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#### **PAPER**

# Tailoring the physical characteristics of ScTaPd<sub>2</sub>Sn<sub>2</sub> and ScTaPt<sub>2</sub>Sn<sub>2</sub> double half-Heusler compound for thermoelectric applications

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## Abstract

Due to its potential uses in thermoelectrics, spintronics, and other sectors, double half-Heusler compounds have recently attracted much attention. This study presents the first-ever report on the structural, electronic, optical, elastic, and thermoelectric characteristics of the double half Heusler (DHH) compounds ScTaPd<sub>2</sub>Sn<sub>2</sub> and ScTaPt<sub>2</sub>Sn<sub>2</sub>, employing density functional theory (DFT). Using the EV-GGA approximation, the estimated band structures exhibit a semiconductor behavior with an indirect bandgap of 0.549 eV and 0.851 eV, respectively. In addition, we examined optical characteristics. Our material structural stability and stiffness were confirmed using elastic characteristics. Boltzmann's semiclassical theory attempts to explain a simulation concept in the BoltzTrap software. According to the thermoelectric investigation, these DHH are a p-type material, a candidate for thermoelectric application, specifically when doped.

## 1. Introduction

In a recent study, Anand et al proposed the notion of double half-Heusler (DHH) compounds characterized by their enhanced structural complexity, which leads to substantially reduced K<sub>L</sub> values, this novel group of half Heusler alloys exhibits many chemical formulas, including double quaternary (X'X"Y<sub>2</sub>Z<sub>2</sub>; X<sub>2</sub>Y'Y"Z<sub>2</sub>;  $X_2Y_2Z'Z''$ ), triple  $(X_2'X''Y_3Z_3)$ , and quadruple  $(X_3'X''Y_4Z_4)$  [1]. In the context of this investigation, a significant focal point pertains to the quaternary double half Heusler compounds, which exhibit analogous characteristics. These characteristics encompass behaviors to semiconductors, an indirect and narrow band gap, a substantial contribution to the density of states from the d-orbital, mechanical stability, and a Debye temperature of approximately 400 K [2-6]. Furthermore, the researchers Anand et al conducted an experimental study to showcase the reduced thermal conductivity (K<sub>1</sub>) of the compound Ti<sub>2</sub>FeNiSb<sub>2</sub>, which was synthesized by combining the aliovalent compounds TiFeSb and TiNiSb. The K<sub>L</sub> value of Ti<sub>2</sub>FeNiSb<sub>2</sub> was found to be significantly lower (6–7 W mK<sup>-1</sup>) compared to that of the compound TiCoSb (25 W mK<sup>-1</sup>). This reduction in  $K_L$  can be attributed to the decreased velocity of phonon groups in the high-frequency range. The recent finding [7] simulated thermal conductivity values of  $Hf_2Ni_2InSb$  in its cubic phase range from 13.3 to 3.3 (W mK<sup>-1</sup>), while in its tetragonal phase, the range is from 12.4 to 2.8 (W mK $^{-1}$ ). On the other hand, for the ternary compound HfNiSn, the thermal conductivity ranges from 18.9 to 4.7 (W mK<sup>-1</sup>) within the temperature range of 300-900 K. The thermal conductivity, denoted as  $K_L$ , of  $Zr_2Ni_2InSb$  exhibits a range of values between 17.8 to  $4.3 \, (W \, mK^{-1})$  in the cubic crystal structure and  $12.1 \, to \, 2.9 \, (W \, mK^{-1})$  in the tetragonal crystal structure throughout the temperature range of 300-900 K. In comparison, the thermal conductivity of ZrNiSn within the same temperature range varies between 18.7 to  $4.5 \, (\mathrm{W \, mK}^{-1})$ . The previously reported thermal conductivity values at ambient temperature for ZrNiSn and HfNiSn are 19.6 and 18.5 (W  $\mathrm{mK}^{-1}$ ), respectively. In contrast, for  $\rm Zr_2Ni_2InSb$  and  $\rm Hf_2Ni_2InSb$  in the tetragonal phase, the corresponding values at 300 K are 13.5 and 12.5 (W  $mK^{-1}$ ), respectively at 300 K [1]. Another illustration from the research we've done in previous periods about the DHH ScNbNi<sub>2</sub>Sn<sub>2</sub> which has a lower  $K_L$  5.3 (W mK<sup>-1</sup>). At 300 K and 1.3 (W mK<sup>-1</sup>) at 1000 K [2] however, a parent ScNiSn has 11.7 (W mK $^{-1}$ ) at 300 K [8]

Half-Heusler compounds can be synthesised using a diverse range of elements, wherein each lattice site accommodates a distinct group of elements. Based on the valence balance rule [9], it is expected that the net valence (NV) of the three components in stable compounds should be zero. Therefore, this criterion can be used to choose from the different possible ternary half-Heusler compounds (generally XYZ) based on their constituent elements. Several alloy compositions form disordered solutions that can only be sustained at high temperatures. The stability of double half-Heusler materials  $(X_2Y'Y''Z_2)$  instead of XYZ, with Y' and Y'' being non-isovalent) is established via aliovalent replacement. This notion is derived from the idea of double perovskites, where the term 'double' signifies the duplication of the perovskite formula unit  $(A_2B'B'O_6)$  as compared to ABO<sub>3</sub>). The materials that exhibit aliovalent substitution, such as ScNbNi<sub>2</sub>Sn<sub>2</sub>, are commonly referred to as double half-Heusler compounds. This nomenclature is used to distinguish them from other quaternary configurations that are alloyed isovalently regardless of whether they experience an order—disorder transformation at elevated temperatures or not. In contrast to ternary compositions, the inclusion of quaternary elements in double half-Heusler structures offers a significantly larger phase space for the exploration and discovery of new compounds.

Our study aims to analyze the electronic properties of  $ScTaPd_2Sn_2$  and  $ScTaPt_2Sn_2$ , specifically their band structure and density of states (DOS). Additionally, we seek to investigate its thermoelectric properties, particularly the Seebeck coefficient. The outcomes of electrical conductivity, electronic thermal conductivity, the Seebeck coefficient, and the figure of merit are presented alongside an explanation of the computational framework derived from Boltzmann's semiclassical theory, utilizing the BoltzTrap code. Moreover, the elastic and optical properties are presented in the last part of the article. However, there seems to be a scarcity of similar inquiries in the existing body of literature.

The objective of our research is to analyse the electrical properties of ScTaPd<sub>2</sub>Sn<sub>2</sub> and ScTaPt<sub>2</sub>Sn<sub>2</sub>, specifically their band structure and density of states (DOS). Additionally, we seek to investigate their thermoelectric capabilities, particularly their Seebeck value. The computational model described in Boltzmann's semiclassical theory is utilised to determine the electrical conductivity, electronic thermal conductivity, Seebeck coefficient, and figure of merit. Moreover, the optical properties such as the dielectric function and optical conductivity are determined. Nevertheless, there is a lack of comparable studies in the existing body of literature.

The manuscript is divided into four sections, with the opening portion providing a detailed analysis of the importance and practical uses of DHH. The second section provides an explanation of the approach or computational details. Within the third section, we will proceed to showcase our findings. The final section presents the conclusions drawn from all the results.

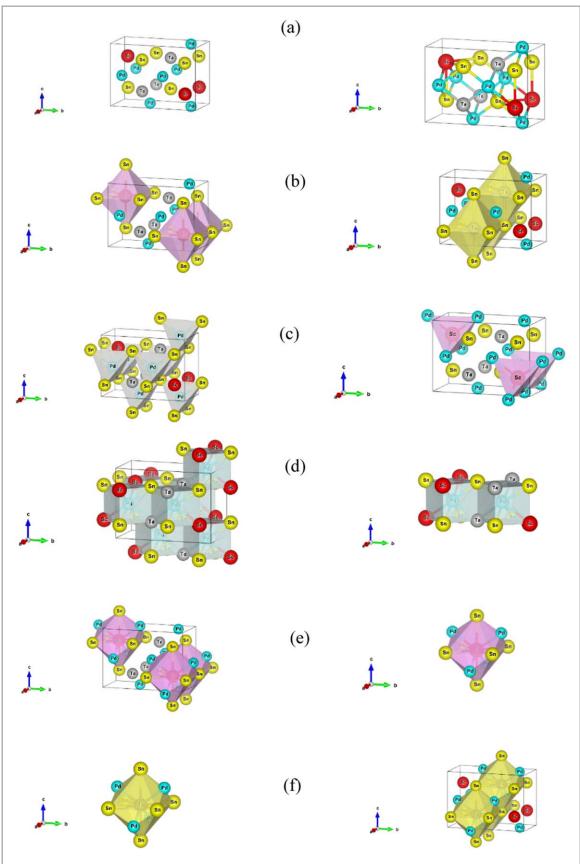
# 2. Computation details

The Wien2k code [10, 11] implements the full linearized augmented plane wave (FP-LAPW) method [12], which relies on density functional theory, or DFT [13]. In our research, we utilize the generalized gradient approximation (GGA-PBE) [14] and the local density approximation (LDA) [15] to handle the exchange–correlation energy. Additionally, we incorporate the modified Beck-Johnson [16] and the Engel-Vosko (EV-GGA) [17] potentials to enhance the accuracy of the electronic structure. The muffin-tin radius (R.M.T) for ScTaPd<sub>2</sub>Sn<sub>2</sub> varies between 2.31 and 2.37 atomic units (a.u.), while for ScTaPt<sub>2</sub>Sn<sub>2</sub> it varies between 2.26 to 2.37 a.u. The highest modulus of the reciprocal vector  $K_{max}$  is found in the first *Brillouin zone*. To establish a relationship, we equate the product of R.M.T ×  $K_{max}$  to 9. The wave function expansion parameters within the muffin tin are  $l_{max} = 10$  and  $G_{max} = 12$ .  $G_{max}$  represents the magnitude of the largest vector in the Fourier expansion of the charge density. By including 1000 and 900 k-points for ScTaPd<sub>2</sub>Sn<sub>2</sub> and ScTaPt<sub>2</sub>Sn<sub>2</sub> respectively in the Brillouin zone, the computations are ensured to be precise. The self-consistent computation is said to have reached convergence when the total energy difference is  $10^{-4}$  Ry. The Boltztrap software [18], which is based on the semi-classical Boltzmann theory, is utilized to assess thermoelectric transport properties. Additionally, the impact of temperature and pressure on the Debye temperature and lattice thermal expansion is estimated using the quasi-harmonic approximation integrated in the Gibbs software [19, 20].

# 3. Result and analysis

# 3.1. Structural properties

The arrangement of atoms or molecules in a material determines its structural properties. These properties can be divided into geometrical properties, which include crystal structure, and lattice parameters, and we can predict physical properties such as electronic, optical, elastic, and thermoelectric based on geometrical properties. Our double half-Heusler ScTaSn<sub>2</sub>Pd<sub>2</sub> and ScTaPt<sub>2</sub>Sn<sub>2</sub> alloys structural characteristics are examined in this section.



**Figure 1.** Crystal structure of the double half-Heusler ScTaPd<sub>2</sub>Sn<sub>2</sub> compound (a) Conventional unit cell, (b) the octahedral X'Z<sub>6</sub>, XZ<sub>6</sub>, (c) the tetrahedral YZ<sub>4</sub>, XY<sub>4</sub> (d) cube YXX'<sub>3</sub>Z<sub>3</sub>, YX<sub>3</sub>X'Z<sub>3</sub> (e) and (f) bicapped square prism XY<sub>4</sub>Z<sub>6</sub>, X'Y<sub>4</sub>Z<sub>6</sub> according to the direction (111)

 $ScTaPd_2Sn_2$  and  $ScTaPt_2Sn_2$  is half-Heusler-derived structured and crystallize in the simple orthorhombic with the space group  $Pmn2_1$  (N°31). The structures depicted in figure 1 is referred to as double half-Heusler structures, which are formed by combining two half-Heusler compounds with a distinct crystal arrangement. In

**Table 1.** The positions of atoms in the DHH of ScTaPd<sub>2</sub>Sn<sub>2</sub> and ScTaPt<sub>2</sub>Sn<sub>2</sub>.

Compound	Atom	x (Bohr)	у	z
ScTaPd <sub>2</sub> Sn <sub>2</sub>	Sc	0.50000000	0.12166132	0.75504239
		0.00000000	0.87833868	0.25594239
	Ta	0.00000000	0.37356296	0.26112829
		0.50000000	0.62643704	0.76112829
	Sn1	0.50000000	0.62678591	0.26624628
		0.00000000	0.37321409	0.76624628
	Sn2	0.00000000	0.87488038	0.75024701
		0.50000000	0.12511962	0.25024701
	Pd1	0.00000000	0.13462731	0.50000000
		0.50000000	0.86537269	0.00000000
	Pd2	0.00000000	0.61538705	0.51745290
		0.50000000	0.38461295	0.01745290
ScTaPt <sub>2</sub> Sn <sub>2</sub>	Sc	0.50000000	0.62243198	0.26900163
		0.00000000	0.37756802	0.76900163
	Ta	0.00000000	0.87556905	0.75938957
		0.50000000	0.12443095	0.25938957
	Sn1	0.00000000	0.37796990	0.27383598
		0.50000000	0.62203010	0.77383598
	Sn2	0.50000000	0.13007003	0.75178726
		0.00000000	0.86992997	0.25178726
	Pt1	0.00000000	0.11402053	0.50000000
		0.50000000	0.88597947	0.00000000
	Pt2	0.00000000	0.63662292	0.52598238
		0.50000000	0.36337708	0.02508238

**Table 2.** Calculated structural parameters ( $A^0$ ) and magnetic moment ( $\mu_B$ ) for ScTaPd<sub>2</sub>Sn<sub>2</sub> and ScTaPt<sub>2</sub>Sn<sub>2</sub> compounds using GGA and LDA approximations.

Compounds 31_Pmn21	App	a(A <sup>0</sup> )	b	с	b/a	c/a	E <sub>min</sub> (Ryd)	B(GPa)	Bʻ	$M_{Tot}(\mu_B)$
ScTaPd <sub>2</sub> Sn <sub>2</sub>	GGA	4.4029	8.8386	6.2475	2.0074	1.4188	-155307.887786	135.0660	4.8254	0.00054
	LDA	4.3655	8.7636	6.1935	2.0074	1.4188	-155307.887760	145.0508	4.9720	0.00034
ScTaPt <sub>2</sub> Sn <sub>2</sub>	GGA	4.4245	8.8617	6.2658	2.0028	1.4161	-262483.766381	152.8098	5.0062	-0.00033
	LDA	4.3975	8.8076	6.2276	2.0028	1.4161	-262400.542383	162.7297	5.0974	0.00139

order to produce completely stable geometrical configurations of the DHH alloys, we extensively optimized the atomic structure by relaxing the cell parameters. The relaxed atomic coordinates discovered are shown in table 1. In order to establish the equilibrium lattice constant and define the stable structure of the examined double half-Heusler alloy, structural optimizations on the  $ScTaPd_2Sn_2$  alloy for ferromagnetic (FM) and paramagnetic (PM) phases are also carried out. The total energy values can be adjusted as a function of volume by employing the Birch-Murnaghan equation of state (EOS) [21] in conjunction with the LDA and GGA approximations.

Table 2 shows the optimized results for the lattice parameter a, bulk modulus B (GPa), and its first derivative B'. Based on the current findings shown in table 2, the calculated optimized b/a and c/a ratios were 2.0074 and 1.4188 for  $ScTaPd_2Sn_2$ , 2.0028,1.4161 for  $ScTaPt_2Sn_2$  and 2.0074/2.0073 in the context of GGA approximation. The aforementioned values suggest a marginal deformation of both compounds. By transitioning from the GGA to the LDA approximation, it becomes apparent that the bulk modulus increases while the lattice parameters a and b decrease. Additionally, the substitution of atoms Pd with Pt causes an increase in the aforementioned parameters. Since  $ScTaPt_2Sn_2$  has a greater modulus of compressibility than  $ScTaPd_2Sn_2$ , it is incompressible in comparison to  $ScTaPd_2Sn_2$ .

According to figure 2, Sc is bonded in a 10-coordinate geometry Sc is attached to four Pd and six Sn atoms to form distorted face -sharing ScPd<sub>4</sub>Sn<sub>6</sub> tetrahedra Sc-Pd lengths chemical bonds are divided into tree shorter (2.72001Å) and one longer (2.84417Å). Three shorter (3.09396Å) and three longer (3.12966Å) bond lengths of Sc–Sn, furthermore we delete the Sc-Pd bond we get octahedral site ScSn<sub>6</sub>. to form the distorted face sharing TaPd<sub>4</sub>Sn<sub>6</sub> tetrahedra Ta is bonded to four Pd and six Sn atoms, four Ta-Pd lengths chemical bonds one is shorter (2.58594Å) and three longer (2.67836Å) (see table 1). Four Ta-Sn bond lengths three are shorter (3.11014Å) and three longer (3.13954Å) Nb–Sn. Two incompatible Pd sites wich shape a body centered cubic geometry to each Sc, Ta, Sn. In the first Pd site, Pd is bonded to one Sc, three equivalent Ta, and four Sn atoms. Four Pd–Sn bonds

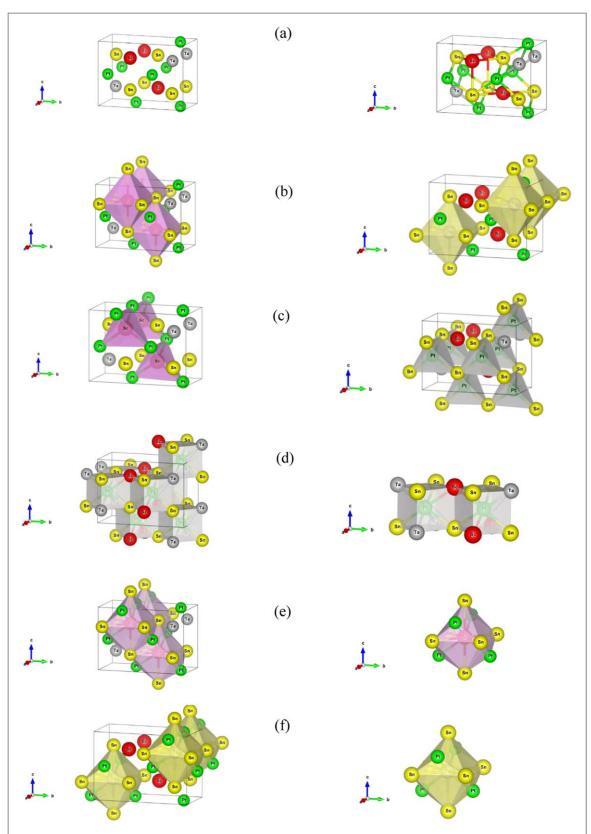
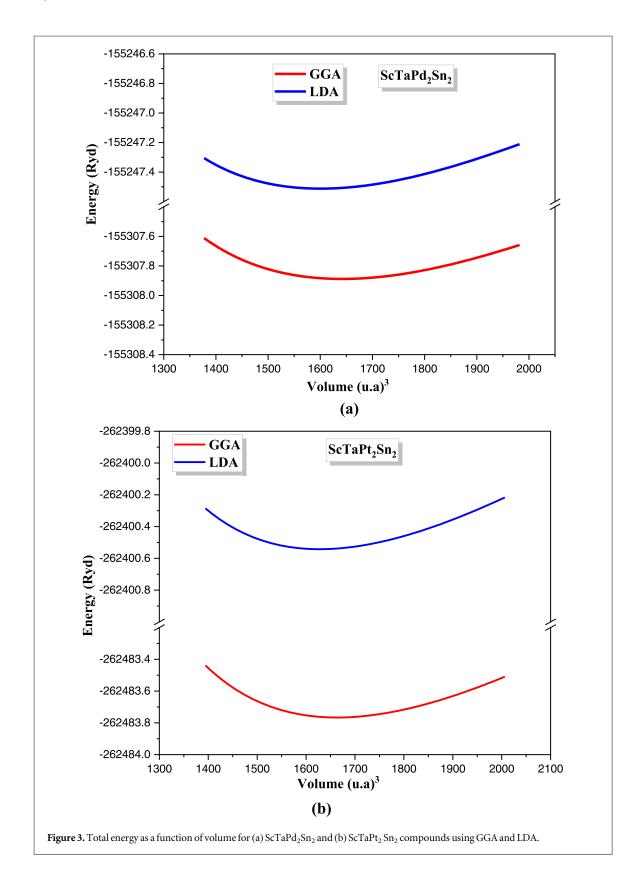


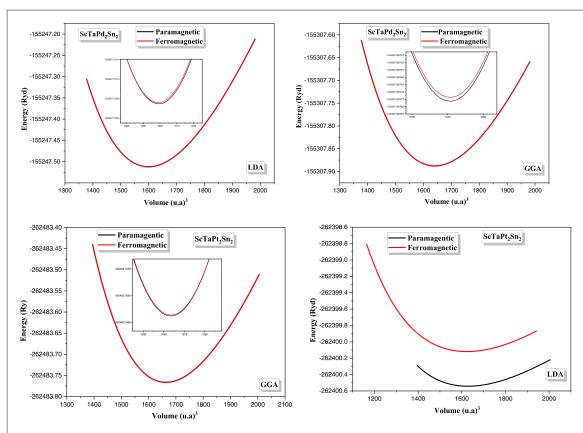
Figure 2. Crystal structure of the double half-Heusler  $ScTaPt_2Sn_2$  compound (a) Conventional unit cell, (b) the octahedral  $X'Z_6$ ,  $XZ_6$ , (c) the tetrahedral  $YZ_4$ ,  $XY_4$  (d) cube  $YXX'_3Z_3$ ,  $YX_3X'Z_3$  (e) and (f) bicapped square prism  $XY_4Z_6$ ,  $X'Y_4Z_6$  according to the direction (111).

one is shorter (2.64530 Å) and three longer (2.70 Å). In terms of the second Pd site, we note that Pd is bonded to three Sc, one Ta, and four Sn atoms. Four Pd–Sn bonds, three are shorter (2.69Å) and one longer (2.77 Å). for the Tin atom has two distinct sites. In the first Sn site, Sn is connected in 10 coordinates to three equivalent Sc, three equivalent Ta, and four Pd atoms. For the second Sn atom, Sn has 10-coordinate 3 Sc, 3 Ta, and 4 Pd atoms.



As an example, consider the interatomic distance Sc-Sn1 3.12 $\mathring{A}$  (3.1 $\mathring{A}$ ) in ScTaPd<sub>2</sub>Sn<sub>2</sub> which indicates quite similarities to ScNbNi<sub>2</sub>Sn<sub>2</sub> using GGA and LDA, respectively (see table 3).

To precisely explain the structural properties of our DHH compounds, we optimized a computation employing two distinct approximations GGA and LDA. Based on figure 3, it appears that the GGA approximation has the lowest energy for both  $ScTaPd_2Sn_2$  and  $ScTaPt_2Sn_2$ , thus we will finish our investigation with it. The calculations indicated that our DHH compounds would exhibit a paramagnetic (PM) state see figure 4.



 $\textbf{Figure 4.} \ Energy \ versus \ unit \ cell \ volume \ in \ the \ Paramagnetic \ and \ Ferromagnetic \ configurations \ of \ ScTaPd_2Sn_2 \ and \ ScTaPt_2\ Sn_2 \ compounds \ using \ GGA \ and \ LDA.$ 

 $\label{eq:Table 3. Interatomic distances (Å) of ScTaPd_2Sn_2, and ScTaPt_2Sn_2 DHH compounds using GGA and LDA approximations.$ 

•			
Compounds	Interatomic distance (A <sup>0</sup> )	GGA	LDA
ScTaPd <sub>2</sub> Sn <sub>2</sub>	d <sub>Sc-Pd1</sub>	2.72001 (x3)	2.69844 (x3)
	$d_{Sc-Pd2}$	2.84417 (x1)	2.83635 (x1)
	$d_{Sc-Sn1}$	3.12966 (x3)	3.10531 (x3)
	$d_{Sc-Sn2}$	3.09396 (x3)	3.06744 (x3)
	$\mathbf{d_{Ta-Pd1}}$	2.58594 (x1)	2.55564 (x1)
	$\mathbf{d_{Ta-Pd2}}$	2.67836 (x3)	2.65271 (x3)
	$d_{Ta-Sn1}$	3.13954 (x3)	3.10793 (x3)
	$d_{Ta-Sn2}$	3.11014(x3)	3.08592 (x3)
	$\mathbf{d_{Sn-Pd1}}$	2.69 (x3);	2.67 (x3);
		2.77(x1)	2.76(x1)
	$\mathbf{d_{Sn-Pd2}}$	2.70 (x3);	2.68 (x3);
		2.64 (x1)	2.61 (x1)
ScTaPt <sub>2</sub> Sn <sub>2</sub>	$d_{Sc-Pt1}$	2.883 (x1)	2.863 (x1)
	$d_{Sc-Pt2}$	2.742(x3)	2.723 (x3)
	$d_{Sc-Sn1}$	3.099(x3)	3.077(x3)
	$d_{Sc-Sn2}$	3.120(x3)	3.099(x3)
	$d_{Ta-Pt1}$	2.68 (x3)	2.66(x3)
	$d_{Ta-Pt2}$	2.576(x1)	2.55(x1)
	$d_{Ta-Sn1}$	3.157 (x3)	3.135 (x3)
	$d_{Ta-Sn2}$	3.163 (x3)	3.139 (x3)
	$d_{Sn-Pt1}$	2.72(x3);	2.70 (x3);
		2.66 (x1)	2.64(x1)
	$d_{Sn-Pt2}$	2.70 (x3);	2.68 (x1);
		2.69 (x1)	2.77 (x1)

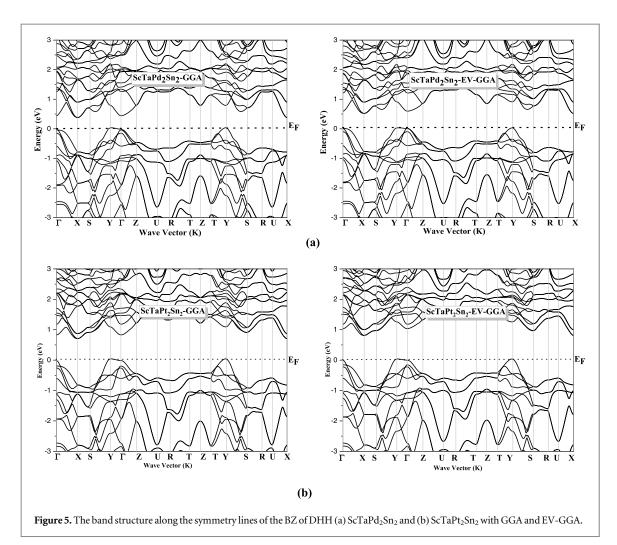


Table 4. Band gap of ScTaSn<sub>2</sub>Pd<sub>2</sub> and ScTa Pt<sub>2</sub>Sn<sub>2</sub> compounds.

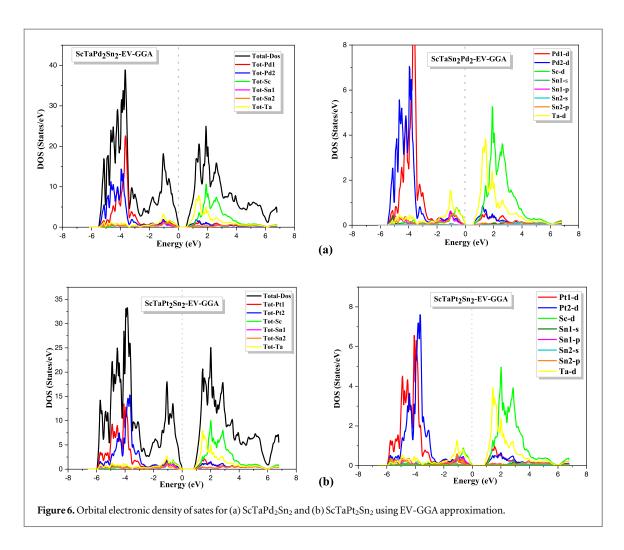
		E <sub>g</sub> (	$Y \to X$ ) (eV)		
ScTaPd <sub>2</sub> Sn <sub>2</sub>	GGA 0.416	LDA 0.447	EV-GGA 0.549	mBJ-GGA 0.425	mBJ-LDA 0.453
ScTa Pt <sub>2</sub> Sn <sub>2</sub>	0.740	0.775	0.851	0.828	0.433

The EV-GGA approximation will be taken into account to describe other results in the parts that follow. We compare our DHH compounds results to those estimated earlier in Mekki *et al* [2]. because there are no other theoretical or empirical data to compare.

#### 3.2. Electronic structure

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We calculate the electronic characteristics of ScTaPd<sub>2</sub>Sn<sub>2</sub> and ScTaPt<sub>2</sub>Sn<sub>2</sub> at their equilibrium lattice parameters, along the high symmetry points in the first *Brillouin zone* of the simple orthorhombic structure ( $\Gamma - X - S - Y - \Gamma - Z - U - R - T - Z - T - Y - S - R - U - X$ ), by utilizing an assortment of exchange–correlation potentials, including EV-GGA and GGA. Figure 5 shows that ScTaPd<sub>2</sub>Sn<sub>2</sub> and ScTaPt<sub>2</sub>Sn<sub>2</sub> exhibit semiconductor behavior with an indirect band gap  $Y_v$ - $X_c$ . The best values for this band gap are 0.549 eV and 0.851 eV, respectively, as determined by the EV-GGA approximation in table 4. The total density of states (DOS) and partial density of states (PDOS) plots for ScTaPd<sub>2</sub>Sn<sub>2</sub> and ScTaPt<sub>2</sub>Sn<sub>2</sub> exhibit significant similarities, as depicted in figure 6. it seems clear that the main contribution occupied by the Pd-d in the energy range of -5.475 eV to -2.128 eV also a few contributions of Ta-d states at the level of the valence band, the other side of the band structure which is the conduction band shows a slight contribution to the Pd-d whereas the Sc/Ta-d appear in strong contribution from the of the energy gap limit 0.542 eV to -6.035 eV. Initial analysis of the electronic properties indicates that our DHH and ScNbNi2Sn<sub>2</sub> demonstrate comparable characteristics.



#### 3.3. Optical properties

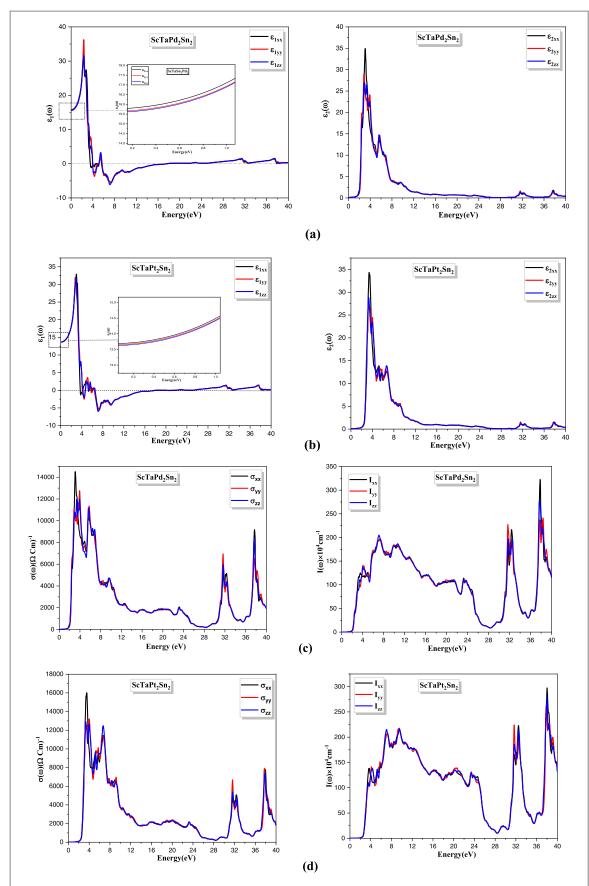
The process by which that light interacts with the electrons in material defines its optical properties as a result of providing a through description of the electronic structure and energy level of materials, DFT enables the prediction and analysis of the optical properties, so in this part of the results the optical properties has been calculated in the energy range from 0 up to 40 eV in the three directions x, y, and z-axes for  $ScTaPd_2Sn_2$  and  $ScTaPt_2Sn_2$  within EV-GGA. The complex dielectric constant describes the relationship between the incident wavelength of light and the optical properties of a material. It can be described as:

$$\varepsilon(\omega) = \varepsilon_1(\omega) + i\varepsilon_2(\omega) \tag{1}$$

Where  $\varepsilon_2(\omega)$  represents the imaginary part, and on the other side  $\varepsilon_1(\omega)$  represents the real part of the dielectric function can be obtained from the imaginary part Kramers-Kroning relation [22].

The dielectric functions' real  $\varepsilon_1(\omega)$  and imaginary  $\varepsilon_2(\omega)$  parts are plotted as a function of the energy of photons along the x, y, and z-axes in figures 7(a) and (b). The computed static values of  $\varepsilon_1(0)$  for ScTaPd<sub>2</sub>Sn<sub>2</sub>/ScTaPt<sub>2</sub>Sn<sub>2</sub> is found to be 15.764/13.687 for  $\varepsilon_{1xx}(0)$ , 15.635/13.652 for  $\varepsilon_{1yy}(0)$  and 15.585/13.623 for  $\varepsilon_{1zz}(0)$ , respectively. The mean static value  $\varepsilon_1(0)$  is 15. 661 for ScTaPd<sub>2</sub>Sn<sub>2</sub> and 13.625 for ScTaPt<sub>2</sub>Sn<sub>2</sub>. Considering that there is a similarity between the two DHH compounds, we will pick ScTaPd<sub>2</sub>Sn<sub>2</sub> for further investigation, as depicted in figure 7(a), the values start to rise from the  $\varepsilon_1(0)$  and reach their peak at 33.372 for  $\varepsilon_{1xx}(\omega)$ , 36.261 for  $\varepsilon_{1yy}(\omega)$  at 2.326 eV, and 31.636 for  $\varepsilon_{1zz}(\omega)$  at 2.351 eV, respectively then They commence decreasing. The negative value of the real part indicates that ScTaSn<sub>2</sub>Pd<sub>2</sub> can reflect incident photon radiations across a wide range of energy levels of 3.554 to 17.673 eV which corresponds to the semiconductor behavior. From figure 7(b) the imaginary part of the dielectric function  $\varepsilon_2(\omega)$  reveal fundamental absorption threshold near 0.542 eV which represents the transition between the maximum valence band and the minimum conduction band the highest peak value is 34.921 for  $\varepsilon_{2xx}(\omega)$ , 29.023 for  $\varepsilon_{2yy}(\omega)$ , and 26.873 for  $\varepsilon_{2zz}(\omega)$  located at about 2.843 eV and 3.081eV respectively, that prove the anisotropy of ScTaPd<sub>2</sub>Sn<sub>2</sub>.

From figure 7(c) the high peaks of optical conductivity  $\sigma(\omega)$  are located at 3.088 eV (14.511 × 10<sup>3</sup>) for  $\sigma_{xx}(\omega)$ ,3.931 eV for  $\sigma_{yy}(\omega)$ (12.759) and  $\sigma_{zz}(\omega)$ (11.981), for ScTaPd<sub>2</sub>Sn<sub>2</sub>. In the other hand the compound ScTaPt<sub>2</sub>Sn<sub>2</sub> has the value of 16 × 10<sup>3</sup> at 3.523eV for  $\sigma_{xx}(\omega)$ ,13.217 at 4.013 for  $\sigma_{yy}(\omega)$  and 12.913 at 3.332 eV for



 $\textbf{Figure 7.} \ \ Variation of (a) \ the \ real, (b) \ the \ imaginary \ parts \ of \ the \ dielectric \ function, (c) \ optical \ conductivity, and (d) \ absorption \ coefficient \ for \ DHH \ ScTaPd_2Sn_2 \ and \ ScTaPt_2Sn_2 \ using \ EV-GGA \ approximation.$ 

 $\sigma_{zz}(\omega)$  respectively. The primary structure extends between the threshold and 28 eV .in addition, intermediate peaks appear up to 40 eV.

From figure 7(d) we can see that the absorption begins from the energy 1.52 eV/2 eV and increase gradually with the increasing of the photon energy then reaching a maximum value  $204.9 \times 10^4$  cm<sup>-1</sup> at 7.082 eV/ $217 \times 10^4$  cm<sup>-1</sup> at 9.152 eV. Subsequently, the intensity rises once again, reaching an exceptionally high value of around 322 at 37.811 eV for ScTaPd<sub>2</sub>Sn<sub>2</sub> and 297.421 at 38 eV for ScTaPt<sub>2</sub>Sn<sub>2</sub>, along the *x*-axis.

Figure 8(a) displays the extinction coefficient  $k(\omega)$  which describe how strongly a DHH reflects the radiation, as we see in the curve the maximum values of  $k_{xx}(\omega)$ ,  $k_{yy}(\omega)$  and  $k_{zz}(\omega)$  are 3.583, 3.451 and 3.365 at 3.193 eV, 4.014 eV and 4.042 eV for ScTaPd<sub>2</sub>Sn<sub>2</sub> respectively while the compound ScTaPt<sub>2</sub>Sn<sub>2</sub> has the values of 3.753, 3.376 and 3.192 at 3.635 eV, 4.121 eV and 4.104 eV . According to the figure 8(b) the refractive index begins at the following values 3.970 for  $n_{xx}(0)$ , 3.954 for  $n_{yy}(0)$  and 3.9478 for  $n_{zz}(0)$ , respectively. The average quantity of n(0) is precisely 3.9574, and it is directly related to  $\varepsilon_1(\omega)$  by  $n(0) = \sqrt{\varepsilon_1(0)} = \sqrt{15.6619} = 3.9574$ . The refractive index rise in the energy range 0–2.353 eV and reaches its maximum values 5.88,6.10 and 5.73 for  $n_{xx}$ ,  $n_{yy}$ , and  $n_{zz}$  directions, respectively Thereafter, it gradually decreases as photon energy rises, this finding is equally evident in the ScTaPt<sub>2</sub>Sn<sub>2</sub> compound see table 5.

The curve in the figure 8(c) represents the reflectance variation, the reflectance value is approximately 35.501% starting from the zero frequency, there are several intense peaks in the energy range 2.301–12.11 eV however the most intense has a reflectivity about 59.921%, while the ScTaPt<sub>2</sub>Sn<sub>2</sub> compound has significantly same behavior as ScTaPd<sub>2</sub>Sn<sub>2</sub>. These compounds has a wide range of applications in the medical and space industries because it has a high reflectivity in the UV region. Figure 8(d) displays the change of the energy loss spectra function. The peaks observed in the spectra correspond to the plasma resonance, and the frequencies at which the resonant energy loss occurs are 17 eV for ScTaPd<sub>2</sub>Sn<sub>2</sub>, 15 eV for ScTaPt<sub>2</sub>Sn<sub>2</sub>, and 38 eV for all directions. The principal peak is located at 25 eV and 40 eV for ScTaPd<sub>2</sub>Sn<sub>2</sub> and ScTaPt<sub>2</sub>Sn<sub>2</sub> respectively.

# 3.4. Elastic properties

We use the IRelast package [23] implemented in Wien2k code to calculate the elasticity constants.in order to determine the compound's mechanical stability, nine independent elastic constants explain how an orthorhombic structure behaves elastically. These constants are:  $C_{11}$ ,  $C_{22}$ ,  $C_{33}$ ,  $C_{12}$ ,  $C_{13}$ ,  $C_{23}$ ,  $C_{44}$ ,  $C_{55}$  and  $C_{66}$ , where  $C_{11}$ ,  $C_{22}$  and  $C_{33}$  reflect the resistance to compression in one direction, respectively. however, the remaining ones show shear stress resistances ( $C_{12}$ ,  $C_{13}$ ,  $C_{23}$ ,  $C_{44}$ ,  $C_{55}$  and  $C_{66}$ ). the GGA approximation was used to calculate the elastic properties. Table 6 show that  $C_{22}$  value of ScTaPd<sub>2</sub>Sn<sub>2</sub> and ScTaPt<sub>2</sub>Sn<sub>2</sub> is larger than  $C_{11}$  and  $C_{33}$  which indicate the strong resistance to deformation on the b-axis than the c-axis and the a-axis, therefore  $C_{11}$  and  $C_{22}$  are higher than those of Mekki et al [2] revealing that both compounds ScTaPd<sub>2</sub>Sn<sub>2</sub> and ScTaPt<sub>2</sub>Sn<sub>2</sub> has strong deformation resistance along a-axis and b-axis, also  $C_{44}$  has higher value comparing to ScNbNi<sub>2</sub>Sn<sub>2</sub>, as well as we can state that these DHH compounds display elastic anisotropy.

$$C_{11} > 0, \quad C_{22} > 0, \quad C_{33} > 0, \quad C_{44} > 0, \quad C_{55} > 0, \quad C_{66} > 0, \quad [C_{11} + C_{22} - 2C_{12}] > 0, \\ [C_{11} + C_{33} - 2C_{13}] > 0, \quad [C_{22} + C_{33} - 2C_{23}] > 0, \quad C_{11} + C_{22} + C_{33} + 2 \quad [C_{12} + C_{13} + C_{23}] > 0, \\ \frac{1}{3} \quad [C_{12} + C_{13} + C_{23}] < B < \frac{1}{3} \quad [C_{11} + C_{22} + C_{33}]$$

$$(2)$$

The calculated elastics constants satisfying the conditions of the stability criteria given in (11), accordingly, these materials are mechanically stable.

The Voigt-Reuss-Hill (VRH) approximation can be used to derive the bulk modulus (B) and shear modulus (G) of polycrystal materials from the obtained constants  $C_{ij}$  of a single crystal

And Hill's compressibility module and shear modulus are [24–26]:

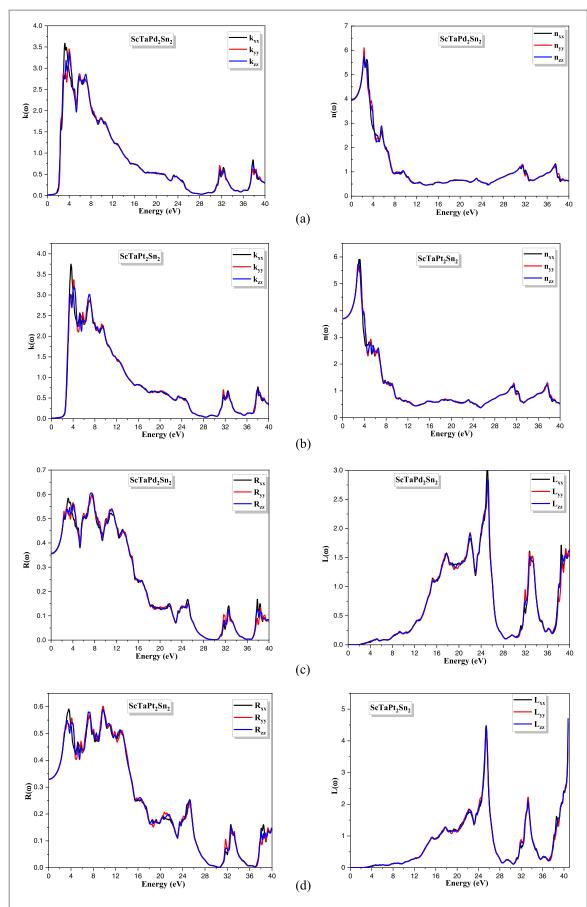
$$G_H = \frac{1}{2}(G_V + G_R) \text{ and } B_H = \frac{1}{2}(B_V + B_R)$$
 (3)

where  $B_V$  and  $G_V$  correspond to Voigt's bulk modulus and shear modulus,  $B_R$  and  $G_R$  are Reuss's bulk modulus and shear modulus, respectively, [27, 28].

The Young's modulus E and Poisson ratio  $\sigma$  [29] are calculated as follows

$$E = \frac{9BG}{(G+3B)} \text{ and } \sigma = \frac{3B-2G}{2(3B+G)} = \frac{1}{2} \left(1 - \frac{E}{3B}\right)$$
 (4)

The bulk modulus reflects a material's resistance to volume change. The shear modulus reflects a material's resistance to shape change. The Young's modulus measures material stiffness, and the higher the value, the stiffer the material. Using the relations up above, all elastic modulus (compressibility modulus, B, shear modulus, G, Young modulus E, Poisson ratio G and G ratio) for G ratio for G ratio G and G ratio for G ratio G and G ratio for G ratio G



 $\textbf{Figure 8.} \ \ Variation \ of (a) \ the \ refractive \ index, (b) \ the \ extinction \ coefficient, (c) \ reflectivity \ spectra, and (d) \ the \ energy \ loss \ function \ as \ a \ function \ of \ photon \ energy \ for \ ScTaPd_2Sn_2 \ and \ ScTaPt_2Sn_2 \ within \ EV-GGA.$ 

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 $\textbf{Table 5.} \ Calculated \ static \ optical \ properties \ for \ ScTaPd_2Sn_2 \ compounds \ within \ different \ approximations.$ 

ScTaSn <sub>2</sub> Pd <sub>2</sub>		$arepsilon_{ m l}(0)$			n(0)			R(0)			L(0)		
Approximation	$\varepsilon_{1xx}$	$arepsilon_{\mathrm{l}yy}$	$arepsilon_{1zz}$	$n_{xx}$	$n_{yy}$	$n_{zz}$	$R_{xx}$	$R_{yy}$	$R_{zz}$	$L_{xx}$	$L_{yy}$	$L_{zz}$	
GGA	16.1305	15.9754	15.9316	4.0163	3.9969	3.9914	0.3615	0.3597	0.3591	$3.9229 \times 10^{-4}$	$3.9034 \times 10^{-4}$	$3.9318 \times 10^{-4}$	
EVGGA	15.7646	15.6355	15.5857	3.9704	3.9542	3.9478	0.3571	0.3555	0.3549	$3.9927 \times 10^{-4}$	$3.9832 \times 10^{-4}$	$4.001 \times 10^{-4}$	
mBJ $GGA$	15.8695	15.6844	15.6406	3.9836	3.9548	3.3954	0.3584	0.3561	0.3556	$4.1623 \times 10^{-4}$	$4.1519 \times 10^{-4}$	$4.1865 \times 10^{-4}$	
LDA	15.6574	15.4761	15.4339	3.9569	3.9339	3.9286	0.3558	0.3536	0.3530	$4.0524 \times 10^{-4}$	$4.0385 \times 10^{-4}$	$4.0675 \times 10^{-4}$	
mBJLDA	15.4768	15.3006	15.2414	3.9340	3.91162	3.9040	0.3536	0.3514	0.3506	$4.1406 \times 10^{-4}$	$4.1332 \times 10^{-4}$	$4.1623 \times 10^{-4}$	

 $\textbf{Table 6.} \ Calculated \ static \ optical \ properties \ for \ ScTaPt_2Sn_2 \ compounds \ within \ different \ approximations.$ 

ScTaSn <sub>2</sub> Pt <sub>2</sub>		$arepsilon_{ m l}(0)$			n(0)			R(0)			L(0)		
Approximation	$\varepsilon_{\mathrm{l}xx}$	$arepsilon_{lyy}$	$arepsilon_{1zz}$	$n_{xx}$	$n_{yy}$	$n_{zz}$	$R_{xx}$	$R_{yy}$	$R_{zz}$	$L_{xx}$	$L_{yy}$	$L_{zz}$	
GGA	14.2820	14.2345	14.1907	3.7791	3.7728	3.7670	0.3381	0.3375	0.3369	$3.797 \times 10^{-4}$	$3.778 \times 10^{-4}$	$3.797 \times 10^{-4}$	
EVGGA	13.687	13.652	13.623	3.6434	3.6388	3.6347	0.3240	0.3236	0.3231	$4.104 \times 10^{-4}$	$4.086 \times 10^{-4}$	$4.105 \times 10^{-4}$	
mBJ GGA	13.179	13.162	13.110	3.633	3.627	3.620	0.3227	0.3224	0.3216	$4.105 \times 10^{-4}$	$4.08 \times 10^{-4}$	$4.109 \times 10^{-4}$	
LDA	13.8641	13.8333	13.7784	3.7234	3.7193	3.7119	0.3324	0.3320	0.3312	$3.957 \times 10^{-4}$	$3.935 \times 10^{-4}$	$3.959 \times 10^{-4}$	
mBJLDA	13.2354	13.1903	13.1555	3.6386	3.6318	3.6270	0.3235	0.3228	0.3223	$4.098 \times 10^{-4}$	$4.086 \times 10^{-4}$	$4.105 \times 10^{-4}$	

**Table 7.** The computed elastic constants for ScTaSn<sub>2</sub>Pd<sub>2</sub>, and ScTaSn<sub>2</sub>Pt<sub>2</sub> using GGA.

Elastic constants (GPa)	ScTaPd <sub>2</sub> Sn <sub>2</sub>	ScTaPt <sub>2</sub> Sn <sub>2</sub>
$C_{11}$	244.5670	275.1153
$C_{22}$	246.9794	277.4895
$C_{33}$	198.7872	226.3281
$C_{44}$	97.0080	108.2926
$C_{55}$	94.0026	104.3802
$C_{66}$	47.3981	54.3908
$C_{12}$	57.8910	66.4894
$C_{13}$	104.2453	115.6720
$C_{23}$	106.8555	120.1886
$A_I$	1.6521	1.6037
$A_2$	1.6203	1.5848
$A_3$	0.5045	0.6772

**Table 8.** Modules of elasticity  $B_V$ ,  $B_R$ ,  $B_H$ ,  $G_V$ ,  $G_R$ ,  $G_H$ ,  $E_V$ ,  $E_E$ ,  $E_H$ ,  $G_V$ ,  $G_R$ ,  $G_H$  for ScTaPd<sub>2</sub>Sn<sub>2</sub> and ScTaPt<sub>2</sub>Sn<sub>2</sub> using GGA approximation.

Elasticity modules (GPa)	ScTaPd <sub>2</sub> Sn <sub>2</sub>	ScTaPt <sub>2</sub> Sn <sub>2</sub>
$B_V$	136.479	153.736
$B_R$	136.472	153.722
$B_H$	136.475	153.729
$G_V$	75.77	85.184
$G_R$	67.391	76.602
$G_H$	71.58	80.893
$E_V$	191.813	215.710
$E_R$	173.598	197.071
$E_H$	182.783	206.464
$\sigma_V$	0.265	0.266
$\sigma_R$	0.287	0.286
$\sigma_H$	0.276	0.276
$B_H/G_H$	1.906	1.9003
$A_B\%$	$2.56 \times 10^{-3}$	$4.55 \times 10^{-3}$
$A_G\%$	5.852	5.3045
$A^{u}$	0.621	0.5602

the ScNbNi<sub>2</sub>Sn<sub>2</sub> compound ( $B_{H\,ScTaPt2Sn2} = 153.72\,GPa > B_{H\,ScTaPd2Sn2} = 136.47\,GPa$ ) which indicate that ScTaPt<sub>2</sub>Sn<sub>2</sub> exhibit elastic stiffness more than the ScTaPd<sub>2</sub>Sn<sub>2</sub>, as well as, The zero-pressure bulk modulus reported in table 8 is close to that from the equation of Murnaghan (EOS) for the ScTaPd<sub>2</sub>Sn<sub>2</sub> and ScTaPt<sub>2</sub>Sn<sub>2</sub> compounds. These compounds have a high shear modulus (71.5 GPa, and 80.8 GPa) it can be notice that ScTaPt<sub>2</sub>Sn<sub>2</sub> display large shear modulus than ScTaPd<sub>2</sub>Sn<sub>2</sub>.

Pugh [30] introduced the bulk to shear modulus (B/G) ratio as an indicator of ductile versus brittle characteristics. If the B/G ratio is more than 1.75, the material is ductile, otherwise, the material is brittle. Both compounds have B/G ratios greater than 1.75, indicating that DHH compounds are ductile. The ratio of tensile stress to tensile strain is defined as the young modulus E which is large for  $ScTaPt_2Sn_2$  compared to the compound  $ScTaPd_2Sn_2$ .

The dimensionless amount the Poisson's ratio is a measure of how well a material resists lateral deformation. For stable compounds, the Poisson's ratio value of a solid is -1 to 0.5 [31]. Table 8 shows the Poisson's ratio value is about 0.276 for the DHH which confirms that these compounds are stable under shear stress, this suggests that  $ScTaPd_2Sn_2$  and  $ScTaPt_2Sn_2$  are contracted by 27.6%.

The Debye temperature  $\theta_D$  is the temperature of a crystal's highest normal mode of vibration, and it correlates the elastic properties with the thermodynamic properties such as phonons, thermal expansion, thermal conductivity, specific heat, and lattice enthalpy [32]. The Debye temperature [33] can be calculated using the average sound velocity, the temperature of Debye  $\theta_D$  can be determined as follows [33, 34].

The computed sound velocity and Debye temperature for DHH compounds are displayed in table 9. All DHH compounds seem to have computed Debye temperatures above 300 K implying that they share the same Debye temperature and melting point.

**Table 9.** Acoustic sound velocities  $(V_bV_b,V_m \text{ in m/s})$ , and Debye temperature  $(\theta_D \text{ in K})$  for ScTaPd<sub>2</sub>Sn<sub>2</sub>, and ScTaPt<sub>2</sub>Sn<sub>2</sub> DHH alloys using *GGA* approximation.

Compounds	V <sub>t</sub> (m/s)	V <sub>l</sub> (m/s)	V <sub>m</sub> (m/s)	$\theta_{\mathrm{D}}(\mathrm{K})$
ScTaPd <sub>2</sub> Sn <sub>2</sub>	2783.84	5010.86	3100.51	338.605
$ScTaPt_2Sn_2$	2652.3	4769.52	2953.78	321.105

Zener anisotropy A is an indicator for the degree of anisotropy in the bonding between atoms in various planes and quantified by the shear anisotropic factors. for the shear anisotropy factor for the  $\{100\}$  shear planes between the  $\langle 011 \rangle$  and  $\langle 010 \rangle$  directions is [35]

$$A_1 = 4C_{44}/(C_{11} + C_{33} - 2C_{13}) (5)$$

For the shear anisotropy factor for the  $\{010\}$  shear planes between the  $\langle 101 \rangle$  and  $\langle 001 \rangle$  directions [35]

$$A_2 = 2C_{66}/(C_{11} - C_{12}) (6)$$

Finally, for the shear anisotropy factor for the  $\{001\}$  shear planes between the  $\langle 110 \rangle$  and  $\langle 010 \rangle$  directions is [35]

$$A_3 = 4C_{66}/(C_{11} - 2C_{12} + C_{22}) (7)$$

As well, for isotropic crystals, the factors  $A_1$ ,  $A_2$  and  $A_3$  must be one. We might confirm that the crystal has elastic anisotropy if any of the three factors is lower or larger than one the crystal represents the degree of the anisotropy.

From table 7 it seems that the shear anisotropy of ScTaPd<sub>2</sub>Sn<sub>2</sub> and ScTaPt<sub>2</sub>Sn<sub>2</sub> is larger for (100) and (010) shear planes

The percentage anisotropy for the bulk modulus A<sub>B</sub> and shear modulus A<sub>G</sub> is given by [36]

$$\begin{cases} A_B = (B_V - B_R)/(B_V + B_R) \times 100 \\ A_G = (G_V - G_R)/(G_V + G_R) \times 100 \end{cases}$$
(8)

The universal anisotropy index (A<sup>U</sup>) is given by [37]:

$$A^{U} = 5\frac{G_{V}}{G_{R}} + \frac{B_{V}}{B_{R}} - 6 (9)$$

The values of  $A_B$ ,  $A_G$ , and  $A^U$  must be zero for an isotropic material. The degree of anisotropy is represented by a deviation larger than zero. From table 8 the percentage of bulk modulus  $A_B$ % for  $ScTaPd_2Sn_2$ , and  $ScTaPt_2Sn_2$  is zero because the Voigt and Reuss approaches anticipate the same values for the bulk modulus, indicating that these compounds are slightly isotropic in compressibility. Likewise, the value of the shear modulus percentage for  $ScTaSn_2Pd_2$  and  $ScTaPt_2Sn_2$  are 5.8% 5.3% shear modulus respectively. Thus, are characterized by significant anisotropy in shear modulus and isotropy in bulk modulus.

The study mentioned above is insufficient to adequately characterize the elastic properties of a crystal. Surface structures that show the linear compressibility and the reciprocal of Young's modulus with directions are useful in fact. The orthorhombic system's linear compressibility is [38]:

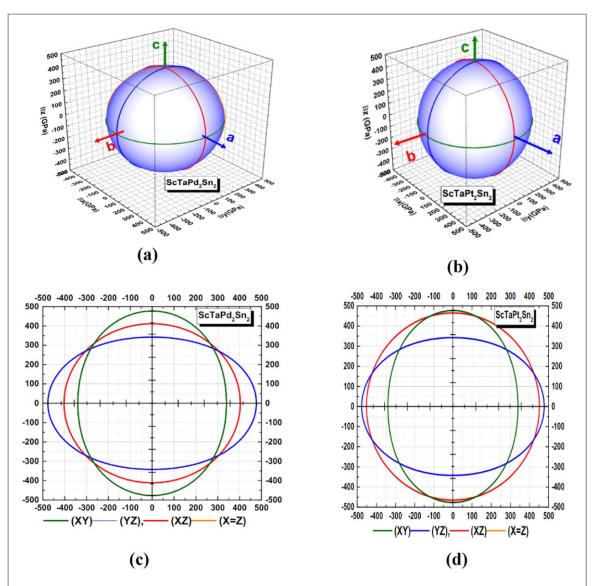
$$\beta = n_1^2 (S_{11} + S_{12} + S_{13}) + n_2^2 (S_{12} + S_{22} + S_{23}) + n_3^2 (S_{13} + S_{23} + S_{33})$$
(10)

 $S_{ij}$  are the elastic constants of deformability, and  $n_1$ ,  $n_2$ ,  $n_3$ : are the directional cosines (in spherical coordinates) following x, y and z, respectively.

The orthorhombic system defines the reciprocal of Young's modulus E in the direction of the unit vector  $n_i$  as [38]:

$$E = \frac{1}{n_1^4 S_{11} + 2n_1^2 n_2^2 S_{12} + 2n_1^2 n_3^2 S_{13} + n_2^4 S_{22} + 2n_2^2 n_3^2 S_{23} + n_3^4 S_{33} + n_2^2 n_3^2 S_{44} + n_1^2 n_3^2 S_{55} + n_1^2 n_2^2 S_{66}}$$
(11)

The linear compressibility of  $ScNbPd_2Sn_2$ , and  $ScNbPt_2Sn_2$  is determined using the theoretical elastic constants depicted in figure 9. The equation (10) establish a three-dimensional surface, where the distance from the origin of coordinates to the surface corresponds to the linear compressibility in a certain direction. The cubic system exhibits isotropic linear compressibility, resulting in a spherical form. The spherical form of figure 9 illustrates the isotropic nature of linear compressibility of our two alloys  $ScTaPd_2Sn_2$ , and  $ScTaPt_2Sn_2$ . To obtain a more detailed and full understanding of the anisotropic features, it is necessary to examine the orientation-dependent linear compressibility in different planes. The elastic anisotropy in the (XY) plane is slightly higher than in the (XZ) plane for  $ScTaPd_2Sn_2$  alloy. Additionally, the (YZ) plane has the highest elastic anisotropy. However, in the  $ScTaPt_2Sn_2$  alloy, the (XZ) plane has a larger elastic anisotropy than both the (XY) and (YZ) planes. Figure 10



 $\textbf{Figure 9.} \ (a) \ \text{and} \ (b) \ \text{represent the orientation dependent linear compressibility of, ScTaPd}_2Sn_2 \ \text{and ScTaPt}_2Sn_2 \ \text{respectively .} \ (c) \ \text{and} \ (d) \ \text{represent linear compressibility projections in different planes of ScTaPt}_2Sn_2 \ \text{and ScTaPd}_2Sn_2 \ \text{respectively.}$ 

depicts the predicted orientation-dependent Young's modulus for  $ScTaPd_2Sn_2$ , and  $ScTaPt_2Sn_2$  alloys by employing the elastic compliance constants. A three-dimensional closed surface is defined by equation (11) when the distance between the surface and the origin of coordinates is equal to Young's modulus in the specified direction. Evidently, the surfaces representing Young's modulus for both alloys  $ScTaPd_2Sn_2$ , and  $ScTaPt_2Sn_2$  exhibited non-spherical shapes Consequently, these two compounds have a significant degree of anisotropy because of the different bonding characteristics between adjacent atomic planes.

The alloys  $ScTaPd_2Sn_2$  and  $ScTaPt_2Sn_2$  display a large elastic anisotropy in the (XY) plane while (X = Y), (XZ) and (YZ) planes are almost identical.

# 3.5. Thermoelectric properties

# 3.5.1. Lattice thermal conductivity

Weak lattice thermal conductivity is a necessary characteristic for possible thermoelectric materials. We investigated the electronic transport properties of ScTaPd<sub>2</sub>Sn<sub>2</sub> and ScTaPt<sub>2</sub>Sn<sub>2</sub> DHH, such as the Seebeck coefficient S, electrical conductivity  $\sigma/\tau$ , the figure of merit ZT and the thermal conductivity ( $\kappa_{Tot} = \kappa_e + \kappa_L$ ) which is equal to the sum of the electronic thermal conductivity  $\kappa_e$  and the lattice thermal conductivity  $\kappa_L$ . The slacks formula is a widely used method for estimating  $\kappa_L$  [39, 40].

The lattice thermal conductivity  $k_L$  was determined using Slack's equation and the BoltzTraP code was used to assess the electronic thermal conductivity  $k_e$ . The Debye temperature has been determined using a quasi-harmonic Debye model that was included in the original Gibbs code [41–44].

Constant relaxation time estimation was utilized in our calculation  $\tau = 3 \times 10^{-14}$  s for temperatures ranging from 50 K to 1000 K [45]. the lattice thermal conductivity  $K_L$  represented in figure 11(a) which reveals a

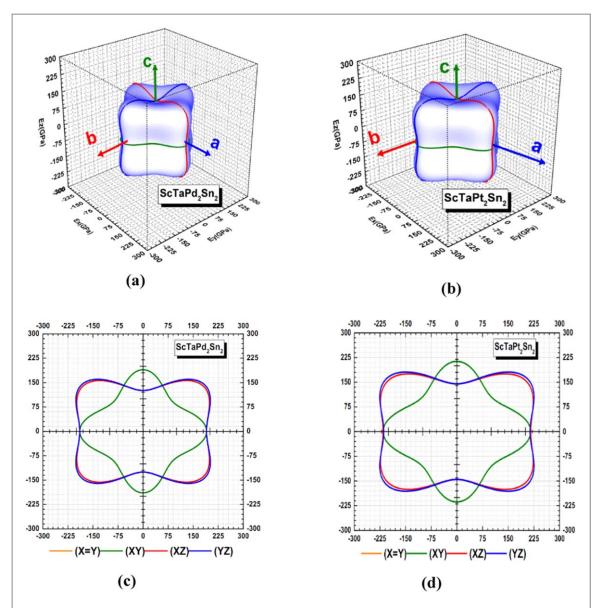


Figure 10. (a) and (b) represent the 3D surface of the Young's modulus for,  $ScTaPd_2Sn_2$  and  $ScTaPt_2Sn_2$ , (c) and (d) their cross sections in different planes respectively.

significant reduction in the low temperature range <300 K, then progressively lowers against temperature due to the phonon interaction in the lattice. On the other hand, at room temperature the magnitude of  $K_L$  is 4.97 W  $K^{-1}$ .m for ScTaPd<sub>2</sub>Sn<sub>2</sub> and 4.98 W  $K^{-1}$ .m for ScTaPt<sub>2</sub>Sn<sub>2</sub> which are relatively larger value comparing to the existing DHH ScNbNi<sub>2</sub>Sn<sub>2</sub> (5.3 W  $K^{-1}$ .m). Thus, low thermal conductivity is desirable in thermoelectric energy conversion materials. The electronic thermal conductivity  $K_{el}$  can be mathematically defined by the Wiedemann–Franz law, which asserts that the ratio of a metal's electronic thermal conductivity to electrical conductivity  $\sigma$  is proportional to temperature T.

 $(k_e/\sigma=LT)$ , where L stands for the Lorentz number. The electronic thermal conductivity of our material is shown in figure 11(b), As the temperature rises, there is definitely a linear increase due to the excitation of the electrons. As previously stated, thermal conductivity is the sum of electronic and lattice thermal conductivity, and  $K_T$  displayed in figure 11(c) appears to have the same behavior as  $K_L$ .

## 3.5.2. Transport properties

The relaxation time approximation can be used to calculate the transport coefficients ( $\sigma$ , S, and  $k_e$ ) of the ScTaSn<sub>2</sub>Pd<sub>2</sub> compound [46–49].

The electronic thermal conductivity components can also be derived using the Wiedemann–Franz law [50],  $K_e = LT\sigma$ , L is the Lorentz number

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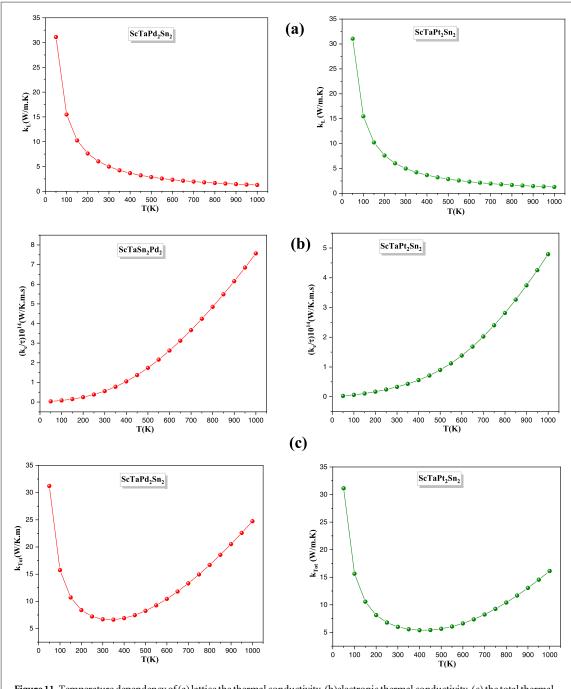
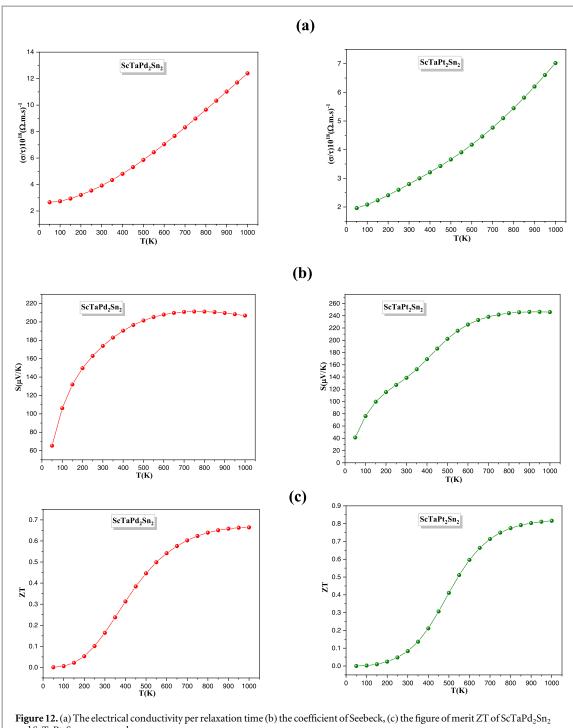


Figure 11. Temperature dependency of (a) lattice the thermal conductivity, (b)electronic thermal conductivity, (c) the total thermal conductivity of  $ScTaPd_2Sn_2$  and  $ScTaPt_2Sn_2$  compounds.

Figure 12(a) depicts the electrical conductivity  $\sigma/\tau$  of the DHH compounds ScTaPd<sub>2</sub>Sn<sub>2</sub> and ScTaPt<sub>2</sub>Sn<sub>2</sub> as a function of temperature. We can clearly see the increase in electronic conductivity as the temperature rises, at room temperature, it has the value of  $3.91 \times 10^{18} \, (\Omega.\text{m.S})^{-1}$  and  $2.79 \times 10^{18} \, (\Omega.\text{m.S})^{-1}$  respectively.

The Seebeck coefficient is a measurement of the voltage produced across a material when a temperature gradient is applied to it, and it is defined as:  $S = V/\Delta T$ , Thus the figure 12(b) shows the Seebeck coefficient as a function of temperature hence, at room temperature ( $T = 300 \, \text{K}$ )  $S_{ScTaPd2Sn2} = 173.8 \, \mu V/K$ ,  $S_{ScTaPt2Sn2} = 138.7 \, \mu V/K$  and reached a maximum value  $S_{ScTaPd2Sn2} = 211.2 \, \mu V/K$ ,  $S_{ScTaPt2Sn2} = 246.35 \, \mu V/K$  at  $T = 750 \, \text{K}$  and  $T = 950 \, \text{K}$  These values are much lower than the previous DHH ScNbNi<sub>2</sub>Sn<sub>2</sub> findings ( $S = 173.2 \, \mu V/K$  at 300 K, Smax = 226.8  $\mu V/K$  at 650 K). So both DHHs materials exhibit a positive Seebeck value across the temperature range, indicating that holes or positive charge carriers are the dominating charge carriers, confirming that our materials are p-type. The band structure degeneracy plays a crucial role in determining the thermoelectric properties of materials such as  $ScTaPd_2Sn_2$  and  $ScTaPt_2Sn_2$ . The presence of multiple energy valleys along the k-path leads to enhanced Seebeck coefficients due to the increased density of states contributing to charge carrier transport. This phenomenon arises from the overlap of electronic bands, resulting in a higher effective mass for

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and ScTaPt2Sn2 compounds.

charge carriers and consequently, a higher Seebeck coefficient. Additionally, the degeneracy of bands can facilitate improved carrier mobility, reducing scattering events and enhancing thermoelectric performance. Understanding and manipulating the band structure degeneracy through techniques such as alloying, doping, and strain engineering offer promising avenues for further enhancing the thermoelectric efficiency of these materials.

The figure of merit (ZT) is a significant measure used to assess a material's thermoelectric efficiency for energy conversion applications. Figure 12(c) illustrates the way ZT increases with temperature. at 900 K ZT are 0.664 and 0.816 for ScTaPd<sub>2</sub>Sn<sub>2</sub> and ScTaPt<sub>2</sub>Sn<sub>2</sub> respectively within EV-GGA. Researchers have recently been working on a new DHHs compounds due to its performance in thermoelectric applications. Figure 13 shows thermoelectric figure of merit ZT as a function of the temperature for DHH compounds ScTaPd<sub>2</sub>Sn<sub>2</sub> and ScTaPt<sub>2</sub>Sn<sub>2</sub> as well as ScNbNi<sub>2</sub>Sn<sub>2</sub> [2] and TiZrCo<sub>2</sub>Bi<sub>2</sub>, TiHfCo<sub>2</sub>Bi<sub>2</sub> and ZrHfCo<sub>2</sub>Bi<sub>2</sub> [5]. We can observe that ZT increases with temperature for all DHH compounds, at 300 K the ZT values are 0.79, 0.76 and 0.8 for TiZrCo<sub>2</sub>Bi<sub>2</sub>

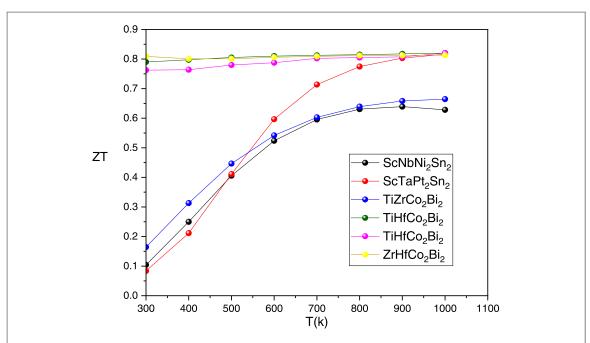


Figure 13. Thermoelectric figure of merit ZT of DHH compounds  $ScTaPd_2Sn_2$  and  $ScTaPt_2Sn_2$ . Data are taken from ref  $ScNbNi_2Sn_2$  [2]  $TiZrCo_2Bi_2$ ,  $TiHfCo_2Bi_2$ ,  $TiZrCo_2Bi_2$  [5].

 $\textbf{Table 10.} \ The \ calculated \ of \ electrical \ conductivity \ tensor, the \ Seebeck \ coefficient \ tensor, and \ the \ electronic \ thermal \ conductivity \ of \ the \ DHH \ alloys \ ScTaPd_2Sn_2, and \ ScTaPt_2Sn_2 \ at \ 300 \ K \ and \ 1000 \ K \ within \ EV-GGA.$ 

Alloys	T(K)	$\sigma/ au^*$	$\sigma/\tau^* 10^{18}  (\text{W.m.s})^{-1}$ $S(\mu V/K)$ $\text{ke}/\tau^* 10^{14}  (\text{W/m.s})^{-1}$			n.K)				
ScTaPd <sub>2</sub> Sn <sub>2</sub>	1000	16.59	6.43	14.15	208.99	197.63	213.83	10.47	3.46	8.76
	300	6.11	1.80	3.83	152.08	198.03	171.36	0.76	0.31	0.58
$ScTaPt_2Sn_2$	1000	12.35	1.48	7.22	197.97	305.35	234.65	7.32	1.64	5.4
	300	5.78	0.18	2.42	116.78	149.55	149.55	0.67	0.02	0.28

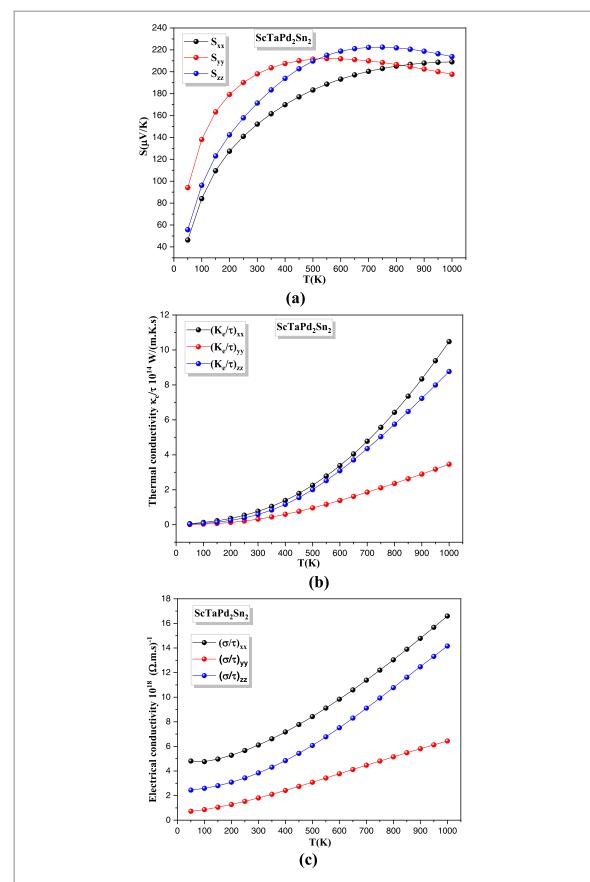
 $TiHfCo_2Bi_2 \ and \ ZrHfCo_2Bi_2 \ DHH \ compounds \ respectively \ all \ the three \ compound \ are \ p-type, likewise \ the \ ZT$  of the parent about 1.42 at 973 K [5], obviously all DHH are suitable for higher temperatures. Further, ZT = 0.99 and ZT = 0.74 at 300 K for TiPtSb HH compound in spin dn and spin up channel respectively with P-type, also 0.95,0.97 and 0.98 for TiRuSb-dn, TiRuSb-up and Ti\_2RuPtSb\_2 with n-type [6], PtHfSn 0.57 and PtZrSn 0.24 in the p-type region [51].

Similarly, we compute the Seebeck coefficient, electrical conductivity and electronic thermal conductivity along the x,y and z axes, considering that Tensors ( $\sigma_{xx} \neq \sigma_{yy} \neq \sigma_{zz}$ ), ( $S_{xx} \neq S_{yy} \neq S_{zz}$ ), ( $k_{e_{xx}} \neq k_{e_{yy}} \neq k_{e_{zz}}$ ) are diagonal.

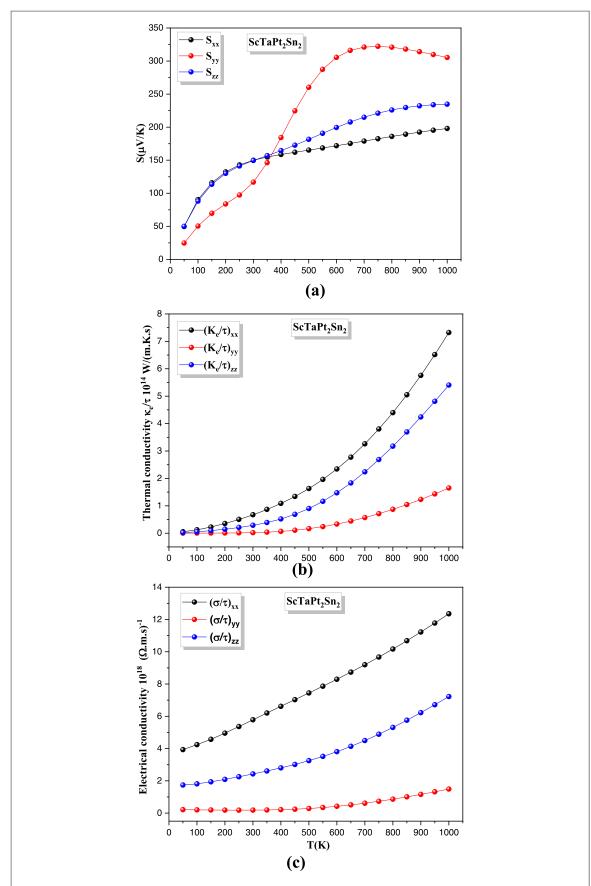
Figure 14(a) display the Seebeck coefficient of ScTaPd<sub>2</sub>Sn<sub>2</sub> compound in the three direction x,y and z,it appears that  $S_{xx}$  and  $S_{yy}$  stratify slightly at 50 K with the values 46.24  $\mu$ V/K and 55.7  $\mu$ V/K then increase linearly with temperature, reaching a maximum value at 750 K for  $S_{zz}$ , However  $S_{zz}$  (222.4  $\mu$ V/K) has a greater value than  $S_{xx}$  (208.9  $\mu$ V/K) and  $S_{YY}$  (212  $\mu$ V/K). The largest value among all tensors has been found to be along the yy axes, as indicated by the highest value of  $S_{yy}$  for temperatures of 1000 K and 300 K, as given in table 10. This suggests that the yy-axis is the dominant factor in transport. In the other hand, figure 15(a) depicts the Seebeck coefficient of ScTaPt<sub>2</sub>Sn<sub>2</sub> compound in the three direction x,y and z, it appears that  $S_{xx}$  and  $S_{zz}$  stratify slightly at 50 K with the values 49.58  $\mu$ V/K and 49.85  $\mu$ V/K then increase linearly with temperature, reaching a maximum value for all directions at 1000 K, However  $S_{yy}$  (305  $\mu$ V/K) has a greater value than  $S_{xx}$  (197.9  $\mu$ V/K) and  $S_{zz}$  (234.6  $\mu$ V/K).

# 3.5.3. Strategies for improving the seebeck coefficient

Figure 16 displays a variation in the Seebeck coefficient within the temperature range of 300–800 K and 300–900 K for ScTaPd<sub>2</sub>Sn<sub>2</sub> and ScTaPt<sub>2</sub>Sn<sub>2</sub>, respectively. The value of Seebeck coefficient for the alloys ScTaPd<sub>2</sub>Sn<sub>2</sub> and ScTaPt<sub>2</sub>Sn<sub>2</sub> are  $S = 211.15 \,\mu\text{V/K}$  at  $T = 800 \,\text{K}$  and  $S = 246.26 \,\mu\text{V/K}$  at  $T = 900 \,\text{K}$  which correspond to a carrier concentration of  $n = 4.92 \times 10^{20} \,\text{Cm}^{-3}$  at and  $n = 3.34 \times 10^{20} \,\text{Cm}^{-3}$  respectively. In order to enhance the thermoelectric properties of double half Heusler alloys, we conducted a study on the effects

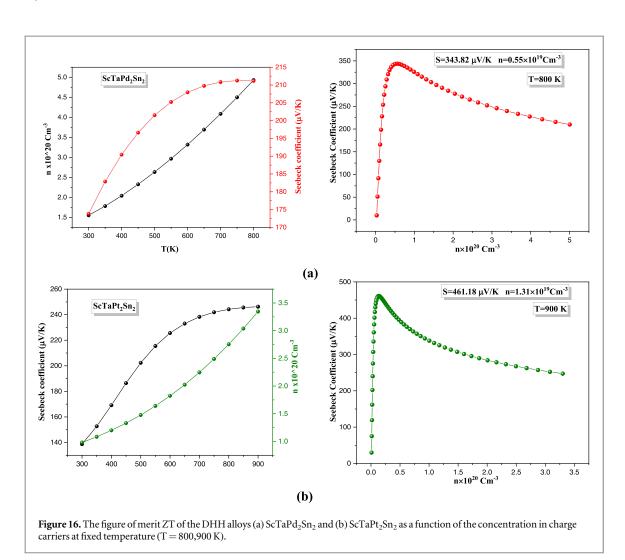


 $\textbf{Figure 14.} Computed the anisotropic transport characteristics of ScTaPd_2Sn_2 compound (a) Seebeck coefficient (b) electronic thermal conductivity and (c) ratio of electrical conductivity tensors component.$ 



 $\textbf{Figure 15.} \ Computed the anisotropic transport characteristics of ScTaPt_2Sn_2 \ compound \ (a) \ Seebeck \ coefficient \ (b) \ electronic thermal \ conductivity \ and \ (c) \ ratio \ of \ electrical \ conductivity \ tensors \ components.$ 

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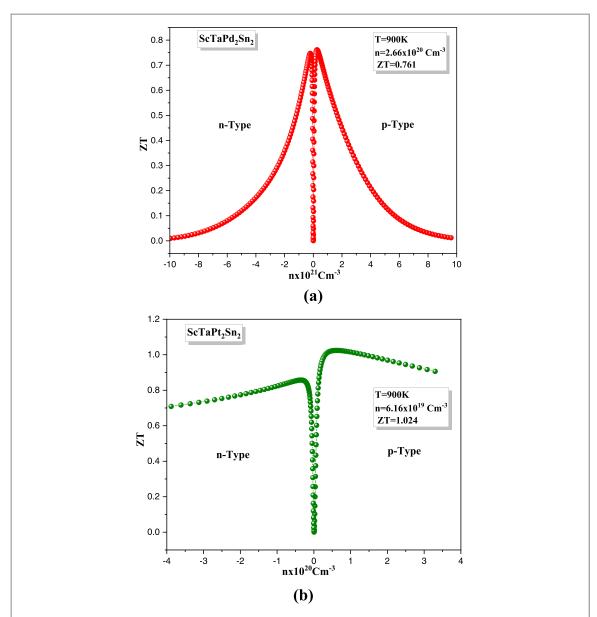
of carrier concentration on the Seebeck coefficient at a temperature of 800 K and 700 K for ScTaPd<sub>2</sub>Sn<sub>2</sub> and ScTaPt<sub>2</sub>Sn<sub>2</sub>, respectively. Based on the data provided in figure 15 it is evident that the maximum values of Seebeck coefficient are 343.82  $\mu$ V/K and 461.18  $\mu$ V/K, these values occur when the concentration of charge carriers is reduced to a point where n = 0.55 × 10<sup>19</sup> Cm<sup>-3</sup> and n = 1.31 × 10<sup>19</sup> Cm<sup>-3</sup>, ScTaPd<sub>2</sub>Sn<sub>2</sub> and ScTaPt<sub>2</sub>Sn<sub>2</sub>, respectively.

# 3.5.4. Methods for enhancing the merit figure ZT

Based on figure 17, the lattice thermal conductivity values for all the DHH alloys  $ScTaPd_2Sn_2$  and  $ScTaPt_2Sn_2$  are low as a result of the described disorder. As the temperature rises, the values of  $k_L$  decrease and are almost equal to zero at extremely high temperatures. In this situation, we can determine an estimated value of without the need to calculate the average value of:

$$ZT \sim T \frac{S^2(\sigma/\tau)}{(k_e/\tau)} = T \frac{S^2\sigma}{k_e}$$
 (12)

The rigid band approximation (RBA) assumes that the band structure of a system remains unchanged whether the temperature or doping is altered [52]. We investigated the relation between the figure of merit and the carrier concentration at a temperature of T=900~K for double half Heusler, respectively. Figure 16 indicates that the ScTaPd<sub>2</sub>Sn<sub>2</sub> alloy has a maximum ZT value 0.74, which is achieved by the n-doped material (n-type semiconductor) with an optimal carrier concentration of  $-2\times10^{20}~Cm^{-3}$  whereas for a p-type semiconductor, the highest ZT value is around 0.76 with carrier concentration  $n=2.66\times10^{20}~Cm^{-3}$ . Regrading to ScTaPt<sub>2</sub>Sn<sub>2</sub> alloy which has the best enhancing figure of merit ZT value comparing the other alloy for the reason that the maximum ZT value is 0.85, which is achieved by the n-doped material (n-type semiconductor) with an optimal carrier concentration of  $-3.5\times10^{19}~Cm^{-3}$  whereas for a p-type semiconductor, the highest ZT value is around 1.024 with carrier concentration  $n=6.16\times10^{19}~Cm^{-3}$  for the ScTaPt<sub>2</sub>Sn<sub>2</sub> alloy (see table 11).



 $\textbf{Figure 17.} \ \ \text{The figure of merit ZT of the DHH alloys (a) ScTaPd}_2Sn_2 \ \ \text{and (b) ScTaPt}_2Sn_2 \ \ \text{as a function of charge carrier concentration at fixed temperature } T = 900 \ \ \text{K}.$ 

 $\label{thm:continuous} \textbf{Table 11.} \ Calculated \ ZT \ and \ corresponding \ transport \ coefficient \ at the \ optimal \ chemical \ potential \ for \ the \ DHH \ alloys \ ScTaPd_2Sn_2, \ and \ ScTaPt_2Sn_2 \ at \ various \ temperatures.$ 

Alloys	μ(eV)	T(K)	n(Cm <sup>-3</sup> )	K <sub>e</sub> (W/m.K)	$\sigma(10^5/\Omega.\text{m})$	S(μV/K)	ZT
ScTaPd <sub>2</sub> Sn <sub>2</sub>	0.68326	1000	$6.82 \times 10^{20}$	23.46	3.8428	206.81	0.664
		900	$5.84 \times 10^{20}$	19.07	3.4159	209.7	0.658
		300	$1.55 \times 10^{20}$	1.72	1.2148	173.8	0.16
	0.68866	900	$2.66 \times 10^{20}$	13.67	1.6281	267.08	0.761
	0.68806	900	$2.91 \times 10^{20}$	14.23	1.7694	260.7	0.69
ScTaPt <sub>2</sub> Sn <sub>2</sub>	0.74039	1000	$4.02 \times 10^{20}$	14.854	2.1765	245.99	0.816
		900	$3.35 \times 10^{20}$	11.6	1.922	246.26	0.803
		300	$9.8 \times 10^{19}$	1.01	8.67745	138.72	0.083
	0.74709	1000	$1.67 \times 10^{20}$	9.56	0.94888	313.73	0.861
	0.75149	900	$6.16\times10^{19}$	4.81	0.3871	376.36	1.024

Hence, the double half Heusler alloy discussed in our study is considered the most suitable material for thermoelectric applications, primarily due to its high figure of merit, a crucial parameter in the field of thermoelectricity.

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#### 4. Conclusion

In this research, a theoretical investigation of  $ScTaPd_2Sn_2$  and  $ScTaPt_2Sn_2$  a new DHH were carried out using Wien2k code, which included several properties such as structural, electronic, optical, elastic and thermoelectric. Based on the obtained calculations, it is evident that our compounds have characteristics of a semiconductor and possesses an indirect band gap. Additionally, the determination of the density of states demonstrates that the d orbitals have the most substantial contribution to the overall density of states. The optical spectra, including the dielectric function, refractive index, and extinction coefficient, exhibit similarities to  $ScNbNi_2Sn_2$ . The analysis of the elastic characteristics suggests that our compounds exhibit stability. The thermoelectric characteristics of our compounds indicate that  $ScTaPd_2Sn_2$  and  $ScTaPt_2Sn_2$  exhibit moderate thermoelectric efficiency

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# Data availability statement

No new data were created or analysed in this study.

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