



CONFÉRENCE INTERNATIONALE

« Statistiques et Analyse Avancées: Domaines d'Interactions et d'Applications 1 »

« SAADIA 1 »



جامعة بجاية
Tasdawit n Bgayet
Université de Béjaïa

☆☆☆

ATTESTATION DE PARTICIPATION

La présidente du comité d'organisation de la Conférence Internationale « SAADIA 1 »

« Statistiques et Analyse Avancées: Domaines d'Interactions et d'Applications 1 » ; organisée par le Laboratoire de Mathématiques Appliquées à l'Université de Béjaïa du 20 au 22 Novembre 2023, atteste que :

BASTI Bilal

a présenté une communication **poster** intitulée :

« Well-posedness of the multidimensional nonlinear free energy equation for modeling many physical phenomena »

Co-auteur: **DJEMIAT Rabah**



LA PRÉSIDENTE DU COMITÉ D'ORGANISATION
LMA
Pr. BECHIR née ZEROUATI Halima

Well-Posedness of the Multidimensional Nonlinear Free Energy Equation for Modeling Many Physical Phenomena

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Abstract

This paper discusses and theoretically studies the existence and uniqueness of radially symmetric solutions for a multidimensional nonlinear time and space-fractional reaction-diffusion/wave equation that enables treating vibration and control, signal and image processing, and modeling earthquakes, among other physical phenomena. Additionally, applying Schauder's and Banach's fixed point theorems facilitates identifying the existence and uniqueness of solutions for the selected equation.

Keywords: Multidimensional nonlinear equation; reaction-diffusion/wave; time and space-fractional order; radially symmetric solutions; existence and uniqueness.

AMS Classification: 35R11; 35A01; 34A08; 35C06; 34K37.

1 Introduction and statement of results

In this work, we shall give an example of a class of fractional-order's PDEs, which helps to describe various complex phenomena; it is a multidimensional nonlinear time and space-fractional reaction-diffusion/wave equation and is written as follows:

$$\partial_t^\alpha u - \kappa^2 \Delta u = F\left(t, x, u, \partial_t^\beta u, (-\Delta)^s u\right), \text{ for } 0 < s \leq 1 < \beta \leq \alpha \leq 2, \quad (1)$$

where $u = u(t, x)$ is a scalar function of the time $t \geq 0$ and space variables $x \in \mathbb{R}^m$, with $m \in \mathbb{N}^*$. Also $F : [0, \infty) \times \mathbb{R}^m \times \mathbb{C} \times \mathbb{C} \times \mathbb{C} \rightarrow \mathbb{C}$ is a nonlinear function, $\kappa \in \mathbb{R}^*$ is a real constant the symbol $(-\Delta)^s$ defines the fractional Laplacian operator [2].

1.1 The significance of the equation

Equation (1) is a representation of a large class of linear and nonlinear equations. Note that, for $F \equiv 0$ and $\alpha = 1$ (resp. $\alpha = 2$), the PDE (1) represents the standard heat equation (resp. the wave equation). In addition to that, it becomes the Klein-Gordon equation when we

choose $F = \kappa u$, $|\kappa| = 1$ and $\alpha = 2$. All these equations fall under the name of the fractional reaction-diffusion/wave equation (see table 1).

Obviously, the development of accurate mathematical models for the description of complex anomalous systems depends on swapping the fractional Laplacian with integer-order Laplacian, e.g.

Fractional equation	Formula
Reaction-diffusion/wave [3, 4, 5, 7, 8, 9, 10]	$\partial_t^\alpha u + \kappa^2 (-\Delta)^s u + c(t, x) u = 0$
Quasi-geostrophic [11]	$\partial_t v + \theta \cdot \nabla v + \kappa (-\Delta)^s v = f$
Cahn-Hilliard [12, 13, 14]	$\partial_t w + (-\Delta)^s (-\varepsilon^2 \Delta w + f(w)) = 0$
Porous medium [12, 13, 14, 15]	$\partial_t u + (-\Delta)^s (u ^{m-1} \text{sign} u) = 0$
Schrödinger [16]	$i\hbar \partial_t \psi = \partial_t^\alpha (-\hbar^2 \Delta)^s \psi + c(t, x) \psi$
Ultrasound [17, 18]	$\frac{1}{c_0^2} \partial_t^2 \theta = \nabla^2 \theta - \left\{ \tau \partial_t (-\Delta)^s + \eta (-\Delta)^{s+\frac{1}{2}} \right\} \theta$

Table 1: Significant equations involving fractional Laplacian

1.2 Problem statement and main results

Let $0 < s \leq 1$, $1 < \beta \leq \alpha \leq 2$, $\varepsilon, \ell > 0$, and $T_\varepsilon = \ell \varepsilon^{\frac{2}{\alpha}}$ be such that $\Omega = [0, T_\varepsilon] \times [\varepsilon/\sqrt{m}, +\infty)^m$. We consider:

$$\begin{cases} \partial_t^\alpha u - \kappa^2 \Delta u = F(t, x, u, \partial_t^\beta u, (-\Delta)^s u), & (t, x) \in \Omega, \kappa \in \mathbb{R}^*, \\ u(0, x) = |x|^\delta u_0, \quad \frac{\partial u}{\partial t}(0, x) = 0, & \delta, u_0 \in \mathbb{C}, \end{cases} \quad (2)$$

where $F : \Omega \times \mathbb{C} \times \mathbb{C} \times \mathbb{C} \rightarrow \mathbb{C}$ is a nonlinear function.

This paper's contribution regards determining the existence, uniqueness, and main properties of the general solution of stability problems obtained through replacing classical rules with fractional quadrature rules of the radially symmetric solution (see [6, 3, 4, 5, 7, 19, 20, 21]),

$$u(t, x) = |x|^\delta f\left(|x|^{-\frac{2}{\alpha}} t\right), \text{ for } |x| = \sqrt{x_1^2 + \dots + x_m^2}, \text{ and } \delta \in \mathbb{C}, \quad (3)$$

the basic profile f is not known in advance and is to be identified.

For the forthcoming analysis, we impose the following hypotheses:

(hyp.1) $F : \Omega \times \mathbb{C} \times \mathbb{C} \times \mathbb{C} \rightarrow \mathbb{C}$ is a continuous function that is invariant by the change of scale (3). It gives us:

$$F\left(t, x, u, \partial_t^\beta u, (-\Delta)^s u\right) = |x|^{\delta-2} \left(J\left(\eta, f(\eta), f'(\eta), {}^C \mathcal{D}_{0+}^\beta f(\eta)\right) - \frac{4\kappa^2}{\alpha^2} \eta^2 f''(\eta) \right), \quad (4)$$

where $\eta = |x|^{-\frac{2}{\alpha}} t$ and $J : [0, \ell] \times \mathbb{C} \times \mathbb{C} \times \mathbb{C} \rightarrow \mathbb{C}$ is a continuous function.

(hyp.2) There exist three positive constants $\omega_1, \omega_2, \omega_3 > 0$ so that the continuous function J given by (4) satisfies:

$$\left| J(\eta, f, g, h) - J(\eta, \tilde{f}, \tilde{g}, \tilde{h}) \right| \leq \omega_1 |f - \tilde{f}| + \omega_2 |g - \tilde{g}| + \omega_3 |h - \tilde{h}|,$$

for any $f, g, h, \tilde{f}, \tilde{g}, \tilde{h} \in \mathbb{C}$.

(hyp.3) There exist four positive functions $a, b, c, d \in C([0, \ell], \mathbb{R}_+)$ such that the continuous function J given by (4) satisfies:

$$|J(\eta, f, g, h)| \leq a(\eta) + b(\eta)|f| + c(\eta)|g| + d(\eta)|h|,$$

for any $f, g, h \in \mathbb{C}$ and $\eta \in [0, \ell]$.

λ denotes the positive constant defined by

$$\lambda = \max \left\{ \frac{\alpha \ell^{\beta-1} (|q| + c^*) + d^*}{\ell^{\beta-\alpha} \Gamma(\alpha - \beta + 1)}, \frac{\alpha \ell^{\beta-1} (|q| + \omega_2) + \omega_3}{\ell^{\beta-\alpha} \Gamma(\alpha - \beta + 1)} \right\},$$

where $q = -\frac{2\kappa^2}{\alpha^2} (\alpha(2\delta + m + 2) + 2)$ and

$$a^* = \sup_{\eta \in [0, \ell]} a(\eta), \quad b^* = \sup_{\eta \in [0, \ell]} b(\eta), \quad c^* = \sup_{\eta \in [0, \ell]} c(\eta) \quad \text{and} \quad d^* = \sup_{\eta \in [0, \ell]} d(\eta).$$

Now, we give the main theorems of this work.

Theorem 1. Assume the hypotheses (hyp.1)–(hyp.3) hold. If we put $\lambda \in (0, 1)$ and

$$\frac{T_\varepsilon^\alpha (|\delta \kappa^2 (\delta + m - 2)| + b^*)}{\Gamma(\alpha + 1)(1 - \lambda)} < \varepsilon^2, \quad (5)$$

then, there is at least one solution to the problem (2) on Ω in the radially symmetric form (3).

Theorem 2. Assume the hypotheses (hyp.1), (hyp.2) hold. We give $\lambda \in (0, 1)$ and

$$\mathcal{K} = \left(\frac{\Gamma(\alpha + 1)(1 - \lambda)}{|\delta \kappa^2 (\delta + m - 2)| + \omega_1} \right)^{\frac{1}{\alpha}}.$$

If we put

$$T_\varepsilon < \varepsilon^{\frac{2}{\alpha}} \mathcal{K}, \quad (6)$$

then the problem (2) admits a unique solution in the radially symmetric form (3) on Ω .

References

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CONFÉRENCE INTERNATIONALE « SAADIA 1 »



Statistiques et Analyse Avancées: Domaines d'Interactions et d'Applications
Université de Béjaia, du 20 au 22 Novembre 2023
Programme préliminaire

Lundi : 20/11/2023

08h30 9h30	Ouverture		
09h30 10h30	Président : Pr. Fatah BOUHMILA		
	Conférence	Pr. Nadia MOHDEB Differential equations and population dynamics	U. de Béjaia
10h30 11h00	Pause-café		
11h00 12h00	Conférence	Pr. Leila RAHMANI On the mathematical modeling of thin layers in mechanical structures	U. Tizi-Ouzou
12h00	Déjeuner		
14h00 15h00	Présidente : Pr. Megdouda OURBIH		
	Conférence	Pr. Hédi NABLI Modèles fluides modulés par un processus markovien	U. Sfax, Tunisie
	Pause-café		
15h30 17h00	Sessions de communications Orales		
	Auditorium (Analyse) Président : Pr. Abdelkrim MOUSSAOUI	Salle 1 (Méthodes Statistiques) Présidente : Pr. Louiza BERDJOU DJ	Salle2 (Statistiques Non paramétriques) Président : Pr. Elias OULD SAID
	1- Meriem CHABEKH : Asymptotic behavior of a thermo elastic laminated Timoshenko Beam with nonlinear delay and nonlinear structural damping. 2- Farida MADJOUR : Modélisation asymptotique et conditions aux limites approchées Pour une plaque de mindlin-timoshenko entourée d'une couche mince. 3- Nassima BOUSSAHOUA : On a hartman-grobman theorem for scalar reaction-diffusion Equations. 4- BOURENI Dalila Résolution de quelques problèmes non linéaires. 5- Nouredine MEDJOU DJ A comparison type principle for a class of quasilinear elliptic Systems : critical and subcritical cases.	1- Khedidja BOUGHAFENE : In the context of imprecise Probabilities (p-boxes) using taylor series expansion. 2- Amel BELHADJ : Anticipating stochastic integral with respect to mixed Fractional brownian motion. 3- Ahlem BOUCHERIKHA : Interaction mode system for the tropical atmosphere. 4-Samira AIT MEKIDECHE : Uncertainty quantification in the classical risk models. 5- Aimen DEHIMI : The m/m/c/n with vacation interruption and Impatient customers.	1- Yasmina ZITOUT: Estimation of availability density function using Beta prime kernel. 2- Meriem MEDJIDER : Nonparametric stress strength reliability estimation using Asymmetric kernels based on log-symmetric distributions. 3- Massilva BOUROUINA : Comparaison entre les estimateurs à noyaux de Densité basé sur trois familles de noyaux asymétriques. 4. Khaoula AIDI Goodness of fit tests for linear exponentiel models. 5- Imane METMOUS Nonparametric estimation of the conditional Distribution function for surrogate data by the regression model.

Mardi : 21/11/2023

8h30 10h00	Président : Pr. Idir ARAB		
	Conférence	Pr. Paulo Eduardo OLIVEIRA Asymptotic for sums of random variables	U. Coimbra, Portugal
	Présidente: Pr. Leila RAHMANI		
	Conférence	Pr. Abdelkrim MOUSSAOUI Multiples solutions pour le système de Gierer-Meinhardt	U. de Béjaia
10h30 11h00	Pause-café		
11h00 12h00	Président : Pr. Rachid BENABIDALLAH		
	Conférence	Pr. Abdenacer MAKHLOUF Rota-Baxter operators on algebras and Beyond	U. Mulhouse, France
12h00	Déjeuner		
14h00 15h00	Président : Pr. Hamid LOUNI		
	Conférence	Pr. Idir ARAB Comparing Random Variables: The Theory and Applications of Stochastic Orders	U. Coimbra, Portugal
15h00 15h30	Pause-café		
15h30 17h00	Sessions de communications Orales		
	Auditorium (Analyse) Présidente : Pr. Nadia. MOHDEB	Salle 1 (Analyse) Présidente : Pr. Leila RAHMANI	Salle 2 (Statistiques et Simulation) Présidente : Pr. Megdouda OURBIH
	1- Fatiha TALBI : <i>Pseudo almost periodic solution for a Nicholson's blowflies model with Stepanov pseudo almost periodic coefficients and a linear harvesting term.</i> 2- Elhabib HADJADJ : <i>Existence results for a problem differential fractional inclusion on an unbounded domain.</i> 3- Hacene GAROUT : <i>Cycles of a symmetrically decoupled three-dimensional map.</i> 4- Mohammed CHEBBAH : <i>Résolution primale Duale et déploiement de logiciels d'un Problème Min-Max en Contrôle Optimal et extensions aux cas Quadratiques linéaires en contrôle.</i> 5- Lydia BOUCHAL <i>Contribution to the fixed point theory for sum of operators with application to a second order bvp</i>	1-Hassina ARROUDJ: <i>Identification du coefficient de Manning en hydrodynamique.</i> 2- Abderrahmane YOKANA : <i>General and optimal decay for a quasilinear parabolic viscoelastic system.</i> 3- Naima MAHNAOUI : <i>Decay of the local energy for the solutions of The critical klein-gordon equation.</i> 4- Faiza ZAAMOUNE <i>Basins of attraction Analysis in hidden Bifurcation Method to Multiscroll chaotic attractors Produced by Parallel.</i> 5- Fatiha ZATOUCHE <i>Singular quasilinear elliptic systems with gradient dependence.</i>	1-Farid Graiche : <i>Tests de rupture épidémique dans la variance.</i> 2- Omar TIMERIDJINE : <i>Méthodes et Analyse de la sensibilité des modèles de simulation.</i> 3- Atika BOULKROUNE : <i>Quelques résultats théoriques et le théorème central limite pour la méthode EDA.</i> 4-Farida HAMRANI : <i>Consistency of the relative error regression estimator under left truncated and dependent data.</i> 5- Joweyda GHOUIL <i>Estimation Bayésienne des paramètres d'un modèle AR(1) par la méthode d'échantillonnage descriptif améliorée.</i>

Mercredi : 22/11/2023

8h30 - 09h30	Présidente : Pr. Karima TIMERIDJINE		
8h30- 09h30	Conférence	Pr. Elias OULD SAID Strong uniform consistency of the local linear relative error regression estimator under left truncation	U. Littoral Côte d'Opale, France
09h30- 10h30	Sessions de communications orales		
	Auditorium (Analyse) Présidente : Pr.Fatiha TALBI	Salle 1 (Algèbre) Président : Pr. Abdenacer MAKHLOUF	Salle 2 (Processus Stochastiques) Présidente : Pr. Karima LAGHA
	1-Mohamed Tahar MEZDDEK: <i>Newton's binomial theorem application for compute a square root of tiangular Toeplitz matrices.</i> 2- Mohamed HELAL: <i>Global Uniqueness Results for Partial Hadamard Fractional Integral Equations.</i> 3- Yousra DJABRI : <i>Stepanov-orliczergodic functions</i>	1- Zohir CHEBEL : <i>Rota-baxter weak bialgebras.</i> 2- Naima KHELOUFI : <i>Hom-vector space.</i> 3- Safia SEFFAH : <i>Repdigits as products of k-fibonacci and k-lucas.</i>	1-Djilali SEBA : <i>Hybrid model for modelling crude oil production in Algeria.</i> 2- Nesrine BENAKLEF: <i>Estimation in short-long memory time series.</i> 3- Mahdia AZZAL: <i>Nards algorithm for reduce the variance of kaplan meier estimator.</i>
10h30 11h00	Pause-café		
	Table ronde		
11h00 12h00	Remise des attestations et clôture		
12h00	Déjeuner		
13h00 17h00	Sortie		

Communications Poster

Lundi : 20/11/2023

Sessions de communications Poster	
10h 30 – 11h	1- Rabah DJEMIAT <i>Thermo dynamical problem of high-frequency ultra sound waves Equation with caputo's fractional operators.</i> 2- Chahira GUENOUNE <i>Analysis for unilateral contact problem with friction in thermovisco-Plasticity.</i> 3- Houria BOUZIANE <i>Sur l'estimation des modèles spatiaux Non linéaires et applications.</i> 4- Bilal BASTI <i>Well-posedness of the multidimensional nonlinear free energy Equation for modeling many physical phenomena.</i> 5- Fatima KHANCHOUCHE <i>Anarchy price in congestion game and nash equilibria.</i> 6- Djamila Zirem <i>Application of M/G/1 queues in stocks management.</i> 7-Sonia IDRES <i>Solutions approximative d'un probleme inverse léniaire à operateur inconnu.</i>
15h00- 15h30	1- Thiziri CHERGUI <i>Ljusternik-Schnirelman theory for Schrodinger systems with Critical exponent.</i> 2- Lydia AMZAL <i>Kiefer-wolfowitz algorithm under quasi-associated random errors in the functional case.</i> 3- Hadjila TABTI <i>Asymptotic normality of the conditional cumulative distribution Function estimator.</i> 4- Kheira MECHERI <i>The asymptotic properties of the conditional quantile using the kernel method in the functional case.</i> 5- Sonia Medjbar <i>On some nonlinear and singular elliptic systems.</i> 6- Bahia BARACHE <i>Mann's stochastic scheme with random errors for solving Fredholm's integro-differential equations.</i> 7- Samiha DJAMAI <i>Degenerate parabolic reaction-diffusion system appearing in Biological science.</i>

Mardi : 21/11/2023

	Sessions de communications Poster
10h 30 – 11h	1- Malika BOUTIBA <i>Three approximations of numerical solutions by fem for Resolving space-time partial differential equations involving fractional derivative order .</i> 2- Abir KADI <i>Analysis of m/m/1/k queue with working vacation and impatient customer suner n-policy.</i> 3- Kahina OUADHI <i>Monte Carlo methods and linear regression.</i> 4- Meriem BOUBALOU <i>Development of an rds software component under matlab.</i> 5- Siham KEBAILI <i>Refined descriptive sampling with dependent variables.</i> 6-Amazigh Zehnati <i>Landweber iterative method to solve an inverse problem with unknown operator.</i> 7- Taous Douali <i>Numerical study of Saint-Venant equations.</i>
15h00- 15h30	1- Chahira ALLOUTI <i>Etude de points fixes à erreurs associées.</i> 2- Akila NASRI <i>Etude de l'existence de solution périodique pour un systeme différentiel d'ordre 3.</i> 3- Khalidja IDJIS <i>Comparaison du générateur getRDS et le générateur Mersenne Twister en utilisant le modèle de file d'attente M/M/1.</i> 4- Lamia LAKAOUI <i>Approximation method for performance analysis of a retrial Queuing system with collisions and transmission errors for Wireless networks.</i> 5- Nabil AIT YALA <i>Drift condition and strong stability with application to Queueing model.</i> 6- Samir HAMCHAOU <i>Analyse statistique appliquée à un service d'eau potable -Cas des réclamations des clients.</i> 7- Djamel ABID <i>Fractional elliptic problem involving critical exponent via nehari Manifold method.</i> 8- Tinhinane MEZIANI <i>Dynamics of ratio-dependent predator-prey model with non-smooth prey harvesting.</i>