Performance analysis of parameter estimation for compound Gaussian with log-normal texture clutter in the presence and absence of thermal noise

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***Abstract*— Accurate parameter estimation plays a pivotal role in enhancing radar detection performance. This paper addresses parameter estimation for the CG-LNT (Compound Gaussian with Log-Normal Texture) distribution. The study meticulously examines both situations involving the presence and absence of thermal noise, comparing the performance of estimation methods, HOME (Higher-Order Moments Estimation) , FOME (Fractional Order Moment Estimation), [zlog(z)], and curve fitting, using real radar data. The estimation performance is evaluated by the MSE (Mean Squared Error criterion). The results show the ability of the CG-LNT in presence of thermal noise to model high-resolution sea clutter.**

Keywords sea clutter, parameter estimation, CG-LNT distribution.

1. INTRODUCTION

Radar detection is the process of detecting interest target emerged in undesirable signals produced by clutter, thermal noise and interference targets. This leads us towards statistical modelling. In high resolution radars and/or for a low grazing angle, clutter can no longer be modelled by a Gaussian law and clutter has a non-Gaussian nature [1]. The statistical models of clutter are characterized by parameters, the estimation of these parameters is an essential task in the detection process.

In this paper, our attention is focused on the estimation of the parameters of the CG-LNT distribution [1]. After a reminder of noise-free estimation methods, namely; the HOME estimator (Higher Order Moment Estimator), this method is based on higher order moments, the FOME estimator (Fractional Order Moment Estimator) based on fractional order moments and the estimator [zlog(z)] based on logarithmic moments [2]. In the case of the presence of thermal noise, the mathematical difficulties increase and we are oriented towards the curve fitting method based on the Nelder-Mead optimization algorithm [3]. The estimation performance is assessed through a real radar database [4].

2. CG-LNT MODEL

The CG-LNT distribution is defined by two components. The first component, known as texture, represents the local mean level of clutter and follows a Log-Normal distribution. The second component is called speckle, which obeys a Rayleigh distribution. Therefore, the total PDF of the CG-LNT distribution is obtained by averaging the speckle component over all possible values of the texture component as follows:[1]

$$p(z) = \int_0^{\infty} p(z\,/y)p(y)dy \qquad (1)$$

Where p(y) is the texture and p (z / y) is the speckle.

In the case of a quadratic detector and in the absence of thermal noise, the PDFs of the texture and the speckle are, respectively, given by:[1]

$$p(y) = \frac{1}{y\sqrt{2\pi\sigma^2}} \exp\left(-\frac{ln^2\left(\frac{y}{\delta}\right)}{2\sigma^2}\ by\right) \qquad (2)$$

$$p(z|y) = \frac{2z}{y} exp\left(-\frac{z^2}{y}\right) \qquad (3)$$

By substituting (2) and (3) into (1) and after some mathematical simplifications, we obtain the total PDF of the CG-LNT distribution without noise in the following form:[1]

$$p(z) = \frac{z}{\sqrt{2\pi\sigma^2}} \int_0^{+\infty} \frac{2}{y^2} exp\left(-\frac{\left[ln\left(\frac{y}{\delta}\right)\right]^2}{2\sigma^2} - \frac{z^2}{y}\right) dy \qquad (4)$$

The parameters δ and σ represent the mean value of y and the standard deviation of $\ln(y^2)$, which is related to the clutter area of the radar (i.e., shape parameter). The parameter σ is associated with the sea state (spickness), where large values of σ resulting non-Gaussian distribution with heavy tails, as illustrated in Figure 1.

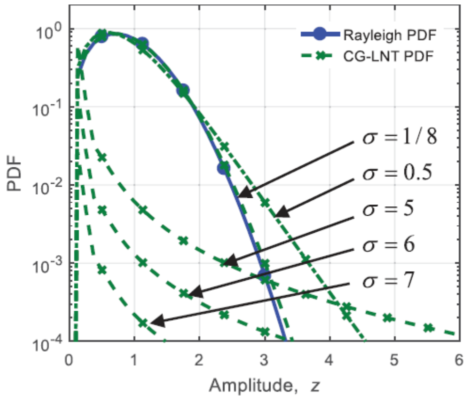

Figure 1 Comparison of CG-LNT and Rayleigh Distributions

The expression for the n^{th} moment by:[1]

$$\langle z^n \rangle = \frac{1}{M}\sum_{i=1}^{M} z_i^n = \delta^{\frac{n}{2}}\Gamma\left(1+\frac{n}{2}\right) exp\left[\frac{1}{2}\left(\frac{n\sigma^2}{2}\right)\right] \quad (5)$$

In the presence of thermal noise, the PDF of speckle in the presence of thermal noise follows the Rayleigh distribution, given by:

$$p(x|y) = \frac{x}{p_n + 4y^2/\pi} exp\left(-\frac{x^2}{p_n + 4y^2/\pi}\right) \quad (6)$$

Where P_n is the power of the thermal noise.

The total PDF of the CG-LNT-plus-noise model is given by:

$$p(x) = \int_0^{+\infty} \frac{x^2}{p_n/2 + 2y^2/\pi} exp\left(-\frac{x^2}{p_n + 4y^2/\pi}\right) \frac{1}{\sigma y\sqrt{2\pi}} exp\left(-\frac{(\ln(y)-\mu)^2}{p_n}\right) dy \quad (7)$$

The expression for the moments of the CG-LNT-plus-noise clutter is obtained by replacing (7) in the theoretical expression of the n^{th} order moments given by:

$$\langle z^n \rangle = \int_0^{+\infty} z^n p(z) dz \quad (8)$$

By substituting (7) into (8), we obtain:

$$\langle z^n \rangle = \int_0^{\infty} \frac{\Gamma(n+1)(p_n + y)^n}{\sigma y\sqrt{2\pi}} exp\left(-\frac{[\ln(y/\delta)]^2}{2\sigma^2}\right) dy \quad (9)$$

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3. ESTIMATION METHODS IN THE ABSENCE OF THERMAL NOISE

A. HOME

This method is based on the two first even orders moment as.

$$\hat{\sigma} = \sqrt{ln\left(\frac{\langle z^4 \rangle}{2\langle z^2 \rangle^2}\right)}$$
$$\hat{\delta} = \langle z^2 \rangle exp\left(-\frac{\hat{\sigma}^2}{2}\right) \quad (10)$$

B. FOME

This method is relied on fractional order moments. The expression of FOME is as follows [2]:

$$\hat{\sigma} = 2\sqrt{\frac{1}{n} ln\left(\frac{\langle z^{n+1} \rangle}{\langle z \rangle \langle z^n \rangle} \frac{0{,}5n\sqrt{\pi}\Gamma(0{,}5n)}{(n+1)\Gamma(0{,}5n+0{,}5)}\right)} \quad (11)$$

C. [zlog(z)]

This method is developed in [2] and is based on the partial derivative of the moment expression with respect to the order n, the [zlog(z)] estimator is given by

$$\hat{\sigma} = 2\sqrt{\frac{\langle zlog(z) \rangle}{\langle z \rangle} - \langle \log(z) \rangle - 1 + \log(2)} \quad (12)$$

To estimate the scale parameter δ, we can replace $\hat{\sigma}$ in the expression of the second-order moment. The estimator of the scale parameter is then written as follows:

$$\hat{\delta} = \langle z^2 \rangle exp\left(-\frac{\hat{\sigma}^2}{2}\right) \quad (13)$$

4. ESTIMATION METHOD FOR CG-LNT IN THE PRESENCE OF THERMAL NOISE

parameter estimation becomes more complex due to the addition of the third parameter, which is the noise power p_n. In this case, the expression of moments becomes more complicate, making direct manipulation of moments challenging. This leads to the use of alternative estimation methods. The curve-fitting method based on the Nelder-Mead optimization algorithm [3] will be employed to estimate the parameters of the CG-LNT-plus-noise clutter model.

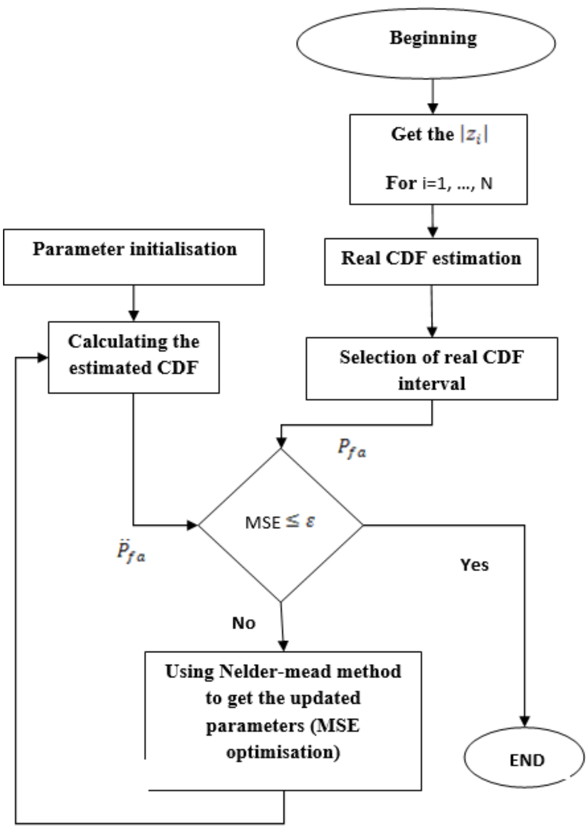


Figure 2: Schema-block of CG-LNT-plus-noise Distribution Parameter Estimation

5. RESULTS AND DISCUSSION

In this section, we present an analysis of the parameter estimation performance of the Compound Gaussian distribution with a log-normal texture.

The estimation methods presented in the presence and absence of thermal noise are evaluated using real sea clutter data collected by the experimental radar system, IPIX, operating in the X-band [4]. The IPIX radar is a high-resolution system that utilizes four different polarizations: HH, HV, VH, and VV. Located in Grimsby, Ontario, Canada, the radar operates at an altitude of 20 metersabove the level of Lake Ontario. It functions in the X-band frequency range (8-12 GHz) with a beam width of 9 degrees. The pulse repetition frequency is 1 KHz. The database consists of 34 resolution cells and 60,000 pulses. The data is organized into three files corresponding to resolutions of 3, 15, and 30 meters.

To illustrate the quality of parameter estimation using different methods, we compare the theoretical Probability Density Functions (PDFs) and Complementary Cumulative Distribution Functions (CCDFs) calculated based on the parameters estimated by the various methods with the real PDF and CCDF obtained directly from real samples using the MATLAB routine (ksdensity) [5].

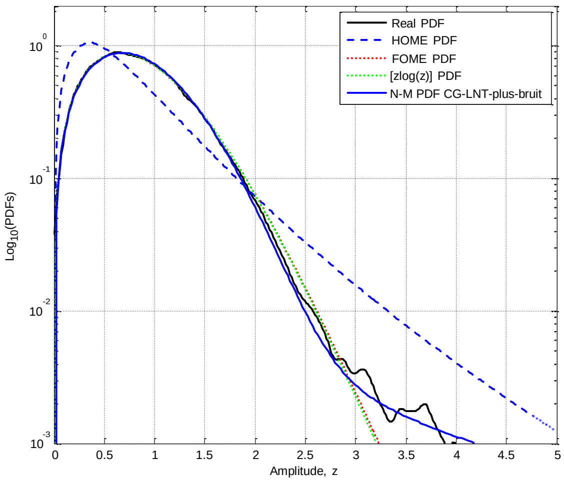
The Figure 3, it represents the curves of the PDFs (a) and CCDFs (b) corresponding to the estimated values of the parameters by the HOME, FOME and [zlog(z)] methods and the curve fitting method based on the Nelder-Mead (N-M) algorithm. The PDF curves are obtained through the use of the 16th distance cell, HH polarization and 3m resolution. It is observed that the two PDF and CCDF curves obtained by the N-M method give almost similar fits to the real PDF and CCDF comparing with the noise-free methods HOME, FOME and [zlog(z)]. This confirms the effectiveness of the curve fitting method as well as the presence of thermal noise in the real data. We also notice from table 3.2 that the N-M estimator gives the low value of MSE. In addition, the curves obtained by the HOME method deviate considerably from the real PDF and CCDF.

Figure 4 is obtained from the 15th cell distance and resolution 15m, we notice that the PDFs and CCDFs curves obtained by the N-M estimator give the best fits to the real data. This is clearly visible in the values of the MSEs obtained (see table 1).

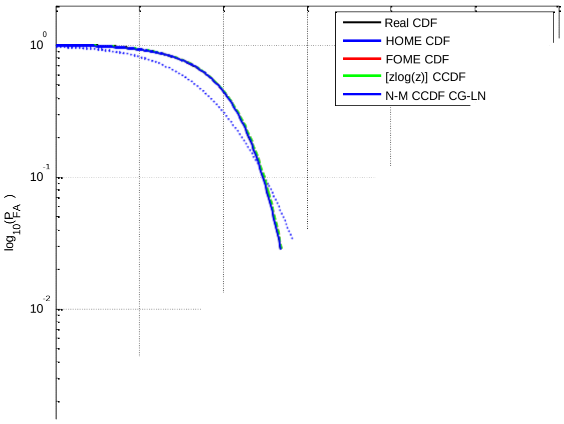
For both cases the N-M method offers the low MSE values and the best fit to the real data. This confirms the presence of thermal noise as well as the robustness of the N-M estimator.

Table 1 - Estimation Results of HOME, FOME, [zlog(z)], and N-M Methods Using Real Data

		HOME	FOME	[zlog(z)]	N-M Plus bruit
The data IPIX: HH polarization, 3m resolution, and 16th cell distance.	PDF MSE	0.0219	7.0184e-05	7.5054e-05	2.4925e-05
	CCDF MSE	0.0054	1.1732×10^{-5}	1.3554×10^{-5}	2.2174e-06
The IPIX data: Polarization HH, 15m resolution, and 15th range cell.	PDF MSE	0.0059	0.0085	0.0080	9.6073 e-05
	CCDF MSE	9.7381e-04	2.1923e-04	2.1703e-04	6.6413 e-06

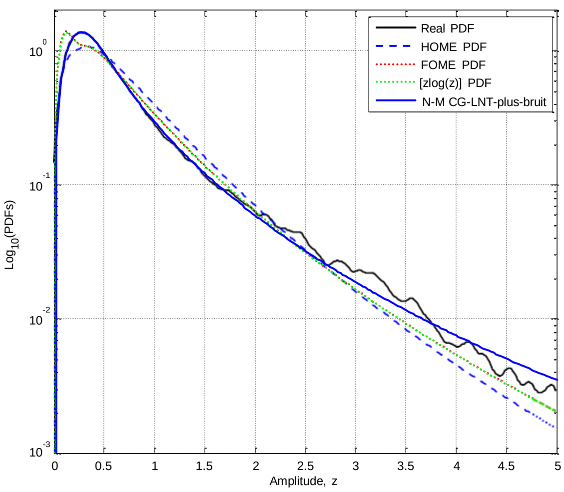


(a)



(b)

Figure 3 – PDF (a) and CCDF (b) Curves Obtained by HOME, FOME, [zlog(z)], and N-M Methods Using the 16th Range Cell, HH Polarization, and 3m Resolution .



(a)

(b)

Figure 4- PDF (a) and CCDF (b) Curves Obtained by HOME, FOME, [zlog(z)], and N-M Methods Using the 15th Range Cell, HH Polarization, and 15m Resolution

6. Conclusion

This paper considers parameter estimation for the Compound Gaussian model with Log-Normal Texture, with and without thermal noise. The HOME, FOME, [zlog(z)], and curve-fitting methods are applied to estimate the parameters using real IPIX radar data. The results emphasize the effectiveness of the curve-fitting method, which demonstrates the best fit to the real data, characterized by the lowest MSE values. Additionally, it becomes evident that the real IPIX radar data inherently incorporates thermal noise, affirming the suitability of the CG-LNT-plus-noise distribution for modelling real sea clutter data.

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