1. Zoulikha DOUIOU[1,2], 2. Izzeddine CHALABI[1,2], 3. Abdelkader DJERAD[1]

Electronic Department, Faculty of Technology, University of M'sila, PO Box 166 Ichebilia, 28000 M'sila, Algeria. (1),
Laboratory of Electrical Engineering (2),
ORCID: 1. 0009-0002-6110-2265; 2. 0000-0001-9252-3076; 3. 0000-0002-1593-168X

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Fractional Negative Order Moment Parameter Estimator of Compound-Gaussian Clutter with Lognormal Texture

***Abstract**. parameter estimation of Compound-Gaussian with lognormal texture (CG-LNT) distribution is considered. The CG-LNT distribution is non-Gaussian heavy-tailed distribution, it used in radar domain to describe the variation of high-resolution sea clutter. The probability density function (PDF) is characterized by two parameters, the standard deviation and the mean. This paper proposes a closed form estimator of CG-LNT distribution parameters based on fractional negative order moment (FNOME). Comparative study is established against the existing estimators existing in the literature to evaluate the efficiency of the proposed estimator using CG-LNT simulated and real sea clutter data, the mean square error (MSE) criterion is also used to measure the estimation accuracy.*

***Streszczenie.** uwzględniono estymację parametrów złożonego Gaussa z rozkładem lognormalnym tekstury (CG-LNT). Rozkład CG-LNT jest rozkładem ciężkoogonowym niegaussowskim i jest używany w domenie radarowej do opisu zmienności zakłóceń morskich o wysokiej rozdzielczości. Funkcja gęstości prawdopodobieństwa (PDF) charakteryzuje się dwoma parametrami: odchyleniem standardowym i średnią. W artykule zaproponowano zamknięty estymator parametrów rozkładu CG-LNT oparty na ułamkowym momencie rzędu ujemnego (FNOME). Przeprowadzono badanie porównawcze z istniejącymi estymatorami istniejącymi w literaturze w celu oceny efektywności proponowanego estymatora przy użyciu symulowanych i rzeczywistych danych dotyczących zakłóceń morskich CG-LNT. Kryterium błędu średniokwadratowego (MSE) jest również wykorzystywane do pomiaru dokładności estymacji. (**Ułamkowy estymator parametrów momentu ujemnego rzędu złożonego bałaganu gaussowskiego z teksturą lognormalną**)*

Keywords: Sea clutter, parameters estimation, CG-LNT distribution.
Słowa kluczowe： Bałagan morski, estymacja parametrów, dystrybucja CG-LNT.

Introduction

Many radars constant false alarm rate (CFAR) detectors are developed according to the statistical model of the clutter and require the knowledge of their parameters [1-3]. Several non-Gaussian models have been used to describe high-resolution sea clutter such as lognormal, Weibull, compound K, CG-LNT, Pareto, CIG (compound inverse Gaussian) [4-9]. In the other hand, to estimate parameters of radar clutter, many methods have been proposed in literature based on the maximum likelihood estimation (MLE), higher order moment estimator (HOME), fractional order moment estimator (FOME) and [zlog(z)] methods [10-14]. In this paper, we are interesting about parameters estimation of the CG-LNT distribution. The performance modelling of this distribution is validated in [7], where the authors carried out a statistical analysis of real sea clutter, this experimental database had been collected by Ka-band radar with high-resolution from south coast of Spain. Results of this study show the ability ant the suitability of CG-LNT distribution to fit heavy-tailed sea clutter.

Previous studies have considered the parameter estimation of the CG-LNT and various estimation procedures with various degrees of precision have been proposed in [7,13,15]. In this context, in statistical analysis high-resolution sea clutter in [7], the parameters of CG-LNT distribution have been estimated by the HOME exploiting the first two higher order moments and their empirical counterpart, the accuracy of the HOME estimation requires the use of very large data samples. Good estimation performance is achieved in [13], where two estimation methods are proposed, the FOME based on fractional order moment and the [zlog(z)] estimator based on the logarithmic moment. In [15], the authors proposed outlier-robust tri-percentile estimator for the CG-LNT clutter in presence of outliers by the use of look-up table method.

In order to enhance the estimation accuracy of CG-LNT distribution, we propose a closed form FNOME estimator which is based on the use of fractional negative order moment, the proposed method provides good estimation accuracy comparing to the existing methods for very spiky clutter.

CG-LNT Model

CG-LNT distribution is heavy-tailed non-Gaussian model, this distribution is used to describe high-resolution sea clutter amplitude variation. CG-LNT is defined as a mixture of two components; the texture component which represents the average local level of the clutter which follow lognormal distribution and the speckle component which obeys Rayleigh distribution. The global PDF of the CG-LNT model is given as [7]

(1) $$p(z)=\frac{z}{\sqrt{2\pi\sigma^2}}\int_0^{+\infty}\frac{2}{y^2}exp\left(-\frac{[ln(y/\delta)]^2}{2\sigma^2}-\frac{z^2}{y}\right)dy$$

Where σ and δ are the standard deviation and the mean respectively. For high values of σ the CG-LNT model becomes heavy-tailed and for small values of σ it tends towards a Rayleigh law [13].

Using the PDF in (1), the CCDF can be written as a function of normalized threshold T as

(2)
$$CCDF(T)=\int_0^{+\infty}exp\left(-\frac{T^2}{y}\right)\frac{1}{y\sqrt{2\pi\sigma^2}}exp\left(-\frac{(ln(y/\delta))^2}{2\sigma^2}\right)dy$$

Proposed FNOME estimator

The proposed FNOME estimator in this paper is developed by manipulating moments with fractional negative and positive order. The moment expression of the CG-LNT is given as [7]

(3) $$\langle z^n\rangle=\delta^{\frac{n}{2}}\Gamma\left(1+\frac{n}{2}\right)exp\left(\frac{1}{2}\left(\frac{n\sigma}{2}\right)^2\right)$$

Where n represent the order and $\Gamma(.)$ denote the gamma function.

The empirical moments are obtained from clutter samples as

(4) $$\langle z^n\rangle=\frac{1}{M}\sum_{i=1}^{M}z^n$$

To eliminate the mean δ in (3), we propose the use of use of fractional negative order $-n$ given as

(5) $$\langle z^{-n}\rangle = \delta^{\frac{-n}{2}}\Gamma\left(1-\frac{n}{2}\right) exp\left(\frac{1}{2}\left(\frac{n\sigma}{2}\right)^2\right)$$

Multiplying (3) and (5) side by side, the mean δ is eliminated and we obtain

(6) $$\langle z^{n}\rangle\langle z^{-n}\rangle = \Gamma\left(1+\frac{n}{2}\right)\Gamma\left(1-\frac{n}{2}\right) exp\left(\left(\frac{n\sigma}{2}\right)^2\right)$$

The closed form of the proposed FNOME is given as

(7) $$\hat{\sigma} = \frac{2}{n}\sqrt{log\left(\frac{\langle z^{n}\rangle\langle z^{-n}\rangle}{\Gamma(1+n/2)\Gamma(1-n/2)}\right)}$$

In the other hand, to obtain the estimate of the standard deviation δ, we divide (3) and (5) side by side and after simplifications a closed form of $\hat{\delta}$ is given as

(8) $$\hat{\delta} = \left(\frac{\langle z^{n}\rangle\Gamma(1-n/2)}{\langle z^{-n}\rangle\Gamma(1+n/2)}\right)^{\frac{1}{n}}$$

To assess the estimation accuracy of the proposed FNOME method in the next section, we will compare it with the existing HOME, FOME and [zlog(z)] methods for the CG-LNT clutter [13].

The HOME is based on the two first even order moments as

(9) $$\begin{cases}\hat{\sigma} = \sqrt{log\left(\frac{\langle z^4\rangle}{2\langle z^2\rangle^2}\right)}\\ \hat{\delta} = \langle z^2\rangle exp\left(-\frac{\hat{\sigma}^2}{2}\right)\end{cases}$$

The FOME method is obtained by exploiting the statistical ratio $\langle z^{n+1}\rangle/\langle z\rangle\langle z^{n}\rangle$, this estimator is given as:

(10) $$\hat{\sigma} = 2\sqrt{\frac{1}{n}log\left(\frac{\langle z^{n+1}\rangle}{\langle z\rangle\langle z^{n}\rangle}\frac{0.5n\sqrt{\pi}\Gamma(0.5n)}{(n+1)\Gamma(0.5n+0.5)}\right)}$$

The [zlog(z)] method is obtained by the derivatives of the moment expression (3) with respect to the order n. The expression of the [zlog(z)] estimator is

(11) $$\hat{\sigma} = 2\sqrt{\frac{\langle z\, log(z)\rangle}{\langle z\rangle} - \langle log(z)\rangle\text{-}1 + \log(2)}$$

The logarithmic moments are obtained from the data as

(12) $$\langle log(z)\rangle = \frac{1}{M}\sum_{i=1}^{M} log(z_i)$$

and

(13) $$\langle z\, log(z)\rangle = \frac{1}{M}\sum_{i=1}^{M} z_i\, log(z_i)$$

The estimate of the mean $\hat{\delta}$ is obtained for the FOME and [zlog(z)] methods as function of fractional order moment $\langle z^{n}\rangle$ and the estimate $\hat{\sigma}$ as

(14) $$\hat{\delta} = \left(\frac{\langle z^{n}\rangle}{\Gamma(1+n/2)} exp\left(-\frac{n^2\hat{\sigma}^2}{8}\right)\right)^{\frac{2}{n}}$$

Performance analysis

The performance analysis of the proposed FNOME estimator in this paper is performed by several comparisons. The estimation accuracy of the proposed FNOME method is compared to the existing HOME, FOME, and [zlog(z)] estimators, which utilize both simulated CG-LNT data and real sea clutter datasets.

First, based on simulated data generated according to the CG-LNT distribution, the estimation accuracy is evaluated by MSE criteria with L=10000 independent trials. The samples number M is 1000 and the value of fractional order moment n is 0.1 for the FOME and FNOME methods. The Fig.1 show MSEs values according to the standard deviation σ and the mean δ respectively. From the Figure 2-a, it is clear that the FNOME provides the best estimation outcomes especially when the clutter spikiness increases, which is the consequence of the increase in the standard deviation σ, for low values of the standard deviation σ the CG-LNT distribution tends to the Rayleigh distribution. Whereas, the estimation performance in terms of δ are assessed for $\sigma = 2$. We observe that the MSE curves of FNOME, FOME and [zlog(z)] methods are nearly closed and yield the best estimation performance compared with HOME estimator as shown in Figure 2-b.

Now, the estimation performance is also assessed by exploiting the IPIX (Intelligent PIxel Processing) real data [16], where of the real PDF and CCDF will be compared with the theoretical CG-LNT, the MSE will also calculate for each estimator. The curves plotted in the Fig.2 are carried out using HH polarization, 8th range cell and resolution 30m, the results show clearly the ability of fitting of the FNOME, this is validated by the lowest MSE values displayed in Table 1. For VV polarization, 6th range cell and resolution 30m. The curves of the PDFs and the CCDFs are plotted in Fig.3. Here, the estimated CG-LNT PDF and CCDF of the proposed FNOME method offer better fit to the empirical PDF and CCDF with the least MSEs values as shown in Table 1.

Table 1. The parameters of the sensor

IPIX data	MSE	HOME	FOME	[zlog(z)]	FNOME
8th cell range of resolution 30m and polarization HH	PDFs MSE	0.0021	0.0014	0.0014	0.0010
	CCDFs MSE	9.3262×10^{-5}	7.5925×10^{-6}	6.9164×10^{-6}	1.7607×10^{-6}
6th cell range of resolution 30m and polarization VV	PDFs MSE	0.0022	7.6210×10^{-4}	7.6087×10^{-4}	7.6040×10^{-4}
	CCDFs MSE	1.1279×10^{-5}	1.1963×10^{-6}	1.1805×10^{-6}	1.1744×10^{-6}

a) b)

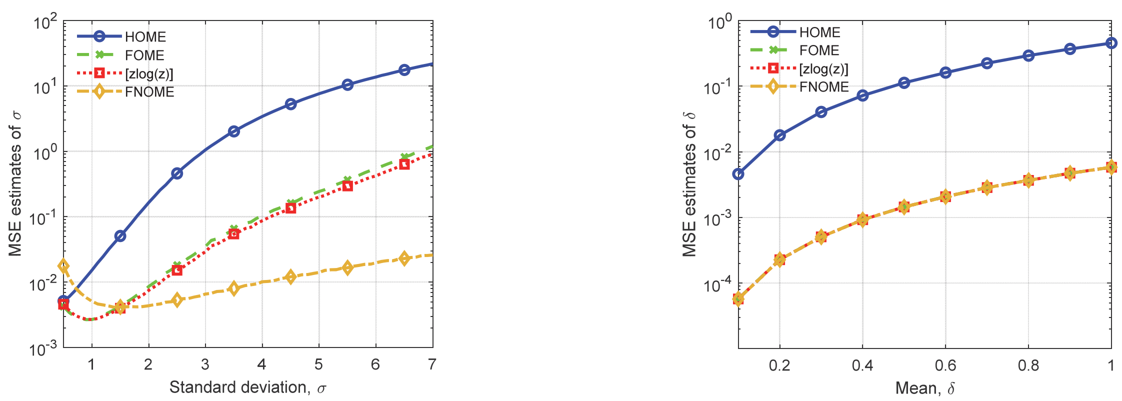

Fig.1. Estimation results, MSEs of the standard deviation σ (a) and the mean δ (b).

a)

b)

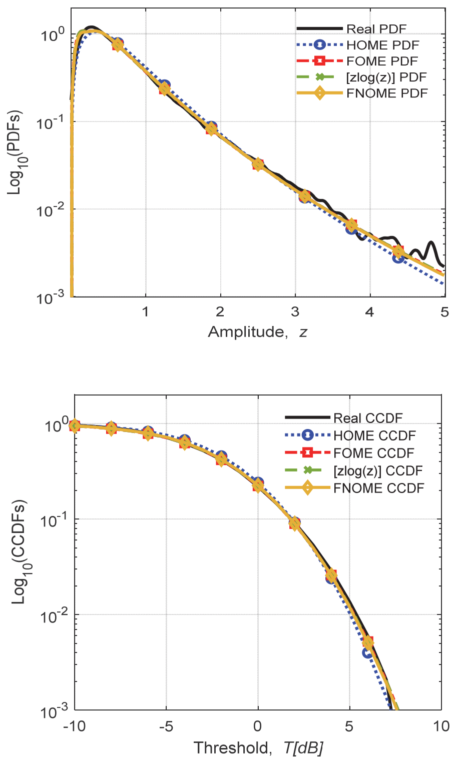

Fig.2. PDFs (a) and CCDFs (b) of CG-LNT obtained by HOME, FOME, [zlog(z)] and FNOME using IPIX data of the 8th cell range, resolution 30m and polarization HH.

a)

b)

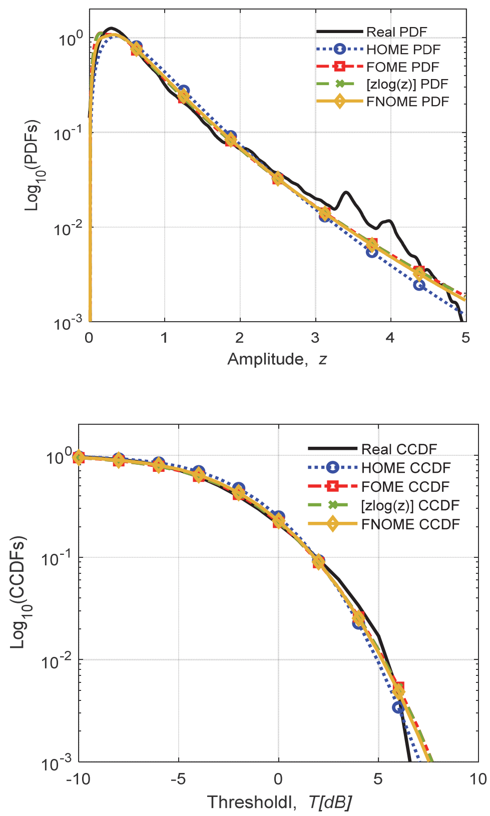

Fig.3. PDFs (a) and CCDFs (b) of CG-LNT obtained by HOME, FOME, [zlog(z)] and FNOME using IPIX data of the 6th cell range, resolution 30m and polarization VV.

Conclusion

In this paper, closed form parameter estimator for CG-LNT has been developed based on fractional negative order moment. Using both simulated and real IPIX data from high-resolution sea clutter, the proposed FNOME estimator's estimation accuracy is evaluated. According to the results, the FNOME estimator provides the best accuracy with the lowest MSE values and the better fitting to the real data compared to HOME, FOME and [zlog(z)] estimators especially for very spiky clutter.

Authors:
Ms. Zoulikha DOUIOU, Faculty of Technology, University of M'sila, PO Box 166 Ichebilia, 28000 M'Sila Algeria, E-mail: zoulikha.douiou@univ-msila.dz; Dr. Izzeddine CHALABI, Faculty of Technology, University of M'sila, PO Box 166 Ichebilia, 28000 M'Sila Algeria, E-mail: izzeddine.chalabi@univ-msila.dz (corresponding author); Dr. Abdelkader DJERAD, Faculty of Technology, University of M'sila, PO Box 166 Ichebilia, 28000 M'Sila Algeria, E-mail: abdelkader.djerad@univ-msila.dz.

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