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Traveling Profile Solutions for Parabolic Equations Describing Diffusion Phenomena

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Abstract. This paper aims to investigate and derive new exact solutions for a degenerate parabolic partial differential equation, specifically a nonlinear diffusion equation that is not in divergence form. We propose an approach inspired by the traveling profile method to obtain a general form of self-similar solutions to this equation. The behavior of these solutions depends on certain parameters, which determine whether their existence is global or local in a given time T.

Keywords: Traveling profiles · Blow-up · Global · Exact Solutions.

1 Introduction

Partial differential equations (PDEs) often present significant challenges in theoretical construction, particularly in the case of nonlinear versions. This complexity is evident in equations that describe diffusion phenomena, such as the parabolic PDE known as the nonlinear diffusion equation, which is not in divergence form. This equation is expressed as:

$$\frac{\partial u}{\partial t} = u^m \frac{\partial^2 u}{\partial x^2},\ m > 0, \tag{1}$$

where $u = u(x,t)$ is a non-negative scalar function of spatial variable $x \in \mathbb{R}$ and time $t \geq 0$.

The nonlinear diffusion equation not in divergence form (1) represents a large class of nonlinear parabolic equations (see [1,13,23,24,25]). These studies have provided results on the existence and uniqueness of global solutions, as well as solutions that blow up in finite time, for certain classes of the function u that satisfy specific sufficient conditions.

For $m \in (0,1)$, we consider:

$$\begin{cases} \frac{\partial u}{\partial t} = u^m \frac{\partial^2 u}{\partial x^2},\ m \in (0,1) \\ u(x,0) = u_0(x). \end{cases} \tag{2}$$

This model was studied by Wenshu and Zhengan [25], who proved the existence of a unique solution for any initial condition $u_0 \in C(\mathbb{R},\mathbb{R}) \cap L^\infty(\mathbb{R},\mathbb{R})$.

Similarly, Hulshof and Vàzquez [17] studied viscosity solutions of the problem (2). However, they only established the existence of a maximal viscosity solution with nonnegative, continuous, and compactly supported initial data, leaving the uniqueness of viscosity solutions unresolved.

On the other hand, for $m = 1$, the equation (1) was investigated by Ferreira et al. [13], on the half-line $x \in (0, \infty)$ with $t \in (0, T)$. Their study particularly focused on the behavior of solutions near $T > 0$, where T represents the maximal existence time for the solution u, which may be finite or infinite.

1. If $T = +\infty$, i.e. $t \in (0, \infty)$, the existence of the solution is global in time.
2. If $T < +\infty$ and
$$\lim_{t \to T} \|u(\cdot, t)\|_{L^\infty} = +\infty,$$
the solution u becomes unbounded in finite time, and we say that it blows up.

In general, for certain PDEs characterized by symmetries [4,5,6,7,8,10,11,12,18,19,20,21], exact solutions can be determined through specific finite transformations. These transformations reduce the PDE to an ordinary differential equation (ODE), yielding what are known as "self-similar solutions" [4,5,7,14,15,16,19,24]. Self-similar solutions are paramount in the analysis of PDEs because they provide a unified framework for solving the equations locally and globally. This equivalence underscores their central role in the study of PDEs, as they often reveal fundamental insights into the behavior and properties of the solutions.

Wang and Jingxue [24] thoroughly investigated the existence and uniqueness of a shrinking self-similar solution to the equation (1) for $m \geq 1$. The proposed solution was:
$$u(x, t) = \frac{1}{(t+1)^{\beta}} \omega\left((t+1)^{\alpha} x^2\right),$$
where ω is a positive function satisfying some properties and
$$\alpha \geq 0,\ \beta = \frac{1+\alpha}{m},$$
are constants selected so that the solutions exist.

Our objective in this work is to study the parabolic PDE (1) and to find exact solutions in the general self-similar form:
$$u(x, t) = c(t) f\left(\frac{x - b(t)}{a(t)}\right), \text{ with } a, c \in \mathbb{R}^*,\ b \in \mathbb{R}.$$
Here $f > 0$ is the "base profile," a, b, and c are functions of time t to be determined.

The approach presented next is inspired by the method known as the "Traveling Profile Method" [2,7,8,10]. This method enables us to obtain many exact solutions for large classes of nonlinear partial differential equations.

2 Traveling Profile Solutions

In this section, we consider $f \in H^2(\mathbb{R},\mathbb{R}) \cap C^2(\mathbb{R},\mathbb{R})$, a nonnegative scalar function referred to as the base profile. The Sobolev space H^2 is defined as follows:

$$H^2(\mathbb{R},\mathbb{R}) = \left\{ f \in L^2(\mathbb{R},\mathbb{R}),\ f' \in L^2(\mathbb{R},\mathbb{R}),\ f'' \in L^2(\mathbb{R},\mathbb{R}) \right\}.$$

Now, let

$$u(x,t) = c(t)\, f\left(\frac{x-b(t)}{a(t)}\right),\ \text{with } a, c \in \mathbb{R}^*,\ b \in \mathbb{R}. \tag{3}$$

be the traveling profile solution for the PDE (1). If we set $\eta = \frac{x-b(t)}{a(t)}$, then $u(x,t) = c(t)\, f(\eta)$ and

$$\frac{\partial u}{\partial t} = \dot{c}(t)\, f - \frac{\dot{a}(t)}{a(t)} c(t)\, \eta f' - \frac{\dot{b}(t)}{a(t)} c(t)\, f'. \tag{4}$$

Similarly, we obtain:

$$\frac{\partial^2 u}{\partial x^2} = \frac{c(t)}{a^2(t)} f''. \tag{5}$$

Substituting (4) and (5) into (1), we get the following equation:

$$\frac{\dot{c}(t)}{c(t)} f - \frac{\dot{a}(t)}{a(t)} \eta f' - \frac{\dot{b}(t)}{a(t)} f' = \frac{c^m(t)}{a^2(t)} f^m f''.$$

This equation involves several unknown parameters, and our objective is to determine the coefficients a, b, and c along with the base profile f.

In principle, the coefficients a, b, and c are functions determined by solving the following minimization problem [8]:

$$\min_{\dot{a},\dot{b},\dot{c}} \int_{-\infty}^{+\infty} \left| \frac{\partial u}{\partial t} - u^m \frac{\partial^2 u}{\partial x^2} \right|^2 dx.$$

Therefore, we obtain the following three orthogonality equations (see [7,8])

$$\begin{cases} \left\langle \frac{\partial u}{\partial t} - u^m \frac{\partial^2 u}{\partial x^2}, f \right\rangle = 0, \\ \left\langle \frac{\partial u}{\partial t} - u^m \frac{\partial^2 u}{\partial x^2}, \eta f' \right\rangle = 0, \\ \left\langle \frac{\partial u}{\partial t} - u^m \frac{\partial^2 u}{\partial x^2}, f' \right\rangle = 0, \end{cases} \tag{6}$$

where $\langle \cdot\,, \cdot \rangle$ denotes the inner product in the space $L^2(\mathbb{R},\mathbb{R})$, defined by

$$\langle u\,, v \rangle = \int_{\mathbb{R}} u(\eta)\, v(\eta)\, d\eta.$$

The PDE (1) is thus transformed into a set of three coupled ODEs:

$$\begin{cases} \frac{\dot{c}}{c}\langle f,f\rangle - \frac{\dot{a}}{a}\langle \eta f',f\rangle - \frac{\dot{b}}{a}\langle f',f\rangle = \frac{c^m}{a^2}\langle f^m f'',f\rangle \\ \frac{\dot{c}}{c}\langle f,\eta f'\rangle - \frac{\dot{a}}{a}\langle \eta f',\eta f'\rangle - \frac{\dot{b}}{a}\langle f',\eta f'\rangle = \frac{c^m}{a^2}\langle f^m f'',\eta f'\rangle \\ \frac{\dot{c}}{c}\langle f,f'\rangle - \frac{\dot{a}}{a}\langle \eta f',f'\rangle - \frac{\dot{b}}{a}\langle f',f'\rangle = \frac{c^m}{a^2}\langle f^m f'',f'\rangle . \end{cases} \tag{7}$$

We denote by $(z)_+$ the positive part of z, which is z if $z>0$ and zero otherwise.

Theorem 1. *Let a, b, and c be three real functions of $t \geq 0$, which satisfy:*

$$a(0)=1,\ b(0)=0 \text{ and } c(0)=1. \tag{8}$$

Then, for $f \in H^2(\mathbb{R},\mathbb{R}) \cap C^2(\mathbb{R},\mathbb{R})$, the equation (1) admits an exact solution in the form (3), if the base profile f is a solution of following differential equation:

$$f^m f'' = \alpha f + \beta \eta f' + \gamma f',\ \text{with } \alpha,\beta,\gamma \in \mathbb{R}. \tag{9}$$

We then consider the following cases:

- *If $\alpha m + 2\beta \neq 0$, the coefficients a, b, and c are given by:*

$$\begin{cases} a(t) = (1-(\alpha m+2\beta)t)_+^{\frac{\beta}{\alpha m+2\beta}} \\ b(t) = \frac{\gamma}{\beta}(1-(\alpha m+2\beta)t)_+^{\frac{\beta}{\alpha m+2\beta}} - \frac{\gamma}{\beta} \\ c(t) = (1-(\alpha m+2\beta)t)_+^{-\frac{\alpha}{\alpha m+2\beta}} \end{cases},\ 0 \leq t < T. \tag{10}$$

The moment T represents the maximal existence value of the coefficients a, b, and c, such that

$$\begin{cases} T = \frac{1}{\alpha m+2\beta}, & \text{if } \alpha m + 2\beta > 0, \\ T = +\infty, & \text{if } \alpha m + 2\beta < 0. \end{cases}$$

- *If $\alpha m + 2\beta = 0$, the coefficients a, b and c, in this case, are given by:*

$$\begin{cases} a(t) = e^{-\beta t} \\ b(t) = \frac{\gamma}{\beta}e^{-\beta t} - \frac{\gamma}{\beta} \\ c(t) = e^{\alpha t} \end{cases},\ \forall t \geq 0. \tag{11}$$

Proof. The proof of this theorem is based on the results shown presented in [8] for an evolution differential operator. In the following, we recall these results in a special case where the differential operator $A_x u = \frac{\partial^2 u}{\partial x^2}$ is multiplied by the nonlinear term u^m for $m>0$. Let

$$V_t = \{f, \eta f', f'\},$$

be a subspace of $L^2(\mathbb{R},\mathbb{R})$ generated by the associated function f at the moment t. This implies that the functions f, $\eta f'$, and f' are linearly independent and their inner products are non-zero.

From (6), we deduce that the equation $\frac{\partial u}{\partial t} - u^m \frac{\partial^2 u}{\partial x^2}$ is orthogonal to the subspace V_t. In particular, we have $\frac{\partial u}{\partial t} \in V_t$, thus

$$\left\langle \frac{\partial u}{\partial t} - u^m \frac{\partial^2 u}{\partial x^2}, \frac{\partial u}{\partial t} \right\rangle = 0.$$

Therefore, if $u^m \frac{\partial^2 u}{\partial x^2}$ also belongs to V_t, the method provides us a weak exact solution in the form (3).
According to the principle of the method's evaluation, if $u^m \frac{\partial^2 u}{\partial x^2} = \frac{c^{m+1}}{a^2} f^m f''$ belongs to V_t, then

$$u(x,t) = c(t)\, f\left(\frac{x - b(t)}{a(t)}\right),$$

is an exact solution for the equation (1). In this case, the term $f^m f''$ can be expressed as a linear combination of the functions f, $\eta f'$, and f'. That is,

$$f^m f'' = \alpha f + \beta \eta f' + \gamma f', \text{ with } \alpha, \beta, \gamma \in \mathbb{R}.$$

The coefficients a, b, and c are obtained as follows:
If we replace $f^m f''$ by the combination $\alpha f + \beta \eta f' + \gamma f'$ in the system (7), we obtain:

$$MX = \frac{c^m}{a^2} MY, \tag{12}$$

with

$$M = \begin{pmatrix} \langle f, f\rangle & \langle \eta f', f\rangle & \langle f', f\rangle \\ \langle f, \eta f'\rangle & \langle \eta f', \eta f'\rangle & \langle f', \eta f'\rangle \\ \langle f, f'\rangle & \langle \eta f', f'\rangle & \langle f', f'\rangle \end{pmatrix}, \; X = \begin{pmatrix} \frac{\dot{c}}{c} \\ -\frac{\dot{a}}{a} \\ -\frac{\dot{b}}{a} \end{pmatrix}, \; Y = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$$

The matrix M in system (12) is symmetric and invertible, thus it becomes:

$$X = \frac{c^m}{a^2} M^{-1} M Y = \frac{c^m}{a^2} Y,$$

which can be written as:

$$\begin{cases} \dot{a} = -\beta \frac{c^m}{a}, \\ \dot{b} = -\gamma \frac{c^m}{a}, \\ \dot{c} = \alpha \frac{c^{m+1}}{a^2}. \end{cases} \tag{13}$$

Then $\frac{\dot{a}}{a} = -\frac{\beta}{\alpha} \frac{\dot{c}}{c}$, and we deduce:

$$a(t) = K c^{-\frac{\beta}{\alpha}}(t), \; K \in \mathbb{R}. \tag{14}$$

Assuming the conditions (8) are satisfied, thus $K = 1$. Substituting (14) in (13), we obtain:

$$c^{-\frac{\alpha m + 2\beta}{\alpha} - 1} dc = \alpha dt. \tag{15}$$

If $\alpha m + 2\beta \neq 0$, the solution of (15) is:

$$c(t) = \left(1 - (\alpha m + 2\beta)\, t\right)_+^{-\frac{\alpha}{\alpha m + 2\beta}}.$$

Similarly, we get:

$$\begin{cases} a(t) = (1-(\alpha m+2\beta)t)_+^{\frac{\beta}{\alpha m+2\beta}} \\ b(t) = \frac{\gamma}{\beta}(1-(\alpha m+2\beta)t)_+^{\frac{\beta}{\alpha m+2\beta}} - \frac{\gamma}{\beta}. \end{cases}$$

We deduce that the coefficients a, b, and c are defined globally if $\alpha m + 2\beta < 0$, and the coefficients are maximal functions if $\alpha m + 2\beta > 0$, and well defined if and only if

$$0 < t < T = \frac{1}{\alpha m + 2\beta}.$$

If $\alpha m + 2\beta = 0$, the coefficients a, b, and c are defined globally. In this case, we obtain:

$$\begin{cases} a(t) = e^{-\beta t} \\ b(t) = \frac{\gamma}{\beta}e^{-\beta t} - \frac{\gamma}{\beta} \\ c(t) = e^{\alpha t} \end{cases}, \ \forall t \geq 0.$$

We observe from this theorem that there are two distinct time behaviors of the coefficients a, b, and c. These behaviors depend on the similarity parameters α, β and γ.

3 Global Existence and Blow-up of Solutions

In this section, we provide the sufficient conditions on the parameters α, β, and γ to determine whether the solutions under the traveling profile form are global or exhibit blow-up behavior.

Theorem 2. *Let a, b, and c be three real functions of $t \geq 0$, which satisfy the condition (8), and let $f \in H^2(\mathbb{R},\mathbb{R}) \cap C^2(\mathbb{R},\mathbb{R})$ be a solution of the following differential equation:*

$$f^m f'' = \alpha f + \beta\eta f' + \gamma f'.$$

If

$$\alpha m + 2\beta \leq 0,$$

then the equation (1) admits a global solution in time under the traveling profile form, defined for any $t \geq 0$. Moreover, if the profile f is a bounded function and $\alpha < 0$, we have:

$$\lim_{t\to+\infty} u(x,t) = 0, \ \textit{for all } x \in \mathbb{R}.$$

Conversely, if

$$\alpha m + 2\beta > 0 \ \textit{and} \ \alpha > 0,$$

the equation (1) admits a solution under the traveling profile form, which blows up in finite time. The solution is defined for all $t \in [0,T)$, the moment T represents the blow-up time of the solution such that:

$$\textit{for all } x \in \mathbb{R}, \ \lim_{t\to T^-} u(x,t) = +\infty, \ \textit{with } T = \frac{1}{\alpha m + 2\beta} > 0.$$

Proof. We have already proved that the traveling profile solution (3) is an exact solution of equation (1), provided that the base profile f is a solution of differential equation (9). Assuming the conditions (8) are satisfied. If $\alpha m + 2\beta < 0$, then the coefficients a, b, and c are given by:

$$\begin{cases} a\left(t\right) = \left(1-\left(\alpha m+2\beta\right)t\right)_+^{\frac{\beta}{\alpha m+2\beta}} \\ b\left(t\right) = \frac{\gamma}{\beta}\left(1-\left(\alpha m+2\beta\right)t\right)_+^{\frac{\beta}{\alpha m+2\beta}} - \frac{\gamma}{\beta} \\ c\left(t\right) = \left(1-\left(\alpha m+2\beta\right)t\right)_+^{-\frac{\alpha}{\alpha m+2\beta}} \end{cases}, \ \forall t \geq 0$$

and if $\alpha m + 2\beta = 0$, the coefficients a, b, and c are given by:

$$\begin{cases} a\left(t\right) = e^{-\beta t} \\ b\left(t\right) = \frac{\gamma}{\beta}e^{-\beta t} - \frac{\gamma}{\beta} \\ c\left(t\right) = e^{\alpha t} \end{cases}, \ \forall t \geq 0.$$

In each case, the coefficients a, b, and c are defined globally in time. Now, if $\alpha < 0$, we have $\lim\limits_{t\to+\infty} c\left(t\right) = 0$, thus:

$$\lim_{t\to+\infty} u\left(x,t\right) = \lim_{t\to+\infty} c\left(t\right) f\left(\frac{x-b\left(t\right)}{a\left(t\right)}\right) = 0,$$

if and only if the profile f is a bounded function for all $x \in \mathbb{R}$.
Now, we recall that the solution blows up in finite time if there exists $T < +\infty$, which we call the blow-up time, such that the solution is well-defined for all $0 \leq t < T$, while

$$\sup_{x\in\mathbb{R}} \left|u\left(x,t\right)\right| \to +\infty, \text{ when } t \to T^-.$$

In the case $\alpha m + 2\beta > 0$, $c\left(t\right)$ is well-defined if and only if

$$0 \leq t < T = \frac{1}{\alpha m + 2\beta}.$$

If $\alpha > 0$, the value T represents the blow-up time of the solution, thus $\lim\limits_{t\to T^-} c\left(t\right) = +\infty$, and

$$\lim_{t\to T^-} u\left(x,t\right) = \lim_{t\to T^-} c\left(t\right) f\left(\frac{x-b\left(t\right)}{a\left(t\right)}\right) = +\infty.$$

4 New Explicit Solutions for the Nonlinear Diffusion Equation

Now, we present some new explicit solutions on the traveling profile form for equation (1), where the profile f is an integrable function on $\mathbb{R}$, with

$$\int_{\mathbb{R}} f\left(\xi\right) d\xi = M, \text{ with } M > 0.$$

Therefore, for $0 < m \neq 1$ and

$$\int_{\mathbb{R}} u(s,t)\, ds = c^m(t),$$

we can explicitly find a new exact solution for the PDE (1). In fact,

$$\int_{\mathbb{R}} u(s,t)\, ds = \int_{\mathbb{R}} c(t)\, f\left(\frac{s-b(t)}{a(t)}\right) ds = a(t)\, c(t) \int_{\mathbb{R}} f(\xi)\, d\xi = c^m(t),$$

this implies that

$$\frac{c^{m-1}(t)}{a(t)} = M. \tag{16}$$

According to the formulas of the functions a and c in (10) and (11), the equality (16) implies that:

$$\beta = \alpha(1-m). \tag{17}$$

In this case, the equation (9) becomes:

$$f'' = \left(\frac{1}{1-m}(\beta\eta+\gamma)\, f^{1-m} + k_0\right)', \text{ where } k_0 \in \mathbb{R}.$$

If we assume $f'(0) = \frac{\gamma}{1-m} f^{1-m}(0)$, after integration, we get:

$$f^{m-1} f' = \frac{1}{1-m}(\beta\eta+\gamma),$$

or

$$\frac{1}{m} f^m = \frac{1}{1-m}\left(\frac{\beta}{2}\eta^2 + \gamma\eta\right) + k_1,$$

for some $k_1 \in \mathbb{R}$. Finally the solution is written as follows:

$$f(\eta) = \left(\frac{\beta m}{2(1-m)}\eta^2 + \frac{\gamma m}{1-m}\eta + k\right)_+^{\frac{1}{m}}, \text{ with } k \in \mathbb{R}.$$

Now, we determine the coefficients a, b, and c. If $\alpha m + 2\beta = \alpha(2-m) \neq 0$, i.e., $m \neq 2$, then

$$\begin{cases} a(t) = (1-\alpha(2-m)\,t)_+^{\frac{m-1}{m-2}} \\ b(t) = \frac{\gamma}{\beta}(1-\alpha(2-m)\,t)_+^{\frac{m-1}{m-2}} - \frac{\gamma}{\beta} \\ c(t) = (1-\alpha(2-m)\,t)_+^{\frac{1}{m-2}} \end{cases}, \; 0 < t < T.$$

Where T represents the maximal existence value:

$$\begin{cases} T = \frac{1}{\alpha(2-m)}, & \text{if } \alpha(2-m) > 0, \\ T = +\infty, & \text{if } \alpha(2-m) < 0. \end{cases}$$

We obtain the exact solution of (1) as follows:

$$u\left(x,t\right)=c\left(t\right)\left(\frac{\beta m}{2\left(1-m\right)}\left(\frac{x-b\left(t\right)}{a\left(t\right)}\right)^{2}+\frac{\gamma m}{1-m}\left(\frac{x-b\left(t\right)}{a\left(t\right)}\right)+k\right)_{+}^{\frac{1}{m}}. \quad (18)$$

If $m=2$, the coefficients a, b, and c are given by (11), and in this case, we obtain:

$$u\left(x,t\right)=e^{\alpha t}\left(k-\frac{1}{\beta}\left[\left(\beta x+\gamma\right)e^{\beta t}-\gamma\right]^{2}-2\gamma\left[\left(\beta x+\gamma\right)e^{\beta t}-\gamma\right]\right)_{+}^{\frac{1}{2}}.$$

In each case, $k\in\mathbb{R}$ is an arbitrary constant.

For the coefficients a, b, and c that satisfy the parameters of the classical self-similar case, i.e.,

$$c\left(t\right)=t^{\alpha},\ a\left(t\right)=t^{-\beta},\ b\left(t\right)=0,$$

the solution of the equation (1) is written as follows:

$$u\left(x,t\right)=t^{\alpha}f\left(\eta\right),\ \text{with}\ \eta=xt^{\beta},$$

where α and β are exponents that satisfy the similarity condition [18,19,20,21]

$$\alpha m+2\beta+1=0, \quad (19)$$

and the function f is the self-similar solution determined by the solution of the following differential equation:

$$f^{m}f''=\alpha f+\beta\eta f'.$$

If $\beta=\alpha\left(1-m\right)$, the similarity condition (19) gives us:

$$\alpha=\frac{1}{m-2}\ \text{and}\ \beta=\frac{1-m}{m-2}.$$

For $m\in(0,1)$ we get $\alpha,\beta<0$. Thus, the solution (18) for $\gamma=0$ and $k>0$, is written as follows:

$$\begin{aligned}\mathcal{U}\left(x,t\right)&=t^{\alpha}\left(k+\frac{\beta m}{2\left(1-m\right)}\left(\frac{x}{t^{-\beta}}\right)^{2}\right)_{+}^{\frac{1}{m}}\\&=\left[\left(\frac{1}{1-m}\right)^{\frac{1-m}{m}}t^{\alpha(1-m)}\left(k\left(1-m\right)+\frac{\beta m}{2}x^{2}t^{2\beta}\right)_{+}^{\frac{1-m}{m}}\right]^{\frac{1}{1-m}}.\end{aligned}$$

If we set $m\in(0,1)$,

$$p=\frac{1}{1-m}>1,\ \alpha_{0}=-\frac{\alpha}{p},\ \text{and}\ \beta_{0}=-\beta,$$

then the solution of the equation (1) in the classical self-similar case is written as follows:

$$\mathcal{U}(x,t) = \left[p^{\frac{1}{p-1}} B_C(x,t;\alpha_0,\beta_0,A)\right]^p,$$

where

$$B_C(x,t;\alpha_0,\beta_0,A) = t^{-\alpha_0}\left(C - A\frac{x^2}{t^{2\beta_0}}\right)_+^{\frac{1}{p-1}}, \tag{20}$$

and $C = \frac{k}{p} > 0$ is a free constant. The parameters α_0, β_0, and A have precise values:

$$\alpha_0 = \beta_0 = \frac{1}{p+1}, \text{ and } A = \frac{\beta_0(p-1)}{2p}. \tag{21}$$

After the implicit change of variables:

$$v = p^{\frac{1}{p-1}} u^{\frac{1}{p}}, \text{ with } p = \frac{1}{1-m} \text{ where } m \in (0,1),$$

the equation (1) can be written in the divergence form of the porous media equation (see [3,17,22])

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v^p}{\partial x^2}, \ p > 1. \tag{22}$$

The porous media equation (22) admits the properties of similarity. There are several known fundamental families of self-similar solutions, perhaps the most important one is formed by the Barenblatt solutions, discovered independently by Barenblatt in [3] and by Zeldovich and Kompaneets in [26], which are written under the form (20)–(21) for $p > 1$ and gived by:

$$B_C = t^{-\frac{1}{p+1}}\left(C - \frac{p-1}{2p(p+1)} x^2 t^{-\frac{2}{p+1}}\right)_+^{\frac{1}{p-1}},$$

where $C > 0$ is a free constant.

5 Conclusion

In this work, we have discovered new solutions to a nonlinear diffusion equation that is not in divergence form. The behavior of these solutions depends on certain parameters that satisfy specific conditions, determining whether the existence is global or local in time T. The method employed is inspired by the traveling profile method. This method is based on the decomposition of the differential operator in a subspace of L^2 which is generated by functions associated with the base profile f. We have generalized the families of self-similar solutions of the porous media equation, which are represented by the Barenblatt solutions.

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Program CARI'2024
23-26 November - Bejaïa, Algeria

Sunday 24 November				
8:30-9:00	**Welcome participants**			
9:00 – 10:00	**Opening session**			
10:00 – 11:00	**Invited Talk** **Prof. Salissou Moutari**, Université Belfast, United Kingdom ***«Optimisation in Cataract Surgery: Estimating personalised intraocular lens power »*** **Auditorium – Chair: Prof. SIDJE**			
11:00 – 11:30	Coffee break			
11:30 – 12:30	**Parallel sessions**			
	Session 1CS - *Optimization and Heuristics* 1	**Session 2CS - Cluster Analysis**	**Session 1AM - *Control, Optimization and Stochastic Methods 1***	**Session 2AM - *Fractional Dynamics for Epidemic Models I***
	Chair person : **Prof. RADJEF**	**Chair person :** **Prof. MOHDEB**	**Chair person :** **Prof. AISSANI**	**Chair person :** **Prof. DEBBOUCHE**
	Auditorium	**Room A**	**Room B**	**Room C**
	Lillia Ouali, Kamal Amroun, Madani Bezoui and Zineb Younsi : ***New heuristic order based on tree-decomposition for solving CSPs*** Israël Tankam Chedjou: ***Combinatorial optimization of banana clean seed reliance*** Moussa Diallo, Abdoulaye Sidibé and Djibril Diarra : ***Imbalanced Data Classification using Synthetic Minority Oversampling Technique in stages for a Rice Dataset***	Idris Si-Ahmed, Leila Hamdad and Sophie Dabo-Niang: ***Functional Sparse Data Clustering using Conditional Expectation PACE Method*** Siham Hadjout, Kahina Ouazine and Mohamed Essaid Khanouche: ***Clustering for performance optimization in Vehicular Ad-hoc NETworks :A comprehensive survey*** Abdoul Wahab Diallo, Mory Ouattara, François Kaly and Ndèye Niang: ***Hierarchical Sparse Subspace K-Means***	Boudehane Kheireddine and Taleb Samira : ***A Markov Regenerative Stochastic Petri Net for Performance and Reliability analysis of a 3-out-of-(2 + 2 + 1): G Repairable Redundant System*** Marwa Ouni and Abderrahmane Habbal: ***A classical control-type approach to solve the quasi-newtonian Cauchy-Stokes problem***	Fatiha Mesdoui and Naziha Belmahi : ***Linear Control Design for Synchronization of Fractional Predator-Prey Systems with Group Defense and Harvesting*** M. Djouassoum, A. Hamadjam, Banbeto Gouroudja Sylvain and Effa Joseph Yves : ***Fractional dynamics and projection of Tuberculosis in Adamaoua-Cameroon*** Fandio Rubin, A. Hamadjam, S. Banbeto Gouroudja and Ekobena Fouda Paul Henri : ***Fractional dynamics of a Cholera model with Atangana-Baleanu operator***
12:30 – 14:00	Lunch			
14: 00 - 15:00	**Invited Talk 2** **Prof. Ismael Gonzalez Yero**, Universidad de Cadiz, Spain ***« Graph theory and application to social networks »*** **Auditorium: Chair: Prof. SLIMANI**			
15:00 – 15:30	Coffee break			
15:30 – 18:00	**Session Short Papers** **CS**: 12,39,58,62,81,98,111,160,196,200 **AM:** 5,23,25,30,31,44,48,63,82,94,125,126,127, 191 **Oral presentation (10 minutes for each short paper** **(Discussions and exchanges in front of posters)** **Auditorium Hall**			
18:00	**Social Event (Musical Concert)**			

Monday 25 November				
9:00 – 10:00	Invited Talk 3 **Prof. Farouk Toumani** , Université Clermont Ferrand, LIMOS- CNRS, France « *Ontology-based query answering* » **Auditorium: Chair: Prof. BARKAOUI**			
10:00 – 10:30	Coffee break			
	Parallel sessions			
10:30 – 12:30	**Session 3CS - Optimization and Heuristics 2**	**Session 4CS - Reactive Systems**	**Session 3AM - Predator - Prey Systems 2**	**Session 4AM - Partial Differential Equations Modeling**
	Chair person: **Prof. BENTOBACHE**	Chair person: **Prof. KIRCHNER**	Chair person: **Prof. MELATAGIA**	Chair person: **Prof. GMATI**
	Room B	Room C	Room A	Auditorium
	R. Tiyo Melong, J. Lacmou Zeutouo, M. Zekeng Ndadji and M. Tchoupé Tchendji: **A DSL for Dynamic Programming: Generating Sequential Code** Oumarou Abdou Arbi, Jeremie Bourdon and Anne Siegel: **Modelling Behavior of Microbiota Metabolic Network Subject to Diets** Aimen Said Mezabiat, Lyes Abada and Tarek Gacem: **Improved photometric stereo based on Bee Swarm Optimization BSO algorithm** Lydia Sonia Bendimerad, Habiba Drias and Naila Aziza Houacine: **GPU-based Support Vector Machine using Artificial Orca Algorithm for Intrusion Detection Systems**	M.D Nguedia Momo, M. Zekeng Ndadji, M. Tchendji and Franck Tonle Noumbo: **Expressiveness of the LSAWfP Workflow Language Compared to BPMN** Joskel Ngoufo Tagueu, Adrian Puerto Aubel and Maurice Tchoupe Tchendji: **A Service Composition Engine for Incremental Computation** Ameur-Boulifa Rabéa and Sarah Chabane Mechiouri: **Addressing Input/Output composition through Open Automata representation** Razika Lounas and Mezghiche Mohamed: **Formal Verification of Deep Learning Matrix Calculus**	Abboubakar Hamadjam, Banbeto Gouroudja Ardo Sylvain and Rashid Jan: **Global of a SEIRD-B epidemic model with convex incidences** Tewfik Sari and Mohamed Dellal: **A model of plasmid-bearing, plasmid-free competition in a chemostat** Radhwane Benkhaled and Karim Yadi: **On a slow-fast SEIRS Model structured in age** Amina Hammoum and Karim Yadi: **Global asymptotic stability results for a general predator-prey model with variable mortality rate**	Simon Rodrigue Tega II, Ivric Valaire Yatat-Djeumen, Jean Jules Tewa and Pierre Couteron : **Traveling wave and critical wave speed in a nonlocal dispersal fire-mediated tree-grass interactions system in humid tropical savannas** Bilal Basti : **Traveling Profile Solutions for Parabolic Equations Describing Diffusion Phenomena** Abderrahmane Layati and Ali Rimouche: **Positive solutions for elliptic problems with critical Hardy-Sobolev exponent and singular weight** Nikolay Zaitsev and Zinaida Zhuravlova: **Solving the anti-plane problem of elasticity for an infinite strip weakened by a crack**
12:30 – 14:00	Lunch			
	Parallel sessions			
14:00 – 15:30	**Session 5CS - Knowledge Discovery, Natural Language and Speech Processing**	**Session 6CS - Information Security**	**Session 5AM - Fractional Dynamics for Epidemic Models 2**	**Session 6AM - Control, Optimization and Stochastic Methods 2**
	Chair person : **Prof. ROCHE**	Chair person : **Prof. BOUALLOUCHE**	Chair person : **Prof. HAMADOUCHE**	Chair person : **Prof. AIDER**
	Room A	Auditorium	Room B	Room C
	Go Issa Traore and Borlli Michel Jonas Some: **A low-ressource languages dataset: Moore Natural Emotions Speech Data Set** Go Issa Traore and Borlli Michel Jonas Some: **Automatic pure speech files detection using MFCC+F0 features and LSTM classifier in multilingual context for low-resource languages** Apollinaire Batoure Bamana, Yannick Sokdou Bila Lamou and Alioum Abdoulaye: **Benchmark Analysis of Time Series Models for Malaria Trends in the Adamawa Region**	Nesrine Guenfoud, Mohamed Issad and Mohamed Debyeche: **HW/SW Co-design Implementation of Elliptic Curve Digital Signature Algorithm for Blockchain Network** Berlin Djionang, Gilbert Tindo and Roger Atsa Etoundi: **Intrusion recognition coupled with a heuristic attributes selection method using neural networks**	Banbeto Gouroudja Sylvain, Abboubakar Hamadjam, Manasse Djouassoum and Effa Joseph Yves: **Fractional dynamics of a new trachoma transmission model** Fandio Rubin, Abboubakar Hamadjam, Banbeto Gouroudja Ardo Sylvain and Ekobena Fouda Henri Paul: **Fractional-order dynamics of a Typhoid epidemic model**	Takieddine Lombarkia, Kamel Marir and Naila Marir: **Multi-Aggregation Strategies in Ensemble-Based Machine Learning and Deep Learning Models for Cough-Based COVID-19 Detection** Steeve Gael Koula and Stéphane Bouka: **A Note on Additive Model Building for Spatial Functional Regression** Hassaini Katia and Bibi Mohand Ouamer: **Quadratic Programming Problems with Preprocessing and a Diagonally Dominant M-matrix**
15:30	Social Event (Guided tour of Bejaia and surrounding area)			

Tuesday 26 November				
8:30 – 9:30	**Invited Talk 4** **Prof. Houari Sahraoui**, Université de Montréal, Canada *« Generative AI for Software Engineering »* **Auditorium: Prof. ROCHE**			
9:30 – 10:00	Coffee break			
10:00 – 12:00	**Parallel sessions**			
	Session 7CS - *Deep and Machine Learning*	**Session 8CS - *Image Processing and Computer Vision***	**Session 7AM - *Predator - Prey Systems 1***	**Session 8AM - *Performance and Reliability Evaluation***
	Chair person : **Prof. SAHRAOUI**	Chair person : **Prof. NORCY**	Chair person: **Prof. SARI**	Chair person: **Prof. HADJADJ-AOUL**
	Auditorium	**Room A**	**Room B**	**Room C**
	Samiha Ait Taleb, Abderrazak Sebaa, Rafik Bouzera, Randa Ladlani, Dalil Hadjout and Anais Aksouh : ***Natural gas consumption forecasting based on weighted ensemble learning*** Yanni Ammar Khodja, Karim Akilal and Samia Chibani Sadouki: ***Machine learning trust prediction using localized nodes characteristics*** Mikaël Ange Mousse : ***Enhancing the management of enterprise processes using Deep Learning : Case of the pension management process at the National Social Security Fund of Benin*** *Naila Aziza Houacine, Habiba Drias, Fethi Azibi and Serine Nada Bendjebbar:* ***A Data-Driven Wildfire Forecasting: Case study of Algeria***	Loubna Bougheloum, Mounir Bousbia Salah and Maamar Bettayeb : ***An Advanced Object Detection for the Visually Im-paired by using YOLOv5+*** Hocine Attoumi, Hachem Slimani and Fatah Bouchebbah: ***New approach based on alliances in graphs for image segmentation : application to breast MR images*** Amos Mbietieu, Tapamo Kenfack Hyppolite Michel and Kouamou Georges Edouard *ShipFPN:* ***New Feature Pyramid Network Architecture for Object Detection in High-Resolution Satellite Images: Application to Ship Detection*** Youcef Sklab, Hanane Ariouat, Edi Prifti, Jean-Daniel Zucker and Eric Chenin: ***Identification of Non-Plant Elements in Herbarium Images Using YOLO***	Tsidikaina Nirilanto, Stefana Tabera Tsilefa, Angelo Raherinirina and Nikolaos Limnios: ***Modeling and analysis of covid'19 patients hospitalization dynamic reliability by a semi-markovian model*** Hayat Berhoune, Mustapha Lakrib and Tewfik Sari: ***The operating diagram of an SIS model in the chemostat*** Radhouane Fekih Salem : ***On a competition model of two toxin-producing species in the chemostat***	Yasmina Zitout and Karima Lagha : ***Beta Prime Kernel Availability Density Estimation of a Repairable Series System*** Ait Mokhtar El Hassene and Laggoune Radouane : ***Imperfect maintenance of series systems*** Bernine Nassima, Nacer Hassina, Adel-Aissanou Karima and Aissani Djamil : ***Performance Evaluation of a Web services System*** Mohand Moktefi, Louiza Bouallouche-Medjkoune, Soraya Touloum and Mohand Yazid : ***The fair OFDMA-FD MAC for 802.11be and 802.11ax***
12:00 - 12:30	**Closing session**			
12:30 - 14:00	Lunch			
14:00 - 16:30	**ASDS General Assembly**			