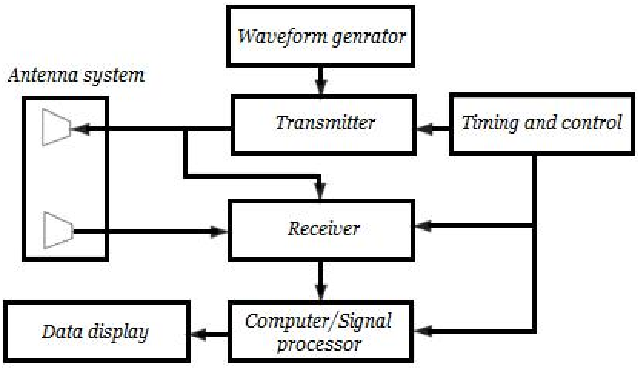

Fig. 1 Basic block diagram of a radar system

3. RADAR DETECTION PROBABILITIES

The purpose of this section is to illustrate the mathematical model that determines the generated radar signal and to show mathematically how the detection problem can be described in terms of two decision hypothesis (detection, or false alarm). Then, it gives different methods of radar detection which used either for a fluctuating or non-fluctuating targets.

We consider the basic problem of detecting the presence or absence of a complex signal s(t) with envelope A in a set of measurements $y(t) = y_I(t) + i\, y_J(t)$ corrupted by a sum of independent additive complex noises corresponding to the clutter echoes c(t) and white Gaussian thermal noise n(t). Mathematically, we describe this problem in terms of a hypothesis test between the following pair of statistical hypothesis [11]:

$$H_0 \;: y(t) = n(t) + c(t) \qquad (1)$$

$$H_1 \;: y(t) = s(t) + n(t) + c(t) \qquad (2)$$

If we note $pH_0(r)$ the probability density of the noise envelope $|n(t) + c(t)|$, the detection threshold T is fixed by the value of the given probability of false alarm P_{fa}.

$$P_{fa} = \int_T^{+\infty} pH_0(r)dr \quad (3)$$

While, denoting $pH_1(r)$ the probability-density function (PDF) of the envelope of the complex signal embedded in noise $|s(t) + n(t) + c(t)|$, the detection probability P_D is classically given by :

$$P_D = \int_T^{+\infty} pH_1(r)dr \quad (4)$$

Generally, target signal is modeled either as a random variable in the single pulse case or as a very simple stochastic process in the pulse train case [12]. In the latter case only completely correlated or completely uncorrelated pulse-to-pulse fluctuations can be considered. So, target signals can be classified into fluctuating target models and non-fluctuating target models.

Target fluctuation models

Swerling extended Marcum's works which are methods for predicting the detection performance of non-fluctuating targets to incorporate what has become known as the four Swerling models which concerned the fluctuating targets [13].

- **Detection of Swerling I**

The Swerling I model signifies fluctuating amplitude, constant within a scan, but uncorrelated from scan to scan. The probability of detection for this type targets was derived by Swerling as the following formula:

$$P_D = e^{-T/(1+SNR)} \quad for \quad N = 1 \quad (5)$$

$$P_D = 1 - \Gamma(T, N-2) + \left(1 + \frac{1}{N.SNR}\right)^{N-1} . \Gamma\left(\frac{T}{1 + 1/N.SNR}, N-2\right) e^{-T/(1+N.SNR)} \quad for \quad N > 1 \quad (6)$$

Where T is the threshold, N is the number of integrated pulses, and Γ represent is the incomplete gamma function defined as:

$$\Gamma(a, N) = \int_0^a \frac{x^N e^{-x}}{N!} dx \quad (7)$$

- **Detection of Swerling II**

The formula for the probability of detection for Swerling II type targets is given by:

$$P_D = 1 - \Gamma\left(\frac{T}{1 + SNR}, N\right); \quad N \leq 50 \quad (8)$$

In this model, the PDF is as for Swerling I case, but the fluctuations are more rapid and are taken to be independent from pulse to pulse.

- **Detection of Swerling III**

In the case of Swerling III targets, the probability of detection is given by the following expression:

$$P_D = \left(1 + \frac{2}{N.SNR}\right)^{N-2} \left(1 + \frac{T}{1 + N.SNR/2} - \frac{2(N-2)}{N.SNR}\right) . e^{-T/(1+N.SNR/2)} \quad (9)$$

The fluctuations are independent from scan to scan as in case Swerling I.

- **Detection of Swerling IV**

The probability of detection for Swerling IV model is given by the following expression:

$$P_D = 1 + \left(\frac{SNR}{SNR + 2}\right)^N \sum_{k=0}^{N} \frac{N!}{k!\,(N-k)!} \left(\frac{SNR}{2}\right)^{-k} . \Gamma\left(\frac{2T}{2 + SNR}, 2N - k - 1\right) \quad (10)$$

The PDF is as for case Swerling III, but the fluctuations are independent from pulse to pulse.

Target non-fluctuation models

In this class of non-fluctuation we have two main models

- **Detection of Swerling V**

A common, fifth, target type is a constant RCS (Radar Cross Section) target. This is termed a Swerling 0 target by some and a Swerling V target by others. It is the simple case implies constant amplitude or no fluctuation.

$$P_D = P_{D1}(10log(N.SNR), e^{-T}) - e^{-T-N.SNR} \sum_{m=2}^{N} \left(\frac{T}{N.SNR}\right)^{\frac{m-1}{2}} . I_{m-1}(2\sqrt{T.N.SNR}) \quad (11)$$

P_{D1} is the single pulse:

$$P_{D1} = \frac{1}{2}\left(1 - erf\left(\sqrt{T} - \sqrt{SNR}\right)\right) + \frac{e^{-(\sqrt{T}-\sqrt{SNR})^2}}{4\sqrt{\pi}\sqrt{SNR}} . \left(1 - \frac{\sqrt{T} - \sqrt{SNR}}{4\sqrt{SNR}} + \frac{1 + 2\left(\sqrt{T} - \sqrt{SNR}\right)^2}{16SNR} - \cdots\right) \quad (12)$$

Where $erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$ is one form of the error function.

$I_m(x)$ is the modified Bessel function of the first kind and order m.

- **Detection of Albersheim**

Walter Albersheim [14] derived a simple formula for signal to noise ratio, which is required to achieve a given level of performance for envelope detection of non-fluctuating signal, in narrow band noise. The Albersheim equation is given as:

$$SNR_{dB} = -5log_{10}N + \left(6.2 + \frac{4.54}{\sqrt{N + 0.44}}\right) . log_{10}(A + 0.12AB + 1.7B) \quad (13)$$

Where $A = ln(0.62/P_{fa})$, $B = ln(P_D/(1 - P_D))$, and P_{fa} means the false alarm probability.

However, it can be rearranged to obtain a solution for detection probability (P_D) in terms of the other parameters using the following sequence of calculations [15]:

$C = \frac{SNR_{dB}+5log_{10}N}{6.2+4.54/\sqrt{N+0.44}}$, and $D = \frac{10^{C}-A}{1.7+0.12A}$, then the Albersheim detection probability is:

$$P_D = \frac{1}{1 + e^{-D}} \quad (14)$$

4. NUMERICAL SIMULATIONS

The purpose of this section is to investigate the performances of radar system which are, mainly, based on the probability of detection curves. We have used the Matlab software to program and develop the different types of targets which are theoretically described and studied in the above section. We can show firstly the plots of detection probabilities versus signal-to-noise ratio for given false alarm probability for all the six target cases, then the plots of the probability of detection value versus the signal to noise ratio for several values of probability of false alarm are depicted, and finally, 3D representations are given in order to illustrate the variation of the SNR as function of detection probability and number of integrated samples for a fixed value of false alarm probability.

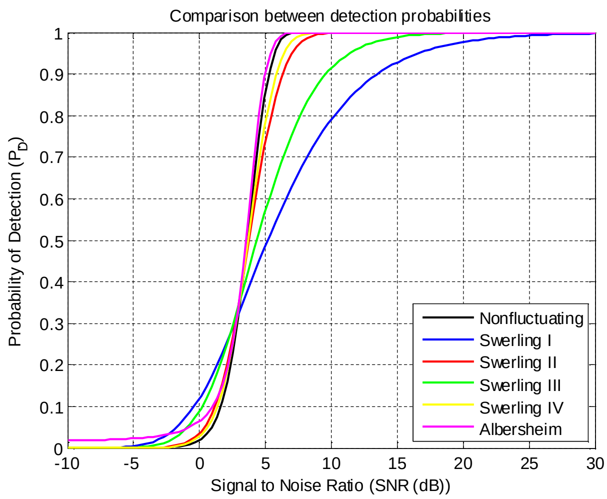

Fig. 2 Probability of detection as a function of SNR for different target models when N = 10, and Pfa = 10^{-6}.

The results illustrated in fig.2 and fig.3 lead us to make the following observation; for high detection probabilities, more signal-to-noise is required for all target models.
The fig.2 and fig.3 show also a comparison between the six detection probabilities with 10 and 100 pulses integrated respectively. For all these models, greater number of pulses integrated requires low signal-to-noise ratio to yield a good probability of detection. For low number of pulses integrated, on the other hand, the reverse of this relationship is correct. In addition, the comparison between fig.2 and fig.3 indicate also that when the number of integrated pulses is larger, the more likely it will be for the fluctuations to average out, and the curves of Swerling II and Swerling IV will approach to the constant target case which represents non-fluctuating (Swerling V) and Albersheim models.

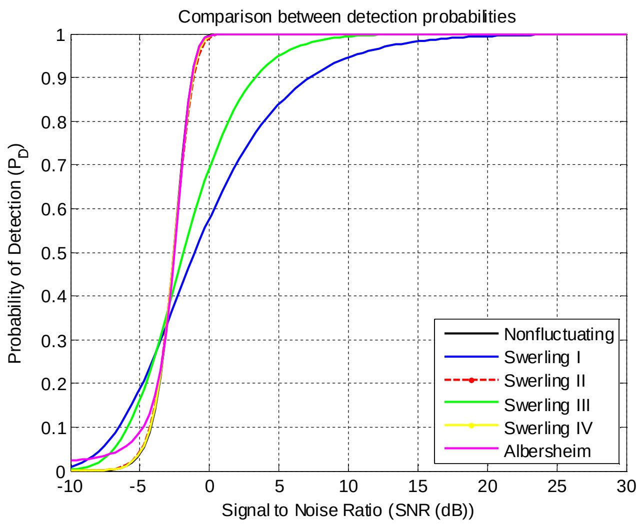

Fig. 3 Probability of detection as a function of SNR for different target models when N = 100, and Pfa = 10^{-6}.

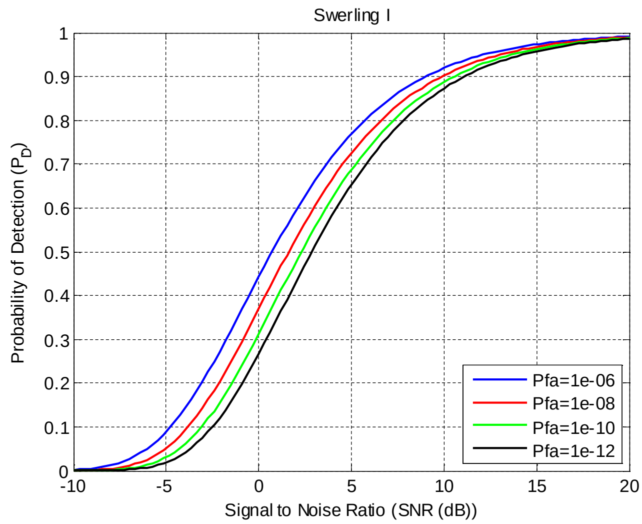

Fig. 4 Probability of detection for Swerling I versus SNR, for various values of Pfa when N=50.

Fig.4 shows plots for the probability of detection value of Swerling I as scan-to-scan fluctuation versus the signal to noise ratio for several values of probability of false alarm. In other part, fig.5 shows plots for the probability of detection value of Swerling IV as pulse-to-pulse fluctuation versus the signal to noise ratio for several values of probability of false alarm.

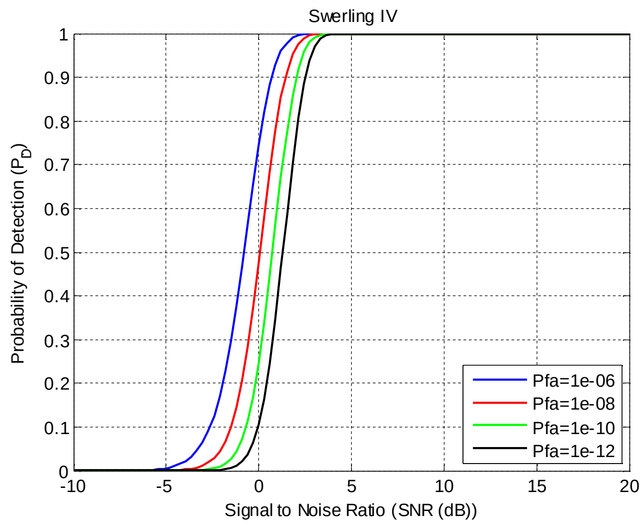

Fig. 5 Probability of detection for Swerling IV versus SNR, for various values of Pfa when N=50.

The performance comparisons showed through these figures allows us to say, for a given SNR, the detection probability is inversely proportional to the value of the false alarm probability. This statement is true either for scan-to-scan detectability (i.e. Swerling I) or pulse-to-pulse detectability (i.e. Swerling IV).

The following figures show an example of the variation of the required SNR as a function of probability of detection (Pd), probability of false alarm (Pfa), and number of integrated pulses (N) for various target models (the four Swerling fluctuation models, the non-fluctuation model, and Albersheim's model) . In this example, the value of Pfa is fixed to 10^{-6} and Pd and N vary.

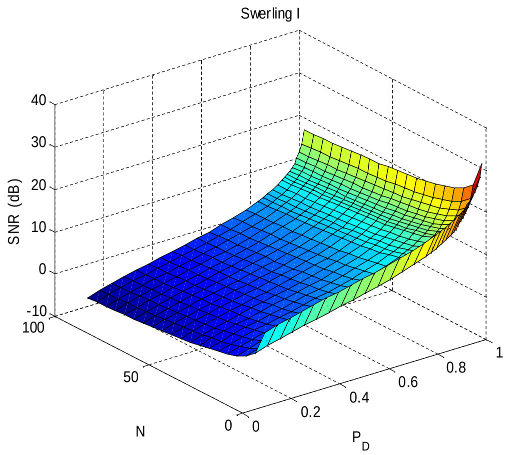

Fig. 6 Signal to noise ratio as a function of probability of detection and number of integrated pulses for Swerling I targets.

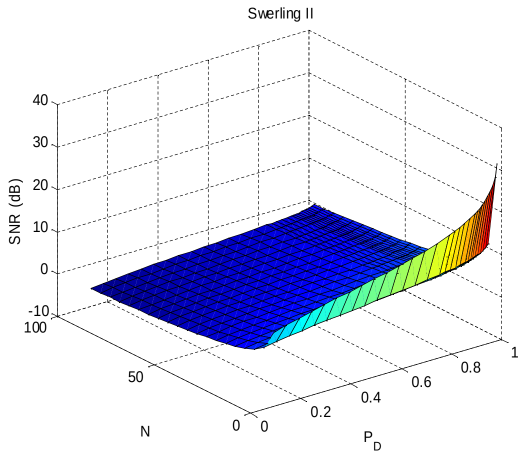

Fig. 7 Signal to noise ratio as a function of probability of detection and number of integrated pulses for Swerling II targets.

By visualizing fig.6 and fig.8, we can not notice a great difference because Swerling I and Swerling III are both scan-to-scan fluctuating targets. The comparison between the figure of Swerling II and that of Swerling IV allows us to deduce the similarity because Swerling II Swerling and IV are of unique type which is pulse-to-pulse fluctuating targets. We cannot almost distinguish the difference between the representations of fig.10 and fig.11 because both are from the same class of targets.

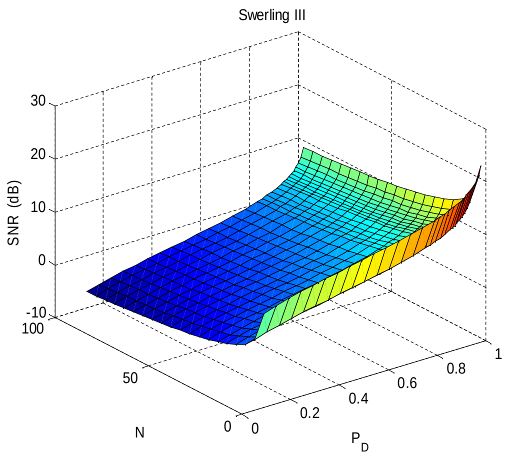

Fig. 8 Signal to noise ratio as a function of probability of detection and number of integrated pulses for Swerling III targets.

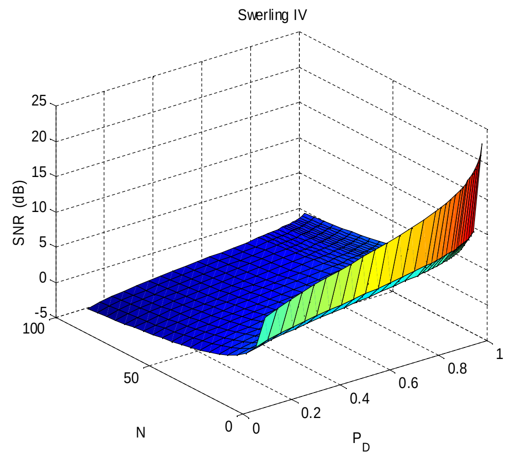

Fig. 9 Signal to noise ratio as a function of probability of detection and number of integrated pulses for Swerling IV targets.

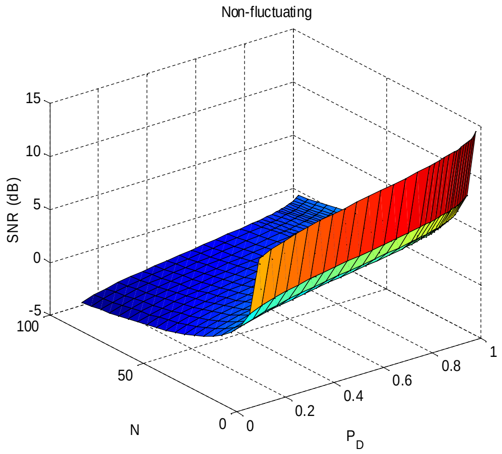

Fig. 10 Signal to noise ratio as a function of probability of detection and number of integrated pulses for non-fluctuating (Swerling V) targets.

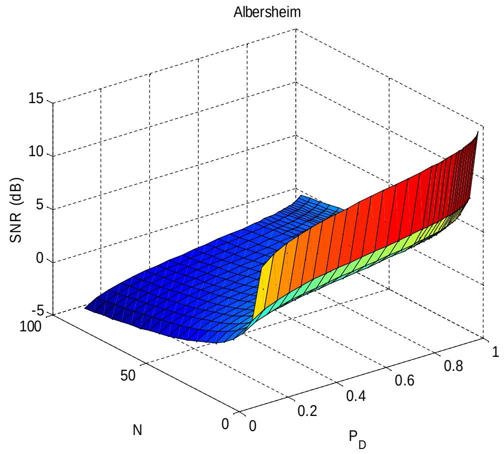

Fig. 11 Signal to noise ratio as a function of probability of detection and number of integrated pulses for Albersheim targets.

Finally, the following general remarks can be drawn through our simulation's analytical work:

1. For detection probability upper than 50%, non-fluctuating targets (Swerling V and Albersheim) are easier to detect than any fluctuating targets which can make detection more difficult by requiring a higher SNR for example.
2. The way in detection of the fluctuating targets is not the same. In fact, pulse-to-pulse fluctuations (Swerling II and Swerling IV) are easier to detect than scan-to-scan fluctuations (Swerling I and Swerling III) for detection probabilities higher than 50%.
3. The converse of the above two remarks is occurred for detection probabilities less than 50%.

5. CONCLUSION

A radar tracks a target, measures its range and velocity, and sometimes can identify it, only because there is, before all, a detection of an echo signal carried out by the system radar itself.
The radar detection probabilities can be calculated by Swerling for four different fluctuation models of cross section. In two of the four cases (Swerling.I and Swerling.III), it is assumed that the fluctuations are completely correlated during a particular scan but are completely uncorrelated from scan to scan. In the other two cases (Swerling.II and Swerling.IV), the fluctuations are assumed to be more rapid and uncorrelated pulse to pulse. The models presented and described in this paper are simple algorithms which give quick solutions for the radar detection of targets. They are also important due to their fast computation speed and simple solution performance.

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