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MATHEMATICAL INSIGHTS TO CONTROL EPIDEMICS USING FRACTIONAL OPERATORS

BILAL BASTI AND RABAH DJEMIAT

ABSTRACT. In this paper, we discuss and provide some analytical studies for a mathematical model of fractional-order SIRD for COVID-19 in the sense of the Caputo-Katugampola derivative. We compute and derive several stability results based on some parameters that satisfy some conditions that prevent the pandemic from occurring. The paper also investigates the problem of the existence and uniqueness of solutions for the SIRD model. It does so by applying the properties of Schauder's and Banach's fixed point theorems.

1. INTRODUCTION

The coronavirus pandemic was a major worldwide challenge in 2020. The severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) is the virus responsible for COVID-19 infection. Wuhan city in China is reported to be where this virus originated; it then spread throughout the rest of the world, leading to a pandemic outbreak that persisted throughout 2020.

COVID-19 was declared to be a global threat, following its spread outside China as far as we know, and it affected 212 countries around the world. The available antivirals and vaccines at that point in time were ineffective against the new virus, while the new vaccines currently reached the final stage of development and are being tested on larger populations.

A better understanding and evaluation of infectious diseases' existence, stability, and control can be acquired through modeling them mathematically [1, 2]. However, mathematical models' classical approaches are not highly accurate in modeling such diseases; hence, introducing fractional differential equations for handling these problems becomes necessary. We mention optimization problems, artificial intelligence, medical diagnoses, and their various applications in applied fields. For further reading on the subject, readers can refer to the following books (Samko *et al.* 1993 [3], Podlubny 1999 [4], Kilbas *et al.* 2006 [5], Diethelm 2010 [6]).

Our model requires the division of the total population \mathcal{N} into four epidemiological classes:

\mathcal{S} : Susceptible class, \mathcal{I} : Infected class, \mathcal{R} : Recovered class, and \mathcal{D} : Death class.

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Key words and phrases. COVID-19, SIRD model, fractional derivative, existence and uniqueness.

The parameters are positive and could be described as follows:

- β is the average number of contacts per person per time t ,
- γ is the recovery rate,
- κ is the death rate.

For $0 < \alpha < 1$ and $\rho > 0$,

$$(1.1) \quad \begin{cases} {}^C\mathcal{D}_{0+}^{\alpha,\rho}\mathfrak{S}(t) &= -\beta\frac{\mathfrak{S}(t)\mathfrak{I}(t)}{\mathcal{N}_0}, \\ {}^C\mathcal{D}_{0+}^{\alpha,\rho}\mathfrak{I}(t) &= \beta\frac{\mathfrak{I}(t)\mathfrak{S}(t)}{\mathcal{N}_0} - (\gamma + \kappa)\mathfrak{I}(t), \\ {}^C\mathcal{D}_{0+}^{\alpha,\rho}\mathfrak{R}(t) &= \gamma\mathfrak{I}(t), \\ {}^C\mathcal{D}_{0+}^{\alpha,\rho}\mathfrak{D}(t) &= \kappa\mathfrak{I}(t), \end{cases}$$

along with the positive initial conditions

$$(1.2) \quad \mathfrak{S}(0) = \mathfrak{S}_0, \mathfrak{I}(0) = \mathfrak{I}_0, \mathfrak{R}(0) = \mathfrak{R}_0, \mathfrak{D}(0) = \mathfrak{D}_0,$$

where $\mathcal{N}_0 > 0$ is the initial total population at the moment $t = 0$, with $0 \leq t \leq T < \infty$.

It is essential to find the solution of (1.1) with an efficient technique. Considering this point, our main goal in this work is to determine the main properties of the solution for the system of nonlinear fractional differential equations (1.1), checking the considered model's stability necessitates establishing some existence results.

2. PRELIMINARY AND NECESSARY DEFINITIONS

The essential definitions from the fractional calculus theory are introduced in this section. Banach space of continuous functions from $[0, T]$ into \mathbb{R} is denoted by $C([0, T], \mathbb{R})$, with the norm:

$$\|\varphi\|_\infty = \sup_{t \in [0, T]} |\varphi(t)|.$$

Recently, Katugampola proposed a generalized derivative in [7, 8, 9, 10]; moreover, he demonstrated the existence of solutions of Caputo-Katugampola fractional differential equations in [9]. It reads

$$(2.1) \quad {}^C\mathcal{D}_{0+}^{\alpha,\rho}\varphi(t) = \mathcal{I}_{0+}^{1-\alpha,\rho}\left(\tau^{1-\rho}\frac{d}{d\tau}\varphi\right)(t) = \frac{\rho^\alpha}{\Gamma(1-\alpha)}\int_0^t \frac{\varphi'(\tau)}{(t^\rho - \tau^\rho)^\alpha}d\tau,$$

where $\alpha \in (0, 1)$, $\rho > 0$ and

$$(2.2) \quad \mathcal{I}_{0+}^{\alpha,\rho}\varphi(t) = \frac{\rho^{1-\alpha}}{\Gamma(\alpha)}\int_0^t \frac{\tau^{\rho-1}}{(t^\rho - \tau^\rho)^{1-\alpha}}\varphi(\tau)d\tau, \text{ with } \varphi \in C([0, T], \mathbb{R}).$$

At this point, it is worth mentioning that a fractional Cauchy-type problem was solved for an existence and uniqueness theorem in [11], a decomposition formula for the derivative of Caputo-Katugampola among others could, thus, be obtained as follows:

$$(2.3) \quad \mathcal{I}_{0+}^{\alpha,\rho} {}^C\mathcal{D}_{0+}^{\alpha,\rho}\varphi(t) = \varphi(t) - \varphi(0), \text{ for } \varphi \in C([0, T], \mathbb{R}).$$

It follows from (2.1) that if $\rho = 1$, Caputo-Katugampola derivative is found to be the well-known Caputo fractional derivative [4, 5]. On the other hand, by applying

the L'Hôpital rule, when $\rho \rightarrow 0^+$, we have

$$\begin{aligned} \lim_{\rho \rightarrow 0^+} \frac{\rho^\alpha}{\Gamma(1-\alpha)} \int_0^t \frac{\varphi'(\tau)}{(t^\rho - \tau^\rho)^\alpha} d\tau &= \frac{1}{\Gamma(1-\alpha)} \int_0^t \lim_{\rho \rightarrow 0^+} \frac{\rho^\alpha \varphi'(\tau)}{(t^\rho - \tau^\rho)^\alpha} d\tau \\ &= \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\varphi'(\tau)}{(\ln t - \ln s)^\alpha} d\tau, \end{aligned}$$

which is Caputo-Hadamard fractional derivative [12, 10, 8].

3. MAIN RESULTS

Lemma 3.1. *The solution of model (1.1) is restricted to the feasible region given by*

$$u = \{(\mathfrak{S}, \mathfrak{I}, \mathfrak{R}, \mathfrak{D}) \in \mathbb{R}_+^4, 0 \leq \mathcal{N}(t) \leq \mathcal{N}_0\},$$

and the pandemic will occur if $\mathfrak{S}_0 > \frac{\gamma+\kappa}{\beta} \mathcal{N}_0$, where $\frac{\gamma+\kappa}{\beta}$ is called threshold phenomenon or critical community size for the pandemic.

Theorem 3.2. *The disease-free equilibrium point of (1.1) is*

$$u^* = \left(\frac{\gamma+\kappa}{\beta} \mathcal{N}_0, 0, \mathfrak{R}_0, 0 \right).$$

Theorem 3.3. *If the susceptible class $\mathfrak{S}(t) < \mathfrak{S}^*$, the pandemic free equilibrium point of (1.1) is locally asymptotically stable and is unstable if $\mathfrak{S}(t) > \mathfrak{S}^*$.*

Let $u = (\mathfrak{S}, \mathfrak{I}, \mathfrak{R}, \mathfrak{D}) \in E$, where $E = [C([0, T], \mathbb{R}_+)]^4$ is a Banach space equipped with the norm

$$\|u\|_E = \|\mathfrak{S}\|_\infty + \|\mathfrak{I}\|_\infty + \|\mathfrak{R}\|_\infty + \|\mathfrak{D}\|_\infty$$

and let $f = (f_1, f_2, f_3, f_4)$, be such that

$$\begin{cases} f_1(t, u(t)) &= -\beta \frac{\mathfrak{S}(t)\mathfrak{I}(t)}{\mathcal{N}_0}, \\ f_2(t, u(t)) &= \beta \frac{\mathfrak{I}(t)\mathfrak{S}(t)}{\mathcal{N}_0} - (\gamma + \kappa) \mathfrak{I}(t), \\ f_3(t, u(t)) &= \gamma \mathfrak{I}(t), \\ f_4(t, u(t)) &= \kappa \mathfrak{I}(t), \end{cases}$$

it is clear that the function $f \in ([0, T] \times E)^4$ is continuous.

By applying the fractional integral to both sides of the system (1.1), we get

$$\begin{cases} \mathfrak{S}(t) &= \mathfrak{S}_0 + \frac{\rho^{1-\alpha}}{\Gamma(\alpha)} \int_0^t \frac{\tau^{\rho-1}}{(t^\rho - \tau^\rho)^{1-\alpha}} f_1(\tau, u(\tau)) d\tau, \\ \mathfrak{I}(t) &= \mathfrak{I}_0 + \frac{\rho^{1-\alpha}}{\Gamma(\alpha)} \int_0^t \frac{\tau^{\rho-1}}{(t^\rho - \tau^\rho)^{1-\alpha}} f_2(\tau, u(\tau)) d\tau, \\ \mathfrak{R}(t) &= \mathfrak{R}_0 + \frac{\rho^{1-\alpha}}{\Gamma(\alpha)} \int_0^t \frac{\tau^{\rho-1}}{(t^\rho - \tau^\rho)^{1-\alpha}} f_3(\tau, u(\tau)) d\tau, \\ \mathfrak{D}(t) &= \mathfrak{D}_0 + \frac{\rho^{1-\alpha}}{\Gamma(\alpha)} \int_0^t \frac{\tau^{\rho-1}}{(t^\rho - \tau^\rho)^{1-\alpha}} f_4(\tau, u(\tau)) d\tau, \end{cases}$$

By choosing $u_0 = (u_1, u_2, u_3, u_4) = (\mathfrak{S}_0, \mathfrak{I}_0, \mathfrak{R}_0, \mathfrak{D}_0)$, we get

$$(3.1) \quad u(t) = u_0 + \frac{\rho^{1-\alpha}}{\Gamma(\alpha)} \int_0^t \frac{\tau^{\rho-1}}{(t^\rho - \tau^\rho)^{1-\alpha}} f(\tau, u(\tau)) d\tau.$$

In what follows, we present the principal theorems:

Theorem 3.4. Let $\beta, \gamma, \kappa, \alpha, \rho, T \in \mathbb{R}_+$, be such that $\alpha \in (0, 1)$ and

$$(3.2) \quad T < \left(\frac{\rho^\alpha \Gamma(\alpha + 1)}{4(\beta + \gamma + \kappa + 3)} \right)^{\frac{1}{\rho\alpha}},$$

then the problem (1.1)–(1.2) has at least one solution on $[0, T]$.

Theorem 3.5. Let $\alpha \in (0, 1)$ and $\beta, \gamma, \kappa, \rho, \mu \in \mathbb{R}_+^*$, be such that

$$\mu = \max \{ \beta + \gamma + \kappa + 2, \gamma + 3, \kappa + 3 \}.$$

If

$$(3.3) \quad \frac{4\mu T^{\rho\alpha}}{\rho^\alpha \Gamma(\alpha + 1)} < 1,$$

then the problem (1.1)–(1.2) admits a unique solution on $[0, T]$.

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INTERNATIONAL CONFERENCE
ON MATHEMATICS AND ITS APPLICATIONS
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ICMAST' 2024

Setif, December 15-16, 2024

Auditorium : Mouloud Kacem Nait Belkacem- El Bez

CONFERENCE PROGRAM

Sunday, December 15th, 2024 - Morning

8h 00 – 9h 00	WELCOME - REGISTRATION		
9h 00 – 9h 30	OPENING CEREMONY		
9h 30 – 10h 30	Chairman: Pr. Bachir YALAOUI Speaker : Pr. Abdellah SALHI - Title " Strawberry Plants will get you there: Plant Propagation-based Optimization "		
10h 30 – 11h 00	COFFEE BREAK		
11h 00 – 12h 00	Chairman : Pr. Abdellah SALHI Speaker : Pr. Haslinda IBRAHIM – Title: " Explore, Discover & Develop (ExDID): A Combinatorial Path to Problem Solving and Nature-Inspired Design "		
10h 00 – 12h 00	POSTER SESSION		
	<ul style="list-style-type: none"> • Moufida Goutali • Nor Sahnoune • Bochra Zeghad • Nassima Anane • Hadjer Laraba • Choubeila Souli • Raouf Ziadi 	<ul style="list-style-type: none"> • Rania Benmenni • Nour el islam Hiber • Djaouida Guettal • Asma Maiza • Barkahoum Chebabhi • Amel Noui • Ahmed Ayoub Boukelia 	<ul style="list-style-type: none"> • Aicha Kraria • Merzaka Khaldi • Assma Leulmi • Soumaya Belabbes • Fatima Khanchouche • Baya Takhedmit • Razika Sait
12h 00 – 14h 00	LUNCH BREAK		

Sunday, December 15th, 2024 – Afternoon

	ORAL SESSION		
	SESSION 1: Differential Equations and its Applications Chairman : H. Benseridi	SESSION 2: Dynamic Systems and Mathematical Biology Chairman : S. Mesbahi	SESSION 3: Algebra, Cryptography and Information Security Chairman : N. Trabelsi
14h 00 – 14h 20	Sara Hadi Some results on the asymptotic behaviour of Allen-Cahn equation	Bilel Elbetch Effect of nonlinear diffusion and spatial heterogeneity on the dynamics of a population and on the coexistence of species	Zineb Hebbache On negacyclic codes over mixed alphabet $\mathbb{Z}_p(\mathbb{Z}_p + v\mathbb{Z}_p)$
14h 20 – 14h 40	Hamza Brahim Boulares Maximal, Calderon-Zygmund and sublinear operators on variable weak Herz spaces	Mohamed Cherif Benkara Mostefa Study of a differential-algebraic system with harvesting prey	Ahlem Hamani Some results on minimal non-finite-by-(locally nilpotent) groups
14h 40 – 15h 00	Houd Khireddine On a dynamic frictional contact problem with tresca's friction	Nesrine Benmehenni Enhancing watermark security through Hénon-based digit shuffling	Zoulikha Benhelal Groups whose proper subgroups of infinite rank are (locally finite)-by-engel
15h 00 – 15h 20	Imed Heddar Stability via Lyapunov pairs for nonlinear differential equations	Yassine Adjabi Lyapunov and Hartman type inequalities for a class of mixed nonlinear generalized Caputo fractional differential equations with a forcing term	Boualem Sadaoui A general recursion formula for the multiple Hurwitz zeta values at non positive integers
15h 20 – 15h 50	COFFEE BREAK		
	SESSION 1: Differential Equations and its Applications Chairman : S. Boutechebak	SESSION 2: Numerical Analysis and Simulation Chairman : Dj. Benterki	SESSION 3: Mathematical Finance and Economics Chairman : N. Daili
15h 50 – 16h 10	Bilel Madjour General decay of solutions for a von Karman plate equations with nonlinear fractional damping	Bachir Merikhi Inverse barrier method for nonlinear programming	Amel Belhadj An extension of rough volatility model in financial markets
16h 10 – 16h 30	Amirouche Laadjel Some perov fixed points results in generalized tow metric spaces	Nawel Boudjellal A theoretical and numerical study of a new weighted interior point method based on algebraic equivalent transformation for LCCO	Amira Rami Stabilization of fractional order duopoly game between an heterogeneous player and boundedly rational player with adaptive expectation
16h 30 – 16h 50	Lamia Chouchane Variational analysis of an electro-viscoelastic contact problem with normal compliance and unilateral constraint	Mostafa Bahi Study the uniform convergence of multigrid methods for parabolic variational inequality noncoercive	Aicha Ghanem Parametric curves in Isaac's pursuit-evasion game

14h 30– 16h 30	POSTER SESSION		
	<ul style="list-style-type: none"> • Chamseddine Bouaziz • Abdelmounaim Hamdi • Samiha Djemai • Housseem Eddine Kadem • Safa Benghebrid • Mohamed Meziani • Nedjma Boulelli 	<ul style="list-style-type: none"> • Bachir Fridjat • Karima Laidoune • Bilal Basti • Lina Chetoui • Fatima Hathat • Roufaida Ketfi • Meriem Guechi 	<ul style="list-style-type: none"> • Imane Zouak • Riadh Hedli • Tarek Rouabhi • Fatiha Barache • Leila Younsi née Abbaci • Lamia Djerroud • Zina Hamoudi

Monday, December 16th, 2024 – Morning

08h 30 – 09h 30	Chairman : Pr. Aissa AIBECH Speaker : Pr. Abderrahmene ZIAD - Title " Strong Nash implementation in finite and infinite fair allocation problems "		
9h 30 – 10h 30	Chairman: Pr. Abderrahmene ZIAD Speaker : Pr. Said Houari Belkacem - Title " Reaction diffusion systems and applications "		
10h 30 – 11h 00	COFFEE BREAK		
	ORAL SESSION		
	SESSION 1: Differential Equations and its Applications Chairman : I. Boukaroura	SESSION 2: Data Science and Analytics Chairman : Y. Bencheikh	SESSION 3: Algebra, Cryptography and Information Security Chairman : T. Rouabhi
11h 00 – 11h 20	Soumia Hamdi Well-posedness and stability result for a flexible satellite system under unknown distributed disturbances during attitude maneuvering	Fella Berrimi Mathematical tools for image processing and artificial intelligence	Kheir Saadaoui On intuitionistic and neutrosophic fuzzy sets
11h 20 – 11h 40	Dalila Bitat Some generalizations of integral inequalities of wendroff type	Ikram Bouras Path based formulation for the maximum influence problem in signed social networks	Raouda Chettouh The intersection of the disk and the rectangle containing the numerical range of a bounded linear operator
11h 40 – 12h 00	Ghada Boutara A modified quasi-boundary value method for a class of inverse problem with involution	Massilva Bourouina Nonparametric reliability estimation with three families of asymmetric kernels	Salah Eddine Rihane Complete solutions of the exponential Diophantine equation $P_n^x + P_{n+1}^x = P_m^y$
10h 00 – 12h 00	POSTER SESSION		
	<ul style="list-style-type: none"> • Abbes Ourahmoun • Samira Zahar • Abla Boulaouad • Sihem Smata • Hamid Achab • Rim Benzatat • Khaoula imane Saffidine 	<ul style="list-style-type: none"> • Bisma Founas • Sabrina Guidoum • Narimene Achour • Abdelaaziz Bouziane • Meriem Chabekh • Imane Boudrissa • Imane Aouina 	<ul style="list-style-type: none"> • Slimane Hassaine • Fouzia Bouzeghaya • Chaima Benarab • Yasmina Kadri • Taklit Hana Lahlah • Aziza Aib
12h 00 – 14h 00	LUNCH BREAK		

Monday, December 16th, 2024 – Afternoon

	ORAL SESSION		
	SESSION 1: Numerical Analysis and Simulation + Algebra, Cryptography and information Security Chairman : N. Chougui	SESSION 2: Numerical Analysis and Simulation Chairman : M. Achache	SESSION 3: Fractional Calculus + Mathematical Methods in Engineering + History of science Chairman : A. Kadem
14h 00 – 14h 20	Amel Dilmi Groups whose proper subgroups of infinite rank are finite-by-hypercentral or hypercentral-by-finite	Allaoua Mehri <i>A new error estimation of a finite difference Scheme for a transport-advection equation with fractional order derivative</i>	Mongia Khlifi On the Super Catalan numbers and their applications
14h 20 – 14h 40	Raouf Bouchelaghem On the linear independence measures of logarithms of rational numbers. ii	Youcef Elhamam Hemici Study of a modified PRP method using a diagonal hessian approximation approach	Abdelhadi Safsaf Well posedness of coupled nonlinear wave equations with logarithmic source terms and blow-up
14h 40 – 15h 00	Welid Grimes Polynomial interior-point algorithms for linearly constrained convex optimization	Mohamed Lamine Ouaoua Comparative numerical study between differents variants of conjugate gradient methods of the Dai-Liao type	Kheireddine Boudehane A markov regenerative stochastic petri net for performance and reliability analysis of a 3-out-of-(2+2+1):G repairable redundant system
15h 00 – 15h 20		Yasmina Bendaas A numerical study of an infeasible interior-point algorithm for convex quadratic semi-definite optimization	Brahim Moussaouali The critical analysis of Euclidean geometry in the arab tradition
15h 20 – 15h 50	COFFEE BREAK		
15h 50 – 16h 20	CLOSING CEREMONY		