

Certificate of participation

This is to certify that: **Somia GUECHI**

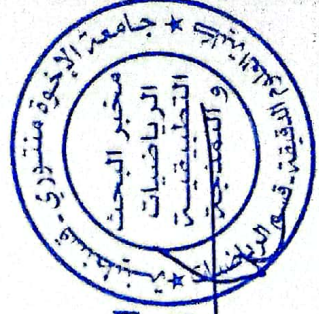
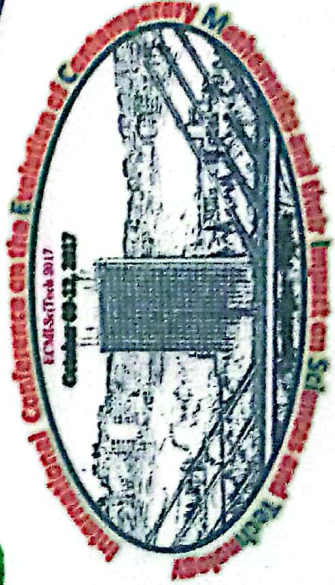
Has participated as "oral presenter" and presented the following paper entitled
"Taylor series for solving linear and Hammerstein ill-posed Volterra problems via iterations method"

during the

First international conference on the
"Evolution of Contemporary Mathematics and their Impact in Sciences and Technology"
(ECMI-SciTech2017)



held on October 09-12, 2017
Freres Mentouri University,
Constantine – Algeria.
<http://ecmi2017.labomam.net>



مدير مختبر البحث
الرياضيات التطبيقية والنمذجة
الأستاذ/بصيلة خالد
Chairman: Dr. Khaled BESBITA
Head of Applied Mathematics
and Modeling Laboratory



**Abstracts book
of
The First International Conference
on the Evolution of Contemporary
Mathematics and their Impact in
Sciences and Technology**

ECMISciTech2017

October 09-12, 2017

Freres Mentouri University- Constantine, Algeria.



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Conference Program - ECMI-SciTech 2017.

October 09, 2017.

08h00: Inscriptions.

08h30: Opening Ceremony.

09h00: Conference Plenary 1.

10h00: Coffee-Break.

11h00: Starting of the different Workshops.

12h30: Lunch.

13h30: Afternoon sessions 1 for the programmed workshops.

15h00: Coffee-Break & Poster session 1.

16h00: Afternoon sessions 2 for the programmed workshops.

October 10, 2017.

08h00: Inscriptions.

09h00: Conference Plenary 2.

10h00: Coffee-Break & Poster session 2.

11h00: Starting of the different Workshops.

12h30: Lunch.

13h30: Afternoon sessions 1 for the programmed workshops.

15h00: Coffee-Break & Poster session 3.

16h00: Afternoon sessions 2 for the programmed workshops.

October 11, 2017.

08h00: Inscriptions.

09h00: Conference Plenary 3.

10h00: Coffee-Break & Poster session 4.

11h00: Starting of the different Workshops.

12h30: Lunch.

13h30: Afternoon sessions 1 for the programmed workshops.

15h00: Coffee-Break.

16h00: Afternoon sessions 2 for the programmed workshops.

17h00: Closing ceremony

& and distribution of participation certificates.

The number of workshops is three.

October 12, 2017:

Social Program: half-day excursion / Tour

Plenary Speakers:

- Professor Said BOUSSAKTA : “**The Role of Mathematics in Modern Communications, Signal Processing and Information Security.**”, School of Electrical and Electronic Engineering, Newcastle University, United Kingdom.
- Professor Fethi BIN MUHAMMAD BELGACEM, “**New Trends in Applicative Theory with the Smudu Transform.**”, Department of Mathematics, Faculty of Basic Education, PAAET, Al - Ardhiya, Kuwait.
- Professor Mohamed DALAH : “**An Antiplane Contact Problem with Long-Term Memory: Case Electro-Visco-Elastic Materials.**”, Department of Mathematics, Faculty of Exact Sciences, Freres Mentouri University Constantine. Algeria.



Workshop 1

Day 1: October 09, 2017

- Mohammed CHEBBAH. Simulations for Efficient combination of two lower bound functions in univariate global optimization. Mouloud Mammeri University, Tizi Ouzou, Algeria. [P08](#)
- Houda HASSOUNA. The proof of method of spatial contextual Gaussian process of image classification. Mohamed Khider University, Biskra, Algeria. [P07](#)
- Badredine ISSAADI. A new bounds for approximation of invariant measures in countable space Markov chains. University of boumerdes, Laboratory LaMOS, Targa Ouzemour, 06000 Bejaia, Algeria. [P08](#)
- Rebiha ZEGHDANE. Bernoulli polynomials for the numerical solution of Volterra -Fredholm-Hammerstein stochastic integral equations. Mohamed El Bachir El Ibrahimi University, Bordj Bou Arréridj, Algeria. [P09](#)
- Mohammed Nor FRIOUL. Age Structured Model Of Virus Dynamics With General Nonlinear Infection Response. Aboubekr Belkaid University, Tlemcen, Algeria. [P10](#)
- Bachir ALILI. Reliability modeling of industrial gas turbine using Weibull distribution. Ziane Achour University, Djelfa, Algeria. [P11](#)
- Fatima Zohra SIDI ALI. Stabilization of the wave equation with a delay term in the nonlinear boundary feedback. Mostefa Ben Boulaïd University, Batna, Algeria. [P12](#)
- Moussa BOULOUDENE. On the Limit Distributions Of Multi-state Homogeneous Consecutive-out-of-n:G Systems. Freres Mentouri University, Constantine, Algeria. [P13](#)

Day 2: October 10, 2017

- Nizar MANNAI. Optimal replacement time for a stochastically degrading system with a random initial age. Polytechnic School of Tunisia, Tunisia. [P14](#)
- Souâd YACHEUR. Mathematical analysis of a Malaria model. Aboubekr Belkaid University, Tlemcen, Algeria. [P15](#)
- Assia OUTAMAZIRT. A New Analytical formula for Computing the Delay Probability in Cloud Center Modeling as M/G/c/k Queue. Abderrahmane Mira University, Bejaia, Algeria. [P16](#)
- Soumaya GHNIMI. Probabilistic modeling of imperfect repair effects for a repairable system. University of Tunis El Manar, Tunisia. [P17](#)
- of difference equations. Mohamed Seddik Ben Yahia University, Jijel, Algeria. [P18](#)
- Imène KHELIFA. Inverse boundary value problem in a perturbed strip. Badji Mokhtar University, Annaba, Algeria. [P19](#)
- Wassila GHECHAM. Stabilization of the wave equation with a delay term in the nonlinear boundary feedback. Mostefa Ben Boulaïd University, Batna, Algeria. [P20](#)
- Lamia CHOUCANE. A history-dependent problem for elastic-viscoplastic materials with internal state variable. Ferhat Abbas University, Setif, Algeria. [P21](#)



Workshop 1

Day 3: October 11, 2017

- Lounes AMEUR. Robust Analysis of Retrial Queues. Abderrahmane Mira University, Bejaia, Algeria. [P22](#)
- Abdelkader LAIADI . Free surface flow over trapezoidal obstacle under surface tension and gravity. Mohamed Khider University, Biskra, Algeria. [P23](#)
- Fethi SOUNA. Bifurcation and global dynamics of a predator-prey model with herd behavior and quadratic predator harvesting. Djillali Liabes University, Sidi Bel Abbes, Algeria. [P24](#)
- Tengaa Peter. Roles of Mathematics Reasearches towards industrial Economic growth, case of Tanzania. College of Business Education(CBE) , Tanzania. [P25](#)
- Faïrouz AFROUN. Matrix method for the analysis and performance evaluation of the unreliable M/M/1/N queueing model. Mouloud Mammeri University, Tizi Ouzou, Algeria. [P26](#)
- Nedjma AIANE. Strong Stability Estimate in a Periodic Review Inventory Model. Abderrahmane Mira University, Bejaia, Algeria. [P27](#)
- Somia GUECHI. Taylor series for solving linear and Hammerstein ill-posed Volterra problems via iterations method. Mohamed Boudiaf University, M'Sila, Algeria. [P28](#)
- Soumia LATRECHE. History-dependent frictional contact problems. Ferhat Abbas University, Setif, Algeria. [P29](#)

Simulations for Efficient combination of two lower bound functions in univariate global optimization

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Abstract: We propose a new method for solving univariate global optimization problems by combining a lower bound function given in α BB method [1], with the improved lower bound function of the method developed in [5]. The new lower bound function is better than the two lower bound functions by its construction. The complementarity of the two lower bound functions allows us to derive the convex/concave test and the pruning step which accelerate the convergence of the proposed method. Illustrative examples are treated efficiently

Keywords: Global optimization, α BB method, quadratic lower bound function, Branch and Bound, pruning method

References

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- [5] Le Thi Hoai An and Ouanes Mohand, *Convex quadratic underestimation and Branch and Bound for univariate global optimization with one nonconvex constraint*, RAIRO Oper. Res. (2006) 40: 285-302.
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Proof of the method of spatial contextual Gaussian process of image classification

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Abstract: Recently, a new machine learning approach, based on the Gaussian process theory, was introduced. Though it is shown that, for numerous other classification approaches, the exploitation of spatial contextual information can be potentially attractive to increase the classification accuracy, it still remains to adequately address the issue of integrating such information. In this paper, we are interested in image classification with special attention to the Gaussian process based one. Specifically, we will be looking at the proof of the iterative method by reformulating the standard Gaussian process classification learning model so as to integrate spatial contextual information, in order to improve the accuracy of such classification. In conclusion, this work, by closely examining the proof of spatial contextual classification method, sheds new light on the effectiveness of the latter on image classification.

Keywords: Gaussian processes; image classification; spatial contextual information

References

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A new bounds for approximation of invariant measures in countable space Markov chains

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Abstract: This work investigate the approximation of invariant distributions for countable space Markov chains using truncations of the transition matrix.

Keywords: Truncation; Queueing System; weak Stability; Markov chain; Algorithm.

MSC 2010: 60M20; 60K25; 60J10.

References

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Bernoulli polynomials for the numerical solution of Volterra-Fredholm-Hammerstein stochastic integral equations

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Abstract: The aim of this paper is to use a new operational matrix for integration of Bernoulli polynomials. By using this new operational matrix of integration and the so-called collocation method, nonlinear Volterra-Fredholm-Hammerstein stochastic integral equation is reduced to nonlinear system of algebraic equations with unknown Bernoulli coefficients. Some illustrative error estimations and examples are provided and included to show the accuracy and applicability of the technique.

Keywords: Nonlinear Volterra-Fredholm-Hammerstein integral equations; Bernoulli polynomials; Operational matrix; Collocation method.

MSC 2010: 65R20.

References

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AGE STRUCTURED MODEL OF VIRUS DYNAMICS WITH GENERAL NONLINEAR INFECTION RESPONSE.

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Abstract: We consider an age-structured virus dynamics model with very general nonlinear infection function. We prove the global asymptotic stability of equilibria depending only on the basic reproductive number R_0 . That is, if $R_0 \leq 1$ the infection free equilibrium is globally asymptotically stable, whereas, if $R_0 > 1$; the infection equilibrium is globally asymptotically stable. The proof is based on constructing suitable Lyapunov functionals; using to this end, the existence of compact attractor, total trajectory and the strongly uniform persistence..

Keywords: Age structure; virus dynamics model; general nonlinear infection function; Compact attractor; Total trajectories; Lyapunov functional; Global stability.

References

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Reliability modeling of industrial gas turbine using Weibull distribution

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Abstract: This work propose an approach for reliability optimization applied to gas turbine, to ensure there availability. we propose to develop the tools and sustainability issues, competitiveness, safety and security, instrumentation on this complex industrial systems. Because the increasing complexity of industrial systems has a major impact on the proper operation of these oil installations. In the gas and oil industry, rotating machinery have great interest, they are usually strategic, the aim of this work is increasing the efficiency in gas compression station, for prevention against failures of the gas turbines is using a reliable monitoring approach, which will ensure the optimum availability of these machines. In this context, the quantitative models to assess maintenance performance are essential to optimize decision making and implementation of advanced maintenance policies. The objective of this work is to study and use the right tools of reliability using Weibull distribution to ensure the optimum availability of these machines.

Keywords: Reliability, Weibull distribution, process engineering, gas turbine, availability, dependability, dynamic reliability.

MSC 2010:

References

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Stabilization of the wave equation with a delay term in the nonlinear boundary feedback

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Abstract: The purpose of this paper is to study the stability of the multidimensional wave equation with a delay term in the nonlinear boundary feedback. This problem has been investigated in [3] in the case of linear boundary feedback.

Keywords: Wave equation; non-linear boundary feedback; time delay; stabilization; multiplier method

MSC 2010:

References

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On the Limit Distributions Of Multi-state Homogeneous Consecutive k -out-of- n : G Systems

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Abstract: In this paper, we focus on the limit distributions of consecutive k -out-of- n : G system with multi-state identical components. First, we establish a formula for the reliability of a multi-state consecutive k -out-of- n : G system. Then, we treat the case of identical components where we discuss their limit distributions. In the end, examples are given to motivate the results.

Keywords: Reliability; Limit distributions; Multi-state system; Consecutive k -out-of- n : G system

MSC 2010: 1988a; 9455b; 5549c; 8891d

References

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Optimal replacement time for a stochastically degrading system with a random initial age

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Abstract: This paper study the optimal age replacement policy for used systems, which start their second life-cycle in a more severe environment with an initial age that is uncertain. This uncertain age is modelled as a random variable following continuous probability distributions. A mathematical model is developed to minimize the total expected cost per unit of time for these systems on an infinite time horizon. Optimality and existence conditions for a unique optimal solution are derived and used in a numerical procedure to solve the problem. Numerical experiments are provided to demonstrate the added value and the impacts of the random initial age on the optimal replacement policy.

Keywords: Random age; Stochastic Degradation; Optimization; Reliability

MSC 2010: 60G57; 97K60; 80M50; 90B25

Mathematical analysis of a Malaria model

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Abstract: One of the major problem that North African countries might face is the possible reappearance of the vector-borne diseases like Malaria. This due to the high migration of the sub-Saharan populations to the North African countries, as well as the climate change factor that create new maps of the mosquitoes, responsible for transmission of the disease, distribution in the region.

The fact the north African countries are Malaria free is not any more guarantee that it will stay at this status for ever. There several report of countries that lost the free Malaria status by WHO because of several reasons, among which the human movement and climate change.

In this work, we aim to study the possible transmission of imported Malaria in North African populations (example Algeria and Morocco), called local population, by the sub-Saharan populations, called non-local population, via two patches ODE model and on strain of mosquitoes.

The goal is to find the main factors that control this possible transmission and explore the resource possible to control such infectious from migration policies point of view. Model is set in manner to consider the carrying capacity for the non-local population to mimic the restriction of the number of immigrant that could be allowed to come the country.

Keywords: Malaria dynamic, local and non-local populations, reproduction number, global stability.

MSC 2010:

References

- [1] Sehjeong Kim, Abdessamad Tridane & Dong Eui Chang. Human migrations and mosquito-borne diseases in Africa. *SIAM Journal of Applied Mathematics*, 23:2, 123-146, 2016.
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- [3] Sandip Mandal, Ram Rup Sarkar & Somdatta Sinha. Mathematical models of malaria - a review. *Malaria Journal*, 10:202, 2011.

A New Analytical formula for Computing the Delay Probability in Cloud Center Modeling as $M/G/c/k$ Queue

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Abstract: Successful development of cloud computing paradigm has revived the queuing theory. Based on this theory, we provide an analytical formula to compute the delay probability in cloud computing center modeling as $M/G/c/k$ queue. This probability is used as a cloud performance measure in this paper, because it is usually the main factor to affect customer's decisions; so, it can happen that an arrival customer will leave the system without obtaining service due to long queuing length, which will directly result in revenue losses for a cloud provider.

Keywords: Queuing Theory; Loss Models; Delay Probability; Cloud Computing

MSC 2010: 60K25; 68M20; 90B22

References

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Probabilistic modeling of imperfect repair effects for a repairable system

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Abstract: We consider in this paper a parallel system consisting of η identical components. Each component works independently of the others and has a Weibull distributed inter-failure time. When the system fails, we assume that the repair maintenance is imperfect according to the Arithmetic Reduction of Age models (ARAm) proposed by Doyen and Gaudoin. The purpose of this paper is to generate a simulated failure data of the whole system in order to forecast the behavior of the failure process. Besides, we estimate the maintenance efficiency and the reliability parameters of an imperfect repair following ARAm models using Maximum Likelihood Estimation (MLE) method. Our method is tested with several data sets available from related sources. The real data set corresponds to the time between failures of a compressor which is tested by Likelihood Ratio Test (LR). An analysis of the importance and the effect of the memory order of imperfect repair classes (ARAm) will be discussed using Likelihood Ratio Test.

Keywords: Exponentiated Weibull distribution; Maximum likelihood estimation; likelihood ratio test; imperfect repair

MSC 2010: 97K60; 62H12; 62F03; 62N05

Existence and global attractivity of periodic solutions in a max-type system of difference equations

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Abstract: We consider in this paper the following system of difference equations with maximum

$$\begin{cases} x(n+1) = \max\{f_1(n, x(n)), g_1(n, y(n))\} \\ y(n+1) = \max\{f_2(n, x(n)), g_2(n, y(n))\} \end{cases}, n = 0, 1, 2, \dots,$$

where $f_i, g_i, i = 1, 2$, are real functions with periodic coefficients. We use the Banach fixed point theorem to get a sufficient condition under which this system admits a unique periodic solution. Moreover, we show that this periodic solution attracts all the solutions of the current system. Some examples are also given to illustrate our results.

Keywords: Max-type difference equation; Periodic solutions; Banach fixed point theorem; Global attractivity.

MSC 2010: 39A10; 40A05.

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Inverse boundary value problem in a perturbed strip

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Abstract: We consider the inverse problem to recover a part γ of the boundary of a perturbed strip Ω from a pair of Cauchy data of an harmonic function u in Ω . This problem arises in electrostatic, thermal imaging (detecting a corrosion surface, nondestructive testing...)

Keywords:

MSC 2010:

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Stabilization of the wave equation with a delay term in the nonlinear internal feedback

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Abstract: The aim of this paper is to study the stability of the multidimensional wave equation with a delay term in the nonlinear internal feedback.

Keywords: Wave equation; Nonlinear internal feedback; Time delay; Stabilization

MSC 2010: 35L05; 93D15

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A history-dependent problem for elastic-viscoplastic materials with internal state variable

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Abstract: We consider a mathematical model which describes the quasistatic contact between an elastic-viscoplastic body and a deformable obstacle. The contact is frictional and it is modelled with normal compliance. The material's behavior is modelled by an elastic-viscoplastic constitutive law with internal state variable. We derive a variational formulation to the problem and we prove that it has a unique weak solution. The proof is based on a recent result on history-dependent quasivariational inequalities. Finally, a fully discrete scheme is proposed and error estimates are derived.

Keywords: Contact Problem.; History-dependent quasivariational inequalities; Elastic-viscoplastic materials ; internal state variable; Numerical scheme

MSC 2010: 65K15, 74D10, 74S05, 74S20

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Robust Analysis of Retrial Queues

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Abstract: In this paper, we investigate the M/M/1 retrial queue with finite size orbit, working vacation interruption and classical retrial policy, where we show how parameter uncertainty can be incorporated into the model. We will assume that the retrial rate to be a random variable of distribution which obtained from sample statistic. Therefore, we illustrate the impact of computing the performance measures of the model with an analysis of parameter uncertainty. Specifically, we will provide an approach based on Taylor series expansion for Markov chains for evaluating the uncertainty in the performance measures of the considered queueing model. We develop an algorithm for evaluating the expected value and the variance of different performance measures under the assumption that the retrial rate is computed with uncertainty. Several numerical examples are carried out to illustrate the accuracy of the proposed approach.

Keywords: Retrial queues; Taylor-series expansions; Parameter uncertainty; Robustness analysis; Algorithm.

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free surface flows over a trapezoidal obstacle with surface tension and gravity

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Abstract: Steady two-dimensional flows over a trapezoidal obstacle under the effects of gravity and surface tension in water of a constant depth, and in the presence of a uniform stream. The fluid is treated as inviscid and incompressible. The flow is assumed irrotational. The problem is solved numerically by using the boundary integral equation technique, based on Cauchy integral formula. It is shown that, for both supercritical and sub-critical flows, solutions depend on three parameters: (i) the *Froude* number F , (ii) the *Weber* number α (iii) the angle of obstacle γ . Multiple families of solutions are found to exist for particular values of the precedent three parameters.

Keywords: Free surface flow; potential flow; Weber number; surface tension; Froude number; integro-differential equations

MSC 2010: 76B10; 76C05; 76M25

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bifurcation and global dynamics of a predator-prey model with herd behavior and quadratic predator harvesting.

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Abstract:

Animals grouping together is one of the most interesting phenomena in population dynamics and different functional responses as a result of prey-predator forming groups have been considered by many authors in their model, in this work we investigate the dynamics of predator- prey model with quadratic predator harvesting and in the presence of a herd behavior. We examine the boundedness of the system and the related dynamical behavior of the model. We also study the global stability of the axial and interior equilibria. After the analysis of the model, we observe the presence of a transcritical bifurcation for the axial equilibrium and a Hopf bifurcation occur for the interior equilibrium in this case a limit cycles appear, we will give some numerical simulations to illustrate our theoretical analysis.

Keywords: Herd behavior; hopf bifurcation; Groupe defense; Limit cycles; Quadratic predator harvesting; supercritical Hopf bifurcation; global dynamics.

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Use of Statistics in Record keeping for improving Small Business Enterprises in Tanzania

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Abstract: Despite the fact that it has been truly said that what is wrong with education cannot be fixed with technology; but rather Integrating statistics in record keeping has become a great concern for many people in developing countries like Tanzania, there is no doubt that current life is surely dominated by technology. There is widespread acknowledgment of the need to use statistical tools in education as we enter the era of globalization where the free flow of information via satellite and the internet is embracing the world. A great deal of instructional and managerial work for many small business owners in Tanzania is still carried out without any track of proceedings. This paper, for that basis, identifies benefits a small business drives from proper records keeping. Furthermore this study also critically examines record keeping basis for small business and highlights the record keeping tools. The major challenge on the use of statistics in Tanzania high cost of tools like computer; lack of human skills and knowledge in statistics and lack of relevant software. Sampling techniques were used (systematic, stratified and random samples) and Sample size of 60 respondents were used as Questionnaire and interviews were used to collect data whilst on data analysis mathematical software (matrix laboratory, mathematica 9 and excel) were employed also Statistical package (SPSS) were employed to analyze the data. The preliminary results of the study show that, the most business are using simple manila folders for record keeping whilst most of small business has almost little knowledge on the online electronic system for record keeping.

Keywords: Use of Statistics, Record keeping, Record keeping tools and software.

MSC 2010:

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Q–matrix method for the analysis and performance evaluation of unreliable $M/M/1/N$ queueing model

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Abstract: In the present work, we study an $M/M/1/N$ queueing system with multiple vacations, Bernoulli feedback, balking, reneging and retention of the impatient customers and the possibility of a server breakdown and repair. Firstly, by using of the Q -matrix (infinitesimal generator matrix) method we have obtained the steady-state probabilities of the system. After that, some useful performance measures are obtained. Finally, the influence of the reliability parameters on the performance measures of the system has been examined numerically.

Keywords: Queueing models; Reliability; Markov process; Performance measures; Infinitesimal generator.

MSC 2010: 60K25; 90B22; 68M20.

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Strong Stability Estimate in a Periodic Review Inventory Model

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Abstract: Real-life inventory control problems are often very complicated and are solved only by approximations. Therefore, it is very important to justify this approximation and to estimate the resulting error. This paper presents an approximation method in a periodic inventory control model (R, s, lQ) with instant delivery, based on a strong stability approach. By disturbing the distribution of demand, we obtain quantitative estimates of the stationary distribution. Then, a numerical example can be presented to illustrate the performance of the approach used.

Keywords: Inventory control; Strong Stability; Markov chain ; perturbation; Approximation.

MSC 2010: 60J05; 15A51; 60J10; 65F35; 65F20; 12X34; 56Y78

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TAYLOR SERIES FOR SOLVING LINEAR AND HAMMERSTEIN ILL-POSED VOLTERRA PROBLEMS VIA ITERATIONS METHOD

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Abstract: There exist a number of problems arising in many areas of mathematics, science and technology belong to a class of ill-posed problems, and the integral equations of the first kind are considered ill-posed problems. In this work, we convert linear and nonlinear Volterra integral equations of the first kind to well-posedness problems by simple technique, then we apply the variational iteration method for giving the solutions of these ill-posed problems. Finally, we present two examples which show the performance and efficiency of our technique. .

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History-dependent frictional contact problems

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Abstract: We consider two mathematical models describing the frictional contact between an elastic-viscoplastic body and a foundation. In both models the process is assumed to be quasistatic. The contact is described with a normal compliance or with normal damped response associated to a version of Coulomb's law of dry friction. We derive a variational formulation of the models, which is in the form of a history-dependent quasivariational inequality for the velocity field. Then we prove the unique weak solvability of each problem, the proof is based on arguments of history-dependent quasivariational inequalities. We also study the dependence of the solution of the first problem with respect to the data and prove a convergence result.

Keywords: History-dependent operator; Elastic-viscoplastic material; Coulomb's law friction; normal compliance; normal damped response; weak solution

MSC 2010: 74M10; 74M15; 74G30; 74D10

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Workshop 2

Day 1: October 09, 2017

- M'hamed KESRI. On the solution of an integro-partial differential system in spherical symmetry. University of Sciences and Technology Houari Boumediene, BAB EZZOUAR Algiers, Algeria. [P32](#)
- Allal MEHAZZEM. Dynamic of Discrete Nonlinear Schroedinger Equation with Long-Range Interaction. Larbi Tebessi University, Tebessa, Algeria. [P33](#)
- Abdelhakim NINOUEH. Existence of optimal controls for systems driven by FBDSDEs. Mohamed Khider University, Biskra, Algeria. [P34](#)
- Rima GUERBANE. Primal-dual method of linear programming with hybrid direction. Abderrahmane Mira University, Bejaia, Algeria. [P35](#)
- GUESBA Messaoud. Some singular value inequalities for compact normal operators. Hamma Lakhdar University, El Oued, Algeria. [P36](#)
- Amina ZERARI. Feasible interior point method for linear semidefinite programs. Ferhat Abbas University, Setif, Algeria. [P37](#)
- Amirouche BERKANI. Existence of solutions and stabilization of a viscoelastic rotating Euler-Bernoulli beam. University of Sciences and Technology Houari Boumediene, BAB EZZOUAR Algiers, Algeria. [P38](#)
- Leila AIT KAKI. Some results for a fixed point theory in Banach algebras. ENS Constantine, Algeria. [P39](#)

Day 2: October 10, 2017

- Omar BENNICHE. M-dissipative differential inclusions in arbitrary Banach spaces. Jilali Bounaama University, Khemis Miliana, Algeria. [P40](#)
- Bilal SAOUDI. Factorization of p-adic meromorphic functions. Mohamed Seddik Ben Yahia University, Jijel, Algeria. [P41](#)
- Selma KOUICEM. Singularities of some transmission problems. Abderrahmane Mira University, Bejaia, Algeria. [P42](#)
- Hamza REBAL. Weak Solutions for Boundary Value Problem of Anti-Periodic Fractional Differential Equation in Banach Spaces. University of Sciences and Technology Houari Boumediene, BAB EZZOUAR Algiers, Algeria. [P43](#)
- Nadjet ABADA. Existence Results for Systems of Integral Equations. ENS Constantine, Algeria. [P44](#)
- Sihame BRAHIMI. A Well Posedness Result of a Characteristic Initial Boundary Value Problem with Lipschitz Coefficients. Mostefa Ben Boulaïd University, Batna, Algeria. [P45](#)
- Samira LECHEHEB. Existence of solutions to a Elliptic p -Laplace Equation. August 20, 1955 University, Skikda Algeria. [P46](#)
- Soraya LABIDI. Blow up of solution for a Kirchhoff equation with fractional boundary dissipation. Badji Mokhtar University, Annaba, Algeria. [P47](#)



Workshop 2

Day 3: October 11, 2017

- Abdallah MENAD. On Diffusion models for population dynamics with individual behavior at boundaries. Abdelhamib Ibn Badis University, Mostaganem, Algeria. [P48](#)
- Zohra BOUTEFFAL. Global Uniqueness Results for Impulsive Fractional Partial Hyperbolic Differential Equations with Finite Delay. Mustapha Stambouli University, Mascara, Algeria. [P49](#)
- Mohamed BEZZIOU. Near, approximate and exact viability for fractional differential equation. Jilali Bounaama University, Khemis Miliana, Algeria. [P50](#)
- Ibtissem DJERRAR. Radially symmetric inverse heat conduction problem. Badji Mokhtar University, Annaba, Algeria. [P51](#)
- Abdelkrim ZEKRI. On the boundary value problem with integral conditions. Freres Mentouri University, Constantine, Algeria. [P52](#)
- Mohamed el Amine BENCHEIKH LE HOCINE. An L^∞ -error estimate for an approximation of the solution of a parabolic variational inequality with nonlinear source terms. Amine Elokkel El Hadj Moussa Egakhamouk University Centre, Tamanrasset, Algeria. [P53](#)
- Abdelkarim KELLECHE. Controllability of Some Degenerate Wave Equations. Jilali Bounaama University, Khemis Miliana, Algeria. [P54](#)
- Soraya REKKAB. An Analysis of gradient compensation with minimum energy. Freres Mentouri University, Constantine, Algeria. [P55](#)

On the solution of an integro-differential system in spherical symmetry

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Abstract: Consider a set of particles with the same mass, which they interact without collision. To describe the problem, we introduce a time dependant distribution function f on the phase space, satisfying a continuity equation, the Vlasov or Liouville equation coupled to the field equation with source term:

$$\frac{\partial f}{\partial t} + \vec{v} \frac{\partial f}{\partial \vec{r}} - \frac{\partial V}{\partial \vec{r}} \frac{\partial f}{\partial \vec{v}} = 0 \quad (1)$$

$$\Delta V = 4\pi G \rho \quad (2)$$

where $f = f(\vec{r}, \vec{v}, t)$, \vec{r} the position vector, with module r , $\vec{v} \in \mathbb{R}^3$ the kinetic vector, $t \in \mathbb{R}^+$ the time, $V = V(\vec{r}, t)$ the potential of gravitation, G the gravitational constant. $\rho = \int_{\vec{v}} f d\vec{v}$ represent mass density of the particles.

The objective of this work is to study the nonlinear integro-differential system in case of symmetrical symmetry and giving the numerical solutions.

Keywords: Integro-differential system; Liouville-Boltzmann equation; spectral equation; Poisson equation.

MSC 2010: 45K05; 76P05; 42B10; 85A05; 45G15.

Dynamic of Discrete Nonlinear Schroedinger Equation with Long-Range Interaction

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Abstract: We consider a non-linear oscillator chain with a long-range interaction through a potential proportional to $1/l^{1+\alpha}$ with fractional $0 < \alpha < 2$ and l as a distance between oscillators. In the continues limit the systems dynamics is described by the Fractional Nonlinear Schroedinger equation (called FNLS). The considered chain consists of torsionally coupled, damped, parametrically driven pendulums. We study different spatio-temporal patterns of the dynamics and show transition to chaos.

Keywords: Discrete Shroedinger Equation; Long Range Interacion; Spatiotemporal Chaos; Solitons

MSC 2010:

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Existence of optimal controls for systems driven by FBDSDEs

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Abstract: We prove the existence of optimal relaxed controls as well as strict optimal controls for systems governed by non linear forward-backward doubly stochastic differential equations (FBDSDEs). Our approach is based on weak convergence techniques for the associated FBDSDEs in the Jakubowski S-topology and a suitable Skorokhod representation theorem.

Keywords:

MSC 2010: non linear forward-backward doubly stochastic differential equations; relaxed control; strict control; weak convergence; Jakubowski S-topology

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Primal-dual method of linear programming with hybrid direction .

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Abstract: In linear programming, the conjugation of primal and dual methods is of capital importance in the search for efficient algorithms of resolution. In this work, we are interested in the adaptive method with hybrid direction for the resolution of linear and bounded variables. Using this hybrid direction, we propose to study the increase of the dual function in order to obtain new conditions of optimality.

Keywords: Linear Programming; duality; Hybrid primal direction; Adapted primal-dual method.

MSC 2010: 90C05

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Some singular value inequalities for compact normal operators

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Abstract: In this work we will prove singular value inequality for compact normal operator, which states that if A is compact normal operator on a complex separable Hilbert space H , where $A = A_1 + iA_2$ is the cartesian decomposition of A , then

$$\frac{1}{\sqrt{2}}s_j(A_1 + A_2) \leq s_j(A) \leq s_j(|A_1| + |A_2|), \text{ for } j = 1, 2, \dots$$

Moreover, we prove inequality which states that if A is compact normal operator, then

$$\sqrt{2}s_j(A_1 + A_2) \leq s_j(A + iA^*) \leq 2s_j(A_1 + A_2), \text{ for } j = 1, 2, \dots$$

Also, application of the inequalities are given.

Keywords: Compact operator, Normal operator, Singular value, Inequality

MSC 2010: 90C51; 65K05

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Feasible interior point method for linear semidefinite programs

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Abstract: This work concerns the numerical performances study of a feasible primal polynomial algorithm with two phases for semidefinite programming. The iteration cost depends on the economic computation dominated by the displacement direction, whereas the convergence rate depends on the value of the displacement step. In this sense, we give three new alternatives giving an effective displacement step. The numerical experimentations established consolidate the efficiency of the proposed algorithm.

Keywords: Semidefinite programming; interior point methods; projective algorithm

MSC 2010: 90C51; 65K05

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Existence of solutions and stabilization of a viscoelastic rotating Euler-Bernoulli beam

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Abstract: In this work, we consider a viscoelastic beam which has one end fixed to a rotated motor in a horizontal plane and to a tip mass at the other end. We establish the well-posedness using the Faedo-Galerkin method and, then investigate the exponential stability of system under a suitable control force, for a large class of kernels with weaker conditions used usually in viscoelasticity.

Keywords: Vibration control; rotating Euler-Bernoulli beam; stability; viscoelasticity.

MSC 2010: 35L20; 65N30; 74G25; 74G30; 93D15; 93D20.

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Some results for a fixed point theory in Banach algebras

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Abstract: We introduce a class of Banach algebras satisfying certain sequential condition (P) and we prove fixed point theorems for the sum and the product of nonlinear weakly sequentially continuous operators. In order to obtain existence results for equations in question, we apply the technique of measures of noncompactness and extend the the results given in [J. Garcia-Falset, K. Latrach, E. Moreno-Galvez and M. A. Taoudi, Schaefer-Krasnoselskii fixed point theorems using a usual measure of weak noncompactness, J. Diff. Equ., 252 (2012), 3436-3452.] to Banach algebra. An application on some nonlinear functional integral equations in the Banach algebra of real functions is given.

Keywords: Banach algebra, Weakly compact set, Weakly sequentially continuous operator, Measure of weak noncompactness, Fixed point theorem.

MSC 2010: 47H08, 47H09

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M-dissipative differential inclusions in arbitrary Banach spaces

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Abstract: We consider perturbations of m-dissipative equations in Banach spaces, which are continuous and satisfy a one sided Perron condition. We define the limit solutions and investigate some qualitative properties for them. Existence of limit solutions to nonlocal problems is shown. As application, we investigate an optimal control result for differential equations in abstract Banach spaces

Keywords: differential equations in abstract Banach spaces; one sided Perron condition; nonlocal problems

MSC 2010: Primary 34C25; Secondary 34A60; 49J21

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FACTORIZATION OF p -ADIC MEROMORPHIC FUNCTIONS

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Abstract: In this paper, we study primeness and pseudo primeness of p -adic meromorphic functions. We also consider left primeness of these functions. We give, in particular, sufficient conditions for a meromorphic function to verify such properties.

Keywords: meromorphic function, primeness, left-primeness

MSC 2010: Primary 30D05; Secondary 30D35.

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Singularities of some transmission problems

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Abstract: In our work we give some regularity results concerning the transmission problem for the Laplace operator in a polygonal domain with data in negative fractional order Hilbertian Sobolev space, the transmission heat equation and the transmission problem for the Laplace operator in a polyhedron domain in L^p -Sobolev spaces. We use the characterization of the real interpolation spaces between the domain of the Laplace operator with transmission conditions and L^p in order to prove that the solutions of these problems can be decomposed into a regular part and an explicit singular part.

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Weak Solutions for Boundary Value Problem of Anti-Periodic Fractional Differential Equation in Banach Spaces

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Abstract: This paper is devoted to the existence of weak solutions for boundary value problem of anti-periodic fractional differential equations in Banach spaces. The results are obtained using Mönch's fixed point theorem combined with the technique of measures of weak noncompactness.

Keywords: Caputo fractional derivative, Boundary value problem, Pettis integrals, measure of weak noncompactness, fixed point, Banach space.

MSC 2010: 26A33, 34B15, 34G20

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Existence Results for systems of Integral Equations

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Abstract: In this paper, we study the existence of solution of a nonlinear integral equation in the space $L^1([0, +\infty))$. With the help of Schaefer-Krasnoselskii fixed point theorem and the theory of measure of weak noncompactness, we prove an existence result for a functional integral equation. our results extend and generalize some previous works. An example is given to support ours results.

Keywords: Schaefer-Krasnoselskii fixed point theorem, Measure of weak noncompactness, Non-linear integral equation.

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A Well Posedness Result of a Characteristic Initial Boundary Value Problem with Lipschitz Coefficients

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Abstract: We study the well-posedness of a hyperbolic characteristic Initial boundary value problem. Assuming more general boundary assumptions than those of maximally dissipativeness, we deal with a Lipschitz coefficients hyperbolic system of first order satisfying a minimal structure boundary condition, the so-called Uniform Keiss-Lopatinskii Condition. We show that an a priori estimate of the solution occurs in weighted L^2 norms, with the aid of the paradifferential calculus.

Keywords: hyperbolic boundary value problem, characteristic boundary, Uniform Kreiss-Lopatinski Condition, Paradifferential Operators.

MSC 2010: 35L04 ; 35L40; 35L45;35L50

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EXISTENCE OF SOLUTIONS TO A ELIPTIC p -LAPLACE EQUATION

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Abstract: In this article, we consider the nonlinear elliptic equation

$$A(u) = f(u)$$

where $A(u) = -\operatorname{div}\left(g(|\nabla u|)\nabla u\right) - \frac{1}{\lambda^p} \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ with the Neumann boundary conditions, and under suitable growth condition on the nonlinear term f . The existence of solutions is given via the Brouwer degree theory and the passage to the limit. We finish this article with some numerical result in image denoising and show that our result is the best.

Keywords: Topological degree; elliptic equation; homotopy

MSC 2010: 35J62; 58B05.

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On the decay of the energy for the wave equation with fractional boundary

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Abstract: We consider a wave equation with boundary feedback damping . This feedback involves derivatives of fractional order of the solution. We give sufficient conditions guaranteeing existence of global solutions and decay to the equilibrium state.

Keywords: Exponential decay; Fractional derivative; Weakly singular kernel

MSC 2010: 35B40; 35L70; 35L53

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models for population dynamics with individual behavior at boundaries

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Abstract: The goal of this work is to study a problem of Diffusion with interfaces coming from (concrete) situations in population dynamics. In stationary mode this problem is reduced to an operational form of the type:

$$(P1A) \begin{cases} \text{(Equats)} & u''(x) + Au(x) = G(x) \text{ dans }]-l, 0[\cup]0, 2L[\cup]2L, 2L+l[\\ \text{(Bounda C.)} & u_-(-l) = f_- , \quad u_+(2L+l) = f_+ \\ \text{(Transmission. C.)} & u_-(0) = u_0(0) , \quad u_0(2L) = u_+(2L) \\ \text{(Skewness C.)} & (1-p)du'_-(0) = pdu'_0(0), \quad pdu'_0(2L) = (1-p)du'_+(2L). \end{cases}$$

The operator A verifies the following ellipticity hypothesis:

$$\rho(A) \supset [0, +\infty[\text{ et } \exists C > 0 : \forall \lambda \geq 0, \|(A - \lambda I)^{-1}\|_{L(E)} \leq \frac{C}{1 + |\lambda|}$$

Where $\rho(A)$ denotes the resolvent set of A . The operator $B := -(-A)^{\frac{1}{2}}$ generates

an analytic semigroup which makes it possible to find a representation of the solution. The analysis of this representation makes it possible to find necessary and sufficient conditions on the data for existence, uniqueness and regularity.

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Global Uniqueness Results for Impulsive Fractional Partial Hyperbolic Differential Equations with Finite Delay

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Abstract: This paper deals with the global existence and uniqueness of solutions for impulsive partial functional differential equations with delay, involving the Caputo fractional derivative. Our works will be conducted by using a nonlinear alternative of Leray-Schauder due to Frigon-Granas type for contraction maps on Fréchet spaces

Keywords: Impulsive partial hyperbolic differential equations, fractional order, solution, left-sided mixed Riemann-Liouville integral, Caputo fractional-order derivative, Fréchet spaces, fixed point.

MSC 2010: 26A33, 34K30, 34K37, 35R11.

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Near, approximate and exact viability for fractional differential equation

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Abstract: Let X be a real Banach space and I a nonempty interval. Let $G : I \rightsquigarrow X$ be a set-valued function with the graph \mathcal{K} . We give here a characterization for \mathcal{K} to be approximate/near viable with respect to the fractional differential inclusion $D^\alpha y(t) \in F(t, y(t))$, where $\alpha \in (0, 1)$ and D^α is the Caputo fractional derivative of order α , by means of an appropriate tangency concept and Lipschitz conditions on F . The tangency concept introduced here extends in a natural way the quasi-tangency concept introduced in [5] (see also [4]) in the case when F is independent of t . As an application, we give some results concerning the set of solutions for the differential inclusion $D^\alpha y(t) \in F(t, y(t))$.

Keywords: differential equations in abstract Banach spaces; one sided Perron condition; nonlocal problems

MSC 2010: Primary 34C25; Secondary 34A60; 49J21

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Radially symmetric inverse heat conduction problem

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Abstract: We consider an axisymmetric inverse problem for the heat equation inside the cylinder $0 \leq r \leq 1$. We wish to determine the temperature $f(t) = u(1, t)$ from measured temperature $g^\delta(t)$ in the interior point $0 < r_1 < 1$, this problem is ill-posed. Using Laplace transform we solve the direct problem. Then the inverse problem is reduced to a Volterra integral equation of the first-kind. A standard Tikhonov regularization method is applied to the approximation of this integral equation when the data is noised. Numerical examples are given to illustrate the stability of the proposed method.

Keywords: Ill-posed problem; Radially symmetric heat equation; Laplace transform; Tikhonov regularization.

MSC 2010: 35K05; 65N06; 35R30

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On the boundary value problem with integral conditions

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Abstract: We consider a second-order ordinary differential operator with a spectral parameter and homogeneous integral conditions containing the unknown function and its derivative. We obtain an a priori estimate of the solution for sufficiently large values of the parameter. Also we prove the Fredholm solvability of the problem.

Keywords: Differential equation; Integral condition; Fredholm property; Priori estimate.

MSC 2010: 31A25; 31B20; 34B05; 34G10.

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An L^∞ –error estimate for an approximation of the solution of a parabolic variational inequality with nonlinear source terms

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Abstract: This paper deals with the numerical analysis of the problem of parabolic variational inequalities with nonlinear source terms. Existence and uniqueness of the solution is provided by using a Banach's fixed point theorem. An optimally L^∞ –asymptotic behavior is proved using the semi-implicit time scheme combined with the finite element spatial approximation. The approach is based on Bensoussan-Lions algorithm for evolutionary free boundary problems using the concepts of subsolutions.

Keywords: Parabolic variational inequality, finite element methods, an L^∞ –error estimate.

MSC 2010: 65N30; C.R: G 1.8.

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Controllability of Some Degenerate Wave Equations

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Abstract: This paper is devoted to a study of the null controllability problems for one-dimensional linear degenerate wave equations through a boundary controller. First, the well-posedness of linear degenerate wave equations is discussed. Then the controllability of some degenerate wave equations is established, when a control acts on the non-degenerate boundary. Different from the known controllability results in the case that a control acts on the degenerate boundary, any initial value in state space is controllable in this case. Also, an explicit expression for the controllability time is given. Furthermore, a counterexample on the controllability is given for some other degenerate wave equations.

Keywords: Control of degenerate wave equations.

MSC 2010: 35L05; 35L80; 93B05; 93B07; 93B52; 93D15.

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GRADIENT REMEDIABILITY IN LINEAR DISTRIBUTED PARABOLIC SYSTEMS ANALYSIS, APPROXIMATIONS AND SIMULATIONS

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Abstract: The aim of this paper is the introduction of a new concept that concerned the analysis of a large class of distributed parabolic systems. It is the general concept of gradient remediability. More precisely, we study with respect to the gradient observation, the existence of an input operator (gradient efficient actuators) ensuring the compensation of known or unknown disturbances acting on the considered system. Then, we introduce and we characterize the notions of exact and weak gradient remediability and their relationship with the notions of exact and weak gradient controllability. Main properties concerning the notion of gradient efficient actuators are considered. The minimum energy problem is studies, and we show how to find the optimal control, which compensates the disturbance of the system. Approximations and numerical simulations are also presented.

Keywords: actuators efficient; disturbance; gradient; parabolic systems; remediability; sensors.

MSC 2010: 93B05; 93B07; 93C73

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Workshop 3

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- Ahmed Tageldin Abdelhafiz Ahmed Abdelaal . Bio-mathematical Model Testing the Effects of Bee Honey and Royal Jelly on the Mechanical Properties of the Fetal Membranes. Asiut University Hospital, Asiut, Egypt. **P58**
- Malika DALI- KORSO FECIANE. Goodness of fit tests in a nonlinear diffusion process with delay. Aboubekr Belkaid University, Tlemcen, Algeria. **P59**
- Abdu Masanawa SAGIR. An Efficient Intelligence Approach for Pattern Classification. Hassan Usman Katsina Polytechnic, Katsina, Nigeria. **P60**
- Lynda HARFOUCHE. Bayesian bandwidth selection using two multiplicative bias correction estimators for discrete multivariate associated kernel. Abderrahmane Mira University, Bejaia, Algeria. **P61**
- Ines SADEG. Simulation of the unreliable M/G/1 model with reparation delay and verifications. University of Sciences and Technology Houari Boumediene, BAB EZZOUAR Algiers, Algeria. **P62**
- Oualid SACI. Comparing between empirical distribution functions from a refined descriptive sample and a simple random sample. Abderrahmane Mira University, Bejaia, Algeria. **P63**
- Oualid SACI. Comparing between empirical distribution functions from a refined descriptive sample and a simple random sample. Abderrahmane Mira University, Bejaia, Algeria. **P64**
- Oussama BAH. Estimation in GPD models with interval censoring. Freres Mentouri University, Constantine, Algeria. **P65**

Day 2: October 10, 2017

- Mouloud CHERFAOUI. Influence du pôle d'une densité sur les performances locales de son estimateur à noyaux gamma. Mohamed Khider University, Biskra, Algeria. **P66**
- Fatima BENZIADI. The Application of Kolmogorov's theorem in the one-default model. Tahar Moulay University, Saida, Algeria. **P67**
- Sonia AMROUN. Smoothing parameter selection methods in spline estimate for the regression curve. Abderrahmane Mira University, Bejaia, Algeria. **P68**
- Darel Zerdazi. Analogy between traditional statistical methods and neural networks theory. Laboratory of Applied Mathematics and Modeling, Freres Mentouri University. Constantine. Algeria. **P69**
- Ferhat LOUNIS. An Unreliable Retrial Queue with Impatience and Preventive Maintenance. Mouloud Mammeri University, Tizi Ouzou, Algeria. **P70**
- Ouerdia AREZKI .Estimation Of Missing Value In Random Fields. Mouloud Mammeri University, Tizi Ouzou, Algeria. **P71**
- Boulakhras GHERBAL. Existence of optimal solutions for optimal control problem of linear FBSDEs of mean-field type. Mohamed Khider University, Biskra, Algeria. **P72**
- THARA BELHAMRA. Graphical Approach for claims reserving. Badji Mokhtar University, Annaba, Algeria. **P73**



Workshop 3

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- Ahlem MELAKHESSOU. $Z_q(Z_q + vZ_q + \dots + v^{(m-1)}Z_q)$ – Linear Cyclic, Skew Cyclic and Constacyclic Codes. Mostefa Ben Boulaïd University, Batna, Algeria. [P74](#)
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- Chahrazade MATMAT. Topological invariant for a class of morphisms and some applications. Freres Mentouri University, Constantine, Algeria. [P76](#)
- Maroua MEZGHICHE. Epsilon steepest descent method on Riemannian submanifold of R^n . Badji Mokhtar University, Annaba, Algeria. [P77](#)
- Mohamed Salah BOUDELLIOUA. Equivalence and Similarity of a Class of Matrices over a Commutative Ring. Sultan Qaboos University, Oman. [P78](#)
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Bio-mathematical Model Testing the Effects of Bee Honey and Royal Jelly on the Mechanical Properties of the Fetal Membranes

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Background

Premature rupture of fetal membranes (PROM) carries high risk for preterm labor. Improving the mechanical efficiency of the fetal membranes might be of prophylactic value. The effects of bee honey (H) and royal jelly (RJ) on the mechanical properties of fetal membranes were in-vitro tested.

Material/Methods

Amnion (A) and amnion/chorion complexes (B) were collected from 138 deliveries. Comparative membrane pieces were treated with either H, RJ, H/RJ mixture, or physiologic saline. These were subsequently evaluated by: a) manometric device for their mechanical properties, and, b) histological examination for collagen content.

Results

1. Tearing pressure and elastic extension yield were significantly improved for both A (pressure of 105.7 with H and 146.5 for RJ versus 50.3 mm Hg for the controls; and elastic extension yield of 1.73 with H and 1.93 for RJ versus 1.46 cm for the controls); and B membranes (pressure of 190.7 mm Hg with H and 246.5 for RJ versus 121.2 mm Hg for the controls; elastic extension yield of 2.01 with H, 2.05 with RJ, versus 1.83 cm for the controls).
2. Histological examination and image-analysis quantification revealed significantly increased collagen staining pattern.

Conclusion

H, H/RJ had positive effect on the mechanical properties of fetal membranes. This may be through "collagen promoting action". Further studies are required to: 1) define the exact mechanism involved (e.g. effects on proline/hydroxyproline content, collagen cross-linkage), and, 2) test the efficacy of different methods for clinical extrapolation; i.e. whether ingesting these nutrients, or using them in some form of "local application".

Goodness of fit tests in a nonlinear diffusion process with delay

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Abstract: We consider the problem of goodness of fit tests for a stochastic process with small noise. i.e. We observe complete trajectories $X^\varepsilon = \{X_t, 0 \leq t \leq T\}$ from solution of the stochastic differential equation

$$dX_t = S(X_{t-\theta})dt + \varepsilon dW_t, \quad X_0 = x_0, \quad 0 \leq t \leq T$$

with the deterministic initial value x_0 and the known diffusion coefficient $\varepsilon > 0$. The statistical inference concerns the drift coefficient $S(\cdot)$ only. We propose goodness of fit tests to check whether the model is actually delayed or not. We consider the problem with the asymptotic of a small noise; $\varepsilon \rightarrow 0$.

Keywords: Diffusion process; Goodness of fit tests; Kolmogorov-Smirnov tests; Cramer-Von Mises tests.

MSC 2010: Statistics; Probability theory and stochastic processes.

1 Introduction

First, we consider the case where the basic hypothesis is simple and described by the known drift coefficient up to the unknown delay parameter. We propose a Cramer-von Mises type test and asymptotic behavior of the power function is studied for nonparametric alternatives. The second problem more general, concerns a composite parametric basic hypothesis in which case we propose a test based on a statistic depending on an estimator of the unknown delay parameter with good asymptotic properties. We use for this step the maximum likelihood estimator. The vicinity of power function under nonparametric alternatives is also discussed. The approach is similar to that used for the classical test of Cramer-von Mises (see, for example, Lehmann and Romano [1]). The Cramer-von Mises test has the good quality that its Law does not depend on the basic hypothesis (Distribution free).

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An Efficient Intelligence Approach for Pattern Classification

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Abstract:

This research paper aims to develop an efficient intelligence approach for pattern classification by incorporation the capability of fuzzy logic and artificial neural network learning algorithm that can be used by experts for analysis of data. The proposed method maximises the correctly classified data and minimise the number of incorrectly classified patterns. In addition, an attempt was done to specify the effectiveness of the performance measuring accuracy, sensitivity and specificity. The obtained classification results of the proposed method are comparable and show superiority to classification results obtained by conventional method and other related existing methods.

Keywords: Artificial neural network; Classification; Fuzzy logic; Intelligence approach

MSC 2010: 62J05; 62M10; 92B20; 97R20

Bayesian bandwidth selection using two multiplicative bias correction estimators for discrete multivariate associated kernel

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Abstract: This paper proposed two new estimators called MBC techniques in the context of estimating the multivariate discrete probability mass function. For the choice of the vector bandwidths, we proposed the Bayes global method against the unbiased cross-validation method. We used the Bayesian Markov chain Monte Carlo (MCMC) method for deriving the global optimal bandwidth and we have compared the two proposed method. The performance of both methods is evaluated under the integrated square error criterion. The obtained results show that the Bayes global method performs better than cross-validation.

Keywords: Bayesian global approach; Cross validation; Multiplicative bias correction.

MSC 2010: 62G07; 62G99.

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Simulation of the unreliable $M/G/1$ model with reparation and verification delay

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Abstract: We consider an $M/G/1$ queue with breakdowns, repair and verification delay. The customer whose service is interrupted stays in the service, waiting for delay of verification and repair. After the repair delay this customer completes his service and after this completion of service he leaves the system. We note that the discipline of the service is according with *F.C.F.S.* Then, we concentrate our attention on the limiting distribution of the system state. We obtain simplified expressions for the partial generating functions of the server state and the number of customers in the system and some results of simulation are also investigated, the objective of our study is the optimization of performance in case of failure (reducing the average waiting time).

Keywords: Random breakdowns; Repair time; Verification time; Supplementary variable method

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Comparing between empirical distribution functions from a refined descriptive sample and a simple random sample

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Abstract: In this paper, we compare the empirical cumulative distribution functions obtained by refined descriptive sample and simple random sample of same size through their variances and show that the empirical cumulative distribution function from refined descriptive sample is more efficient than the one obtained from simple random sample.

Keywords: Sampling; Monte Carlo Method; Variance; Empirical distribution function

MSC 2010: 91G60, 62G05

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Existence of optimal controls for systems governed by mean-field forward–backward stochastic differential equations

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Abstract: In this work, we study the existence of optimal control for systems, governed by non linear forward–backward stochastic differential equations of mean-field type. The proof of the main result is based on tightness results of the distributions of the processes defining the control problem and the Skorokhod representation theorem on the Skorokhod space, equipped with the S-topology of Jakubowski . Furthermore, when the Roxin convexity condition is fulfilled, we prove that the optimal relaxed control is in fact strict....

Keywords: Mean-field; Forward backward stochastic differential equation; Relaxed control; Strict control; Existence; Tightness; Weak convergence; The Jakubowski S-topology..

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Estimation in GPD models with interval censoring

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Abstract: In this work, we are interested in the estimation of parameters of a GPD model with interval censoring. First, in a general parametric model with interval censoring we show that the parameter estimates obtained by the method of maximum pseudo-likelihood still possess the properties of efficiency and asymptotic normality. Then, using numerical methods relying on the Barzilai-Borwein algorithm we test the robustness of the parameter estimates of a simulated GPD sample.

Keywords: Interval censoring; pseudo-likelihood; GPD model; asymptotic normality; BB-Algorithm

MSC 2010: 62N01; 62N02

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Influence of a pole-density on the performances of its gamma-kernel estimator

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Abstract: In this paper, in order to highlight the influence of the pole-density on the performances of its gamma-kernel estimator, we first stated some theoretical properties of these estimators for three chosen targeted densities that present different situations of the pole. In a second step, we performed a comparative study of the performance (Bias, variance, MSE, ISE) of the gamma-kernel estimators with those provided by other bias effect correction techniques at the bounds, using simulation technique for the three previous target densities. Moreover, our results confirm the results obtained in the literature and show that in some cases the normalization of the gamma estimators can considerably improve local and global performances of the gamma-kernel estimators.

Keywords:

MSC 2010: 62G07; 62G20.

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The Application of Kolmogorov's theorem in the one-default model

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Abstract: We are interested essentially to the so-called natural model. This model is expressed by a stochastic differential equation called \natural -equation, the latter plays an essential role in this work, but its application has been submitted to a hypothesis of continuity. Then it is important to know under what conditions this hypothesis is satisfied. So, to have this result, we applied the lemma of Kolmogorov.

Keywords: Credit risk; Kolmogorov's continuity criterion; stochastic flow.

MSC 2010: Primary 60G17; Secondary 60H05.

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Smoothing parameter selection methods in spline estimate for the regression curve

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Abstract: The smoothing spline is the one of the most popular methods for estimating the regression function. The most important problem in smoothing spline is the choice of the smoothing parameter λ . In this work we present smoothing parameter selection methods and a bayesian approach in the spline estimate for the regression curve.

Keywords: regression curve; spline smoothing estimate; smoothing parameter; baayesian approach.

MSC 2010: 62G08; 62G05.

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Analogy between traditional statistical methods and neural networks theory.

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Abstract:

This work is an attempt to contribute, even slightly, in situating the neural networks theory into the framework of applied statistics. The central issue of statistical inference was studied under the light of neural approach. A lot of attention was payed to the notion of generalization, with the aim to conceive an unified approach, ghattering together traditional statistical methods with those resulting from neural networks theory, and presenting them as emerging from the same principle. The competitor estimators to the least squares one are surveyed, this is also done for the different neural techniques conceived for the needs of regression and prediction. A comparative study was done with the aim to show that the fundamental concept, at the level of the roots, of these different methods can be seen as unique. A real application case is presented to illustrate the predictive power of neural nets versus the classical methods. .

Keywords: Linear regression ; Ridge regression ; Shrinkage estimators ; Neural nets and related approaches.

MSC 2010: 62J05 ... 62J07 ... 62J12 ... 62M45

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An Unreliable Retrial Queue with Impatience and Preventive Maintenance

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Abstract: In this paper, we analyze an unreliable retrial queue with persistent and impatient customers having different general service distributions. The server is subject to active and passive breakdowns. A persistent customer whose service is interrupted enters the orbit while an impatient one leaves the system. The considered model takes into account two types of arbitrarily distributed maintenances: preventive for improving system performances and preventing breakdowns, and corrective for restoring the service when a failure occurs. If a preventive maintenance occurs in a busy period, then it is postponed to an ulterior date. We give the necessary and sufficient condition for the system to be stable. We obtain the joint probability distribution of the server state and the number of customers in orbit in terms of Laplace and z -transforms. Some performance measures are also given.

Keywords: Retrial queues, Performance measures, Maintenance, z -transforms.

MSC 2010: 60B10, 60F05, 62G30, 60K25.

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Estimation of missing value in random fields

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Abstract: We study a nonstandard prediction problem where a number of observations are added to the quarter-plane past of a stationary random field. The goal is to provide a informative and explicit prediction error variance formulas in terms of moving average parameters of the random fields.

Keywords: random fields; moving average representation; missing value; interpolation.

MSC 2010: 62-02

References

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Existence of optimal solutions to an optimal control problem of linear FBSDEs of mean-eld type

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Abstract: In this work, we prove existence of strict optimal controls for linear forward backward stochastic differential equations (FBSDEs), in which the coefficient depend not only on the state process, but also on the distribution of the state process. The cost functional is also of mean-field type. The control domain and the cost function were assumed convex.

Keywords: Mean-field, Forward backward stochastic differential equation, Strict control, Existence

MSC 2010: 60H10

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Graphical Approach for claims reserving

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Abstract: In this paper, we give for the first time a geometrical aspect of the standard theory of claim reserving using incremental claims. We show how to estimate the graphical development factors (k_j) using graphical technique, for that we need to calculate the tangents for each development year j , then we estimate the lower circles using observed circles and tangents. This leads to estimations of the yearly and total provisions.

Keywords: IBNR; Chain Ladder; Incremental Approach; Graphical Approach; provision.

MSC 2010: 62P05; 91G70

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$\mathbb{Z}_q(\mathbb{Z}_q + v\mathbb{Z}_q + \dots + v^{m-1}\mathbb{Z}_q)$ – Linear Cyclic, Skew Cyclic and Constacyclic Codes

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Abstract: The purpose of this work is to study cyclic, skew cyclic and constacyclic codes over the ring \mathbb{Z}_qR , where $q = p^s$, p is a prime and $v^m = v$. We give the definition of these codes as subsets of the ring $\mathbb{Z}_q^\alpha R^\beta$.

Keywords: Skew cyclic codes; Constacyclic codes; Gray map.

MSC 2010: 11TXX; 11T71; 14G50; 94B05

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Classification of Translation Surfaces in Lorentz Heisenberg 3–Space With Plat Metric

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Abstract: In this work we study the characterization of some types of minimal translation surfaces in the 3– dimensional Lorentz Heisenberg space \mathcal{H}_3 endowed with the plate metric

$$g_3 = dx^2 + (xdy + dz)^2 - [(1-x)dy - dz]^2$$

Keywords: Lorentz Heisenberg 3–Space; Plate metric; Mean curvature; Translation surfaces; Minimal surfaces

MSC 2010: 53A10; 53A45; 53C20

References

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Topological invariant for a class of morphisms and some applications

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Abstract: This paper is devoted to build and study a topological degree for a class of morphisms f, g in the category Top of topological spaces and continuous single valued maps.

This described degree is obtained by the use of the homotopic methods, and it is a tool in coincidence point theory and the Borsuk-Ulam theorem. Some properties are given.

Keywords: Homotopy, Homology, topological invariant, Borsuk-Ulam theorem.

MSC 2010: 55M20; 55P99

Epsilon steepest descent on Riemannian submanifolds of R^n

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Abstract: In this paper we propose the epsilon steepest descent method to solve optimization problems in Riemannian manifolds. This algorithm accelerate the convergence of steepest descent method on Riemannian manifolds. We establish the convergence of our new algorithm with Armijo line search.

Keywords: Optimization on Riemannian manifolds; epsilon steepest descent method; Armijo inexact line searches; geodesics.

MSC 2010: 65M10; 53C21; 90C30

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Equivalence and Similarity of a Class of Matrices over a Commutative Ring

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Abstract: In this paper, the problem of equivalence and similarity of certain classes of matrices over a commutative ring is considered. Such matrices arise for example in the study of linear systems of delay-differential equations. Necessary and sufficient conditions are given under which a class of matrices is similar to the companion form. These conditions turn out to be equivalent to the corresponding characteristic matrices to be equivalent to the Smith normal form.

Keywords: Matrices, Equivalence, Similarity, Companion form, Smith form.

MSC 2010: 11Cxx; 15A21; 15A23

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On Almost Clean Rings

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Abstract: A ring R is *almost unit-clean* provided that every element in R is equivalent to the sum of an idempotent and a regular element. The subject of almost clean rings (in particular, in commutative case) is interested for so many mathematicians, e.g., [1,2,3] and [4,5], as they are related to the well-studied clean rings of Nicholson. In this work, we shall seek to remedy this by looking at an interesting generalization of almost clean rings. We investigate conditions under which a ring is almost unit-clean.

Keywords: almost unit-clean ring; unit regular ring; π -regular ring

MSC 2010: 16E50; 13G99

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Poster Sessions 1 Program (15h00:16h00, October 09, 2017)

Nacera BAARA : "Green's Function for Dispersive Wave Equation in Stratified Media.",
University of Sciences and Technology Mohamed Boudiaf, **Oran**, Algeria.

Izzessalam TELITEL : "A Simple mathematical model for the treatment of a lung cancer.",
Freres Mentouri University, **Constantine**, Algeria.

Sarra BRIHI : "Error estimate of the projection method for the unsteady Darcy-Brinkman-Forchheimer equations.", University of Caen Normandy, **France**.

Abdelali Makhfi : "Brusselator system and non-standard analysis",
University of Sciences and Technology Houari Boumediene, BAB EZZOUAR **Algiers**, Algeria.

Ahmed DIAR : "Ultimate bound and dynamical behavior for a new complex chaotic system",
Larbi Ben M'hidi University, **Oum El Bouaghi**, Algeria.

Nesrine TABCHOUCHE : "Interior-point methods of primal-dual central-path type for solving some classes of linear complementarity problems over symmetric cones.", Ferhat Abbas University, **Setif**, Algeria.

Djamel DEBBABI : "Periodic solutions of some classes of continuous second-order differential equations.", Badji Mokhtar University, **Annaba**, Algeria.

Djamila DJEDID : "Limit cycle of the generalized Michelson system.",
Badji Mokhtar University, **Annaba**, Algeria.

Chahrazed MESSIKH : "Finite volume method for a Keller-Segel problem.", Badji Mokhtar University, **Annaba**, Algeria.

Khaoula ROUIBEH : "Iterative collocation method for solving nonlinear Volterra integral equation.",
Abdelhafid Boussouf University Center, **Mila**, Algeria.

Nidal DIB : "On a stochastic generalized burgers equation in a bounded domain.",
August 20, 1955 University, **Skikda** Algeria.

Ala Eddine DRAIFIA : "Existence of the solution for a quasi-linear parabolic equation with non-local condition.", Larbi Tebessi University, **Tebessa**, Algeria.

Malik BELAID : "Existence and stability for totally nonlinear neutral dynamic equations with variable delay on time scales.", Badji Mokhtar University, **Annaba**, Algeria.

Meneceur BEKKAR : "Non-Existence Of Global Solutions Of Nonlinear Fractional Reaction-Diffusion Equation On The Heisenberg Group.", Freres Mentouri University, **Constantine**, Algeria.



Poster Sessions 1 Program (15h00:16h00, October 09, 2017)

Samira DEHAIMI : "Estimation of parameters of extreme values.",
Badji Mokhtar University, **Annaba**, Algeria.

Houda BOUREZAZ : "Prediction-Satisfaction index in experimental design for Poisson-Gamma model.", Freres Mentouri University, **Constantine**, Algeria.

Nadia KADIRI : "Strong uniform consistency rates of conditional quantiles estimation in the single functional index model for Dependant Functional Data Under Random Censorship.", Djillali Liabes University, **Sidi Bel Abbes**, Algeria.

Amel ZERARI : "Sequential analysis in clinical trials.",
Abdelhafid Boussouf University Center, **Mila**, Algeria.

Besma BENNOUR : "Bayesian Reliability Analysis of a Linear and Circular Consecutive-k-out-of-n System under a Shock Model.", Freres Mentouri University, **Constantine**, Algeria.

Farouk METIRI : "On weighted balanced loss function under the Esscher principle and credibility premiums.", Badji Mokhtar University, **Annaba**, Algeria.



Poster Sessions 2 Program (10h00:11h00, October 10, 2017)

Achouak BEKKAI : "Critical exponent for nonlinear hyperbolic system with spatio-temporal fractional derivatives.", Larbi Tebessi University, **Tebessa**, Algeria.

Feriel Bouhadjera : "Wavelet analysis of market volatility.",
Badji Mokhtar University, **Annaba**, Algeria.

Zineb BARHOUM : "Approche Solution of Population Balances for Growth and Aggregation Processes.", Mohamed Khider University, **Biskra**, Algeria.

Zouhair Diab : "Limit Cycles for the Class of D -Dimensional Polynomial Differential Systems.",
Larbi Tebessi University, **Tebessa**, Algeria.

Zineb ACHOURI : "The Euler-Bernoulli beam equation with boundary dissipation of Caputo's fractional derivative type", Djillali Liabes University, **Sidi Bel Abbes**, Algeria.

Abdallah BEDDANI : "Approche Bolza type problems in infinite dimensional in discrete time.",
Ahmed Zabana University Center, **Relizane**, Algeria.

Hakima DEGAICHIA : "The wolfe epsilon steepest descent algorithm.",
Larbi Tebessi University, **Tebessa**, Algeria.

Aicha SAKHRI : "A study of a class of a dynamic system of Fitzhugh-Nagumo type.",
Badji Mokhtar University, **Annaba**, Algeria.

Selma ELLAGGOUNE : "Upper bounds for the number of limit cycles of polynomial differential systems.", May 8, 1945 University, **Guelma**, Algeria.

Fatima Zahra ARIQUI : "Black-Scholes Option Pricing Model.",
Djillali Liabes University, **Sidi Bel Abbes**, Algeria.

Nesrine HAMIDI : "Estimation of the conditional quantile for functional stationary ergodic data with responses missing at random."
Djillali Liabes University, **Sidi Bel Abbes**, Algeria.

Meryem Bisma BOUCHENAKI : "Estimation of multiple delays in a nonlinear diffusion process.",
Aboubekr Belkaid University, **Tlemcen**, Algeria.

Malika CHEIKH : "Robust Mahalanobis distance in multivariate analysis."
Mouloud Mammeri University, **Tizi Ouzou**, Algeria.

Nassima BERROUIS : "Stochastic optimal control for linear forward backward doubly stochastic differential equations.",
Mohamed Khider University, **Biskra**, Algeria.



Poster Sessions 2 Program (10h00:11h00, October 10, 2017)

Ikhlasse CHEBBAB : "Exponential inequalities for a LNQD autoregressive process.",
Djillali Liabes University, **Sidi Bel Abbes**, Algeria.

Assia AYACHE : "The optimal interpolation method.",
Freres Mentouri University, **Constantine**, Algeria.

Said KHALDI : "Weak Convergence Methods for Non-linear PDE.",
Djillali Liabes University, **Sidi Bel Abbes**, Algeria.

Ali RIMOUCHE : "Multiple solutions for a semilinear problem with boundary singularities.",
Aboubekr Belkaid University, **Tlemcen**, Algeria.

Fatima FENENICHE : "Limit cycles and integrating factors for second order ODEs.",
University of Sciences and Technology Houari Boumediene, BAB EZZOUAR **Algiers**, Algeria.

Radhouane AOUNALLAH : "Exponential decay of solutions of a nonlinearly damped wave equation.",
Djillali Liabes University, **Sidi Bel Abbes**, Algeria.

Omar Farouk AID : "Propagation of Coherent States in Quantum Mechanics.",
Ahmed BENBELLA University, **Oran**, Algeria.



Poster Sessions 3 Program (15h00:16h00, October 10, 2017)

Selma BOULECHFAR: "Mathematical modeling of a model of communication on the Internet TCP.", August 20, 1955 University, **Skikda**, Algeria.

Chafia KAREK: "Homogenization of a Stokes problem in domains with small holes with Non-Homogeneous Slip Boundary Conditions.", August 20, 1955 University, **Skikda**, Algeria.

Sarra TOUALBIA: "Asymptotic behavior for a viscoelastic problem with no necessarily decreasing kernel.", Larbi Tebessi University, **Tebessa**, Algeria.

Khaoula BOUGUETOFF : "Non existence for the Laplace equation of fractional type.", Larbi Tebessi University, **Tebessa**, Algeria.

Hafida LAIB: "Numerical Solution of Neutral Delay Integro-differential Equations by Using Taylor Collocation Method.", Abdelhafid Boussouf University Center, **Mila**, Algeria.

Meryem BEY: "Limit cycles bifurcating from the period annulus of a uniform isochronous center in a quartic polynomial differential system.", Badji Mokhtar University, **Annaba**, Algeria.

Nawel BOUDJELLAL: "Convex quadratic program; central trajectory method; short-step; long-step.", Ferhat Abbas University, **Setif**, Algeria.

Sihem ZETILI : "Study of the Stability of fractional-order dynamical systems.", Freres Mentouri University, **Constantine**, Algeria.

Widad LAOUIRA: "Feedback Control of Lorenz System.", Freres Mentouri University **Constantine**, Algeria.

Amel HANNACHE: "WAVELET ANALYSIS OF MARKET VOLATILITY.", August 20, 1955 University, **Skikda**, Algeria.

Allaoua BOUDJEDOUR: "Existence of the weak solution of the anti-plane contact problem.", Freres Mentouri University, **Constantine**, Algeria.

Nawel MOUDJEB : "Regularization by a modified quasi-boundary value method of the ill-posed problems". Larbi Ben M'hidi University, **Oum El Bouaghi**, Algeria..

Torkia MEROUAN: "Modelling dependence using the copulas theory.", Djillali Liabes University, **Sidi Bel Abbes**, Algeria.

Abdelouahab RASLAIN: "Supervised and unsupervised classification techniques for the treatment of two real situations.", Freres Mentouri University, **Constantine**, Algeria.

Zaineb MEZDOUD: "The volatility of stock prices with Option pricing: Black-Scholes Made Easy software.", Badji Mokhtar University, **Annaba**, Algeria.

ABOUD NEMOUCHI: "Estimation and tests with bivariate censored data.", Freres Mentouri University, **Constantine**, Algeria.



Poster Sessions 3 Program (15h00:16h00, October 10, 2017)

Mouhamed Amine NIANG: "Testing bivariate normality based on nonlinear canonical analysis.",
Alioune Diop University, Bambey. **Senegal**.

Souheyla CHEMIKH: "Asymptotic study of the robust estimator of the regression function.",
Djillali Liabes University, **Sidi Bel Abbes**, Algeria.

Ilhem LAROUSSI: "B-spline estimators of regression function with twice censored data.",
Freres Mentouri University, **Constantine**, Algeria.

Nadjla ZEHANI: "Stochastic Volatility Model.",
Chadli Bendjedid University, **El Tarf**, Algeria.

Amina BOUSSAID: "Solutions to the equation $A_1X B_1 + A_2X B_2 = C$ for Hilbert space operators.",
Mostefa Ben Boulaïd University, **Batna**, Algeria.

Fairouz SOUILAH: "Solving of a nonlinear parabolic problem with data in L_1 .",
August 20, 1955 University, **Skikda** Algeria. Bien 2page

Manel GOUASMIA: "Study of periodic and nonnegative periodic solutions of nonlinear neutral functional differential equations via fixed points.", Badji Mokhtar University, **Annaba**, Algeria.



Poster Sessions 4 Program (08h00:10h00, October 11, 2017)

Nesrine SEMCHEDINE : "General decay of solutions of a wave equation with a boundary control of memory type.", Ferhat Abbas University, **Setif** , Algeria.

Fayrouz ZOUBAI : "On the mixed problem for the nonlinear elasticity system in a domain regular.", Ferhat Abbas University, **Setif** , Algeria.

Ismahène SEHILI : "Two-Dimensional Legendre Approximation.", Mohamed Khider University, **Biskra**, Algeria.

Dahmane BOUAFIA : "Existence of positive solutions for a second order problem on the half - line via Ekeland's variational principle.", Mohamed Boudiaf University, **M'Sila**, Algeria.

Asma MAADADI : "Numerical solution of nonlinear Fredholm integro - differential equations using Chebyshev polynomials." , Mohamed El Bachir El Ibrahimi University, **Bordj Bou Arréridj**, Algeria.

Dounia BELAKROUM : "A numerical method for the solution of hyperbolic damped wave equation.", Freres Mentouri University, **Constantine**, Algeria.

Sara KASSA : "Detecting periodic orbits in some 3D chaotic quadratic polynomial differential systems.", Badji Mokhtar University, **Annaba** , Algeria.

Imene TOUIL : "A primal - dual interior- point method for semidefinite programming problem based on a new kernel function.", Mohamed Seddik Ben Yahia University, **Jijel**, Algeria.

Nouria ARAR : "Spectral properties of the eigenfunctions of the operator $\varrho^{-1} \Delta$.", Freres Mentouri University, **Constantine**, Algeria.

Rekia MESSIOUENE: "Semi classical Fourier integral operator.", Ahmed Benbella University, **Oran**, Algeria.

Mohamed HARIRI : "On The Stabilization Of Degenerate Differential Systems In Hilbert Spaces.", Djillali Liabes University, **Sidi Bel Abbas** , Algeria.

Mabrouk BRIKI: "Solvability of an impulsive boundary value problem on the half- line via critical point theory.", Ziane Achour University, **Djelfa**, Algeria.

Hafidha SEBBAGH : "The exact number of positive solutions for a class of quasilinear boundary value problems.", Aboubekr Belkaid University, **Tlemcen** , Algeria.

Katia BACHI : "Statistical Analysis of Propagation of Parametric Uncertainty In Stochastic System." , Abderrahmane Mira University, **Bejaia**, Algeria.

Soumia MANAA : "Singular and asymptotic behavior of solutions of boundary value problems in non - homogeneous domains.", Ferhat Abbas University, **Setif**, Algeria.



Poster Sessions 4 Program (08h00:10h00, October 11, 2017)

Alima CHIBANI : "A preconditioned UZAWA method for stabilized mixed Velocity - Pressure Finite Elements. ", Freres Mentouri University, **Constantine**, Algeria.

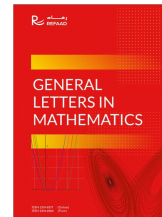
Haizia BOUNADJA : "Decay rates for elastic- thermoelastic star -shaped networks." , Ferhat Abbas University, **Setif**, Algeria.

Ali HAMEIDA : "Optimal regularization for ill posed inverse problems." , Freres Mentouri University, **Constantine**, Algeria.

Okba ZEHROUR : "Optimal and adaptive control for HIV model." , Larbi Ben M'hidi University, **Oum El Bouaghi**, Algeria.

Mohammed El Amine MEKKI : "Ricci Solitons On Sasakian Manifolds. " , Mustapha Stambouli University, **Mascara**, Algeria.

Aziza REZIG : "Groups with many subgroups having soluble - by - finite minimax layers or derived subgroup. ", Ferhat Abbas University, **Setif**, Algeria.



Taylor approximation for solving linear and nonlinear Ill-Posed Volterra equations via an iteration method

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Abstract

In this paper, we present two algorithms for the approximate or exact solution of a class of Volterra integral equations of first kind. As well known, this is an ill posed problem, but we convert it to well-posedness of the second kind Volterra problems, then we apply the variational iteration method. Finally, we present two examples which show the performance and efficiency of our method.

Keywords: Linear Volterra integral equation of the first kind -Hammerstien integral equation of the first kind series, variational iteration method.

2010 MSC: 65N20, 34A12, 30K05.

1. Introduction and preliminaries

Two of the most standard forms of linear and nonlinear integral equations of the first kind are:

$$\lambda \int_0^t k(t, x) \varphi(x) dx = f(t), \quad (1.1)$$

and

$$\lambda \int_0^t k(t, x) F(\varphi(x)) dx = f(t), \quad (1.2)$$

here, both, the function $k(t, x)$, and the function $f(t)$ are known, k on the square $0 \leq x; t \leq 1$, and f on the interval $0 \leq t \leq 1$. The quantity λ is a given constant parameter. The function φ is to be determined on $[0, 1]$ and $F(\varphi(t))$ is a nonlinear function of $\varphi(t)$ (note that we can take $0 \leq t \leq 1$, since every interval such as $[a, b]$ can be transformed into this interval by a linear transformation (see in [10])).

As a classical ill-posed problem, the Volterra integral equations of the first kind have been investigated by many references (see, for instance [12, 8, 11, 3]). These equations arise in many scientific applications such as the population dynamics, spread of epidemics, and semi-conductor devices [12]. For sufficiently smooth f and k , we may differentiate these equations with respect to t to obtain Volterra integral equations

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of the second kind which are known to be well-posed problems. This classical converting is applied in many references (see, for instance [12, 7]).

We recall that in the variational iteration method, it was first proposed in [5] and recently used in the literature for solving both linear and nonlinear problems analytic and numerical. The variational iteration method is based on the general Lagrange's multiplier method is simple and powerful method. Let us take the differential equation

$$L\varphi + N\varphi = g,$$

where L is a linear operator, N is an operator which can be nonlinear, and g is an inhomogeneous term. Using method of successive approximations, we can write the $(n+1)^{\text{th}}$ approximation of the solution as the n^{th} plus some corrections. According to the variational iteration method (see, for instance [13, 12, 10, 2, 9, 1, 4]), we can construct a correction functional as follows

$$\varphi_{n+1}(t) = \varphi_n(t) + \int_0^t \lambda(\varepsilon) (L\varphi_n(\varepsilon) + N\tilde{\varphi}_n(\varepsilon) - g(\varepsilon)) d\varepsilon, \quad 0 \leq \varepsilon \leq t \leq 1$$

where the correction functional contains a general Lagrange multiplier λ which can be identified in an optimal way by the variational theory, noting that in this method λ may be a constant or a function, φ_n is the n^{th} approximate solution, and $\tilde{\varphi}_n$ denotes a restricted variation, i.e. $\delta\tilde{\varphi}_n = 0$, where δ is the variational derivative.

In this article, we intend to combine the variational iteration method and Taylor approximation to obtain a systematic and efficient method for solving previous ill-posed Volterra equations (1.1) and (1.2). We will transform these equations to an equivalent integral equations of the second kind defined in $[0, 1]$.

Let $A(t)$ be an operator with derivatives of first order with respect to t in an interval $[0, 1]$ than for $0 < t-h < t < t+h < 1$, with $h \rightarrow 0$. The Taylor series given by

$$A(t+h) = A(t) + \frac{h}{1!} A'(t) + O(h), \quad (1.3)$$

$$A(t-h) = A(t) - \frac{h}{1!} A'(t) + O(h), \quad (1.4)$$

where $O(h)$ is an unknown error term of approximation.

The presentation and the analysis of our method, based on the idea of Taylor approximation, is the main goal of this paper.

2. Solving linear Volterra integral equation of the first kind

Now, let $A(t)$ be an operator integral defined by

$$A(t) = \int_0^t k(t, x) \varphi(x) dx, \quad 0 \leq x \leq t \leq 1$$

and by using the Taylor series of the first order and Leibnitz rule, we find

$$A(t+h) = A(t) + hk(t, t)\varphi(t) + h \int_0^t \frac{\partial k(t, x)}{\partial t} \varphi(x) dx + O(h).$$

Then, the approximate Volterra integral equation of the first kind (1.1) given by

$$f(t) + hk(t, t)\varphi(t) + h \int_0^t \frac{\partial k(t, x)}{\partial t} \varphi(x) dx = f(t+h) + O(h),$$

where $k(t, t) \neq 0$ and $\frac{\partial k(t, x)}{\partial t} \neq 0$ for $t \in [0, 1]$, we obtain the linear Volterra integral equation of the second kind given by

$$\varphi_h(t) + \int_0^t K(t, x) \varphi_h(x) dx = f_h(t), \quad h \rightarrow 0 \quad (2.1)$$

where

$$K(t, x) = \frac{\frac{\partial k(t, x)}{\partial t}}{k(t, t)}, \quad f_h(t) = \frac{f(t+h) - f(t)}{hk(t, t)} \text{ and } \varphi_h = \varphi \text{ if } h \rightarrow 0.$$

Substituting $t = 0$ into (2.1) gives the initial condition $\varphi_h(0) = \varphi_h^0$.

Now, using Leibnitz rule to differentiate both sides of (2.1) gives

$$\varphi'_h(t) + K(t, t) \varphi_h(t) + \int_0^t \frac{\partial K(t, x)}{\partial t} \varphi_h(x) dx = f'_h(t). \quad (2.2)$$

We apply variational iteration method for equation (2.1) and take $\varphi_h(t) = \psi(t)$. According to this method correction functional can be written in the following form

$$\psi_{n+1}(t) = \psi_n(t) + \int_0^t \lambda(\varepsilon) \left(\psi'_n(t) + K(t, t) \psi_n(t) + \int_0^\varepsilon \frac{\partial K(t, x)}{\partial t} \psi_h(x) dx - f'_h(t) \right) d\varepsilon.$$

Imposing the stationary condition ($\partial \psi_{n+1} = 0$) on the correction functional, the Lagrange multiplier $\lambda(\varepsilon) = -1$.

As a result, we obtain the following iteration formula

$$\psi_{n+1}(t) = \psi_n(t) - \int_0^t \left(\psi'_n(t) + K(t, t) \psi_n(t) + \int_0^\varepsilon \frac{\partial K(t, x)}{\partial t} \psi_h(x) dx - f'_h(t) \right) d\varepsilon. \quad (2.3)$$

By starting from $\psi_0(t) = \varphi_h^0(t)$, we can obtain the exact solution or an approximate solution $\varphi_h(t) = \lim_{n \rightarrow \infty} \psi_n(t)$ of the equation (2.1).

Moreover, the solution $\varphi_h(t)$ of the equation (2.1) converges to the solution $\varphi(t)$ of Volterra integral equation (1.1) of the first kind as $h \rightarrow 0$; or written

$$\varphi(t) = \lim_{h \rightarrow 0} \varphi_h(t).$$

Here, we emphasize the two important remarks related to this converting technique for linear Volterra integral equation. First, if we use the Taylor series of the first order and Leibnitz rule, we obtain the Volterra integral equation of the second kind given by

$$\varphi_h(t) + \int_0^t \frac{\frac{\partial k(t, x)}{\partial t}}{k(t, t)} \varphi_h(x) dx = \frac{f(t) - f(t+h)}{hk(t, t)}, \quad h \rightarrow 0$$

Second, if we use Leibnitz rule and the difference of Taylor series between (1.3) and (1.4), we obtain the well-posed Volterra integral equation given by

$$\varphi_h(t) + \int_0^t \frac{\frac{\partial k(t, x)}{\partial t}}{k(t, t)} \varphi_h(x) dx = \frac{f(t+h) - f(t-h)}{2hk(t, t)}, \quad h \rightarrow 0$$

3. Solving Volterra-Hammerstein integral equation of the first kind

In this section, we shall study a nonlinear integral equation. Let $A(t)$ be a nonlinear operator defined by

$$A(t) = \int_0^t k(t, x) F[\varphi(x)] dx, \quad 0 \leq x \leq t \leq 1$$

and by using the Taylor series of the first order and Leibnitz rule, we find

$$A(t+h) = A(t) + hk(t, t)F(\varphi(t)) + h \int_0^t \frac{\partial k(t, x)}{\partial t} F(\varphi(x)) dx + O(h),$$

then, the Volterra-Hammerstein integral equation of the first kind (1.2) given by

$$f(t) + hk(t, t)F(\varphi(t)) + h \int_0^t \frac{\partial k(t, x)}{\partial t} F(\varphi(x)) dx = f(t+h) + O(h).$$

Then, if $k(t, t) \neq 0$ and $\frac{\partial k(t, x)}{\partial t}$ we obtain

$$F(\varphi_h(t)) + \int_0^t \frac{\frac{\partial k(t, x)}{\partial t}}{k(t, t)} F(\varphi_h(x)) dx = \frac{f(t+h) - f(t)}{hk(t, t)}, \quad (3.1)$$

where $\varphi_h(t) = \varphi(t)$ if $h \rightarrow 0$, After to use the change of variables, we obtain the linear Volterra integral equation of the second kind given by

$$\Psi_h(t) + \int_0^t K(t, x) \Psi_h(x) dx = \frac{f(t+h) - f(t)}{hk(t, t)}, \quad (3.2)$$

where

$$\Psi_h(t) = F(\varphi_h(t)), \quad K(t, x) = \frac{\frac{\partial k(t, x)}{\partial t}}{k(t, t)} \text{ and } f_h(t) = \frac{f(t+h) - f(t)}{hk(t, t)}.$$

Assuming that $F(\varphi_h(t))$ is invertible, then we can set

$$\varphi_h(t) = F^{-1}(\Psi_h(t)).$$

Substituting $t = 0$ into (3.2) gives the initial condition $\Psi_h(0) = F(\varphi(0)) = \Psi_h^0$.

Now, using Leibnitz rule to differentiate both sides of (3.2) gives

$$\Psi'_h(t) + K(t, t) \Psi_h(t) + \int_0^t \frac{\partial K(t, x)}{\partial t} \Psi_h(x) dx = f'_h(t). \quad (3.3)$$

We apply variational iteration method for equation (3.2) and take $\Psi_h(t) = \psi(t)$. According to this method, correction functional can be written in the following form

$$\psi_{n+1}(t) = \psi_n(t) + \int_0^t \lambda(\varepsilon) \left(\psi'_n(t) + K(t, t) \psi_n(t) + \int_0^\varepsilon \frac{\partial K(t, x)}{\partial t} \psi_n(x) dx - f'_h(t) \right) d\varepsilon.$$

Imposing the stationary condition ($\delta\psi_{n+1} = 0$) on the correction functional, the Lagrange multiplier $\lambda(\varepsilon) = -1$.

As a result, we obtain the following iteration formula

$$\psi_{n+1}(t) = \psi_n(t) + \int_0^t \psi'_n(t) + K(t, t) \psi_n(t) + \int_0^\varepsilon \frac{\partial K(t, x)}{\partial t} \psi_h(x) dx - f'_h(t) d\varepsilon. \quad (3.4)$$

By starting from $\psi_0(t) = \Psi_h^0(t)$, we can obtain the exact solution or an approximate solution $\Psi_h(t) = \psi(t) = \lim_{n \rightarrow \infty} \psi_n(t)$ of the equation (3.2). Using the transformation

$$\varphi_h(t) = F^{-1}(\Psi_h(t)).$$

Moreover, the solution $\varphi_h(t)$ of equation (3.2) converges to the solution $\varphi(t)$ of Volterra integral equation (1.2) of the first kind as $h \rightarrow 0$, or write

$$\varphi(t) = \lim_{h \rightarrow 0} \varphi_h(t).$$

Two important remarks related to this converting technique for Volterra-Hammerstein integral equation can be made here. First, if we use the Taylor series of the first order and Leibnitz rule, we obtain the Volterra integral equation of the second kind given by

$$\Psi_h(t) + \int_0^t \frac{\partial k(t, x)}{\partial t} \Psi_h(x) dx = \frac{f(t) - f(t+h)}{hk(t, t)}. \quad h \rightarrow 0$$

Second, if we use Leibnitz rule and the difference of Taylor series (1.3) and (1.4), we obtain the linear Volterra integral equation of the second kind given by

$$\Psi_h(t) + \int_0^t \frac{\partial k(t, x)}{\partial t} \Psi_h(x) dx = \frac{f(t+h) - f(t-h)}{2hk(t, t)}. \quad h \rightarrow 0 \text{ and } k(t, t) \neq 0.$$

We note that if we use another iterations method (like, Adomian decomposition method, the modified decomposition method, ...) the resulting systems for solving the Volterra integral equations of the first kind is slightly different from (2.3) and (3.4).

4. Illustrative examples

This method for linear and nonlinear Volterra integral equations will be illustrated by discussing the following examples.

We consider the linear Volterra integral equation of the first kind

$$\int_0^t (t-x+10^2) \varphi(x) dx = \frac{t^2}{2} + 10^2 t. \quad (4.1)$$

It's equivalent to the Volterra integral equation of the second kind given by

$$\varphi_h(t) + \frac{1}{10^2} \int_0^t \varphi_h(x) dx = \frac{2t+h}{2 \times 10^2} + 1 + O(h), \quad (4.2)$$

for $t \in [0, 1]$ with the boundary condition $\varphi_h(0) \simeq \frac{h}{2} 10^{-2} + 1$.

If we pose $\psi(t) = \varphi_h(t)$ and use Leibnitz rule to differentiate both sides of (4.2) the it gives

$$\psi'(t) + 10^{-2}\psi(t) = 10^{-2}.$$

Using the variational iteration method, the iteration formula for equation (4.2) is

$$\psi_{n+1}(t) = \psi_n(t) - \int_0^t \psi'_n(\varepsilon) + \psi_n(\varepsilon) - 10^{-2} d\varepsilon. \quad (4.3)$$

As stated before, we can use the initial condition to select

$$\psi_0(t) = \psi(0) = \frac{h}{2}10^{-2} + 1.$$

Using this selection into (4.3) gives the following successive approximations ($n \geq 1$) :

$$\begin{aligned} \psi_0(t) &= 1 + \frac{h}{2}10^{-2}, \\ \psi_1(t) &= \psi_0(t) - \int_0^t \psi'_0(\varepsilon) + \psi_0(\varepsilon) - 10^{-2} d\varepsilon = 1 + \frac{10^{-2}}{2}h - \frac{(10^{-2})^2}{2}ht, \\ \psi_2(t) &= \psi_1(t) - \int_0^t \psi'_1(\varepsilon) + \psi_1(\varepsilon) - 10^{-2} d\varepsilon = 1 + \frac{10^{-2}}{2}h - \frac{(10^{-2})^2}{2}ht + \frac{(10^{-2})^3}{4}ht^2, \\ \psi_3(t) &= \psi_2(t) - \int_0^t \psi'_2(\varepsilon) + \psi_2(\varepsilon) - 10^{-2} d\varepsilon = 1 + \frac{10^{-2}}{2}h - \frac{(10^{-2})^2}{2}ht + \frac{(10^{-2})^3}{4}ht^2 \\ &\quad - \frac{(10^{-2})^4}{12}ht^3, \\ &\vdots \\ \psi_n(t) &= \psi_{n-1}(t) - \int_0^t \psi'_{n-1}(\varepsilon) + \psi_{n-1}(\varepsilon) - 10^{-2} d\varepsilon = 1 + \frac{h}{2} \sum_{i=1}^{n+1} (-1)^{i+1} \frac{(10^{-2})^i}{(i-1)!} t^{i-1}. \end{aligned}$$

The variational iteration method admits the use of

$$\varphi_h(t) = \psi(t) = \lim_{n \rightarrow \infty} \psi_n(t),$$

that gives the exact solution of the Volterra integral equation (4.1) by

$$\varphi(t) = \lim_{h \rightarrow 0} \varphi_h(t) = 1.$$

Use the variational iteration method to solve the nonlinear Volterra-Hammerstein integral equation of the first kind

$$\int_0^t (10t - 10x + 6) \log |\varphi(x)| dx = 5t^3 + 9t^2. \quad (4.4)$$

It's equivalent to the Volterra integral equation of the second kind given by

$$\log |\varphi_h(t)| + \frac{5}{3} \int_0^t \log |\varphi_h(x)| dx = \frac{5}{2}t^2 + \left(3 + \frac{3}{2}h\right)t + \frac{9h + 5h^2}{6} + O(h), \quad (4.5)$$

for $t \in [0, 1]$ with the boundary condition $\varphi_h(0) \simeq \frac{9h+5h^2}{6}$.

We first set

$$\varphi_h(t) = \exp(\omega_h(t)),$$

to carry out the nonlinear equation (4.5) to the linear Volterra integral equation

$$\omega_h(t) + \frac{5}{3} \int_0^t \omega_h(x) dx = \frac{5}{2}t^2 + \left(3 + \frac{3}{2}h\right)t + \frac{9h+5h^2}{6}, \quad h \rightarrow 0 \quad (4.6)$$

if we pose $\psi(t) = \omega_h(t)$, using Leibnitz rule to differentiate both sides of (4.5) it gives

$$\psi'(t) + \frac{5}{3}\psi(t) = 5t + 3 + \frac{3h}{2},$$

Using the variational iteration method, the iteration formula for equation (4.5) is

$$\psi_{n+1}(t) = \psi_n(t) - \int_0^t \psi'_n(\varepsilon) + \psi_n(\varepsilon) - 5\varepsilon - 3 - \frac{3h}{2} d\varepsilon. \quad (4.7)$$

As stated before, we can use the initial condition to select

$$\psi_0(t) = \psi(0) = \frac{9h+5h^2}{6}$$

Using this selection into (4.7) gives the following successive approximations:

$$\begin{aligned} \psi_0(t) &= \frac{9h+5h^2}{6}, \\ \psi_1(t) &= \psi_0(t) - \int_0^t \psi'_0(\varepsilon) + \psi_0(\varepsilon) - 5\varepsilon - 3 - \frac{3h}{2} d\varepsilon = \frac{9h+5h^2}{6} - \\ &\quad \left(\frac{5(9h+5h^2)}{18} - 3 - \frac{3h}{2} \right) t + \frac{5}{2}t^2, \\ \psi_2(t) &= \psi_1(t) - \int_0^t \psi'_1(\varepsilon) + \psi_1(\varepsilon) - 5\varepsilon - 3 - \frac{3h}{2} d\varepsilon = \frac{9h+5h^2}{6} - \\ &\quad \left(\frac{5(9h+5h^2)}{18} - 3 - \frac{3h}{2} \right) t + \left(\frac{5(9h+5h^2)}{18} - 3 - \frac{3h}{2} \right) t^2 + \frac{25}{182}t^3, \\ &\quad \vdots \\ \psi_n(t) &= \psi_{n-1}(t) - \int_0^t \psi'_{n-1}(\varepsilon) + \psi_{n-1}(\varepsilon) - 10^{-2} d\varepsilon = \dots \end{aligned}$$

The variational iteration method admits the use of

$$\omega_h(t) = \psi(t) = \lim_{n \rightarrow \infty} \psi_n(t),$$

that gives the exact solution of linear equation (4.6) by

$$\omega(t) = \lim_{h \rightarrow 0} \omega_h(t) = 3t.$$

Finally, the exact solution $\varphi(t)$ of the Volterra-Hammerstein integral equation (4.4) can be obtained by

$$\varphi(t) = \exp(\omega(t)) = \exp(3t).$$

5. Conclusion

In this work, we studied the new technique that converted Volterra integral equation of the first kind to well-posed Volterra integral equation. This technique is combined with a variational iteration method for solving these ill-posedness equations. Two examples have been presented; the method is very useful and reliable for many types of linear and nonlinear Volterra integral equations of the first kind.

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