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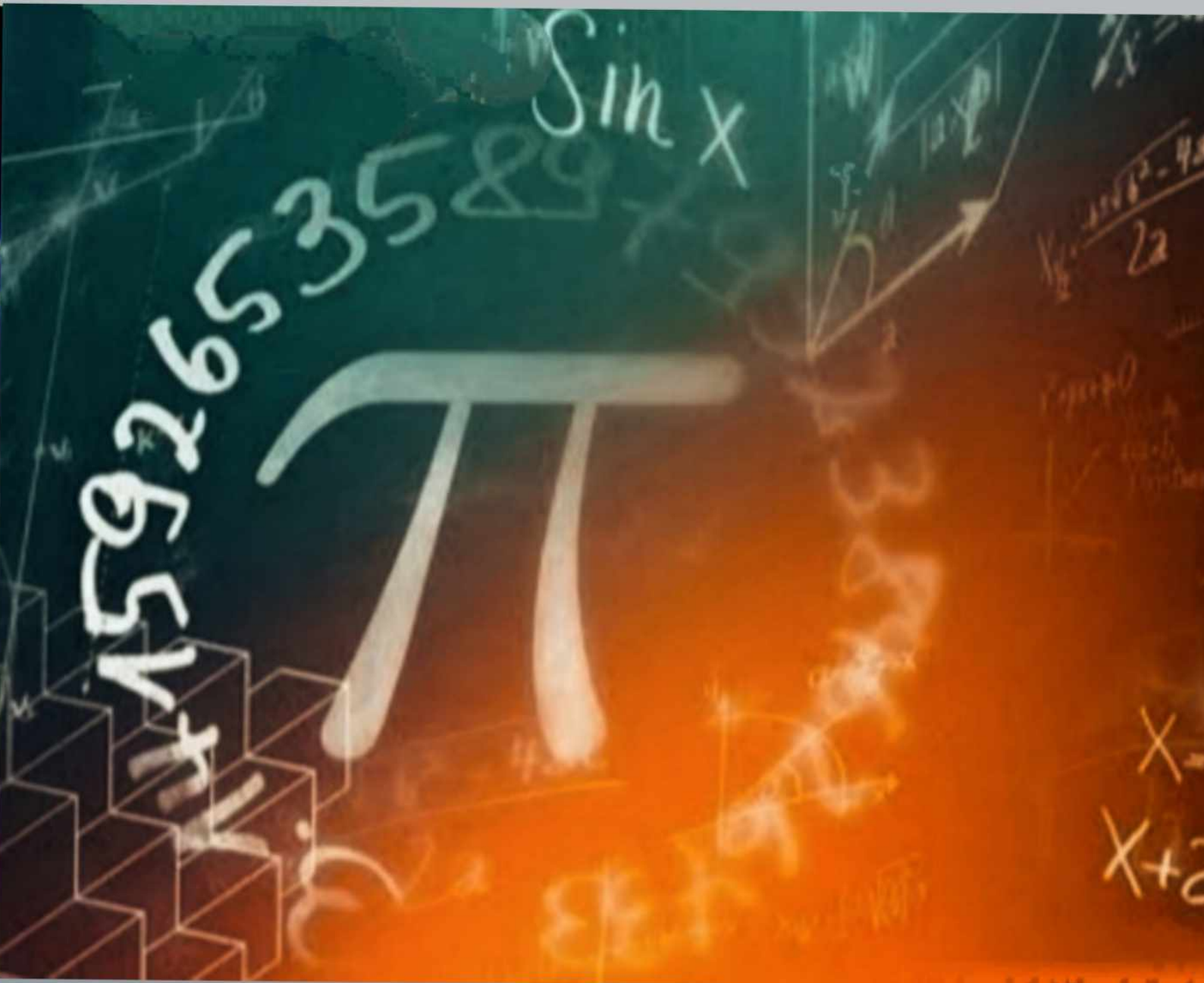
كلية الرياضيات و الإعلام الآلي

*INTERNATIONAL CONFERENCE ON NONLINEAR
MATHEMATICAL ANALYSIS AND ITS
APPLICATIONS*



IC-NMAA'24 MAI 14-15 2024,

BORDJ BOU ARRERIDJ UNIVERSITY, ALGERIA



BOOK OF ABSTRACTS

Preface

The First International Conference on Nonlinear Mathematical Analysis and Its Applications, during which six sessions including "[Partial differential equations and Functional analysis](#)", "[Differential equations and Dynamical systems](#)", "[Probability statistics, Modeling and Control](#)", "[Operational research and Optimization](#)", "[Physical Mathematics](#)" and "[Algebra, number theory and Geometry](#)" were organized in parallel for the presentation of the papers.

The aim of the conference is to bring together as many mathematics scientists as possible to present their latest research and to provide a space for communication between young PhD students and applied mathematics researchers for share experiences, discuss new ideas in applied mathematics, strengthen cooperation and increase scientific exchanges between different researchers.

The conference focuses on advancing faculty and research experts in various fields of research and provides a common forum for leading applied mathematicians and scientists active in various fields of mathematics to advance knowledge and debate about future research challenges.

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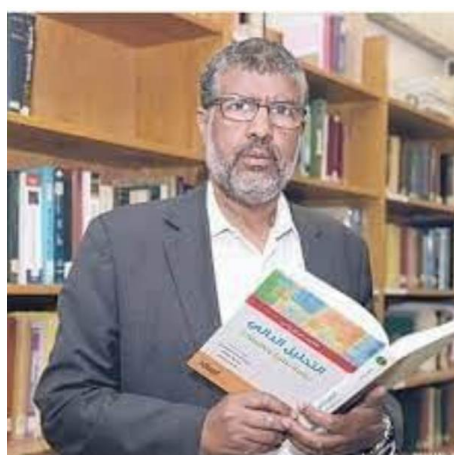
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Plenary Talks

Exponential decay for the quintic wave equation with locally distributed damping

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Abstract We study the stabilization and the wellposedness of solutions of the quintic wave equation with locally distributed damping. The novelty of this talk is that we deal with the difficulty that the main equation does not have good nonlinear structure amenable to a direct proof of a priori bounds and a desirable observability inequality. It is well known that observability inequalities play a critical role in characterizing the long time behaviour of solutions of evolution equations, which is the main goal of this study. In order to address this, we approximate weak solutions for regular solutions for which it is possible to obtain a priori bounds and prove the essential observability inequality. The treatment of these approximate solutions is still a challenging task and requires the use of Strichartz estimates and some microlocal analysis tools such as microlocal defect measures

On a truncated thermoelastic Timoshenko system: Existence and stability

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Abstract In this work we study the well-posedness and the asymptotic stability of a linear one-dimensional truncated thermoelastic Timoshenko system, where the heat conduction is given by Cattaneo's law. . In order to establish the well-posedness, we first solve an auxiliary problem using the semigroup theory and some nontraditional operators. Then, we use this result to solve our main problem. After that, we prove that the presence of the thermal effect in one equation only is strong enough to drive the system exponentially to rest, irrespective to any relation between the coefficients. By the end of the work, we present some numerical tests to illustrate our theoretical findings

Keywords: Cattaneo's law, Exponential decay, truncated, Timoshenko system.

Well-posedness and stability results for thermoelastic Bresse and Timoshenko type systems with Gurtin-Pipkin's law through the vertical displacements

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Abstract The main objective of this work is to study the stability of a linear one-dimensional thermoelastic Bresse system in a bounded domain, where the coupling is given through the first component of the Bresse model with the heat conduction of Gurtin-Pipkin type. Two kinds of coupling are considered; the first coupling is of order one with respect to space variable, and the second one is of order zero. We state the well-posedness and show the polynomial and strong stability of the systems for regular and weak solutions, respectively, where the polynomial decay rates depend on the smoothness of the initial data. Moreover, in case of coupling of order one, we prove the equivalence between the exponential stability and some new conditions on the parameters of the system. However, when the coupling is of order zero, we prove the non-exponential stability independently of the parameters of the system. Applications to the corresponding particular Timoshenko models are also given, where we prove that both couplings lead to the exponential stability if and only if some conditions on the parameters of the systems are satisfied, and both couplings guarantee the polynomial and strong stability for regular and weak solutions, respectively, independently of the parameters of the systems. The proof of the well-posedness result is based on the semigroups theory, whereas a combination of the energy method and the frequency domain approach is used for the proof of the stability results.

For the details, see the following paper:

A. Guesmia, Well-posedness and stability results for thermoelastic Bresse and Timoshenko type systems with Gurtin-Pipkin's law through the vertical displacements, SeMa J., DOI : 10.1007/s40324-023-00333-5.

Lie triple systems: basics and recent developments on cohomology, deformations and related structures

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Abstract Lie triple systems are very important in the study of symmetric spaces. The aim of this talk is to provide some basics, their relationships and some recent and relevant results about representations, cohomology and deformations. Moreover, we discuss O-operators and generalized Reynolds operators on Lie triple systems and related structures.

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Statistical Analysis and Fitting Method for Analytical Estimation of Empirical $\sigma_{K_{\alpha}}$ X-ray Fluorescence Cross Sections for Elements ranging from $_{30}Zn$ to $_{40}Zr$ by Photon Impact at 59.5 keV.

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Partial differential equations and Functional analysis

Generalized Solutions for Elliptic Problems with Measure as Data in $\mathbb{L}_0^{1,p(\cdot)}(\Omega)$

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Abstract: The present work, is devoted to the study of an elliptic problem with variable exponents and measure data, motivated by their applications in the description of many phenomena in applied sciences (physics in nonhomogeneous materials, electro-rheological fluids and image processing). In this context, we generalize the notions of T -sets introduced by Rakotoson Jean Michel to a larger framework, and we establish the existence of a generalized solution for elliptic problems involving variable exponents and measure data.

Keywords: Generalized solutions, elliptic problem, measure data, variable exponents.

Introduction

The present work, is devoted to the study of an elliptic problems with variable exponents and measure data, motivated by their applications in the description of many phenomena in applied sciences (physics in nonhomogeneous materials, electro-rheological fluids and image processing).

Let Ω be a bounded open subset of $\mathbb{R}^N$ ($N \geq 2$) with Lipschitz boundary $\partial\Omega$. We consider the following elliptic problem

$$\begin{cases} -\operatorname{div}(a(x, u, \nabla u)) = \mu & \text{in } \mathcal{D}'(\Omega), \\ u = 0 & \text{on } \partial\Omega, \end{cases} \quad (1)$$

where $\mu \in M(\Omega)$ is a bounded Radon measure. Here, we suppose that $a : \Omega \times \mathbb{R} \times \mathbb{R}^N \rightarrow \mathbb{R}^N$, is a carathéodory function and satisfies, for a.e. $x \in \Omega$, $\forall s \in \mathbb{R}$ and $\forall \xi, \xi' \in \mathbb{R}^N$, the following assumptions

$$\begin{aligned} a(x, s, \xi) \cdot \xi &\geq c_1 |\xi|^{p(x)}, \\ |a(x, s, \xi)| &\leq c_2 (l(x) + |s|^{p(x)-1} + |\xi|^{p(x)-1}), \quad l \in L^{p'(x)}(\Omega), \\ (a(x, s, \xi) - a(x, s, \xi')) \cdot (\xi - \xi') &> 0, \quad \xi \neq \xi', \end{aligned}$$

where c_1, c_2 are strictly positive real numbers, and $p : \overline{\Omega} \rightarrow (1, +\infty)$ be a continuous function satisfying the following condition

$$1 < p^- = \min_{x \in \overline{\Omega}} p(x) < p^+ = \max_{x \in \overline{\Omega}} p(x) < N.$$

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To find a solution in the Sobolev space $W_0^{1,q(\cdot)}(\Omega)$, then the function $q(\cdot)$ must satisfy the condition $1 \leq q(x) < \frac{N(p(x)-1)}{N-1}$ for all $x \in \overline{\Omega}$. This condition is discussed in [6] and implies that $p^- > 2 - \frac{1}{N}$. Therefore, when $p^- \in (1, 2 - \frac{1}{N})$ one cannot anticipate solutions to be part of $W^{1,1}(\Omega)$. As a result, the notions of weak derivatives and distributional solutions become problematic. This problem is studied in the literature using the notion of entropy/renormalized solutions.

In the constants case (i.e $p(x) = p > 2 - \frac{1}{N}$, The authors in [11] proved the existence of solution $u \in W_0^{1,q}(\Omega)$ with $1 \leq q < \frac{N(p-1)}{N-1}$. The general case where $1 < p < N$ was treated by Rakotoson in [1, 2, 3], the author have shown the existence of generalized solutions to (1) by introducing the notion of $L_0^{1,p}$ - sets.

Our aim in this paper is to extend the notion of $L_0^{1,p}$ - sets and define a new class of solution in which the problem (1) is well posed. The main difficulty in solving problem with measures lies in obtaining an a priori estimate in Lebesgue space $L^{s(\cdot)}(\Omega)$. However, we overcome this difficulty by using some properties achieved by a new type of sets.

Main results

The principal result of our paper is the following.

Theorem 1. *Let $\mu \in M(\Omega)$, $p : \overline{\Omega} \rightarrow (1, +\infty)$ be a continuous function, let us assume that*

$$1 < p^- = \min_{x \in \overline{\Omega}} p(x) < p^+ = \max_{x \in \overline{\Omega}} p(x) < N.$$

and $|\nabla p| \in L^\infty(\Omega)$ hold true. Then, there exists at least one solution to problem (1).

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Anisotropic degenerate elliptic problem with singular gradient lower order term

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Abstract: In this paper, we investigate the existence and regularity results of anisotropic elliptic equations with degenerate coercivity and a singular lower order term exhibiting natural growth with respect to the gradient. The model problem we consider is

$$\begin{cases} -\sum_{i=1}^N D_i \left[\frac{|D_i u|^{p_i-2} D_i u}{(1+|u|)^\gamma} \right] + \sum_{i=1}^N \frac{|D_i u|^{p_i}}{|u|^\theta} = f & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \quad (2)$$

where Ω is a bounded domain in $\mathbb{R}^N$, $\gamma > 0$, $0 \leq \theta < 1$ and $2 \leq p_1 \leq p_2 \leq \dots \leq p_N$. The results of our study will rely on the summability of f in specific Lebesgue spaces and the value of γ .

Keywords: Anisotropic elliptic problem. Singular gradient lower order term. Degenerate coercivity. L^m data. Harnack Inequality. Comparison principle

2020 Mathematics Subject Classification: 35J60; 35J70; 35B45; 35D30; 35B65

Introduction

The central aim of this article is to study the following problem

$$\begin{cases} -\sum_{i=1}^N D_i \left[\frac{|D_i u|^{p_i-2} D_i u}{(1+|u|)^\gamma} \right] + b(x) \sum_{i=1}^N \frac{|D_i u|^{p_i}}{|u|^\theta} = f & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \quad (3)$$

where Ω is a bounded open subset of $\mathbb{R}^N$ ($N \geq 3$), f is a positive function (i.e., $f(x) \geq 0$ and not zero almost everywhere) belonging to $L^m(\Omega)$ ($m \geq 1$), $\gamma > 0$, $0 \leq \theta < 1$, and p_i satisfies the following conditions for all $i = 1, \dots, N$:

$$2 \leq p_1 \leq p_2 \leq \dots \leq p_{N-1} \leq p_N \quad \text{and} \quad 2 \leq \bar{p} < N, \quad (4)$$

where $\bar{p}$ is the harmonic mean of p_i , defined as $\frac{1}{\bar{p}} = \frac{1}{N} \sum_{i=1}^N \frac{1}{p_i}$. In this study, we assume that b is a measurable function satisfying the following conditions for some positive numbers α and β

$$0 < \alpha \leq b(x) \leq \beta. \quad (5)$$

Anisotropic equations arise in various mathematical models, finding applications in different fields. For instance, they provide a mathematical description of fluid dynamics in the presence of

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varying conductivities along different directions [1]. Additionally, these equations find relevance in biology, where they serve as models for studying the spread of epidemic diseases in heterogeneous environments [2].

We will outline the characteristics of Problem (3) and highlight the main difficulties encountered. Firstly, let us consider the operator $A : W_0^{1,(p_i)}(\Omega) \rightarrow \left(W_0^{1,(p_i)}(\Omega)\right)^*$ defined as

$$A(u) = - \sum_{i=1}^N D_i \left[\frac{|D_i u|^{p_i-2} D_i u}{(1+|u|)^\gamma} \right],$$

where $\left(W_0^{1,(p_i)}(\Omega)\right)^*$ denotes the dual space of $W_0^{1,(p_i)}(\Omega)$. However, if we take, for example, $u_n(x) = |x|^{\frac{n(p_N-N)}{(n+1)p_N}} - 1$ for every $|x| \leq 1$, the operator A is not coercive (see [5], [7]). Due to the lack of coercivity, the classical theory for elliptic operators acting between spaces in duality (see [5]) cannot be applied, even when the data f is sufficiently regular (see [6]).

Furthermore, the lower order term exhibits growth with respect to the gradient and becomes singular in the variable u . The existence and summability of solutions to problem (3) depend on these characteristics. To overcome these difficulties, we address them by replacing the operator A with another one defined through truncations. Additionally, we approximate the singular term $\frac{|D_i u|^{p_i}}{|u|^\theta}$ with a nonsingular term in such a way that the corresponding approximated problems yield finite energy solutions (see [3, 6]).

Main results

Definition 1. Let f be a positive function belonging to $L^m(\Omega)$, with $m \geq 1$. A measurable function u is a distributional solution of (3) if it satisfies the following conditions:

- 1) u is strictly positive on Ω .
- 2) $\sum_{i=1}^N \frac{|D_i u|^{p_i}}{u^\theta} \in L^1(\Omega)$ and

$$\sum_{i=1}^N \int_{\Omega} |D_i u|^{p_i-2} D_i u \cdot D_i \varphi dx + \sum_{i=1}^N \int_{\Omega} \frac{|D_i u|^{p_i}}{u^\theta} \varphi dx = \int_{\Omega} f \varphi dx, \quad (6)$$

for every $\varphi \in C_0^\infty(\Omega)$.

Our main results are as follows:

Theorem 2. Let $f \in L^1(\Omega)$ be a positive function, and assume that (4) and (5) hold true. Then, the problem (3) has at least one distributional solution $u \in W_0^{1,(\eta_i)}(\Omega)$, where

$$\eta_i = p_i \frac{N(\bar{p} - \theta)}{\bar{p}(N - \theta)}. \quad (7)$$

Theorem 3. If hypotheses (4) and (5) hold, and $f \in L^m(\Omega)$ is a positive function with $m > 1$ satisfying

$$\frac{\bar{p}N}{\bar{p}\theta + 2N(\bar{p} - \theta)} < m < \frac{\bar{p}N}{\bar{p}N - \theta(N - \bar{p})} = m_1. \quad (8)$$

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We also assume, for all $i = 1, \dots, N$,

$$\frac{\bar{p}(N - \theta m)}{mN(\bar{p} - \theta)} < p_i < \frac{\bar{p}(N - \theta m)}{\bar{p}(N - \theta m) - Nm(\bar{p} - \theta)}. \quad (9)$$

Then, the problem (3) has at least one distributional solution $u \in W_0^{1,(\sigma_i)}(\Omega)$, where

$$\sigma_i = \frac{mN(\bar{p} - \theta)}{\bar{p}(N - \theta m)} p_i. \quad (10)$$

Theorem 4. Assume that (4) and (5) hold true. Suppose that $f \in L^m(\Omega)$ such that

$$m_1 < m_2 = \frac{N(\bar{p} + 1)}{\bar{p}(N + 1) - \theta(N - \bar{p})} \leq m < \frac{N}{\bar{p}}. \quad (11)$$

Then, the problem (3) has at least one distributional solution $u \in W_0^{1,(p_i)}(\Omega) \cap L^s(\Omega)$, where

$$s = \frac{mN(\bar{p} - \theta)}{N - \bar{p}m}. \quad (12)$$

Theorem 5. Let $0 < \gamma < \bar{p} - 1$. Suppose that assumptions (4) and (5) hold, and let $f \in L^m(\Omega)$ be a positive function, such that $m > \frac{N}{\bar{p}}$. Then, the problem (3) has at least one distributional solution $u \in W_0^{1,(p_i)}(\Omega) \cap L^\infty(\Omega)$.

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Banach contraction principal with two mappings

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Abstract: We prove the existence of a fixed point and common fixed point for two mappings satisfying and generalizing contractive condition. We provide relevant definitions and properties related to contraction mapping for two mappings. Moreover, we establish the analogue to the Banach contraction theorem for two F-contraction. Our results generalize some known ones in the literature.

Keywords: Contraction mapping; fixed point; common fixed point.

2010 Mathematics Subject Classification: Primary 54H25, secondary 47H10, 47H09.

Introduction

Banach contraction principle is important Theorem to solve all type of equation. if we are given two continuous functions f and g defined on the complete metric space such that, they are commuting each other and that g be an f -contraction such that, the range of g be included in the one's of f . Then f and g must posses a unique common fixed point. Note that the inclusion condition in Jungck's theorem is only sufficient but not necessary for the existence of common fixed points. Some some research we were invested in its meaning for founding a fixed point by different methods. In the present presentation we give an important theorem of the fixed point for two mappings not necessary commuting with a different conditions to Junck's Theorem and we conclude by existence of the fixed point under certain conditions for the case that the norm of derivative function is greater than one and illustrated with an example.

Preliminaries

Let us recall some definitions and known results concerning common fixed points.

The following theorems and propositions will be needed in the sequel.

Definition 2. [5] Let (X, d) be a metric space and $f, g : X \rightarrow X$. The mapping g is called a f -contraction if there exists $k \in (0, 1)$ such that $d(g(x), g(y)) \leq kd(f(x), f(y))$.

Theorem 6. ([4]) Let f be a continuous mapping on a complete metric space (X, d) into itself. Then f has a fixed point in X if and only if there exist a real constant $k \in (0, 1)$, a mapping $g : X \rightarrow X$ which commutes with f , satisfies $g(X) \subset f(X)$ and

$$d(g(x), g(y)) \leq kd(f(x), f(y)), \forall x, y \in X. \quad (13)$$

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Indeed, f and g have a unique common fixed point in X if (13) holds.

Remark 1. If we replace f by a identity mapping in the condition 13, we obtain classical contraction mapping and we find the classical Banach theorem see [3].

Definition 3. Let the map f be defined in the metric space (X, d) to the metric space (X, d) . We say that the map f is Cauchy continuous if the range by f of every Cauchy sequence of the space X is a Cauchy sequence in space X .

Lemma 1. [2] Let $(\mathbb{R}, d)$ be the usual real metric space. Suppose that $x_{n \geq 0}$ and $y_{n \geq 0}$ are two Cauchy sequences of the real space $\mathbb{R}$. Then the sequence $z_n = d(x_n, y_n), n \geq 0$ defined in the space $\mathbb{R}$ with the usual distance, is convergent.

Main results and theorems

The proof of our theorems needs the following definitions, proprieties, propositions, and lemmas.

In the sequel, let x_0 be an element of a non-empty complete metric space X . For notations purposes we define $g^0(x_0) = x_0, f^0(x_0) = x_0$ and inductively $g^{n+1}(x_0) = g(g^n(x_0)), f^{n+1}(x_0) = f(f^n(x_0))$, for $n \in \{1, 2, \dots\}$.

The results in this subsection are known as a similar Banach's contraction principle. In this section, we obtained a fixed point theorem for mappings without appealing to commutativity conditions. It follows an result generalises Theorem 3.8 in [2] with a new form of contraction mapping and bounded sequences.

Lemma 2. Let $(\mathbb{R}, d)$ be the usual real metric space. Let $(z_n)_{n \geq 0}$ and $(t_n)_{n \geq 0}$ two sequences. Assume that $0 \leq t_n \leq z_n$ for all n and $(z_n)_{n \geq 0}$ is Cauchy one. Then $(t_n)_{n \geq 0}$ is Cauchy one.

$$d(u_n(x), u_n(y)) \leq k^n d(v_n(x), w_n(y)), \forall x, y \in X, \forall n \in \mathbb{N}^*, \text{ holds.} \quad (14)$$

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On superposition operator on Besov-type spaces

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Abstract: This paper is devoted to the study of composition operator $T_f(g) := f \circ g$ on Besov-type spaces $B_{p,q}^{s,\tau}(\mathbb{R})$ with $s = \frac{n}{p} - n\tau$, $\tau \geq 0$, $1 < p < \infty$ and $1 \leq q \leq \infty$.

We will prove that the operator T_f takes $B_{p,q}^{s,\tau}(\mathbb{R})$ to it self if and only if $f(0) = 0$ and f is globally Lipschitz and belongs to $B_{p,q}^{s,\tau}(\mathbb{R})$.

Keywords: Littlewood-paley, tempered distributions, Besov spaces, Superposition operator.

2020 Mathematics Subject Classification: Primary 46E35; Secondary 42B35.

Introduction

The Composition operators $T_f : g \mapsto f \circ g$ for a Borel measurable function f , where $g \in E$ and E denote a normed space of functions, are straightforward examples of non-linear mappings. It is somewhat surprising that our understanding of these operators is quite limited. One reason for this limitation is that the properties of T_f heavily rely on E . In this paper, we focus on the case where E is the inhomogeneous Besov-type space $B_{p,q}^{s,\tau}$ defined on $\mathbb{R}^n$.

Definition 4. Let $s, \tau \in \mathbb{R}$ and $p, q \in]0, +\infty]$. The inhomogeneous Besov-type space $B_{p,q}^{s,\tau}(\mathbb{R}^n)$ is the set of $f \in \mathcal{S}'(\mathbb{R}^n)$ such that

$$\|f\|_{B_{p,q}^{s,\tau}} := \sup_{k \in \mathbb{Z}} \sup_{v \in \mathbb{Z}^n} 2^{kn\tau} \left(\sum_{j \geq k_+} \left(2^{sj} \|Q_j f\|_{L_p(P_{k,v})} \right)^q \right)^{1/q} < \infty. \quad (15)$$

Where $P_{k,v}$ is the dyadic cubes given by: For $k \in \mathbb{Z}$ and $v = (v_1, \dots, v_n) \in \mathbb{Z}^n$ we set

$$P_{k,v} := \{x = (x_1, \dots, x_n) \in \mathbb{R}^n : v_j \leq 2^k x_j < v_j + 1, j = 1, \dots, n\}.$$

Recall that for $\tau = 0$ the inhomogeneous Besov-type space (see for more details [5]) coincide with the classical inhomogeneous Besov space $B_{p,q}^s(\mathbb{R})$ (see for more details [4]).

Main results

Theorem 7. Let $n < p/(1 - \tau p) < +\infty$ and $1 < q \leq +\infty$. For a function f from $\mathbb{C}$ to $\mathbb{C}$, the following properties are equivalent:

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- (i) f operates, by left composition, on $B_{p,q}^{n/p-n\tau,\tau}(\mathbb{R})$.
- (ii) There exists $1 < v \leq q$ such that for every $g \in B_{p,v}^{n/p-n\tau,\tau}(\mathbb{R})$, $f \circ g \in B_{p,\infty}^{n/p-n\tau,\tau}(\mathbb{R})$.
- (iii) f is globally Lipschitz and $f(0) = 0$.

Theorem 8. Let $1 \leq p < +\infty$, $1 \leq q \leq +\infty$, and $0 < s < \min(1, n/p - n\tau)$ with $0 \leq \tau \leq 1/p$. For any function f from $\mathbb{C}$ to $\mathbb{C}$, the following equivalences hold:

- (i) T_f maps on $A_{p,q}^{s,\tau}(\mathbb{R})$.
- (ii) For every $g \in A_{p,1}^{s,\tau}(\mathbb{R})$, $f \circ g \in A_{p,\infty}^{s,\tau}(\mathbb{R})$.
- (iii) f is globally Lipschitz and $f(0) = 0$.

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A note on a class of linear operators on Hilbert spaces

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Abstract: Our aim is to present the properties of the class of k -quasi- M -hyponormal operators defined on a separable complex Hilbert space by $T^{*k}(M(T-\lambda)^*(T-\lambda))T^k \geq T^{*k}(T-\lambda)(T-\lambda)^*T^k$ for all $\lambda \in \mathbb{C}$. The finite ascent, the Single Valued Extension Property, Bishop's property and other spectral properties will be established. We show that

1. $\ker(T^k) = \ker(T^{k+1})$.
2. $\ker((T-\lambda)^2) = \ker(T-\lambda)$, for all $\lambda \in \mathbb{C}, \lambda \neq 0$.

Also, we prove that the pair (T, S) satisfies the Fuglede-Putnam's Theorem whenever $T, S \in B(H)$ are elements of the considered class of operators.

Keywords: k -quasi- M -hyponormal operators; Single Valued Extension Property; Finite ascent of operators

2020 Mathematics Subject Classification: Primary 47A30; Secondary 47B20.

Introduction

Let H be a separable complex Hilbert space, and let $B(H)$ be the algebra of all bounded linear operators on H . We denote by $N(T)$ and $R(T)$ respectively, for the null space and the range of an operator $T \in B(H)$. An operator T is said to be normal if T commutes with its adjoint T^* , and hyponormal if $T^*T - TT^* \geq 0$. An operator T is said to be M -hyponormal if there is a positive real number $M \geq 1$, such that $MT^*T \geq TT^*$. A closed subspace N of H is T -invariant (or N is invariant under T) if $TN \subset N$.

Main results

Definition 5. An operator $T \in B(H)$ is said to be k -quasi- M -hyponormal for a positive integer k , if there exists $M > 0$ satisfying

$$T^{*k}(M(T-\lambda)^*(T-\lambda))T^k \geq T^{*k}(T-\lambda)(T-\lambda)^*T^k$$

for all $\lambda \in \mathbb{C}$,

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This definition is equivalent to

$$\left\| \sqrt{M}(T - \lambda)T^k x \right\| \geq \left\| (T - \lambda)^* T^k x \right\|$$

for all $x \in \mathcal{H}$. If $k = 1$, T is said to be quasi- M -hyponormal. Clearly,

$$M\text{-hyponormal} \subset \text{quasi-}M\text{-hyponormal} \subset k\text{-quasi-}M\text{-hyponormal}$$

Theorem 9. *Let $T \in B(\mathcal{H})$ be a k -quasi- M -hyponormal operator. If T has dense range, then T is M -hyponormal.*

Lemma 3. *Let T be a nonzero k -quasi- M -hyponormal operator, but not an M -hyponormal. Then T admits at least, a non trivial closed invariant subspace.*

Definition 6. *The smallest integer k satisfying $N(T^k) = N(T)$ is said to be the ascent of T and is denoted by $\alpha(T)$.*

If such integer does not exist, we shall write $\alpha(T) = \infty$

Theorem 10. *Let T be a k -quasi- M -hyponormal operator. Then :*

- (1) $N(T^k) = N(T^{k+1})$
- (2) $N((T - \lambda)^2) = N(T - \lambda)$, for all $\lambda \in \mathbb{C}, \lambda \neq 0$.

Or equivalent, $\alpha(T) = k$ and $\alpha(T - \lambda) = 1, \lambda \in \mathbb{C}, \lambda \neq 0$.

Definition 7. *$T \in B(\mathcal{H})$ is said to have Bishop's property (β) if for each open subset $U \subset \mathbb{C}$ and every sequence of analytic functions $f_n: U \rightarrow \mathcal{H}$ for which $(T - \lambda)f_n(\lambda) \rightarrow 0$ as $n \rightarrow \infty$ locally uniformly on each compact subset of U , $f_n(\lambda) \rightarrow 0$ as $n \rightarrow \infty$ again locally uniformly on U .*

Definition 8. *An operator T in $B(\mathcal{H})$ is said to have the single valued extension property, briefly SVEP at a complex number α , if for each open neighborhood V of α , the operator $(T - \lambda)$ is one-to-one for all $\lambda \in V$.*

If furthermore, T has SVEP at every $\alpha \in \mathbb{C}$, then T is said to have SVEP.

Definition 9. *An operator $T \in B(\mathcal{H})$ is said to be bounded below if there exists $c > 0$ such that $\|x\| \leq c\|Tx\|$ for all $x \in \mathcal{H}$.*

Proposition 1. *Let $T \in B(\mathcal{H})$ be a k -quasi- M -hyponormal operator, and let $S \in B(\mathcal{H})$ be such that the pair (T, S) satisfies the Fuglede-Putnam's Theorem. If $X \in B(\mathcal{H})$ intertwines T and S , then :*

- (i) *If X is one-to-one, then S has SVEP.*
- (ii) *If X is bounded below, then S has Bishop's property (β).*

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PD ^{α} I type Iterative Learning Control Law for a Nonlinear FDE

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Abstract: In this work, we discuss learning control law for nonlinear differential equations of fractional order. We design proportional fractional order derivative (PD ^{α} I-type) iterative learning control laws with initial state learning. In particular, the PD ^{α} I- type law is more flexible due to the impact of a certain fractional order factor. Thereafter, we provide the corresponding convergence characteristics by defining adequate conditions for open-loop and closed-loop iterative learning schemes in terms of the λ -norm.

Keywords: PD ^{α} I Iterative Learning Control, Fractional Differential Equation, Caputo derivative, Open-loop, Closed-loop.

2020 Mathematics Subject Classification: 26A33, 34A08, 39A12, 39A13, 39A21.

Introduction

It is well known that fractional differential equations are gaining more and more attention in different research areas, such as physics, engineering and control. In recent years, there are many important quality analysis and control results for various fractional differential equations.

Investigating the pursuit of target trajectories, a compelling aspect within control theory, remains pivotal in the domain of multi-operation systems. This motivation researchers to study iterative learning control (ILC) problems, initially introduced by Uchiyama (1978), particularly addressing control challenges within defined finite intervals.

Subsequently, Arimoto and Kawamura (1984), along with Arimoto (1985), expanded upon Uchiyama's concepts to develop the ILC updating law. Recently, there are many contribution on P-type, D-type, PD-type and PDI-type iterative learning control for integer order ordinary differential equations and few work on iterative learning control of fractional order differential equations. In this work, we discuss PD ^{α} I type learning control of a differential equations of fractional order.

Main results

The paper is organized as follows. In Section 2, we present fractional integral and derivative of Caputo and the properties of Mittag-Leffler functions $\mathbb{E}_\alpha(z)$ and $\mathbb{E}_{\alpha,\beta}(z)$. In Sections 3 and 4, we give open-loop and closed-loop iterative learning schemes with zero initial error and random but bounded initial error

$$u_{k+1}(t) = f(u_k(t), e_k(t), D^\alpha e_k(t), I e_k(t)),$$

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and

$$u_{k+1}(t) = g(u_k(t), e_{k+1}(t), D^\alpha e_{k+1}(t), Ie_k(t)),$$

where $e_k(t) := y_d(t) - y_k(t)$ is errors of output for the k^{th} iteration.

Example is presented in Section 5 to demonstrate the validity of our results.

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Etude de l'existence globale d'un système de réaction diffusion à matrice pleine avec une propriété de contrôle de masse

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Abstract: Considérons le système de réaction diffusion à matrice pleine suivant:

$$\left\{ \begin{array}{l} \partial_t u - a_{11} \Delta u - a_{12} \Delta v = f(u, v) \text{ dans } \Omega \times]0, T[, \\ \partial_t v - a_{21} \Delta u - a_{22} \Delta v = g(u, v) \text{ dans } \Omega \times]0, T[, \\ u(0, \cdot) = u_0(\cdot) \geq 0, v(0, \cdot) = v_0(\cdot) \geq 0, \\ \frac{\partial u}{\partial n} = \beta_1, \frac{\partial v}{\partial n} = \beta_2 \text{ sur } (0, +\infty) \times \partial\Omega, \\ \text{ou, } u = \beta_1, v = \beta_2 \text{ sur } (0, +\infty) \times \partial\Omega, \end{array} \right.$$

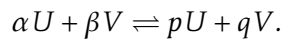
où $a_{ij} \in (0, +\infty)$, $i, j = 1, 2$, $\beta_1, \beta_2 \in [0, +\infty)$ et $f, g : [0, +\infty)^2 \rightarrow \mathbb{R}$ sont de classe C^1 .

Pour $u_0, v_0 \in L^\infty(\Omega)$ avec $u_0, v_0 \geq 0$, on cherche l'existence des solutions bornées non négatives classiques locales sur un intervalle $]0, T^*[$. Puis on s'intéresse à l'étude de l'existence globale en temps en utilisant la méthode des régions invariantes utilisées par S Kouachi [15] en choisissant :

$$f(u, v) = k_1 u^p v^q - k_2 u^\alpha v^\beta, g(u, v) = k_3 u^\alpha v^\beta - k_4 u^p v^q,$$

avec $k_1 > 0, p, q, \alpha, \beta \geq 1$.

Un tel problème peut être un modèle pour la réaction chimique



où u est v sont les concentrations des produits U est V successivement.

Ce type de réactions a été considéré par Michel Pierre [10] qui a prouvé que le problème avec une matrice diagonale vérifie une propriété dite contrôle de masse pour montrer l'existence globale d'une solution.

Keywords: existence globale- réaction diffusion- fonction de Luaponov- contrôle de masse- regions invariantes.

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Comparison of two limiters for solving the one-dimensional scalar conservation law with the discontinuous Galerkin method

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Abstract: The discontinuous Galerkin (DG) methods is a recent numerical method to solve the PDE. For the hyperbolic conservation law, the DG methods mix the features of the finite element method and the finite volume method. One of the main problems with this method is the presence of spurious oscillations. Many limiters have been developed to solve this issue. In this work, we will try to make a comparison between two popular limiters.

Keywords: Discontinuous Galerkin method, Limiters, conservation law.

2020 Mathematics Subject Classification: Partial differential equations and functional analysis.

Introduction

The discontinuous Galerkin (DG) method is a class of the finite element method, where appear for the first time in 1973 in paper of Reed and Hill in the framework of the neutron equation (stead hyperbolic conservation law). The main development of the DG was carried out by Cockburn and Shu et al in a series of papers. They establish the framework to solve the nonlinear time depends hyperbolic conservation law using the explicit nonlinear high order Runge-Kutta time discretization and the DG discretization in space. This method named RKDG method and are energy stable for more details see the review paper [2]. However, The method suffer from the numerical unwanted oscillation especially in the nonlinear problems. This motivated the researchers to create some limiters to fix this issue. In the last years, several limiters have been emerged for example the first limiter is presented by Cockburn and Shu in 1989 [1] based on the total variation bounded TVB technique and the limiter of Zhong and Shu in 2013 [2] is a simple and based in the WENO thechnique ... etc. In this work, we will present the two limiters and we will try to do some compression between them.

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Mathematical model and notations

We consider the one-dimensional scalar hyperbolic conservation law with the appropriate initial condition and periodic boundary condition as:

$$\begin{aligned} u_t + f(u)_x &= 0, & x_{min} \leq x \leq x_{max}, \quad t > 0 \\ u(x, 0) &= u_0(x), & x_{min} \leq x \leq x_{max}, \\ u(x_{min}, t) &= u(x_{max}, t) & t \geq 0. \end{aligned} \quad (16)$$

Let us define some notations which are used to solve the problem mentioned in (16).

We divide the computational domain $[x_{min}, x_{max}]$ into N cells (elements): $0 = x_{\frac{1}{2}} < x_{\frac{3}{2}} < \dots < x_{N+\frac{1}{2}}$ and denote $I_j = (x_{j-\frac{1}{2}}, x_{j+\frac{1}{2}})$, $x_j = \frac{x_{j-\frac{1}{2}} + x_{j+\frac{1}{2}}}{2}$, $\Delta x_j = x_{j+\frac{1}{2}} - x_{j-\frac{1}{2}}$ for all $j = 1, \dots, N$ as the cells, cell center and the spacial step respectively. We assume the mesh is regular (i.e $\exists c > 0, \Delta x_j > ch$, where $h = \max_j \Delta x_j$).

The discontinuous Galerkin finite element space defined as: $V_h^k = \{p : p|_{I_j} \in P^k(I_j), \quad j = 1, \dots, N\}$

$P^k(I_j)$ is the space of polynomials of degree then less or equal to k define on I_j .

Numerical method

To solve the problem (16), one multiply the PDE of the problem with a test function v , then integrate over a cell I_j , and integrate by parts. We get the following weak formulation:

$$\int_{I_j} u_t v dx - \int_{I_j} f(u) v_x dx + f(u_{j+\frac{1}{2}}) v_{j+\frac{1}{2}} - f(u_{j-\frac{1}{2}}) v_{j-\frac{1}{2}} = 0$$

Now, let us consider both of the approximate solution u and the test function v come from the same finite space V_h^k . Then, we take the monotone numerical flux $\hat{f}_{j+\frac{1}{2}}(\hat{u}_{j+\frac{1}{2}}^-, \hat{u}_{j+\frac{1}{2}}^+)$ to replace $f(u_{j+\frac{1}{2}})$ where $\hat{f}(u, u) = f(u)$ (consistency), $\hat{f}(\uparrow, \downarrow)$ (monotonicity) and $\hat{f}$ is lipschitz continuous for both arguments; and for the test function one should take the values inside the cell $v_{j+\frac{1}{2}}^-, v_{j-\frac{1}{2}}^+$.

Hence, the semi-discrete DG scheme is: find $u \in V_h^k$ such that

$$\int_{I_j} u_t v dx - \int_{I_j} f(u) v_x dx + \hat{f}_{j+\frac{1}{2}} v_{j+\frac{1}{2}}^- - \hat{f}_{j-\frac{1}{2}} v_{j-\frac{1}{2}}^+ = 0$$

for all $v \in V_h^k$

For time discretization, the Total variation diminishing Runge-Kutta (TVD) methods has been used for the semi discrete scheme $u_t = L(u)$, where $L(u)$ is a discretization of the spacial operator, see [2], for example the third order:

$$\begin{aligned} u^1 &= u^n + \Delta t L(u^n) \\ u^2 &= \frac{3}{4} u^n + \frac{1}{4} u^1 + \frac{1}{4} \Delta t L(u^1) \\ u^{n+1} &= \frac{1}{3} u^n + \frac{2}{3} u^2 + \frac{2}{3} \Delta t L(u^2) \end{aligned} \quad (17)$$

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Limiters

The approximate solution with the previous numerical method may generate a spurious numerical oscillation especial in the nonlinear case. Here, we should need to implement a nonlinear limiters to control these oscillations. The limiter is considered as a post-processor for the approximate DG solution where change the approximate polynomial in the cell which is thought to have a possible discontinuity (the so-called a troubled cell) by an other polynomial with the degree and less oscillatory. Hence, we can separate the limiter process into two parts:

- Identifying the troubled cell: According [4], the better troubled cell indicator is the indicator proposed by Fu and Shu in 2017. It is defined as:

We denote the indicator stencil as $S = \{I_0, I_1, I_2\}$, where I_0 is the target cell and I_1, I_2 are the left and the right cells respectively. The approximate polynomials on the three cells $p_l(x)$, $l = 0, 1, 2$ respectively. We consider the quantity defined on the target cell as:

$$q_{I_0} := \frac{\sum_{j=1}^2 |\bar{p}_0 - \bar{p}_j|}{\max_{j \in \{0,1,2\}} |\bar{p}_j|}$$

where $\bar{\star}_j$ is the cell average of the function $\star$ on the j^{th} cell and $\bar{\star}_j$ is the cell average of the function $\star$ on the target cell. The target cell I_0 is marked as a troubled cell if $q_{I_0} > C_k$; where C_k is a constant depend only on the polynomial degree k .

- Reconstruct a new polynomial in the troubled cell: We will highlight two of them. The first is the old one proposed by Cockburn and Shu in 1989 and the second is based to the WENO methodology produces by Zhong and Shu in 2013.
 - Cockburn and Shu 1989: The process is defined as: We rewrite the solution in the limits of the target cell as $u_{I_j}(x_{j-\frac{1}{2}}, t) = \bar{u}_{I_j} + \tilde{u}_{I_j}$ and $u_{I_j}(x_{j+\frac{1}{2}}, t) = \bar{u}_{I_j} - \tilde{u}_{I_j}$, where $\bar{u}_{I_j}$ is the cell average in I_j . Then we change the quantities $\tilde{u}_{I_j}$ and $\tilde{u}_{I_j}$ use the modify minmod function define by Shu in 1987, for more details see [1].
 - Zhong and Shu 2013: This limiter change the approximate polynomial in the troubled cell by the following polynomial:
 $p^{new} = \omega_0 p_j + \omega_1 \tilde{p}_{j-1} + \omega_2 \tilde{p}_{j+1}$, where $\tilde{p}_{j-1} = p_{j-1} - p_{j-1}^{\bar{}} + \bar{p}_j$ and $\tilde{p}_{j+1} = p_{j+1} - p_{j+1}^{\bar{}} + \bar{p}_j$ and $p_{j+1}^{\bar{}}$ define in the part of identifying the troubled cell. The constants ω_j , $j = 0, 1, 2$ are defined in the paper, for more details see [2].

Results

In this section, we use tow different numerical test cases (smooth and non smooth) for Burger's equation $f(u) = \frac{u^2}{2}$ to compare between the limiters are presented in the precious section. We take $[0, 1]$ as a computational domain and the following initial conditions:

$$u(x, 0) = \frac{1}{2} + \sin(2\pi x), \quad u(x, 0) = \begin{cases} 0, & \text{if } x < 0, 4 \\ 1, & \text{if } 0, 4 \leq x \leq 0, 6 \\ 0, & \text{other} \end{cases}$$

We notice that all the limiters have been gotten a good result as showing in the figure. However, the old limiter involve to specify a parameter of the modify minmod function for each problem, which has a relationship with the second derivative of the exact solution. The second one is very easier to the old one in the implementation.

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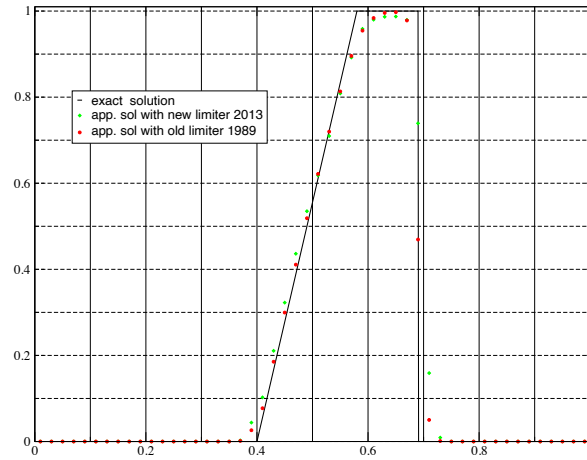


Figure 1: The exact and the approximate solution using DG method for Burger's equation with discontinuity in the initial condition

Conclusion

In this work, we present the Runge-Kutta discontinuous Galerkin method with two separate limiters to control the oscillations. As we notice that all the limiters are good with some preference to the Limiter presented by Zhong and Shu the implementation.

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Global existence of the solution for a viscoelastic wave equation

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Abstract: We consider a coupled system of two viscoelastic wave equations, namely, we investigate the following problem

$$\begin{cases} u_{tt}(x, t) - \Delta u(x, t) + \int_0^t g(t-s) \Delta u(x, s) ds = 0 \\ v_{tt}(x, t) - \Delta v(x, t) = 0. \end{cases}, \quad (18)$$

together with initial conditions and boundary conditions of Dirichlet type. Here $(x, t) \in \Omega \times (0, \infty)$, under suitable assumptions on the kernel g . Our goal is to prove the global existence and the uniqueness of the solution of problem (18) using the Faedo–Galerkin approximations together with some energy estimates.

Keywords: Viscoelasticity, Global existence, Relaxation function, Exponential stability, Lyapunov functionals.

2020 Mathematics Subject Classification: Primary 35Exx, 35R30, 35L45, 35R09, 65J20.

Introduction

It is well-known that the free vibration of membrane can be described by wave equation of the form

$$u_{tt}(x, t) - \Delta u(x, t) = 0,$$

in $\Omega \times (0, \infty)$ subjected to initial conditions and some boundary conditions. Here, $u(x, t)$ represents the displacement of the membrane from its equilibrium position at point x and time t . The Laplacian operator Δ represents the spatial second derivative with respect to x , $\Omega \in \mathbb{R}^2$ is a bounded domain. For the case of high dimension, there are also many discussions in mathematical physics. Studying the existence of solutions to the viscoelastic wave equation involves various mathematical techniques and methods. Some of the common methods are : Semi-Group Theory, Fixed Point Method, Spectral Methods, Faedo-Galerkin Method.

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The aim of the present work is to prove an global existence result of the following problem

$$\begin{cases} u_{tt}(x, y) - \Delta u(x, y) + \int_0^t g(t-s) \Delta u(x, y) ds = 0 \\ v_{tt}(x, y) - \Delta v(x, y) = 0 \\ u(x, t) = v(x, t) = 0 \quad \text{on } \partial\Omega \times (0, T) \\ u(x, 0) = u_0, \quad u_t(x, 0) = u_1 \\ v(x, 0) = v_0, \quad v_t(x, 0) = v_1. \end{cases}$$

where $\begin{cases} u = u(x, t) \\ v = v(x, t) \end{cases}$, $t \geq 0, x \in \Omega$, Δ denotes the Laplacian operator with respect to the x variable,

Ω is a regular and bounded domain of $\mathbb{R}^N$ ($N \geq 1$), (u_0, v_0) and (u_1, v_1) are given functions belonging to suitable spaces, using the Faedo–Galerkin approximations together with some energy estimates.

We set

$$\begin{aligned} (h \star \varphi)(t) &= \int_0^t h(t-s) \varphi(s) ds. \\ (h \diamond \varphi)(t) &= \int_0^t h(t-s) |\varphi(t) - \varphi(s)| ds. \\ (h \circ \varphi)(t) &= \int_0^t h(t-s) \int_{\Omega} |\varphi(t) - \varphi(s)|^2 dx ds. \end{aligned}$$

For the relaxation function g , we assume

B1 $g : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is a C^1 function satisfying

$$g(0) > 0, \quad 1 - \int_0^\infty g(s) ds = l > 0.$$

B2 There exists a positive constant ξ such that $g'(t) \leq -\xi g(t)$, $\forall t \geq 0$.

Main results

The prove of the global existence result consists of carrying out the following three steps:

- Search for approximate solutions.
- Establish a priori estimates based on these approximate solutions.
- The final step involves passing to the limit.

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Existence of solutions for a nonlinear elliptic equation on a bounded domain of $\mathbb{R}^3$

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Abstract: This work is concerned with the following equation:

$$\begin{cases} -\Delta u + q(x)u = f(u) & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded domain of $\mathbb{R}^3$ with smooth boundary, $q \in L^\infty(\Omega)$ and $f \in C(\mathbb{R})$. First of all, we establish the functional energy associated to the above problem and we prove that it is of class C^1 on $H_0^1(\Omega)$. Next, we prove some technical lemmas to prove the main result of this work. Finally, by using the Mountain Pass theorem, we show the existence of solutions to the above problem.

Keywords: Nonlinear elliptic equation, variational methods, critical points theory, Mountain Pass theorem.

Introduction

The nonlinear Schrödinger equation

$$-ih \frac{\partial \psi(x, t)}{\partial t} = -\frac{h^2}{2m} \Delta_x \psi(x, t) + \phi(x) \psi(x, t) - g(x, \psi(x, t)), \quad (x, t) \in \Omega \times \mathbb{R}, \quad (19)$$

is one of the most important equations in physics, which describes quantum particles interacting with the electromagnetic field generated by the motion.

Here, $\Omega \subset \mathbb{R}^3$ is a bounded domain with smooth boundary, h is the Planck constant, m is the particle mass, $\psi : \mathbb{R}^3 \times \mathbb{R}^+ \rightarrow \mathbb{C}$ is the wave function, $\phi : \mathbb{R}^3 \rightarrow \mathbb{R}$ is the electrostatic potential and $g : \mathbb{R}^3 \times \mathbb{R}^+ \rightarrow \mathbb{R}$ is a nonlinear term describing external perturbations.

Equations like (19) arise in many mathematical physics contexts such as

- Quantum mechanics models;
- Semiconductor theory;
- Nonlinear optics and plasma theory.

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We assume that $f(x, e^{i\theta}u) = e^{i\theta}f(x, u)$ for $u \in \mathbb{R}$. Then, the study of standing waves solutions for (19), i.e., solutions of the form

$$\psi(x, t) = e^{-i\omega t}u(x), \quad \omega \in \mathbb{R}$$

leads to following stationary equation

$$-\Delta u + q(x)u + \phi u = f(x, u), \quad x \in \Omega.$$

Moreover, if we assume that the electrostatic potential ϕ is zero in Ω and the particle is constraint to live in Ω , then we get the following problem

$$\begin{cases} -\Delta u + q(x)u = f(x, u) & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega. \end{cases}$$

In what follows, we are interested with the existence of solutions for the following problem

$$\begin{cases} -\Delta u + q(x)u = f(u) & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega, \end{cases} \quad (\mathcal{P})$$

where Ω is a bounded open set of $\mathbb{R}^3$ and we make the following assumptions:

$$(Q) \quad q \in L^\infty(\Omega).$$

$$(F_1) \quad f \in C(\mathbb{R}) \text{ and there exist } a, b > 0 \text{ such that } |f(s)| \leq a + b|s|^5 \text{ for all } s \in \mathbb{R}.$$

$$(F_2) \quad \lim_{|s| \rightarrow \infty} \frac{f(s)}{s} = 0.$$

$$(F_3) \quad \text{There exists } \mu > 2 \text{ such that } f(s)s \geq \mu F(s), \text{ for all } s \in \mathbb{R}, \text{ where } F(u) = \int_0^u f(s) ds.$$

$$(F_4) \quad \lim_{s \rightarrow +\infty} \frac{F(s)}{s^\mu} = \infty.$$

Main results

The main result of this work is the following.

Theorem 11. *Assume that (Q) and $(F_1) - (F_4)$ hold. Then, problem $(\mathcal{P})$ has at least one non trivial solution.*

In order to prove Theorem 11 we will need for the following lemmas.

Lemma 4. *The functional I is of class C^1 on H , and*

$$\langle I'(u), v \rangle = \int_{\Omega} (\nabla u \cdot \nabla v + q(x)uv) dx - \int_{\Omega} f(u)v dx, \quad (20)$$

for all $u, v \in H$.

Lemma 5. *Assume that (F_1) and (F_2) hold. Then, there exist $\rho, \alpha > 0$ such that $I(u) \geq \alpha$ whenever $\|u\| = \rho$.*

Lemma 6 (2). *Assume that (F_1) , (F_3) and (F_4) hold. Then, there exist a function $e \in H$ with $\|e\| \geq \rho$ such that $I(e) < 0$.*

Lemma 7. *Assume that $(F_1) - (F_4)$ hold. Then, the functional I satisfies the (PS) condition.*

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Proof of the main result

We have $I \in C^1(H, \mathbb{R})$ and $I(0) = 0$. By Lemma 2 and 3, the functional I satisfies the geometric property of the Mountain Pass theorem. Lemma 4 implies that the functional I satisfies the $(PS)_c$ condition. Therefore, applying the Mountain Pass Theorem, we deduce that there exists $u_0 \in H$ such that

$$I(u_0) = c_0 \geq \alpha > 0 \text{ and } I'(u_0) = 0.$$

Thus, problem $(\mathcal{P})$ has at least one nontrivial solution.

Example

We consider the following functions

$$f(t) = 4t^4 \arctan(t) + \frac{t^5}{1+t^2} \quad \text{and} \quad q(x) = e^{-|x|^2}$$

Then, it is easy to check that (Q) and $(F_1) - (F_4)$ are satisfied.

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A free boundary problem with fractional Laplacian operator

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Abstract: In this work, we study a class of the free boundary problems with fractional Laplacian operator, with mixed boundary conditions in a bounded domain of $\mathbb{R}^n$ dimensional free boundary problems, with Neuman boundary condition. The free boundary is the interface between two sets $\{u = 0\}$ and $\{u > 0\}$. We show the existence of a solution using mixed of fixed point theorem, monotone operators and other methods.

Keywords: free boundary; boundary condition, fractional Laplacian operator.

2020 Mathematics Subject Classification: Primary 35R35; Secondary 35J15, 35R11.

Introduction

Let Ω be a C^1 bounded domain of $\mathbb{R}^n$, Γ_2, Γ_3 are relatively open connected subsets of $\partial\Omega$, $\Gamma_1 = \partial\Omega \setminus (\Gamma_2 \cup \Gamma_3)$, and $x = (x', x_n)$. For $A \subset \Omega$ we set $\partial A = S_1 \cup S_2 \cup S_3 \cup S_4$, where $S_1 = \Gamma_1, (S_2 \setminus S_4) \subset \Omega, S_3 = \Gamma_3$ and $S_4 \subset (S_2 \cap \Gamma_2)$. $C_{\overline{\Omega}} \mathbb{R}^n \setminus \overline{\Omega}$ is divided into three disjointed parts D_1, D_2 and D_3 such that: $\partial D_1 \cap \partial\Omega = \Gamma_1, \partial D_2 \cap \partial\Omega = \Gamma_2$ and $\partial D_3 \cap \partial\Omega = \Gamma_3$. $\mathbb{R}^n \setminus \overline{A}$ is divided into four disjointed parts E_1, E_2, E_3 and E_4 such that: $E_1 = D_1, \partial E_2 \cap \partial A = S_2, E_3 = D_3$ and $\partial E_4 \cap \partial A = \Gamma_4$. Here we are interested in a problem similar to the one raised in [2] By replacing $-\Delta$ with the operator $(-\Delta)^s$ ($0 < s < 1$), we arrive at the following problem:

$$(P) \begin{cases} u \geq 0, (-\Delta)^s(u + x_n) = 0 & \text{in } A, \\ u = 0 & \text{on } E_2, \\ \mathcal{N}_s u = 0 & \text{on } E_1 \cup E_3, \\ \mathcal{N}_s u \leq 0 & \text{on } E_4, \end{cases}$$

where the operators $(-\Delta)^s$ and $\mathcal{N}_s$ are given by

$$(-\Delta)^s u = c_{n,s} \lim_{\varepsilon \rightarrow 0} \int_{\{|x-y| > \varepsilon\}} \frac{u(x) - u(y)}{|x-y|^{n+2s}} dx dy.$$

$$\mathcal{N}_s u(x) = c_{n,s} \int_A \frac{u(x) - u(y)}{|x-y|^{n+2s}} dy.$$

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Statement of the problem

Assume that A is smooth enough, then for all v smooth enough we get

$$\begin{aligned} \int_A (-\Delta)^s u \cdot v &= \frac{c_{n,s}}{2} \int_{\mathbb{R}^{2n} \setminus (C_A)^2} \frac{(u(x) - u(y) + x_n - y_n)(v(x) - v(y))}{|x - y|^{n+2s}} dx dy \\ &\quad - \int_{\mathbb{R}^n \setminus A} (\mathcal{N}_s u) \cdot v \\ &= \frac{c_{n,s}}{2} \int_{\mathbb{R}^{2n} \setminus (C_A)^2} \frac{(u(x) - u(y) + x_n - y_n)(v(x) - v(y))}{|x - y|^{n+2s}} dx dy \\ &\quad - \int_{E_4} (\mathcal{N}_s u) \cdot v. \end{aligned}$$

Hence, for $v \in H^s(\Omega)$ such that $v \geq 0$ on $E_2 \supset E_4$, we arrive to

$$\frac{c_{n,s}}{2} \int_{\mathbb{R}^{2n} \setminus (C_A)^2} \frac{(u(x) - u(y) + x_n - y_n)(v(x) - v(y))}{|x - y|^{n+2s}} dx dy = \int_{E_4} (\mathcal{N}_s u) \cdot v.$$

Extending u by 0 outside A and taking into account that $\mathcal{N}_s u \leq 0$ on $E_4 \subset E_2$ and putting $\chi = \chi_{\mathbb{R}^{2n} \setminus (C_A)^2}$ we get the following weak formulation of the problem (P)

$$(Q) \left\{ \begin{array}{l} \text{Find } (u, \chi) \in H^s(\Omega) \times L^\infty(\Omega) \text{ such that :} \\ (i) \quad u \geq 0, \quad 0 \leq \chi \leq 1, \quad u(1 - \chi) = 0 \text{ a.e. in } \Omega \\ (ii) \quad u = 0 \quad \text{a.e. on } E_2 \\ (iii) \quad \int_{\mathbb{R}^{2n} \setminus (C_\Omega)^2} \frac{(u(x) - u(y))(v(x) - v(y)) + \chi \cdot (x_n - y_n)(v(x) - v(y))}{|x - y|^{n+2s}} dx dy \leq 0, \\ \quad \forall v \in H^s(\Omega), \quad v \geq 0 \text{ on } E_2. \end{array} \right.$$

Main results

For $\varepsilon > 0$ we consider the following problem:

$$(Q_\varepsilon) \left\{ \begin{array}{l} \text{Find } u_\varepsilon \in H^s(\Omega) \text{ such that :} \\ (i) \quad u = 0 \quad \text{a.e. on } E_2 \\ (ii) \quad \int_{\mathbb{R}^{2n} \setminus (C_\Omega)^2} \frac{(u(x) - u(y))(v(x) - v(y)) + H_\varepsilon(u_\varepsilon(x), u_\varepsilon(y))(x_n - y_n)(v(x) - v(y))}{|x - y|^{n+2s}} dx dy = 0, \\ \quad \forall v \in H^s(\Omega), \quad v = 0 \text{ on } E_2, \end{array} \right.$$

where

$$H_\varepsilon(x, y) = \begin{cases} 1 & : (x \geq \varepsilon \wedge y \geq \varepsilon) \vee (x \geq \varepsilon \wedge y \leq 0) \vee (x \leq 0 \wedge y \geq \varepsilon) \\ \frac{x}{\varepsilon} & : 0 \leq x \leq \varepsilon \wedge y \geq \varepsilon \\ \frac{y}{\varepsilon} & : x \geq \varepsilon \wedge 0 \leq y \leq \varepsilon \\ 0 & : x \geq \varepsilon \wedge y \leq \varepsilon \end{cases}$$

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The following theorem is proving for using the monotone operator theorem and Shoulder fixed point theorem

Theorem 12. For any $\varepsilon > 0$; there exist a solution $u_\varepsilon > 0$ of (Q_ε) , and we have:

$$\int_{\mathbb{R}^{2n} \setminus (C_\Omega)^2} \frac{(u(x) - u(y))(v(x) - v(y)) + H_\varepsilon(u_\varepsilon)(x_n - y_n)(v(x) - v(y))}{|x - y|^{n+2s}} dx dy \leq 0, \forall v \in H^s(\Omega), v \geq 0 \text{ on } E_2.$$

So, we have the following theorem, which ensure the existence of a solution of the problem (Q).

Theorem 13. The sequence (u_ε) tends to u , a solution of problem (Q).

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General Stability Result for a Viscoelastic Plate Equation with a Logarithmic Nonlinearity Source

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Abstract: In a bounded domain of $\mathbb{R}^2$, we consider the fourth-order viscoelastic plate problem involving a logarithmic nonlinear source $h(u) = |u|^{p-2} u \ln |u|$. In addition, it contains a memory boundary condition and a relaxation function g satisfies $g'(t) \leq -\xi(t)H(g(t))$, where H is an increasing and convex function near the origin and ξ is a nonincreasing function. In this work, we establish the decay solution energy in a general case, by relying on two basic methods: the energy and the Lyapunov function. We also used some properties of convex functions. These new results are a generalization of many works in the literature.

Keywords: Viscoelasticity, Logarithmic nonlinearity, Relaxation function, Convex function.

2020 Mathematics Subject Classification: 93D20, 35L55, 74D10, 93D15.

Introduction

It is well known that viscoelastic materials have memory effects, which is due to the mechanical response influenced by the history of the materials themselves. As these materials have a wide application in the natural sciences, their dynamics are interesting and of great importance. From the mathematical point of view, their memory effects are modeled by integro-differential equations. Hence, questions related to the behavior of the solutions for the PDE system have attracted considerable attention in recent years. In the present work, we are concerned with

$$\begin{cases} u_{tt} + \Delta^2 u - \int_0^t g(t-s) \Delta^2 u(s) ds = |u|^{p-2} u \ln |u|, & \text{in } \Omega \times (0, \infty), \\ u = 0, & \text{on } \partial\Omega \times (0, \infty), \\ \mathcal{B}(u) - \mathcal{B}\left(\int_0^t g(t-s) u(s) ds\right) = 0, & \text{on } \partial\Omega \times (0, \infty), \\ u(0) = u_0, \quad u_t(0) = u_1, & x \in \Omega, \end{cases} \quad (21)$$

where Ω is a bounded domain of $\mathbb{R}^2$ with a smooth boundary $\partial\Omega$. Here $n = (\tau_1, \tau_2)$ is the unit outward normal to $\partial\Omega$, $\tau = (-\tau_2, \tau_1)$ is the unit tangent positively oriented on $\partial\Omega$, the integral term in $(21)_1$ is the memory responsible for the viscoelastic damping where g is a positive function called the relaxation function, $p > 2$. The differential operator $\mathcal{B}$ is denoted as follows:

$$\mathcal{B}u = \Delta u + (1 - \nu)Bu,$$

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where

$$Bu = 2\tau_1\tau_2 \frac{\partial^2 u}{\partial x_1 \partial x_2} - \tau_1^2 \frac{\partial^2 u}{\partial x_1^2} - \tau_2^2 \frac{\partial^2 u}{\partial x_2^2}.$$

and $\nu \in (0, \frac{1}{2})$ represents the Poisson coefficient. This system describes the transversal displacement $u = u(x, t)$ of a thin vibrating plate subjected to internal viscoelastic damping.

Our aim in this work is to obtain a general relation between the decay rate of the energy and that of the relaxation function g without imposing restrictive assumptions on the behavior of g at infinity.

Main results

In order to state and prove our main result we first introduce the following:

(H1) The kernel $g \in C^1(\mathbb{R}_+)$ satisfying:

$$1 - \int_0^{+\infty} g(s)ds = l > 0, \text{ and } g(0) > 0.$$

(H2) The function $H \in C^1(\mathbb{R}_+)$, is strictly increasing and strictly convex or linear in $C^2((0, r]; \mathbb{R}_+)$, ($r < 1$) such that

$$H^{(i)}(0) = 0, \quad \forall i \in \{0, 1\} \text{ and } g'(t) \leq -\xi(t)H(g(t)),$$

where the function $g(t)$ is positive and nonincreasing.

We introduce the following functionals

$$\begin{aligned} J(t) &= \frac{1}{2} \left(1 - \int_0^t g(s)ds \right) a(u, u) - \frac{1}{p} \int_{\Omega} |u|^p \ln |u| dx \\ E(t) &= J(t) + \frac{1}{2} \|u_t\|_{L^2(\Omega)}^2 + \frac{1}{2} (g \circ \partial^2 u)(t) + \frac{1}{p^2} \|u\|_{L^p(\Omega)}^p \end{aligned}$$

where

$$(g \circ \partial^2 u)(t) := \int_0^t g(t-s) a(u(t) - u(s), u(t) - u(s)) ds,$$

and

$$a(u, v) = \int_{\Omega} \Delta u \Delta v dx - (1 - \nu) \int_{\Omega} \left\{ \frac{\partial^2 u}{\partial x_1^2} \frac{\partial^2 v}{\partial x_1^2} - 2 \frac{\partial^2 u}{\partial x_1 \partial x_2} \frac{\partial^2 v}{\partial x_1 \partial x_2} + \frac{\partial^2 u}{\partial x_2^2} \frac{\partial^2 v}{\partial x_2^2} \right\} dx.$$

is a bilinear form on $W = \{u \in H^2(\Omega) : u = 0 \text{ on } \partial\Omega\}$.

Theorem 14. Suppose that (H1) and (H2) hold, $u_0 \in H^4(\Omega) \cap W$ and $u_1 \in W$.

There exist four constants c_0, c_1, c_2 and $c_3 > 0$, in order that for all $t \geq t_1$,

If H is linear,

$$E(t) \leq c_1 e^{-c_0 \int_{t_1}^t g(s)ds},$$

If H is nonlinear,

$$E(t) \leq c_4 H_1^{-1} \left(c_3 \int_{t_1}^t g(s)ds \right),$$

provided that H_1 is strictly decreasing in $(0, r]$.

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The weakly coupled system of Cauchy problem for semilinear models with strong damping terms

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Abstract: We study the global (in time) existence of small data Sobolev solutions to the weakly coupled system of Cauchy problem for semilinear models with visco-elastic damping and with a power nonlinearity type, namely,

$$\begin{cases} u_{tt} - \Delta u + u_t - \Delta u_t = |v|^p & \text{for } (t, x) \in (0, \infty) \times \mathbb{R}^n, \\ u(0, x) = u_0(x), \quad u_t(0, x) = u_1(x) & \text{for } x \in \mathbb{R}^n. \\ v_{tt} - \Delta v + v_t - \Delta v_t = |u|^q & \text{for } (t, x) \in (0, \infty) \times \mathbb{R}^n, \\ v(0, x) = v_0(x), \quad v_t(0, x) = v_1(x) & \text{for } x \in \mathbb{R}^n. \end{cases} \quad (22)$$

where $p, q > 1$, and u_0, u_1, v_0, v_1 are given data. We are interested in connections between regularity assumptions for the data and the admissible range of exponents p, q in the power non-linearity.

Keywords: global in time existence; small data solutions; visco-elastic damping; wave equation; frictional damping; power non-linearity; higher regularity of data; fractional chain rule.

Introduction

In the first, we are interested to study the Cauchy problem

$$\begin{cases} u_{tt} - \Delta u + u_t - \Delta u_t = |u|^p & \text{for } (t, x) \in (0, \infty) \times \mathbb{R}^n, \\ u(0, x) = u_0(x), \quad u_t(0, x) = u_1(x) & \text{for } x \in \mathbb{R}^n, \end{cases} \quad (23)$$

Under certain assumptions for the data and the dimension n . Our main goals are to study the influence of regularity parameters $s \in \mathbb{R}^+$ and additional regularity parameter $m \in [1, 2)$ for the data (u_0, u_1) , that is,

$$(u_0, u_1) \in (H^{s_1} \cap L^m) \times (L^2 \cap L^m)$$

whith $s_1 > 0$ on the admissible range of exponents p which allow to prove the global (in time) existence of small data Sobolev solutions or energy solutions with suitable regularity.

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After applying estimates of solutions of linear Cauchy problems

$$\begin{aligned} u_{tt} - \Delta u + u_t - \Delta u_t &= 0 \text{ for } (t, x) \in (0, \infty) \times \mathbb{R}^n, \\ u(0, x) &= u_0(x), \quad u_t(0, x) = u_1(x) \text{ for } x \in \mathbb{R}^n, \end{aligned} \quad (24)$$

to the semilinear Cauchy problem with power nonlinearity $|u|^p$, we find as a result the global (in time) existence of small data solutions to the semilinear Cauchy problem 23 in any space dimension $n \geq 1$. Finally, we derive results about the long time behavior of solutions of 23.

We profit to use the abvious estimate to study the weakly coupled system, we show that there is a loss of decay for v , that is, the decay estimates coincide with those for solutions to the corresponding linear Cauchy problem with vanishing right-hand side.

To prove an existence and uniqueness (globally in time) result for 23, it is necessary to find a family of suitable solution spaces in which an operator N has a unique fixed point, provided that the smallness of the norm of initial data is assumed in a suitable data space.

Main results

Let us state the main results that will be proved in the present paper. First, we are interested in sobolev solutions or energy solutions if $s_1 \geq s_2 \geq 0$.

Theorem 15. *Let $m \in [1, 2)$. with s_1, s_2 be nonegative reals. the data (u_0, u_1) and (v_0, v_1) are asumed to belong $\mathcal{A}_{s_1,0}^m$ and $\mathcal{A}_{s_2,0}^m$ respectievly, then, there exists $\epsilon > 0$ such that, if $\|(u_0, u_1)\|_{\mathcal{A}_{s_1,0}^m} + \|(v_0, v_1)\|_{\mathcal{A}_{s_2,0}^m} \leq \epsilon$, then, there exists a unquety determined globally (in time) sobolev solution to (??) in :*

$$C([0, T], H^{s_1} \cap L^m) \cap C^1([0, T], L^2 \cap L^m) \times C([0, T], H^{s_2} \cap L^m) \cap C^1([0, T], L^2 \cap L^m). \quad (25)$$

For :

$$\begin{cases} \frac{2}{m} \leq p \leq \frac{n}{n-2s_2} & \text{if } s_2 < \frac{n}{2}, \\ 1 \leq p < \infty & \text{if } s_2 > \frac{n}{2}, \end{cases}$$

and

$$\begin{cases} \frac{2}{m} \leq q \leq \frac{n}{n-2s_1} & \text{if } s_2 < \frac{n}{2}, \\ 1 \leq q < \infty & \text{if } s_1 > \frac{n}{2}. \end{cases}$$

With

$$q > \frac{1}{p} \left(1 + \frac{2m}{n}\right) + \frac{2m}{n}.$$

Moreover, then, the solution of (??) and some of its derivatives satisfy that decay estimates:

$$\begin{aligned} \|u(t, \cdot)\|_{L^2} &\lesssim (1+t)^{-\frac{n}{4} - \frac{2-m}{m}} \left(\|(u_0, u_1)\|_{\mathcal{A}_{s_1,0}^m} + \|(v_0, v_1)\|_{\mathcal{A}_{s_2,0}^m} \right), \\ \|u_t(t, \cdot)\|_{L^2} &\lesssim (1+t)^{-1 - \frac{n}{4} - \frac{2-m}{m}} \left(\|(u_0, u_1)\|_{\mathcal{A}_{s_1,0}^m} + \|(v_0, v_1)\|_{\mathcal{A}_{s_2,0}^m} \right), \\ \|u(t, \cdot)\|_{\dot{H}^{s_1}} &\lesssim (1+t)^{-\frac{s_1}{2} - \frac{n}{4} - \frac{2-m}{m}} \left(\|(u_0, u_1)\|_{\mathcal{A}_{s_1,0}^m} + \|(v_0, v_1)\|_{\mathcal{A}_{s_2,0}^m} \right), \\ \|v(t, \cdot)\|_{L^2} &\lesssim (1+t)^{-\frac{n}{4} - \frac{2-m}{m} + \gamma} \left(\|(u_0, u_1)\|_{\mathcal{A}_{s_1,0}^m} + \|(v_0, v_1)\|_{\mathcal{A}_{s_2,0}^m} \right), \\ \|v_t(t, \cdot)\|_{L^2} &\lesssim (1+t)^{-1 - \frac{n}{4} - \frac{2-m}{m} + \gamma} \left(\|(u_0, u_1)\|_{\mathcal{A}_{s_1,0}^m} + \|(v_0, v_1)\|_{\mathcal{A}_{s_2,0}^m} \right). \end{aligned}$$

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$$\|v(t, \cdot)\|_{\dot{H}^{s_2}} \lesssim (1+t)^{-\frac{s_2}{2} - \frac{n}{4} \frac{2-m}{m} + \gamma} \left(\|(u_0, u_1)\|_{A_{s_1,0}^m} + \|(v_0, v_1)\|_{A_{s_2,0}^m} \right).$$

Where

$$\gamma = 1 + \frac{n}{2m} + q\left(\frac{-n}{2m}\right).$$

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Certain type of solutions of conformable fractional plateau type equations

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Abstract: Sometimes, it is not possible to find a general solution for some differential equations using some classical methods, like separation of variables. In such a case, one can try to use theory of tensor product of Banach spaces to find certain solutions, called atomic solutions. The goal is to find atomic solution of fractional plateau type equation.

Keywords: conformable derivative, plateau type equation, tensor product, atomic solution
2020 Mathematics Subject Classification: 26A33, 34A55.

Introduction

In [9], a new definition called α -conformable fractional derivative was introduced:

Let $\alpha \in (0, 1)$, and $f : E \subseteq [0, \infty) \rightarrow \mathbb{R}$. For $x \in E$ let:

$$D^\alpha f(x) = \lim_{\varepsilon \rightarrow 0} \frac{f(x + \varepsilon x^{1-\alpha}) - f(x)}{\varepsilon}$$

If the limit exists, then it is called α -conformable fractional derivative of f at x .

For $x = 0$, if f is α -differentiable on $(0, r)$ for some $r > 0$, and $\lim_{x \rightarrow 0^+} D^\alpha f(x)$ exists then we define $D^\alpha f(0) = \lim_{x \rightarrow 0^+} D^\alpha f(x)$.

Conformable fractional derivative satisfies:

1) $T_\alpha(af + bg) = aT_\alpha(f) + bT_\alpha(g)$, for all $a, b \in \mathbb{R}$.

2) $T_\alpha(\lambda) = 0$, for all constant functions $f(t) = \lambda$.

Further, for $\alpha \in (0, 1]$ and f, g are α -differentiable at a point t , we have:

3) $T_\alpha(fg) = fT_\alpha(g) + gT_\alpha(f)$.

4) $T_\alpha\left(\frac{f}{g}\right) = \frac{gT_\alpha(f) - fT_\alpha(g)}{g^2}$, $g(t) \neq 0$.

We list here the fractional derivatives of certain functions,

1) $T_\alpha(t^p) = pt^{p-\alpha}$, for all $p \in \mathbb{R}$.

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$$2) T_{\alpha}(\sin_{\alpha} \frac{1}{\alpha} t^{\alpha}) = \cos_{\alpha} \frac{1}{\alpha} t^{\alpha}.$$

$$3) T_{\alpha}(\cos_{\alpha} \frac{1}{\alpha} t^{\alpha}) = -\sin_{\alpha} \frac{1}{\alpha} t^{\alpha}.$$

$$4) T_{\alpha}(e^{\frac{1}{\alpha} t^{\alpha}}) = e^{\frac{1}{\alpha} t^{\alpha}}.$$

On letting $\alpha = 1$ in these derivatives, we get the corresponding classical rules for ordinary derivatives.

One should notice that a function could be α -differentiable at a point but not differentiable, for example, take $f(t) = 2\sqrt{t}$, this $T_{\frac{1}{2}}(f)(t) = 1$. Hence, $T_{\frac{1}{2}}(f)(0) = 1$. But $T_1(f)(0)$ does not exist. This is not the case for the known classical fractional derivatives.

For more on fractional calculus and its applications we refer to [1] – [9].

Now, let X and Y be two Banach spaces and X^* be the dual of X . Assume $x \in X$ and $y \in Y$.

The operator $T : X^* \longrightarrow Y$ defined by

$$T(x^*) = x^*(x)y = \langle x^*, x \rangle y, \forall x^* \in X^*.$$

Then T is a bounded linear operator.

we write $x \otimes y$ for T . This is a one rank operator which is called an atom and it has the following properties:

$$\text{i) } (x_1 + x_2) \otimes y = x_1 \otimes y + x_2 \otimes y.$$

$$\text{ii) } y \otimes (x_1 + x_2) = y \otimes x_1 + y \otimes x_2.$$

$$\text{iii) } \alpha(x \otimes y) = (\alpha x \otimes y) = (x \otimes \alpha y).$$

Atoms are among the main ingredient in the theory of tensor products.

Atoms are used in theory of the best approximation in the Banach spaces, see [2].

One of the know result, see [2], that we need is: if the sum of two atoms is an atom, then either the first components are dependant or the second components are dependant..

For more On tensor products of Banach spaces we refer to [8] and [2].

Very often, differential equations can be solve via separation of variables for the reason or another. One of the main reasons of the failure of the method of separation of variables is that the equation is not linear. Here theory of tensor products comes to give us certain solutions of the equation. This is the goal of section 2.

Main results

We are interested in certain atomic solutions of equation (31) given below which is called the Plateau type equation.

A solution of a partial differential equation with variables x and y is called atomic solution if it has of the form

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$$u(x, y) = P(x)Q(y).$$

Consider the following fractional Plateau type equation

$$D_y^\alpha D_x^{2\alpha} u + D_y^{2\alpha} u = D_x^\alpha u, \quad 0 < \alpha < 1. \quad (26)$$

$$u(0, y) = D_x^\alpha u(0, y) = Q(y), \quad u(x, 0) = D_y^\alpha u(x, 0) = P(x).$$

Though this is a linear fractional partial differential equation, yet it is very difficult to use separation of variables to solve it. Hence, we are forced to go to atomic solutions. So, let

$$u(x, y) = P(x)Q(y) = P \otimes Q. \quad (27)$$

be an atomic solution of (26), where $P(x)$ and $Q(y)$ are not constants.

Further, we have the conditions

$$P(0) = P^{(\alpha)}(0) = 1. \quad (28)$$

$$Q(0) = Q^{(\alpha)}(0) = 1. \quad (29)$$

Now, substitute (27) in (26) to get

$$P^{(2\alpha)}(x)Q^{(\alpha)}(y) + P(x)Q^{(2\alpha)}(y) = P^{(\alpha)}(x)Q(y). \quad (30)$$

Equation (30) has the tensorial form:

$$P^{(2\alpha)} \otimes Q^{(\alpha)} + P \otimes Q^{(2\alpha)} = P^{(\alpha)} \otimes Q. \quad (31)$$

So, we have the sum of two atoms is an atom. by the previous lemma, we have two cases:

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Case(i)

$$P^{(2\alpha)} = P = P^{(\alpha)}. \quad (32)$$

Now, considering $P^{(2\alpha)} = P$, and using the result in [6], we get

$$P(x) = c_1 e^{\frac{x^\alpha}{\alpha}} + c_2 e^{-\frac{x^\alpha}{\alpha}}.$$

Using the conditions in (28), we get $c_1 = 1$ and $c_2 = 0$. So,

$$P(x) = e^{\frac{x^\alpha}{\alpha}}.$$

Now, considering $P = P^{(\alpha)}$, and using again the result in [6], we get

$$P(x) = c e^{\frac{x^\alpha}{\alpha}}.$$

Using the conditions in (28), we get $c = 1$. So,

$$P(x) = e^{\frac{x^\alpha}{\alpha}}.$$

Finally, we consider $P^{(2\alpha)} = P^{(\alpha)}$. Another use of [6] provides

$$P(x) = c_1 + c_2 e^{\frac{x^\alpha}{\alpha}}.$$

Using the conditions in (28), we get $c_1 = 0$ and $c_2 = 1$. So,

$$P(x) = e^{\frac{x^\alpha}{\alpha}}.$$

Thus $P(x) = e^{\frac{x^\alpha}{\alpha}}$ satisfies the situations of case (i).

Now, we substitute $P(x)$ in equation (31) to get

$$e^{\frac{x^\alpha}{\alpha}} \otimes Q^{(\alpha)} + e^{\frac{x^\alpha}{\alpha}} \otimes Q^{(2\alpha)} = e^{\frac{x^\alpha}{\alpha}} \otimes Q. \quad (33)$$

Equation (33) is valid for all x . In particular, it is true for $x = 0$. Hence (33) becomes

$$Q^{(\alpha)} + Q^{(2\alpha)} = Q. \quad (34)$$

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The associated characteristic equation of (34) is

$$r^2 + r - 1 = 0.$$

Using the result in [?], we get

$$Q(y) = c_1 e^{r_1 \frac{y^\alpha}{\alpha}} + c_2 e^{r_2 \frac{y^\alpha}{\alpha}}.$$

where $r_1 = \frac{-1 - \sqrt{5}}{2}$ and $r_2 = \frac{-1 + \sqrt{5}}{2}$.

Using the conditions in (29), we get $c_1 = \frac{5 - 3\sqrt{5}}{10}$ and $c_2 = \frac{5 + 3\sqrt{5}}{10}$.

Now, the atomic solution for case(i) is

$$u(x, y) = e^{\frac{x^\alpha}{\alpha}} \left(\left(\frac{5 - 3\sqrt{5}}{10} \right) e^{\frac{-1 - \sqrt{5}}{2} \frac{y^\alpha}{\alpha}} + \left(\frac{5 + 3\sqrt{5}}{10} \right) e^{\frac{-1 + \sqrt{5}}{2} \frac{y^\alpha}{\alpha}} \right). \quad (35)$$

Case(ii)

$$Q^{(\alpha)} = Q^{(2\alpha)} = Q. \quad (36)$$

Now, considering $Q^{(2\alpha)} = Q$, and using the result in [6], we get

$$Q(y) = c_1 e^{\frac{y^\alpha}{\alpha}} + c_2 e^{\frac{-y^\alpha}{\alpha}}.$$

Using the conditions in (29), we get $c_1 = 1$ and $c_2 = 0$. So,

$$Q(y) = e^{\frac{y^\alpha}{\alpha}}.$$

Now, considering $Q = Q^{(\alpha)}$, and using again the result in [6], we get

$$Q(y) = c e^{\frac{y^\alpha}{\alpha}}.$$

Using the conditions in (29), we get $c = 1$. So,

$$Q(y) = e^{\frac{y^\alpha}{\alpha}}.$$

Finally, we consider $Q^{(2\alpha)} = Q^{(\alpha)}$. Another use of [6] provides

$$Q(y) = c_1 + c_2 e^{\frac{y^\alpha}{\alpha}}.$$

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Using the conditions in (29), we get $c_1 = 0$ and $c_2 = 1$. So,

$$Q(y) = e^{\frac{y^\alpha}{\alpha}}.$$

Thus $Q(y) = e^{\frac{y^\alpha}{\alpha}}$ satisfies the situations of case (ii).

Now, we substitute $Q(y)$ in equation (31) to get

$$P^{(2\alpha)} \otimes e^{\frac{y^\alpha}{\alpha}} + P \otimes e^{\frac{y^\alpha}{\alpha}} = P^{(\alpha)} \otimes e^{\frac{y^\alpha}{\alpha}} \quad (37)$$

Equation (37) is valid for all y . In particular, it is true for $y = 0$. Hence (37) becomes

$$P^{(2\alpha)} + P = P^{(\alpha)}. \quad (38)$$

The associated characteristic equation of (38) is

$$r^2 - r + 1 = 0.$$

Using the result in [6], we get

$$P(x) = e^{\frac{1}{2} \frac{x^\alpha}{\alpha}} \left(c_1 \cos \left(\frac{\sqrt{3}}{2} \frac{x^\alpha}{\alpha} \right) + c_2 \sin \left(\frac{\sqrt{3}}{2} \frac{x^\alpha}{\alpha} \right) \right).$$

where $r_1 = \frac{1 + i\sqrt{3}}{2}$ and $r_2 = \frac{1 - i\sqrt{3}}{2}$.

Using the conditions in (28), we get $c_1 = 1$ and $c_2 = \frac{1}{\sqrt{3}}$.

Now, the atomic solution for case(ii) is

$$u(x, y) = e^{\frac{1}{2} \frac{x^\alpha}{\alpha}} \left(\cos \left(\frac{\sqrt{3}}{2} \frac{x^\alpha}{\alpha} \right) + \frac{1}{\sqrt{3}} \sin \left(\frac{\sqrt{3}}{2} \frac{x^\alpha}{\alpha} \right) \right) e^{\frac{y^\alpha}{\alpha}}. \quad (39)$$

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Gevrey regularity for a coupled system of Kadomtsev-Petviashvili II (KP- II) equations

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Abstract: Our goal in this work is to study the Gevrey- 3ρ regularity in time for a coupled system associated to Kadomtsev-Petviashvili II (KP- II) type equations, after discussed the local well-posedness of this initial value problem for a system of KP-II with the initial data in analytical anisotropic Gevrey spaces. Juan Montealegre Scott in [4] proved that the initial value problem is locally well posed in the anisotropic Sobolev spaces with $s_1 > -\frac{1}{3}$ and $s_2 \geq 0$. We prove the existence of solution using the Banach contraction mapping principle by using the linear and bilinear estimates in Gevrey-Bourgain spaces. Grimshaw and Zhu derived the system as a model to describe the oblique strong interaction of weakly, two-dimensional, nonlinear, long internal gravity waves in shallow fluids.

Keywords: Gevrey regularity, Nonlinear dispersive equations, local well-posedness, anisotropic Gevrey space.

Introduction

We consider the ivp for a system of two KP-II equations

$$\begin{cases} \partial_t \psi + \alpha_1 \partial_x^3 \psi + \alpha_2 \partial_x^3 \varphi + \gamma_1 \partial_x (\psi \varphi) + \gamma_2 \psi \partial_x \psi + \gamma_3 \varphi \partial_x \varphi + \partial_x^{-1} \partial_y^2 \psi = 0, \\ \partial_t \varphi + \alpha_3 \partial_x^3 \psi + \alpha_4 \partial_x^3 \varphi + \gamma_4 \partial_x (\psi \varphi) + \gamma_5 \psi \partial_x \psi + \gamma_6 \varphi \partial_x \varphi + \partial_x^{-1} \partial_y^2 \varphi = 0, \\ \psi(x, y, 0) = \psi_0(x, y), \\ \varphi(x, y, 0) = \varphi_0(x, y), \end{cases} \quad (40)$$

where $\psi = \psi(x, y, t)$ and $\varphi = \varphi(x, y, t)$ are unknown functions, $(x, y, t) \in \mathbb{R}^3$ and ψ_0, φ_0 are a given functions. $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5$ and γ_6 are real constants with $\alpha_1 \alpha_2 \alpha_3 \alpha_4 > 0$ and $\alpha_2 \alpha_3 > 0$.

The anisotropic Gevrey spaces are given by $G_{s_1, s_2}^{\delta_1, \delta_2, \rho}(\mathbb{R}^2) = G_{s_1, s_2}^{\delta_1, \delta_2, \rho}$ with $\delta_1, \delta_2 > 0, \rho \geq 1$ and $s_1, s_2 \in \mathbb{R}$

$$\|f\|_{G_{s_1, s_2}^{\delta_1, \delta_2, \rho}} = \left(\int_{\mathbb{R}^2} e^{2\delta_1 |\xi|^{\frac{1}{\rho}}} e^{2\delta_2 |\theta|^{\frac{1}{\rho}}} \langle \xi \rangle^{2s_1} \langle \theta \rangle^{2s_2} |\widehat{f}(\xi, \theta)|^2 d\xi d\theta \right)^{\frac{1}{2}},$$

where $\langle \cdot \rangle = (1 + |\cdot|^2)^{\frac{1}{2}}$, and $\widehat{f}$ is the space Fourier transform of f .

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Main results

The primary outcome focuses on the following auxiliary estimations:

Lemma 8. *Let $b > \frac{1}{2}$, $s_1, s_2 \in \mathbb{R}$, $\delta_1, \delta_2 > 0$, $\rho \geq 1$ and $\beta \geq 0$, there exist a constant $\lambda_1 > 0$ such that*

$$\|w_1(t)S(t)\psi_0\|_{X_{s_1, s_2, b, \beta}^{\delta_1, \delta_2, \rho}} \leq \lambda_1 \|\psi_0\|_{G_{s_1, s_2}^{\delta_1, \delta_2, \rho}}, \quad (41)$$

and

$$\|w_1(t)S(t)\varphi_0\|_{X_{s_1, s_2, b, \beta}^{\delta_1, \delta_2, \rho}} \leq \lambda_1 \|\varphi_0\|_{G_{s_1, s_2}^{\delta_1, \delta_2, \rho}}. \quad (42)$$

for $\psi_0, \varphi_0 \in G_{s_1, s_2}^{\delta_1, \delta_2, \rho}$.

And for $F, G \in X_{s_1, s_2, b-1, \beta}^{\delta_1, \delta_2, \rho}$, there exist $\lambda_1 > 0$ such that

$$\|w_1(t) \int_0^t S(t-t')F(t')dt'\|_{X_{s_1, s_2, b, \beta}^{\delta_1, \delta_2, \rho}} \leq \lambda_1 \|F\|_{X_{s_1, s_2, b-1, \beta}^{\delta_1, \delta_2, \rho}}, \quad (43)$$

and

$$\|w_1(t) \int_0^t S(t-t')G(t')dt'\|_{X_{s_1, s_2, b, \beta}^{\delta_1, \delta_2, \rho}} \leq \lambda_1 \|G\|_{X_{s_1, s_2, b-1, \beta}^{\delta_1, \delta_2, \rho}}. \quad (44)$$

For the nonlinear parts, we have

Lemma 9. *Let $s_1 \in]-\frac{1}{3}, 0[$, $s_2 \geq 0$, $b > \frac{1}{2}$, $\delta_1, \delta_2 > 0$, $\rho \geq 1$ and $\beta > \frac{1}{6}$ such that $b + \beta \leq 1 + s_1$, $b \leq \frac{1}{3}(2 + s_1)$ and $\beta \leq \frac{1}{3}(1 + s_1)$ the following bilinear estimate holds*

$$\|\partial_x(\psi\varphi)\|_{X_{s_1, s_2, b-1, \beta}^{\delta_1, \delta_2, \rho}} \leq \lambda_2 \|\psi\|_{X_{s_1, s_2, b, \beta}^{\delta_1, \delta_2, \rho}} \|\varphi\|_{X_{s_1, s_2, b, \beta}^{\delta_1, \delta_2, \rho}}, \quad (45)$$

for all $\psi, \varphi \in X_{s_1, s_2, b, \beta}^{\delta_1, \delta_2, \rho}$.

The next result addresses the well-posedness of (40).

Theorem 16. *Let $s_1 \in]-\frac{1}{3}, 0[$, $s_2 \geq 0$, $\delta_1, \delta_2 > 0$, $\rho \geq 1$ and $(\psi_0, \varphi_0) \in G_{s_1, s_2}^{\delta_1, \delta_2, \rho} \times G_{s_1, s_2}^{\delta_1, \delta_2, \rho}$, if $b > \frac{1}{2}$ and $\beta > \frac{1}{6}$ satisfy the hypothesis $b + \beta \leq 1 + s_1$, $b \leq \frac{1}{3}(2 + s_1)$ and $\beta \leq \frac{1}{3}(1 + s_1)$, then there exist $T = T(\|\psi_0\|_{G_{s_1, s_2}^{\delta_1, \delta_2, \rho}}, \|\varphi_0\|_{G_{s_1, s_2}^{\delta_1, \delta_2, \rho}})$ and a unique solution (ψ, φ) of (40) on the time interval*

$$(\psi, \varphi) \in C([0, T], G_{s_1, s_2}^{\delta_1, \delta_2, \rho}(\mathbb{R}^2)) \times C([0, T], G_{s_1, s_2}^{\delta_1, \delta_2, \rho}(\mathbb{R}^2)).$$

In the end we show that the solution to (40) has Gevrey regularity in the time variable.

Theorem 17. *Let $-\frac{1}{3} < s_1 < 0$, $s_2 \geq 0$, $\gamma_1, \gamma_2 > 0$, $\rho \geq 1$ and $(\psi, \varphi) \in C([0, T], G_{s_1, s_2}^{\gamma_1, \gamma_2, \rho}(\mathbb{R}^2)) \times C([0, T], G_{s_1, s_2}^{\gamma_1, \gamma_2, \rho}(\mathbb{R}^2))$ be the solution of (40). Then $(\psi, \varphi) \in G^{3\rho}([0, T]) \times G^{3\rho}([0, T])$ in the time variable t .*

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Elliptic Lane-Emden systems

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Abstract: We will present the existence and non-existence results of the following cooperative Lane -Emden system with singular Hardy potential

$$(GS) \begin{cases} (-\Delta)u = \lambda_1 \frac{v}{|x|^2} + f(x, v, \nabla v) & \text{in } \Omega, \\ (-\Delta)v = \lambda_2 \frac{u}{|x|^2} + g(x, u, \nabla u) & \text{in } \Omega, \\ u = v = 0 & \text{on } \partial\Omega \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain containing the origin and $N \geq 3$.

Since we are looking for positive solutions, it is natural to require that f and g are non-negative functions. In particular we will focus our analysis in the cases

$$f = v^p, g = u^q, \quad f = |\nabla v|^p, g = |\nabla u|^q, \quad (46)$$

Keywords: Singular elliptic system, Hardy inequality, LANE-EMDEN system , Cooperative system .

Introduction

The main goal of this paper is to analyze the question of existence and non existence for the cooperative singular system given by

$$(GS) \begin{cases} -\Delta u = \lambda_1 \frac{v}{|x|^2} + f(x, v, \nabla v) & \text{in } \Omega, \\ -\Delta v = \lambda_2 \frac{u}{|x|^2} + g(x, u, \nabla u) & \text{in } \Omega, \\ u = v = 0 & \text{on } \partial\Omega \end{cases}$$

where $\lambda_1, \lambda_2 > 0$.

We will consider two main cases:

1. the case where $f(x, v) = v^p$ and $g(x, u) = u^q$, using suitable process argument, we show the existence of a critical curves in $H(p, q)$ such that the system (S) has a solution according to the sign of H .

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2. the case $f(x, v) = |\nabla v|^p$ and $g(x, u) = |\nabla u|^q$. Here the situation is more complicated and we need to use a suitable weighted Sobolev inequality in order to get the critical curves of existence.

Main results

Our main non-existence results for the *potential case* using an algebraic technique are given by :

Theorem 18. Suppose that $\min\{p, q\} \geq \frac{N-\mu(\sqrt{\lambda_1\lambda_2})}{\mu(\sqrt{\lambda_1\lambda_2})}$, then System (PS) has non weak supersolution.

Theorem 19. Let $\lambda_1, \lambda_2 > 0$ be such that $\sqrt{\lambda_1\lambda_2} \leq \Lambda_N$.

1. $\frac{\mu(\sqrt{\lambda_1\lambda_2})+2}{\mu(\sqrt{\lambda_1\lambda_2})} < p < \frac{N-\mu(\sqrt{\lambda_1\lambda_2})}{\mu(\sqrt{\lambda_1\lambda_2})}$, $q \leq p$ and $\mu(\sqrt{\lambda_1\lambda_2})^{\frac{pq-1}{p+1}} > 2$,
Or
2. $\frac{\mu(\sqrt{\lambda_1\lambda_2})+2}{\mu(\sqrt{\lambda_1\lambda_2})} < q < \frac{N-\mu(\sqrt{\lambda_1\lambda_2})}{\mu(\sqrt{\lambda_1\lambda_2})}$, $p \leq q$ and $\mu(\sqrt{\lambda_1\lambda_2})^{\frac{pq-1}{q+1}} > 2$,

then System (PS) has non weak positive supersolution.

And the existence is the following :

Theorem 20. Let Ω be a bounded domain containing the origin and that $\sqrt{\lambda_1\lambda_2} \leq \Lambda_N$.

1. Suppose that $p, q < \frac{\mu(\sqrt{\lambda_1\lambda_2})+2}{\mu(\sqrt{\lambda_1\lambda_2})}$, namely $(p, q) \in A$. Then System (PS) has a positive supersolution.

For the *gradient case*, the results using an iteration technique are the following :

Theorem 21. Suppose that $\min\{p, q\} \geq \frac{N-\mu(\sqrt{\lambda_1\lambda_2})}{\mu(\sqrt{\lambda_1\lambda_2})+1}$, then System (PS) has non weak supersolution.

Theorem 22. Let $\Omega \subset (R)^N$ be a bounded domain such that $0 \in \Omega$ and $N \geq 3$. Assume that $0 < \lambda_1, \lambda_2 \leq L_N$ and suppose that $p, q > 0$ satisfying the following assumptions:

1. $q < \frac{\mu(\sqrt{\lambda_1\lambda_2})+2}{\mu(\sqrt{\lambda_1\lambda_2})+1} < p < \frac{N-\mu(\sqrt{\lambda_1\lambda_2})}{\mu(\sqrt{\lambda_1\lambda_2})+1}$, $q \leq p$ and $H(p, q) < 0$,
2. $p < \frac{\mu(\sqrt{\lambda_1\lambda_2})+2}{\mu(\sqrt{\lambda_1\lambda_2})+1} < q < \frac{N-\mu(\sqrt{\lambda_1\lambda_2})}{\mu(\sqrt{\lambda_1\lambda_2})+1}$, $p \leq q$ and $H(p, q) < 0$,

Then the System (QS) has no nonnegative weak solution

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Atomic Characterization of Variable Besov-Lorentz Space Associated to Operators

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Abstract: Let (X, d, μ) be a space of homogeneous type and L be a nonnegative self-adjoint operator on $L^2(X)$ with heat kernels satisfying Gaussian upper bounds. In this work, we present the variable Besov- Lorentz space associated to the operator L on spaces of homogeneous type. Then, show that the atomic decomposition of these new spaces are given.

Keywords: Besov–Lorentz space, metric measure, Variable exponents , heat kernel, Atomic characterizations

2020 Mathematics Subject Classification: Primary 46E36, Secondary, 42B35.

Introduction

Recent year, functions spaces associated to operators play an important role not only in the theory of function spaces, but also in the theory of partial differential equations and harmonic analysis, see e.g.[1, 2, 9].

Besov and Triebel-Lizorkin spaces are known for including important function spaces like Lebesgue, Hardy, and Sobolev spaces. Particularly, Besov-Lorentz spaces attracted great of attention and has become a fruitful research topic see e.g [4, 5] and the references therein.

Based on the above article, the main purpose of this paper is to study variable Besov-Lorentz space $B_{p(\cdot), \theta, q(\cdot)}^{s(\cdot), L}(X)$ on a space of homogeneous type.

Main results

In the sequel, we assume (X, d, μ) is a metric space of homogeneous type and μ a Borel measure on X , with satisfies the doubling property:

$$\mu(B(x, 2r)) \leq c_0 \mu(B(x, r)), \quad \text{for all } x \in X \quad \text{and} \quad r > 0, \quad (47)$$

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where $B(x, r)$ is the open ball centered at x with radius r and c_0 is a constant.

Let $\text{Dom}(L)$ signify the domain of the non-negative self adjoint operator L , which is dense in $L^2(X)$, such that the associated semi-group $\{e^{-tL}\}_{t>0}$ consists of integral operators with heat kernel $p_t(x, y)$ satisfy the following : Small-time Gaussian upper bound (GB).

We denote $\mathbb{Z}_+ = \mathbb{N} \cup \{0\}$. We also denote $\mathcal{D}$ the collection of dyadic cubes in X .

Let ϕ and ϕ_0 be two functions in $C^\infty(\mathbb{R}_+)$ satisfying

$$\text{supp } \phi_0 \subset [0, 2], \phi_0^{2\nu+1}(0) = 0, |\phi_0(\xi)| \geq c > 0 \quad \forall \xi \in [0, 2^{\frac{3}{4}}] \quad (48)$$

and

$$\text{supp } \phi \subset [1/2, 2], |\phi(\xi)| \geq c > 0 \quad \forall \xi \in [2^{-\frac{3}{4}}, 2^{\frac{3}{4}}], \quad (49)$$

where c is a constant independent of ξ . Then, for any $\xi \in \mathbb{R}_+, k \in \mathbb{N}$

$$\sum_{k=0}^{\infty} \phi_k(\xi) \geq c > 0,$$

with $\phi_k(\xi) := \phi(2^{-k}\xi)$.

After these preparations, we have the following definitions.

Definition 10. Let $p(\cdot), q(\cdot) \in \mathcal{P}^{\log}(X), s(\cdot) \in C_{\text{loc}}^{\log}(X) \cap L^\infty(X), \theta \in (0, \infty]$ and L be a non-negative self-adjoint operator whose domain is dense in $L^2(X)$ satisfying (GB). The variable Besov-Lorentz space associated with the operator $L, B_{p(\cdot), \theta, q(\cdot)}^{s(\cdot), L}(X)$ is defined to be the set of all $f \in \mathcal{D}'(L)$ such that

$$\|f\|_{B_{p(\cdot), \theta, q(\cdot)}^{s(\cdot), L}(X)} := \left\| \left\{ 2^{ks(\cdot)} \phi_k(\sqrt{L})f \right\}_{k \in \mathbb{Z}_+} \right\|_{\ell^{q(\cdot)}(L^{p(\cdot), \theta}(X))} < \infty,$$

where $\{\phi_0, \phi\} \in C^\infty(\mathbb{R}_+)$ satisfying (48) and (49).

Definition 11. Let $K, S \in \mathbb{Z}_+$, and let Q a dyadic cube in D_i with $i \in \mathbb{Z}_+$. When $k \geq 1$, a function $\alpha_Q \in L^2(X)$ is called (K, S) -atom for Q if α_Q satisfies the following conditions for $m = K$ and $m = -S$

- (a) $\alpha_Q \in \text{Dom}(L^m)$;
- (b) $\text{supp}(L^m \alpha_Q) \subset B(z_Q, (A_1 + 1)2^{-i})$;
- (c) $\sup_{x \in X} |L^m \alpha_Q(x)| \leq 2^{2im} [\mu(Q)]^{-\frac{1}{2}}$;

when $i = 0$, a function $\alpha_Q \in L^2(X)$ is a (K, S) -atom for Q if α_Q satisfies the above conditions only for $m = i$.

Definition 12. Let $p(\cdot), q(\cdot), s(\cdot)$ be positive functions on X and $\theta \in (0, \infty]$. The sequence space $b_{p(\cdot), q(\cdot)}^{s(\cdot), \theta}$ consists all complex sequence $\{\lambda\}_{Q \in \bigcup_{i \geq 0} \mathcal{D}_i}$ such that $\|\lambda\|_{b_{p(\cdot), q(\cdot)}^{s(\cdot), \theta}} < \infty$,

$$\|\lambda\|_{b_{p(\cdot), q(\cdot)}^{s(\cdot), \theta}} := \left\| \left\{ 2^{is(\cdot)} \sum_{Q \in \mathcal{D}_i} |\lambda_Q [\mu(Q)]^{-\frac{1}{2}} \chi_Q| \right\}_{i \in \mathbb{Z}_+} \right\|_{\ell^{q(\cdot)}(L^{p(\cdot), \theta}(X))}. \quad (50)$$

Now we state the main result in this paper.

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Theorem 23. Let $p(\cdot), q(\cdot) \in \mathcal{P}^{\log}(X), s(\cdot) \in C_{\text{loc}}^{\log}(X) \cap L^{\infty}(X), \theta \in (0, \infty]$ and L be a non-negative self-adjoint operator whose domain is dense in $L^2(X)$ satisfying (GB). Let $K, S \in \mathbb{Z}_+$ such that

$$K > \frac{s_+}{2} \text{ and } S > \frac{1}{2} \left(\frac{4d}{\min\{1, p_-, q_-\}} + C_1(s) + C_1(1/q) + s_- \right).$$

Then, there exists a constant $C > 0$ such that for any sequence of (K, S) -atoms $\{\alpha\}_{Q \in \cup_{i \geq 0} \mathcal{D}_i}$,

$$\left\| \sum_{i \in \mathbb{Z}_+} \sum_{Q \in \mathcal{D}_i} \lambda_Q \alpha_Q \right\|_{B_{p(\cdot), \theta, q(\cdot)}^{s(\cdot), L}(X)} \leq C \|\lambda\|_{b_{p(\cdot), q(\cdot)}^{s(\cdot), \theta}}.$$

Conversely, there exists a positive constant C , such that for any function $f \in B_{p(\cdot), \theta, q(\cdot)}^{s(\cdot), L}(X)$ there exist a sequence of (K, S) -atoms $\{\alpha\}_{Q \in \cup_{i \geq 0} \mathcal{D}_i}$ and a sequence of complex scalars $\{\lambda\}_{Q \in \cup_{i \geq 0} \mathcal{D}_i}$ such that $f = \sum_{i \in \mathbb{Z}_+} \sum_{Q \in \mathcal{D}_i} \lambda_Q \alpha_Q$, where the sum converge in D' ; and moreover

$$\|\lambda\|_{b_{p(\cdot), q(\cdot)}^{s(\cdot), \theta}} \leq C \|f\|_{B_{p(\cdot), \theta, q(\cdot)}^{s(\cdot), L}(X)}.$$

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Analysis of a quasistatic problem involving electro-elasto-viscoplastic material with friction and wear

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Abstract: We consider a quasistatic contact problem for electro-elasto-viscoplastic with a following mathematical model.

$$\begin{aligned}\sigma(t) &= \mathcal{A}(\varepsilon(\dot{u}(t))) + \mathcal{B}(\varepsilon(u(t))) + \int_0^t \mathcal{G}(\sigma(s) - \mathcal{A}(\varepsilon(\dot{u}(s))), \varepsilon(u(s))) ds - \xi^* E(\varphi), \\ D &= BE(\varphi) + \xi \varepsilon(u), \text{ in } \Omega \times [0, T],\end{aligned}$$

Where $u : \Omega \times [0, T] \rightarrow \mathbb{R}^d$ is a displacement field, $\sigma : \Omega \times [0, T] \rightarrow \mathbb{S}^d$ is a stress field $\varphi : \Omega \times [0, T] \rightarrow \mathbb{R}$ is the an electric potentiel field and $D : \Omega \times [0, T] \rightarrow \mathbb{R}^d$ is the an electric displacement field.

The bilateral contact with friction and wear is modelad by:

$$\begin{cases} \sigma_\nu = -\alpha |\dot{u}_\nu|, & |\sigma_\tau| = -\mu \sigma_\nu, \\ \sigma_\tau = -\lambda (\dot{u}_\tau - v^*), & \lambda \geq 0, \omega = -kv^* \sigma_\nu, k > 0, \text{ on } \Gamma_3 \times [0, T]. \end{cases}$$

We establish a variational formulation for the model and we prove the existence of a unique weak solution to the problem.

$$\begin{aligned}& (\mathbf{A}(\varepsilon(\dot{u}(t))), \varepsilon(\omega - \dot{u}(t))) + (\mathbf{B}(\varepsilon(u(t))), \varepsilon(\omega - \dot{u}(t)))_{\mathbf{H}} + j(\dot{u}, \omega) - j(\dot{u}, \dot{u}) \\ & + \left(\int_0^t \mathbf{G}(\sigma(s) - \mathbf{A}(\varepsilon(\dot{u}(t))), (\varepsilon(u(t)))) ds - \xi^* E(\varphi), \varepsilon(\omega - \dot{u}(t)))_{\mathbf{H}} \geq (f(t), \omega - \dot{u}(t))_V \right. \\ & \left. (\beta \nabla \varphi(t), \nabla \psi)_{\mathbb{L}^2(\Omega)^d} - (\xi \varepsilon(u(t)), \nabla \psi)_{\mathbb{L}^2(\Omega)^d} = (q(t), \psi)_W \quad \forall \varphi \in W. \dot{\omega} = -kv^* \sigma_\nu, k > 0. \right.\end{aligned}$$

The proof is based on a classical existence and uniqueness result, using arguments of evolutionary variational equalities, the theory of monotonic and Lipschitz friction operators and the fixed point theorem.

Keywords: Piezoelectric, electro-elasto-viscoplastic, a quasistatic process, wear, fixed point
2020 Mathematics Subject Classification: 74M10, 74M15, 74F05, 49J40

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Introduction

In this work, we consider a model for the quasistatic contact between an electro-elasto-viscoplastic body and a foundation. The contact is bilateral with friction and wear and a regularized electrical condition. We derive a variational formulation and prove the existence of a unique weak solution of the problem.

Main results

our main results on existence and uniqueness which state the unique weak solution.

Theorem 24. *Let's make some assumptions about the data the viscosity operator $\mathcal{A}$, the elasticity operator $\mathcal{B}$, the viscoplasticity operator $\mathcal{G}$, the piezoelectric tensor ξ , the electrical permittivity tensor β , while the coefficients of friction μ and α , the surface traction, the density of volumetric and surface electric charges and the initial displacement are verified. So, the problem has a unique solution $(u, \sigma, \varphi, D, \omega)$. In other, the solution satisfies*

$$\begin{aligned}u &\in W^{1,2}(0, T; U) \\ \sigma &\in \mathbb{L}^2(0, T; \mathcal{H}_1) \\ \varphi &\in W^{1,2}(0, T; W) \\ \sigma &\in \mathbb{L}^2(0, T; \mathcal{H}_1) \\ \varphi &\in W^{1,2}(0, T; W) \\ D &\in W^{1,2}(0, T; \mathcal{W}) \\ \omega &\in C(0, T; \mathbb{L}^2(\Gamma_3))\end{aligned}$$

We conclude that the weak solution $(u, \sigma, \varphi, D, \omega)$ the piezoelectric contact problem has regularity.

The theorem is demonstrated in several stages.

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Existence and asymptotic behavior of solution for a nonlinear wave equations type hyperbolic with variable-exponent

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Abstract: The work is devoted to a study of the question of existence (in time) of weak solutions and the derivation of qualitative properties of such solutions for the nonlinear viscoelastic wave equation with variable exponent and a minor damping terms. By using the energy method combined with the Faedo-Galerkin method, the local and global existence of solutions are established. Then, the stability estimate of the solution is obtained by introducing a suitable Lyapunov functional.

Keywords: Faedo-Galerkin, variable exponent, local solution, Lyapunov functional

2020 Mathematics Subject Classification: 35B40, 35L70..

Introduction

We consider the following boundary value problem:

$$\left\{ \begin{array}{ll} u_{tt} - \Delta u + \Delta u_{tt} - \int_0^t h(t-\tau) \Delta u(x, \tau) d\tau \\ + \mu_1 u_t(x, t) + \int_{\tau_1}^{\tau_2} \mu_2(s) u_t(x, t-s) ds = b|u|^{p(x)-2} u, & \text{in } \Omega \times (0, \infty) \\ u(x, t) = 0, & \text{in } \Gamma \times (0, \infty) \\ u(x, 0) = u_0(x), \quad u_t(x, 0) = u_1(x), & \text{in } \Omega \\ u_t(x, -t) = f_0(x, t), & \text{in } \Omega \times (0, \tau_2). \end{array} \right. \quad (51)$$

where Ω is a bounded domain in $\mathbb{R}^n, n \in \mathbb{N}$, with a smooth boundary $\partial\Omega = \Gamma, \rho > 0, \mu_1$ is a positive constant, b is a positive real number and h is a positive nonincreasing function defined on $\mathbb{R}_+$. (u_0, u_1, f_0) are the initial data belonging to a suitable function space. Moreover, $\mu_2 : [\tau_1, \tau_2] \rightarrow \mathbb{R}$ is a bounded function, where τ_1 and τ_2 are two real numbers satisfy $0 \leq \tau_1 \leq \tau_2$. The exponent $p(\cdot)$ is given measurable function on Ω satisfying:

$$2 \leq p_1 \leq p(x) \leq p_2 < \infty,$$

with

$$p_1 = \operatorname{ess\,inf}_{x \in \Omega} p(x), \quad p_2 = \operatorname{ess\,sup}_{x \in \Omega} p(x),$$

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we also assume that $p(\cdot)$ be log-continuous in Ω such that

$$\forall (x, y) \in \Omega^2, |p(x) - p(y)| \leq -\frac{C}{\log|x - y|}, \text{ with } |x - y| < \delta, \quad (52)$$

where $C > 0$, $0 < \delta < \frac{1}{2}$.

Main results

We use the Faedo-Galerkin criterion to confirm the existence of the local and global solution to the wave equation (51). By constructing the appropriate Lyapunov function, we obtain the asymptotic behavior of our problem in the following form:

$$E(t) \leq C_1 e^{-k_1 t} + \frac{\rho}{k_1 B_1}, \quad \forall t > 0. \quad (53)$$

where

$$\lambda = \min\{-1; -\delta\varepsilon; -(\varepsilon + (1 - l))\} < 0,$$

and

$$\begin{aligned} \rho &= [(1 + \varepsilon(1 + \eta))(1 - l) + \varepsilon\delta C_P] \|\nabla u\|_2^2 + (1 + \varepsilon + \mu_1 \frac{\varepsilon}{2\delta}) \|u_t\|_2^2 \\ &+ \frac{2\varepsilon p_1 l}{2p_1 - 2b} CE(0) \\ &+ (1 + \frac{\varepsilon}{4\eta})(h \circ \nabla u)(t) + (1 + \varepsilon) \|\nabla u_t\|_2^2 < C^{te}. \end{aligned}$$

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Application of Riccati-Bernoulli Sub-ODE method to time-fractional KP type equations

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Abstract: In this work, we investigate the soliton solutions to three time fractional (2+1)-dimensional KP type equations.

Fistly, the modified KP equation [1] in the form

$$D_t^\alpha u - 6u^2 u_x + u_{xxx} - 3\lambda(2u_x \partial_x^{-1} u_y - \lambda \partial_x^{-1} u_{yy}) = 0 \quad (54)$$

Secondly, the modified KdV-KP equation [2] in the form

$$(D_t^\alpha u - \frac{3}{2}u_x + 6u^2 u_x + u_{xxx})_x + u_{yy} = 0, \quad (55)$$

and the Gardner - KP Equation [3] in the form

$$D_t^\alpha(u_x) + 6uu_{xx} + 6u_x^2 + 6u^2 u_{xx} + 12uu_x^2 + u_{xxxx} + u_{yy} = 0. \quad (56)$$

Where $\partial^{-1} := \int(\cdot)dx$, $0 < \alpha \leq 1$ and $D_t^\alpha := \frac{\partial^\alpha}{\partial t^\alpha}(\cdot)$ in the Sense of Katugambola Derivative.

The exact solutions for these equations have been presented using Riccati-Bernoulli Sub-ODE method combined with the fractional complex transformation.

The variety of traveling wave solutions (trigonometric, hyperbolic, rational and others) indicates that this powerful method is effective. The availability of Software packages like **Mathematica 11** makes algebraic calculations easier and and plotes surfaces of solutions to interpret the behavior and geometric aspect of the obtained solutions.

Keywords: The modified KP equation, the modified KdV-KP equation, the Gardner - KP Equation, RB Sub-ODE method, Katugambola Derivative

2020 Mathematics Subject Classification: Primary 26A33, 35C07, 35C08.

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Main results

The Katugambola fractional derivative

Definition 13. [4] Let $f : [0, +\infty) \rightarrow \mathbb{R}$ and $t > 0$. Then the fractional derivative of f of order α^{th} is defined by

$$D^\alpha(f)(t) = \lim_{\epsilon \rightarrow 0} \frac{f(te^{\epsilon t^{1-\alpha}}) - f(t)}{\epsilon} \quad (57)$$

for all $t > 0, \alpha \in (0, 1)$. If f is α -differentiable in some $(0, a), a > 0$ and $\lim_{t \rightarrow 0^+} f^{(\alpha)}(t)$ exists then define $f^{(\alpha)}(0) = \lim_{t \rightarrow 0^+} f^{(\alpha)}(t)$.

Description of Riccati-Bernoulli Sub-ODE Method

The general nonlinear Time-Fractional evolution equation, say in $(2+1)$ independent variables x, y and t , is given by

$$P(u, D_t^\alpha u, u_x, D_t^{2\alpha} u, u_{xx}, D_t^\alpha u_x, u_y, u_{yy}, D_t^\alpha u_y, u_{xy}, \dots) = 0, \quad 0 < \alpha \leq 1, \quad (58)$$

where $u = u(x, y, t)$ is an unknown function, P is a polynomial of u and its partial fractional derivatives, in which the nonlinear terms and the highest order derivatives are included. To find the exact solution of Eq. (58) by the proposed method (The RB sub-equation method), we follow the following steps

- **Step 1:** To obtain exact solution, the following fractional complex transformation has been applied

$$u(x, t) = U(\xi), \quad \xi = kx - \omega \frac{t^\alpha}{\alpha}, \quad (59)$$

where k, ω are non-zero constants to be determined.

Then, the Eq (58) is reduced to the following nonlinear ordinary differential equation

$$P(U, -\omega U', kU', \omega^2 U'', k^2 U'', -\omega k U', \dots) = 0, \quad (60)$$

where $U^{(i)} = U_{i\xi}$.

- **Step 2:** Let equation (60) be the solution of the RB equation

$$U'(\xi) = aU(\xi)^{2-m} + bU(\xi) + cU(\xi)^m \quad (61)$$

where a, b, c and m are arbitrary constants.

1. **Case 1:** for $m = 1$, the solution of Eq. (61) is

$$U(\xi) = \mu e^{(a+b+c)\xi} \quad (62)$$

2. **Case 2:** for $m \neq 1, b = 0, c = 0$, the solution of Eq.(61) is

$$U(\xi) = (a(m-1)(\xi + \mu))^{\frac{1}{m-1}} \quad (63)$$

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3. **Case 3:** for $m \neq 1, b \neq 0, c = 0$, the solution of Eq.(61) is

$$U(\xi) = \left(\mu e^{b(m-1)\xi} - \frac{a}{b} \right)^{\frac{1}{m-1}} \quad (64)$$

4. **Case 4:** for $m \neq 1, a \neq 0, \Delta = b^2 - 4ac < 0$, the solution of Eq.(61) is

$$U(\xi) = \left(\frac{\sqrt{-\Delta} \tan\left(\frac{(1-m)\sqrt{-\Delta}}{2}(\xi + \mu)\right)}{2a} - \frac{b}{2a} \right)^{\frac{1}{1-m}}, U(\xi) = \left(-\frac{b}{2a} - \frac{\sqrt{-\Delta} \cot\left(\frac{(1-m)\sqrt{-\Delta}}{2}(\xi + \mu)\right)}{2a} \right)^{\frac{1}{1-m}} \quad (65)$$

5. **Case 5:** for $m \neq 1, a \neq 0, \Delta = b^2 - 4ac > 0$, the solution of Eq.(61) is

$$U(\xi) = \left(-\frac{b}{2a} - \frac{\sqrt{\Delta} \tanh\left(\frac{(1-m)\sqrt{\Delta}}{2}(\xi + \mu)\right)}{2a} \right)^{\frac{1}{1-m}}, U(\xi) = \left(\frac{\sqrt{\Delta} \coth\left(\frac{(1-m)\sqrt{\Delta}}{2}(\xi + \mu)\right)}{2a} - \frac{b}{2a} \right)^{\frac{1}{1-m}} \quad (66)$$

6. **Case 6:** for $m \neq 1, a \neq 0, \Delta = b^2 - 4ac = 0$, the solution of Eq.(61) is

$$U(\xi) = \left(\frac{1}{a(m-1)(\xi + \mu)} - \frac{b}{2a} \right)^{\frac{1}{1-m}}. \quad (67)$$

- **Step 3:** Substituting the derivatives of U into equation (60) gives the equation in terms of U . Collecting terms that belong together and solving for the unknowns constants leads to the solution of equation (58).

Application of the method

Proceeding as above, we obtain various solutions of the three PDEs indicate above.

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On the blowing-up of solutions for a system governed by nonlinear mixed-type fractional differential equations

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Abstract: In this work, we will discuss the existence of sufficient conditions on the parameters with positive initial datas to establish the finite time blow-up of solutions for a system of nonlinear fractional differential equations when we use the test functions method together with some general notions of functional analysis. Our work aims to find new results and generalizes some results previously found in this direction.

Keywords: Fractional differential operator, Blow-up, Test function method, Sufficient conditions

2020 Mathematics Subject Classification: [2010]

Introduction

Fractional derivation is a concept of generalization of classical derivation to a non-integer order. If this order is negative, it is a non-integer integration and if it is positive it is a non-integer derivation. Fractional derivation provides several potentially useful tools for solving integral equations. It also comes naturally in the mechanical modeling of materials which preserve the memory of past transformations. Hence the particular interest in calculus and fractional analysis during the last decades.

Recently, fractional differential equations have added great improvements in the modeling of several engineering works, let us quote as examples: viscoelasticity, fluid mechanics, electrochemistry, electromagnetism, etc. (see [1, 1, 4, 17]). The study of this new type of differential equations drew the attention of many researchers (see for example [3, 9, 5, 13]). Furthermore, a great attention has been paid to the study of the local existence and the uniqueness of the solutions for these systems and their properties such as finite-time explosion, global existence, asymptotic behavior, (see [4, 20, 4, ?]).

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Inspired by the works cited above, we consider the following FDS type problem

$$\begin{cases} {}^c\mathcal{D}_{0+}^{\alpha_1}\mathbf{u}(\mathbf{t}) + \mathbf{A}(\mathbf{t}) {}^c\mathcal{D}_{0+}^{\beta_1}\mathbf{u}(\mathbf{t}) = \lambda_1 e^{|\mathbf{v}(\mathbf{t})|} + \mu_1 \mathcal{D}_{0+}^{\gamma_1}|\mathbf{u}(\mathbf{t})|, & \mathbf{t} > 0, \\ {}^c\mathcal{D}_{0+}^{\alpha_2}\mathbf{v}(\mathbf{t}) + \mathbf{B}(\mathbf{t}) {}^c\mathcal{D}_{0+}^{\beta_2}\mathbf{v}(\mathbf{t}) = \lambda_2 e^{|\mathbf{u}(\mathbf{t})|} + \mu_2 \mathcal{D}_{0+}^{\gamma_2}|\mathbf{v}(\mathbf{t})|, & \mathbf{t} > 0, \\ \mathbf{u}(0) = \mathbf{u}_0 > 0, & \mathbf{v}(0) = \mathbf{v}_0 > 0, \end{cases} \quad (68)$$

where $\lambda_1, \lambda_2, \mu_1, \mu_2 > 0$, $0 < \alpha_i, \beta_i, \gamma_i < 1$ for $i \in \{1, 2\}$, ${}^c\mathcal{D}_{0+}^{\vartheta}, \mathcal{D}_{0+}^{\mu}$ display the derivation operators in the sense of Caputo and Riemann-Liouville respectively with $\vartheta \in \{\alpha_1, \alpha_2, \beta_1, \beta_2\}$, $\mu \in \{\gamma_1, \gamma_2\}$ and $\mathbf{A}(\mathbf{t}), \mathbf{B}(\mathbf{t})$ are two continuous functions.

Blowing-up results

Theorem 25. *Let $\mathbf{A}(\mathbf{t}), \mathbf{B}(\mathbf{t})$ be two positive continuous functions which satisfy respectively $\mathbf{A}_1 \leq \mathbf{A}(\mathbf{t}) \leq \mathbf{A}_2$, $\mathbf{B}_1 \leq \mathbf{B}(\mathbf{t}) \leq \mathbf{B}_2$, where $\mathbf{A}_i, \mathbf{B}_i$ are two positive constants, $\lambda_i, \mu_i > 0$, $0 < \alpha_i, \beta_i, \gamma_i < 1$, for $i \in \{1, 2\}$ and $\lambda_1 \lambda_2 > \frac{1}{4}$. If*

$$\alpha_1 \geq \max\{\alpha_2, \beta_1, \beta_2, \gamma_1, \gamma_2\}, \quad \text{or} \quad \alpha_2 \geq \max\{\alpha_1, \beta_1, \beta_2, \gamma_1, \gamma_2\}.$$

Then the solution of the system (63) blows up in a finite time.

Proof For the proof of our theorem 25, we will proceed by contradiction. For this fact, we suppose that $(\mathbf{u}, \mathbf{v})$ is a weak non-trivial global solution of our fractional initial value problem (63). and we have contradiction ■

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Exponential Decay for the Timoshenko System with Dynamical Boundary Conditions and past history

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Abstract: In this work, we consider the Timoshenko System with Dynamical Boundary Conditions and past history. we show well posedness and the stability exponential decay in the case where the wave propagation are equals . The general decay result is established by the multiplier method.

Keywords: Timoshenko system, Exponential rate decay, Semigroup theory, multiplicative method

2020 Mathematics Subject Classification: Primary 35BP35, 35B40; 35L53

Introduction

Understanding and measuring the vibration of beams has been a standard problem in structural Engineering. Since the seminal book of Timoshenko, different elastic are modeled bars as an infinite large system of particles. Indeed, obtaining a uniform stabilization is one of the most required properties of the system. Among different beam models. Timoshenko model has proved to be simple and it considers enough complexity to include shear deformation.

Main results

In this work, we show that the solution of Timoshenko system with past history and dynamical boundary given by

$$\begin{aligned} \rho w_{tt}(x, t) - \kappa [\varphi(x, t) + w_x(x, t)]_x &= 0, \quad \text{in } (0, L) \times \mathbb{R}_+, \\ I_\rho \varphi_{tt}(x, t) - EI \varphi_{xx}(x, t) + \int_0^\infty g(s) \varphi_{xx}(x, t-s) ds \\ + \kappa [\varphi + w_x](x, t) &= 0, \quad \text{in } (0, L) \times \mathbb{R}_+, \end{aligned} \quad (69)$$

With the following initial conditions

$$w(x, 0) = w_0(x), \quad w_t(x, 0) = w_1(x), \quad \varphi(x, 0) = \varphi_0(x), \quad \varphi_t(x, 0) = \varphi_1(x).$$

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and the boundary conditions

$$\begin{aligned} w(x, 0) = \varphi(x, 0) &= 0, \quad \text{in } (0, L), \\ Mw_{tt}(L, t) - \kappa[\varphi + w_x](L, t) &= 0, \quad \text{in } \mathbb{R}_+, \\ J\varphi_{tt}(L, t) - EI\varphi_x(L, t) + \int_0^\infty g(s)\varphi_x(L, t)ds &= 0, \quad \text{in } \mathbb{R}_+, \end{aligned} \quad (70)$$

where t, x denotes respectively the time variable and the space variable along the beam of length L in its equilibrium configuration, $S = k(w_x + \varphi)$ and $M = \varphi_x$ denote the shear force and the bending moment, respectively. We denote by $w = w(x, t)$ the transversal displacement and $\varphi = \varphi(x, t)$ is the rotation angle of the filament.

decays exponential in the case that the wave speeds of equations are different, our method is based on the semi group technique and the contraction argument of frequency domain method and the multiplier method .

Assumptions on the kernels

We suppose that the kernels $g(t)$ is a $C^1(\mathbb{R}_+, \mathbb{R}_+)$ is nondecreasing function satisfying:

$$\tilde{b} = 1 - \int_0^\infty g(s)ds = 1 - g_\infty = l > 0. \quad (71)$$

$$g(t) > 0, \exists k_0, k_1 > 0 : -k_0 g(t) \leq g'(t) \leq -k_1 g(t), \quad (72)$$

Our result on the exponential stability is based. We have the following theorem

Theorem 26. *Let us suppose that the initial data satisfy*

$$w_0, \varphi_0 \in H^1(0, L), w_1, \varphi_1 \in L^2(0, L),$$

and that the coefficients of system (69) and (70) satisfy

$$\frac{\rho}{I_p} = \frac{k}{EI}.$$

Moreover, assume that the kernel g is of exponential type satisfying (72). Then the energy $E(t)$ decays exponentially as time tends to infinity, that is, there exist positive constant C and α , being independent of the initial data, such that for $t \geq 0$:

$$E(t) \leq CE(0)e^{-\alpha t}. \quad (73)$$

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Nonexistence of global weak solutions of nonlinear wave equation with friction and viscoelastic damping

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Abstract: In this work, we prove the non-existence of global solutions to damped wave equation with viscoelastic damping and nonlinear memory term for any space dimension $n \geq 1$. Our main interest is to study the mixed effect of the term damping and nonlinear memory on blow-up conditions. The problem we consider is parabolic-like from the point of view of the decay of energy at infinity. For this reason we use the test function method to prove our results.

Keywords: Damped wave equation, nonlinear memory, Blow-up, Critical exponent, Test function method.

Introduction

The damped wave equation with nonlinear memory

$$u_{tt} - \Delta u + u_t = \int_0^t (t - \tau)^\gamma |u(\tau, \cdot)|^p d\tau$$

is considered by Fino in 2011 (see[3]) where he studied the global (in time) existence and the blow-up together, In particular, he showed that the critical exponent of this problem is :

$$p^*(\gamma) = \max\{p_\gamma, \frac{1}{\gamma}\},$$

where $p_\gamma = 1 + \frac{2(2-\gamma)}{(n-2+2\gamma)_+}$.

In the other hand, we have the following Cauchy problem for the doubly dissipative wave equation

$$\begin{cases} u_{tt} - \Delta u + u_t - \Delta u_t = 0, & (t, x) \in (0, \infty) \times \mathbb{R}^n, \\ (u, u_t)(0, x) = (u_0, u_1)(x) & x \in \mathbb{R}^n, \end{cases} \quad (74)$$

The profile of the solution to (74) has been recently studied in [4], in the L^2 setting, by assuming data in the energy space.

In 2022, Djaouti and Latif [4].demonstrated the global (in time) existence of small data solutions

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to (75) when the data are supposed to belong to different classes of regularity, but does not investigate blow-up results.

$$\begin{cases} u_{tt} - \Delta u + u_t - \Delta u_t = \int_0^t (t - \tau)^{-\gamma} |u(\tau, \cdot)|^p d\tau, \\ (u, u_t)(0, x) = (u_0, u_1)(x) \end{cases} \quad (75)$$

In this paper, we study the blow-up solutions to the Cauchy problem for the wave equation with frictional and viscoelastic damping and nonlinear memory term to (75). For this reason we apply the test function method for proving our results.

Main results

Theorem 27. Let $0 < \gamma < 1$, $p \in (1, \infty)$ for $n = 1, 2$ and $1 < p < \frac{n}{n-2}$ for $n \geq 3$. Assume that $(u_0, u_1) \in H^1(\mathbb{R}^n) \times L^2(\mathbb{R}^n)$ and satisfy

$$\int_{\mathbb{R}^n} u_0(x) dx > 0, \quad \int_{\mathbb{R}^n} u_1(x) dx > 0. \quad (76)$$

Then,

If $p \leq p^*$ then the solutions of Cauchy problem (75) does not exist globally in time.
where

$$p^* = \max\{p_\gamma, \gamma^{-1}\}, \text{ and } p_\gamma = 1 + \frac{2(2-\gamma)}{(n-2+2\gamma)_+}. \quad (77)$$

Conclusion

In this paper, we add another model that has the same critical exponent with the damped wave equation with nonlinear memory in the paper [3], from it, we can say that the viscoelastic term does not influence the critical exponent.

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Existence Results for Some Nonlinear Parabolic Equations with Degenerate Coercivity and Singular Lower-Order Terms

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Abstract: In this paper, we study the existence results for some parabolic equations with degenerate coercivity, singular lower order term depending on the gradient, and positive initial data in L^1 .

Keywords: singular equation, nonlinear parabolic equations, degenerate coercivity

2020 Mathematics Subject Classification: 35K55, 35K65, 35K67.

Introduction

In this paper, we study the existence and regularity results for following parabolic problem:

$$(P) \quad \begin{cases} \partial_t u + Lu + B \frac{|\nabla u|^p}{u^\theta} = u^r & \text{in } Q_T = (0, T) \times \Omega, \\ u(0, x) = u_0(x) \geq 0 & \text{in } \Omega, \\ u = 0 & \text{on } \Gamma_T = (0, T) \times \partial\Omega. \end{cases}$$

where $T > 0$, $B > 0$ are real numbers, $u_0 \in L^1(\Omega)$, Ω is a bounded open subset of $\mathbb{R}^N$ ($N > 2$) with boundary denoted by $\partial\Omega$, p is a real number such that $p \geq 2$ and L is the operator given by

$$Lu = -\operatorname{div}(A(t, x, u)|\nabla u|^{p-2}\nabla u).$$

Here, we suppose that $A : (0, T) \times \mathbb{R}^N \times \mathbb{R} \rightarrow \mathbb{R}$ are Carathéodory function and satisfying for almost every $(t, x) \in (0, T) \times \Omega$, $\forall s \in \mathbb{R}$, the following:

$$\frac{\beta}{(a(t, x) + |s|)^\rho} \leq A(t, x, s) \leq \alpha, \quad (78)$$

where α, β are strictly positive real numbers and $\rho \geq 0$, $a : (0, T) \times \mathbb{R}^N \rightarrow \mathbb{R}$ is measurable function non-negative verifying

$$a(t, x) \leq \delta, \quad (79)$$

where δ is strictly positive real number. We furthermore suppose that

$$0 < \theta < 1, \quad 0 < r < p - \theta \quad (80)$$

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Bounded initial data ($u_0 \in L^\infty(\Omega)$)

In this section, we prove that there exists a weak solution of problem (P) for u_0 bounded. For this, we will use the result in [1] and then an L^∞ -estimate procedure introduced by [3].

Given a real positive number k , we will define the functions

$$T_k(r) = \begin{cases} k, & \text{if } r \geq k, \\ r, & \text{if } |r| < k, \\ -k, & \text{if } r \leq -k. \end{cases} \quad r \in \mathbb{R}.$$

Its primitive $\Theta_k : \mathbb{R} \rightarrow \mathbb{R}^+$ defined by

$$\Theta_k(r) = \int_0^r T_k(t)dt = \begin{cases} \frac{r^2}{2}, & \text{if } |r| \leq k \\ k|r| - \frac{k^2}{2}, & \text{if } |r| > k \end{cases}$$

we will then use the following results

$$\int_0^T \langle \partial_t v, T_k(v) \rangle dt = \int_\Omega \Theta_k(v(T))dx - \int_\Omega \Theta_k(v(0))dx, \quad (81)$$

and

$$k|r| - \frac{k^2}{2} \leq \Theta_k(r) \leq k|r|, \quad \forall r \in \mathbb{R}. \quad (82)$$

Definition 14. A function u is a weak solution of problem (P) if $u \in L^p(0, T; W_0^{1,p}(\Omega)) \cap L^\infty(Q_T)$, such that $\partial_t u \in L^{p'}(0, T; W^{-1,p'}(\Omega)) + L^1(Q_T)$, $u > 0$, $\frac{|\nabla u|^p}{u^\theta}$ belongs to $L^1(Q_T)$, and

$$\begin{aligned} & \int_\Omega u(T)\varphi(T)dx - \int_\Omega u(0)\varphi(0)dx + \int_{Q_T} u \partial_t \varphi dx dt \\ & + \int_{Q_T} A(t, x, u) |\nabla u|^{p-2} \nabla u \nabla \varphi dx dt + B \int_{Q_T} \frac{|\nabla u|^p}{u^\theta} \varphi dx dt = \int_{Q_T} u^r \varphi dx dt, \end{aligned}$$

for every $\varphi \in L^p(0, T; W_0^{1,p}(\Omega)) \cap L^\infty(Q_T)$ such that $\partial_t \varphi \in L^{p'}(0, T; W^{-1,p'}(\Omega)) + L^1(Q_T)$.

Remark 2. Notice that, because of the fact that $u \in C([0, T]; L^2(\Omega))$ (see [5]), the functions $\varphi(T)$ and $\varphi(0)$ in the above definition have sense and the meaning of the initial condition $u(0) = u_0$ is clear.

Theorem 28. Let $p \geq 2$, $u_0 \in L^\infty(\Omega)$, suppose that (80) hold true. Then, the problem (P) has at least one weak solution $u \in L^p(0, T; W_0^{1,p}(\Omega)) \cap L^\infty(Q_T) \cap C([0, T]; L^1(\Omega))$.

L^1 Initial Data

Theorem 29. Given $u_0 \in L^1(\Omega)$, suppose that (80) hold true. Then the problem (P) has at least a weak solution u , i.e., a function u belonging to $L^q(0, T; W_0^{1,q}(\Omega)) \cap C([0, T]; L^1(\Omega))$, $u > 0$, $\frac{|\nabla u|^p}{u^\theta} \in L^1(Q_T)$, such that

$$\begin{aligned} & - \int_\Omega u(0)\varphi(0)dx + \int_{Q_T} u \partial_t \varphi dx dt + \int_{Q_T} A(t, x, u) |\nabla u|^{p-2} \nabla u \nabla \varphi dx dt \\ & + B \int_{Q_T} \frac{|\nabla u|^p}{u^\theta} \varphi dx dt = \int_{Q_T} u^r \varphi dx dt, \quad q = p - \frac{\theta N}{N+1}, \end{aligned} \quad (83)$$

for every $\varphi \in W^{1,\infty}(0, T; L^\infty(\Omega))$ and such that $\varphi(T) = 0$ in Ω .

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Problem of Caputo fractional differential equations of variable order with boundary value

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Abstract: In this manuscript, the existence, uniqueness, and stability of solutions to the multiterm boundary value problem of Caputo fractional differential equations of variable order are established. All results in this study are established with the help of the generalized intervals and piece-wise constant functions, we convert the Caputo fractional variable order to an equivalent standard Caputo of the fractional constant order. Further, two fixed point theorems due to Schauder and Banach are used, the Ulam–Hyers stability of the given Caputo variable order is examined, and finally, we construct an example to illustrate the validity of the observed results. In literature, the existence of solutions to the variable-order problems is rarely discussed. Therefore, investigating this interesting special research topic makes all our results novel and worthy.

Keywords: fractional differential equations of variable order, boundary value problem, fixed point theorem, Green function, Ulam-Hyers stability.

Introduction

The idea of fractional calculus is to replace the natural numbers in the derivative's order with rational ones. Although it seems an elementary consideration, it has an exciting correspondence explaining some physical phenomena. Especially in the last two decades, significant numbers of papers appeared on this topic, some papers deal with the existence of solutions to problems of variable order, see e.g. ([1, 6, 7, 9, 6]).

In particular, [2] Bai *et al.* studied the following problem

$$\begin{cases} {}^c D_{0+}^u x(t) = f(t, x(t), I_{0+}^u x(t)), & t \in [a, b], \quad u \in]0, 1], \\ x(a) = x_a, \end{cases}$$

where ${}^c D_{0+}^u$ and I_{0+}^u stand for the Caputo-Hadamard derivative and Hadamard integral operators of order u , respectively, f is a given function, $x_a \in \mathbb{R}$, and $0 < a < b < \infty$.

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Inspired [2] and ([1, 6, 7, 9, 6]), we deal with the boundary value problem (BVP)

$$\begin{cases} {}^c D_{0^+}^{u(t)} x(t) + f_1(t, x(t), I_{0^+}^{u(t)} x(t)) = 0, & t \in J := [0, T] \\ x(0) = 0, \quad x(T) = 0, \end{cases} \quad (84)$$

where $1 < u(t) \leq 2$, $f_1 : J \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function and ${}^c D_{0^+}^{u(t)}$, $I_{0^+}^{u(t)}$ are the Caputo fractional derivative and integral of variable-order $u(t)$.

In this paper, we shall look for a solution of (84). Further, we study the stability of the obtained solution of (84) in the sense of Ulam-Hyers (UH).

Preliminaries

This section introduces some important fundamental definitions that will be needed for obtaining our results in the next sections.

The symbol $C(J, \mathbb{R})$ represents the Banach space of continuous functions $\varkappa : J \rightarrow \mathbb{R}$ with the norm

$$\|\varkappa\| = \sup\{|\varkappa(t)| : t \in J\},$$

For $-\infty < a_1 < a_2 < +\infty$, we consider the mappings $u(t) : [a_1, a_2] \rightarrow (0, +\infty)$ and $v(t) : [a_1, a_2] \rightarrow (n-1, n)$. Then, the left Caputo fractional integral (CFI) of variable-order $u(t)$ for function $f_2(t)$ ([4], [5], [5]) is

$$I_{a_1^+}^{u(t)} f_2(t) = \int_{a_1}^t \frac{(t-s)^{u(t)-1}}{\Gamma(u(t))} f_2(s) ds, \quad t > a_1, \quad (85)$$

and the left Caputo fractional derivative (CFD) of variable-order $v(t)$ for function $f_2(t)$ ([4], [5], [5]) is

$${}^c D_{a_1^+}^{u(t)} f_2(t) = \int_{a_1}^t \frac{(t-s)^{n-u(t)-1}}{\Gamma(n-u(t))} f_2^{(n)}(s) ds, \quad t > a_1. \quad (86)$$

As anticipated, in case of $u(t)$ and $v(t)$ are constant, then CFI and CFD are coincide with the standard Caputo fractional derivative and integral, respectively see e.g. [1, 4, 5].

Existence of solutions

Let us introduce the following assumption:

(H1) Let $n \in \mathbb{N}$ be an integer, $\mathcal{P} = \{J_1 := [0, T_1], J_2 := (T_1, T_2], J_3 := (T_2, T_3], \dots, J_n := (T_{n-1}, T]\}$ be a partition of the interval J , and let $u(t) : J \rightarrow (1, 2]$ be a piecewise constant function with respect to $\mathcal{P}$, i.e.,

$$u(t) = \sum_{\ell=1}^n u_\ell I_\ell(t) = \begin{cases} u_1, & \text{if } t \in J_1, \\ u_2, & \text{if } t \in J_2, \\ \cdot & \\ \cdot & \\ \cdot & \\ u_n, & \text{if } t \in J_n, \end{cases}$$

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where $1 < u_\ell \leq 2$ are constants, and I_ℓ is the indicator of the interval $J_\ell := (T_{\ell-1}, T_\ell]$, $\ell = 1, 2, \dots, n$, (with $T_0 = 0$, $T_n = T$) such that

$$I_\ell(t) = \begin{cases} 1, & \text{for } t \in J_\ell, \\ 0, & \text{for elsewhere.} \end{cases}$$

For each $\ell \in \{1, 2, \dots, n\}$, the symbol $E_\ell = C(J_\ell, \mathbb{R})$, indicated the Banach space of continuous functions $x: J_\ell \rightarrow \mathbb{R}$ equipped with the norm

$$\|x\|_{E_\ell} = \sup_{t \in J_\ell} |x(t)|,$$

Then, for any $t \in J_\ell$, $\ell = 1, 2, \dots, n$ the left caputo fractional derivative of variable order $u(t)$ for function $x(t) \in C(J, \mathbb{R})$, defined by (86), could be presented as a sum of left caputo fractional derivatives of constant-orders u_ℓ , $\ell = 1, 2, \dots, n$

$${}^c D_{0+}^{u(t)} x(t) = \int_0^{T_1} \frac{(t-s)^{1-u_1}}{\Gamma(2-u_1)} x^{(2)}(s) ds + \dots + \int_{T_{\ell-1}}^t \frac{(t-s)^{1-u_\ell}}{\Gamma(2-u_\ell)} x^{(2)}(s) ds. \quad (87)$$

Thus, according to (87), (BVP)(84) can be written for any $t \in J_\ell$, $\ell = 1, 2, \dots, n$ in the form

$$\int_0^{T_1} \frac{(t-s)^{1-u_1}}{\Gamma(2-u_1)} x^{(2)}(s) ds + \dots + \int_{T_{\ell-1}}^t \frac{(t-s)^{1-u_\ell}}{\Gamma(2-u_\ell)} x^{(2)}(s) ds + f_1(t, x(t), I_{0+}^{u_\ell} x(t)) = 0, \quad t \in J_\ell. \quad (88)$$

Definition 15. BVP (84) has a solution, if there are functions $x_\ell, \ell = 1, 2, \dots, n$, so that, $x_\ell \in C([0, T_\ell], \mathbb{R})$ fulfilling equation (88) and $x_\ell(0) = 0 = x_\ell(T_\ell)$.

Let the function $x \in C(J, \mathbb{R})$ be such that $x(t) \equiv 0$ on $t \in [0, T_{\ell-1}]$ and it solves integral equation (88). Then (88) is reduced to

$${}^c D_{T_{\ell-1}+}^{u_\ell} x(t) + f_1(t, x(t), I_{T_{\ell-1}+}^{u_\ell} x(t)) = 0, \quad t \in J_\ell.$$

We shall deal with following BVP

$$\begin{cases} {}^c D_{T_{\ell-1}+}^{u_\ell} x(t) + f_1(t, x(t), I_{T_{\ell-1}+}^{u_\ell} x(t)) = 0, & t \in J_\ell \\ x(T_{\ell-1}) = 0, x(T_\ell) = 0. \end{cases} \quad (89)$$

For our purpose, the upcoming lemma will be a corner stone of the solution of BVP (89).

Lemma 10. Let $\ell \in \{1, 2, \dots, n\}$ be a natural number, $f_1 \in C(J_\ell \times \mathbb{R} \times \mathbb{R}, \mathbb{R})$ and there exists a number $\delta \in (0, 1)$ such that $t^\delta f_1 \in C(J_\ell \times \mathbb{R} \times \mathbb{R}, \mathbb{R})$.

Then, the function $x \in E_\ell$ is a solution of BVP (89) if and only if x solves the integral equation

$$x(t) = \int_{T_{\ell-1}}^{T_\ell} G_\ell(t, s) f_1(s, x(s), I_{T_{\ell-1}+}^{u_\ell} x(s)) ds, \quad (90)$$

where $G_\ell(t, s)$ is the Green's function defined by:

$$G_\ell(t, s) = \begin{cases} \frac{1}{\Gamma(u_\ell)} \left[(T_\ell - T_{\ell-1})^{-1} (t - T_{\ell-1})(T_\ell - s)^{u_\ell-1} - (t - s)^{u_\ell-1} \right], & T_{\ell-1} \leq s \leq t \leq T_\ell, \\ \frac{1}{\Gamma(u_\ell)} (T_\ell - T_{\ell-1})^{-1} (t - T_{\ell-1})(T_\ell - s)^{u_\ell-1}, & T_{\ell-1} \leq t \leq s \leq T_\ell, \end{cases}$$

where $\ell = 1, 2, \dots, n$.

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Theorem 30. Let the conditions of Lemma 10 be satisfied and there exist a constants $K, L > 0$, such that,

$t^\delta |f_1(t, y_1, z_1) - f_1(t, y_2, z_2)| \leq K|y_1 - y_2| + L|z_1 - z_2|$, for any $y_i, z_i \in \mathbb{R}, i = 1, 2, t \in J_\ell$. and the inequality

$$\frac{(T_\ell^{1-\delta} - T_{\ell-1}^{1-\delta}) \left((T_\ell - T_{\ell-1}) \left(1 - \frac{1}{u_\ell} \right) \right)^{u_\ell-1}}{(1-\delta)\Gamma(u_\ell+1)} (K + L \frac{(T_\ell - T_{\ell-1})^{u_\ell}}{\Gamma(u_\ell+1)}) < 1. \quad (91)$$

holds.

Then, BVP (89) possesses at least one solution in E_ℓ .

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Comparison between Normal Operators and n-Normal Operators on Hilbert spaces

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Abstract: This paper is devoted to provide a general comparison between normal and n-normal operators, we give some definitions, properties, and applications in Hilbert spaces. Clearly every normal operator is n-normal, the converse is false. But if an operator A is n-normal and hyponormal, then it is normal. Furthermore, if A is an operator normal and A^k is compact for some k , then A is compact, this need not be true in the case of n-normal operators.

Keywords: Normal operator, n-Normal operator, hyponormal operator, normaloid operator, paranormal operator, Hilbert spaces.

2020 Mathematics Subject Classification: Primary 47A05, 47B15, 47B20, 47B02

Introduction

Throughout this paper, $B(H)$ denotes to the algebra of all bounded linear operators acting on a complex Hilbert space H .

Among the various types of operators, normal operators hold a prominent position. An operator T on a Hilbert space H is said to be normal if it commutes with its adjoint T^* . In other words, $TT^* = T^*T$. Normal operators exhibit fascinating properties, such as spectral decomposition, which allows them to be diagonalized by a unitary transformation.

On the other hand, n-normal operators are a generalization of normal operators. An operator T on a Hilbert space H is said to be n-normal if it satisfies the condition $T^n T^* = T^* T^n$ for some positive integer n . This condition implies a weaker form of commutativity compared to normal operators. The study of n-normal operators extends the understanding of normal operators and offers a broader framework for analyzing linear transformations. First, we recall the Fugled theorem which will be needed to prove some of our results.

Theorem 31. *If T is a bounded operator, and N is normal and not necessarily bounded, then*

$$TN \subseteq NT \Rightarrow TN^* \subseteq N^*T.$$

Main results

Proposition 2. *Let $T \in B(H)$. Then T is n-normal if and only if T^n is normal where $n \in \mathbb{N}$.*

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Proof Assume that T is n -normal, then

$$T^n(T^*)^n = T^*T^n(T^*)^{n-1} = T^*(T^nT^*)(T^*)^{n-2} = (T^*)^2T^n(T^*)^{n-2} = (T^*)^nT^n.$$

Then T^n is normal.

Now, assume that T^n is normal, then by Fuglede theorem

$$T^nT = TT^n \Rightarrow T^nT^* = T^*T^n$$

Therefore T is n -normal. ■

It is clear that a bounded normal operator is n -normal for any n . The converse is not true. Indeed

if $T = \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}$ then T is 2-normal which is not normal.

Proposition 3. Suppose T is both k -normal and $(k+1)$ -normal for some positive integer k . Then T is $(k+2)$ -normal. And hence T is n -normal for all $n \geq k$.

Proof Since T is k -normal, $T^kT^* = T^*T^k$. Hence $TT^kT^*T = TT^*T^kT$. So $T^{k+1}T^*T = TT^*T^{k+1}$. Since T is $(k+1)$ -normal, $T^*T^{k+2} = T^{k+2}T^*$. Thus T is $(k+2)$ -normal. ■

Corollary 1. If T is 2-normal and 3-normal, then T is an n -normal for all $n \geq 2$.

The following example shows a 2-normal and 3-normal operator may not be normal.

Example 1. Let $S = \begin{pmatrix} 0 & 0 \\ a & 0 \end{pmatrix}$ be an operator acting on two-dimensional Complex Hilbert space. Then T is 2-normal, 3-normal, and hence it is n -normal for all $n \geq 2$ but it is not normal.

Lemma 11. Let T be K -normal and $(k+1)$ -normal. If either T or T^* is injective, then T is normal.

Proof Since T is $(k+1)$ -normal, $T^{k+1}T^* = T^*T^{k+1}$. And since T is k -normal, $T^{k+1}T^* = T^kT^*T$. Hence $T^k(TT^* - T^*T) = 0$. Since T is injective, $TT^* - T^*T = 0$. Thus T is normal. In case T^* is injective, since T^* is k -normal and $(k+1)$ -normal, T^* is normal. Hence T is normal. ■

And all nonzero nilpotent operators are n -normal operators, for $n \geq k$ where k the index of nilpotance, but they are not normal.

It is well known that if T is normal, then it is hyponormal. And if T is normal and T^k is compact for some k , then T is compact by [4]. The following example shows that these need not be true in case of n -normal operator.

Example 2. Let $H = l^2$ and $e_1, e_2, \dots$ be standard orthogonal basis for l^2 . Define T on H by

$$Te_i = \begin{cases} e_1, & i = 1 \\ e_{i+1}, & i = 2j \\ 0, & i = 2j+1 \end{cases}, j = 1, 2, \dots$$

Where $j = 1, 2, \dots$ P is the orthogonal projection on the space spanned by e_1 . So T is 2-normal but neither T nor T^* is hyponormal.

Now, since T^2 is a projection on one-dimensional space, it is compact. However, since range of T contains an infinite orthonormal set $\{e_i, i = 1, 3, 5, \dots\}$, T is not compact.

Theorem 32. If S, T are commuting n -normal operators, then ST is an n -normal operator.

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Proof Since S, T are commuting n -normal operators, S^n, T^n are commuting normal operator. So $S^n T^n$ is a normal operator. Since $S^n T^n = (ST)^n$, $(ST)^n$ is normal. Hence ST is n -normal. ■

The following example shows that sum of two commuting n -normal operators need not be n -normal.

Example 3. Let $S = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $T = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$. Then S and T are commuting 2-normal. But $S + T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, $(S + T)^2 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ is not normal. Thus $S+T$ is not 2-normal. We note here S is a self adjoint operator.

The following theorem shows that sum of two commuting normal operators is normal.

Theorem 33. If A and B are two normal operators and commute with each other, then $A + B$ is a normal operator.

Proof We have

$$(A + B)(A + B)^* = (A + B)(A^* + B^*) = AA^* + AB^* + BA^* + B^*B$$

and similarly,

$$(A + B)^*(A + B) = (A^* + B^*)(A + B) = A^*A + A^*B + B^*A + B^*B$$

But, since each of A, A^*, B , and B^* commutes with all the others, as a result of the Fugled theorem, the two products are equal. Hence $A + B$ is a normal operator. ■

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Singular $p(x)$ -Laplace Equations with Lower Order Terms and Hardy Potential

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Abstract: The present paper is concerned by the study of a nonlinear elliptic equation which contains a Hardy potential, lower order term, singular term and $L^{m(\cdot)}$ data. Our approach is based on approximating the initial problem with a non-singular problem that is well-posed. We then establish the necessary estimates to pass to the limit.

2020 Mathematics Subject Classification: 35J70, 35J60, 35B65, 35J75.

Introduction

In this paper, we study the existence of solutions for the following nonlinear elliptic equation.

$$\begin{cases} -\Delta_{p(x)} u + \nu |u|^{s(x)-2} u = \mu \frac{|u|^{p(x)-1}}{|x|^{p(x)}} + \frac{f}{u^{\gamma(x)}}, & \text{in } \Omega, \\ u > 0, & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega. \end{cases} \quad (92)$$

Here $\nu > 0$, $\mu \geq 0$ and the operator $\Delta_{p(x)} u$ is defined as

$$\Delta_{p(x)} u = \operatorname{div}(|\nabla u|^{p(x)-2} \nabla u),$$

with Ω being a bounded, open subset of $\mathbb{R}^N$ ($N > 2$) and having a Lipschitz boundary $\partial\Omega$. The functions $p, s, m, \gamma \in C(\overline{\Omega})$ satisfy the following conditions

$$1 < p^- \leq p(x) \leq p^+ < N, \quad 1 < m(x) < \frac{N}{p(x)}, \quad |\nabla m| \in L^\infty(\Omega), \quad (93)$$

$$p^+ = \max_{x \in \overline{\Omega}} p(x), \quad p^- = \min_{x \in \overline{\Omega}} p(x),$$

$$s^+ > s(x) > p^*(x), \quad p^*(x) = \frac{p(x)N}{N - p(x)}, \quad |\nabla s| \in L^\infty(\Omega), \quad (94)$$

and

$$0 \leq \gamma(x) < 1, \quad |\nabla \gamma| \in L^\infty(\Omega). \quad (95)$$

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Problem (92) has been widely applied in various scientific fields such as electro-rheological fluids, image processing ... ect. For this reason this type of equations has received a significant amount of attention in the last years.

In the following, we cite some papers that have discussed the equation (92) .

Firstly, this paper has been motivated by the work in[3]. In that work the authors studied the existence and the regularity of solutions for the problem (92) when $\gamma(x) = 0$ for all $x \in \overline{\Omega}$. Then, in the case when $\mu = 0$ the authors in [2] discussed the existence and regularity of solutions. Also they proved that the lower order term has some regularizing effects on the solutions.

Finally, for constant exponent cases, ie ($p(x) = p, s(x) = r, \gamma(x) = \gamma$ for all $x \in \overline{\Omega}$), Sbair, El Hadfi and Ouardy in their work [5] proved the existence and regularity of the solutions under the assumption $0 \leq f \in L^m(\Omega); 1 < m < \frac{N}{p}$.

In this work, due to the nonlinearity of the $p(x)$ -Laplace operator, the Hardy potential and the singularity term in the setting of Sobolev spaces with variable exponent, the classical methods cannot be applied directly to our problem this is why we use an approximation technique.

Main results

Definition 16. Let $f \in L^{m(\cdot)}(\Omega)$ with $m(x) > 1$, we say that $u \in W_0^{1,1}(\Omega)$ solves problem (92) in the distributional sense if it meets the following conditions

$$|\nabla u|^{p(x)-1} \in L_{loc}^1(\Omega), \quad v|u|^{s(x)-2}u \in L_{loc}^1(\Omega), \quad \mu \frac{|u|^{p(x)-1}}{|x|^{p(x)}} + \frac{f}{u^{\gamma(x)}} \in L_{loc}^1(\Omega),$$

and one has the identity

$$\begin{aligned} & \int_{\Omega} |\nabla u|^{p(x)-2} \nabla u \cdot \nabla \varphi dx + v \int_{\Omega} |u|^{s(x)-2} u \varphi dx \\ &= \int_{\Omega} \left(\mu \frac{|u|^{p(x)-1}}{|x|^{p(x)}} + \frac{f}{u^{\gamma(x)}} \right) \varphi dx, \quad \forall \varphi \in C_c^1(\Omega). \end{aligned} \quad (96)$$

Our main results are as follows:

Theorem 34. Suppose that assumptions (93), (94), and (95) hold true. In addition, suppose that f is a non-negative function in $L^{m(\cdot)}(\Omega)$, where m is a continuous function satisfying this condition for all $x \in \overline{\Omega}$

$$\widehat{m}(x) = 1 + \frac{1 - \gamma(x)}{s(x) - 1 + \gamma(x)} \leq m(x) < \frac{N}{p(x)} \frac{s(x) - p(x)}{s(x) - 1 + \gamma(x)}. \quad (97)$$

Then, there exists a solution u to problem (95), as defined in Definition 16, that satisfies

$$u \in W_0^{1,p(\cdot)}(\Omega) \cap L^{m(\cdot)(s(\cdot)-1+\gamma(\cdot))}(\Omega).$$

Theorem 35. Let us assume that (93), (94) and (95) hold true, and that f is a non-negative function in $L^{m(\cdot)}(\Omega)$, where m is a continuous function satisfying the following condition for all $x \in \overline{\Omega}$:

$$1 < m(x) < \widehat{m}(x). \quad (98)$$

If $p(x)$ and $m(x)$ satisfy one of the following assumptions for every $x \in \overline{\Omega}$:

$$\widehat{p}(x) = 1 + \frac{(1 - \gamma(x))(N - 1)}{N + 1 - \gamma(x)} \leq p(x) < N, \quad 1 < m(x) < \widehat{m}(x), \quad (99)$$

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or

$$1 < p(x) < \widehat{p}(x), \quad \frac{\widehat{m}(x)}{p(x)} < m(x) < \widehat{m}(x), \quad (100)$$

then, there exists a solution u to problem (95), as defined in Definition 16, and it belongs to the function space

$$W_0^{1,q(\cdot)}(\Omega) \cap L^{m(\cdot)(s(\cdot)-1+\gamma(\cdot))},$$

where $q(x)$ is well-defined for all $x \in \overline{\Omega}$

$$q(x) = \frac{m(x)(s(x) - 1 + \gamma(x))}{s(x)} p(x). \quad (101)$$

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Non-Instantaneous Impulsive Stochastic Integro-differential Equations Driven by a Fractional Brownian Motion

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Abstract: This study addresses a class of non-instantaneous impulsive stochastic integro-differential equations (ISIDEs) with nonlocal conditions driven by a fractional Brownian motion (FBM) in a Hilbert space. We employed the Kuratowski's measure of non-compactness, a resolvent operator, and the generalized Darbo's fixed point theorem to prove the existence of mild solutions for a class of non-instantaneous ISIDEs driven by FBM. Examples are given to illustrate the effectiveness of the proposed results.

Keywords: Stochastic integro-differential equations, fractional Brownian motion, non-instantaneous impulses, fixed point theory.

2020 Mathematics Subject Classification: 60J65, 60H20, 37C25.

Introduction

This paper is concerned with the existence of mild solution for a class of non-instantaneous ISIDEs driven by a FBM accompanied by nonlocal conditions. The system under discussion takes the following form:

$$\begin{cases} dy(t) = Ay(t)dt + \int_0^t h(t-s)y(s)ds + G(t, y(t))dt + F(t)dB^H(t), & t \in \cup_{k=0}^N (s_k, t_{k+1}], \\ y(t) = g_k(t, y(t_k^-)), & t \in \cup_{k=1}^N (t_k, s_k], \\ y(0) + \varphi(y) = y_0 \in \mathbb{H}. \end{cases} \quad (102)$$

Where A is the infinitesimal generator of a strongly continuous semigroup $\{T(t)\}_{t \geq 0}$ of bounded linear operators on a separable Hilbert space $\mathbb{H}$ with domain $\mathbb{D}(A)$, $h : \mathbb{D}(h) \rightarrow \mathbb{H}$ is a closed linear operator on $\mathbb{H}$ with domain $\mathbb{D}(h) \subset \mathbb{D}(A)$, G, φ and $F : [0, T] \rightarrow \mathcal{L}_2^0(\mathbb{V}, \mathbb{H})$ are appropriate functions, where $\mathcal{L}_2^0(\mathbb{V}, \mathbb{H})$ denotes the space of all Hilbert-Schmidt operators from $\mathbb{V}$ into $\mathbb{H}$. Also, $B^H(t)$ is a fractional Brownian motion with Hurst parameter $H \in (\frac{1}{2}, 1)$ defined on a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathcal{P})$ with values on a separable Hilbert space $\mathbb{V}$. Let $0 = s_0 = t_0 < t_1 \leq s_1 < t_2 \leq \dots < t_N \leq s_N < t_{N+1} = T$ where $T > 0$ is a constant $g_k : (t_k, s_k] \times \mathbb{H} \rightarrow \mathbb{H}$ is called non-instantaneous impulsive function, for all $k = 1, 2, \dots, N$. Where the state $y(\cdot)$ takes values on a space $\mathbb{H}$.

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Main results

In this section, we prove the existence of mild solutions for the system (102). Consider the Banach space

$$\mathcal{PC}([0, T], \mathbb{H}) = \left\{ \mathcal{F}_t\text{-adapted } \mathbb{H}\text{-valued process } y(t) \text{ is continuous every where except for some } t \neq t_k \right. \\ \left. \text{at which } y(t_k^-) \text{ and } y(t_k^+) \text{ exist and } y(t_k^-) = y(t_k), \quad k = 1, 2, \dots, N \text{ and } \sup_{0 \leq t \leq T} |y(t)|^2 < \infty \right\},$$

with the norm

$$\|y\|_{\mathcal{PC}} = \left(\sup_{0 \leq t \leq T} \|y(t)\|^2 \right)^{\frac{1}{2}}.$$

We also need the followinga generalization of the Darbo fixed point theorem to prove our theorem.

Theorem 36. [2] (Generalized Darbo's fixed point theorem) Let I be a closed and convex subset of a real Banach space $\mathbb{H}$. Suppose that $Q : I \rightarrow I$ is a continuous operator and $Q(I)$ is bounded, for any bounded subset $C \subset I$, $Q^1(C) = Q(C)$, $Q^n(C) = Q(\overline{\text{co}}(Q^{n-1}(C)))$, $n = 2, 3 \dots N$. If there exists a constant $0 \leq \lambda < 1$, and a positive integer n_0 such that for any bounded subset $C \subset I$.

$$\alpha(Q^{n_0}(C)) \leq \lambda \alpha(Q(C)). \quad (103)$$

Then Q has at least one fixed point in C .

We now introduce the concept of mild solution of (102), we present the following definitions

Definition 17. A $\mathcal{F}_t$ -adapted stochastic process $y(t) : [0, T] \rightarrow \mathbb{H}$ is called a mild solution of (102) if $y \in \mathcal{PC}([0, T], \mathbb{H})$, $y(0) + \varphi(y) = y_0$ and

$$y(t) = \begin{cases} \mathcal{R}(t)(y_0 - \varphi(y)) + \int_0^t \mathcal{R}(t-s)G(s, y(s))ds + \int_0^t \mathcal{R}(t-s)F(s)dB^H(s), & t \in [0, t_1], \\ g_k(t, y(t_k^-)), & t \in \cup_{k=1}^N (t_k, s_k], \\ \mathcal{R}(t-s_k)g_k(s_k, y(t_k^-)) + \int_{s_k}^t \mathcal{R}(t-s)G(s, y(s))ds + \int_{s_k}^t \mathcal{R}(t-s)F(s)dB^H(s), & t \in \cup_{k=1}^N (s_k, t_{k+1}]. \end{cases} \quad (104)$$

Now, we make the following assumptions:

(H₁) The operator A is the infinitesimal generator of a strongly continuous semigroup $\{T(t)\}_{t \geq 0}$ on $\mathbb{H}$.

(H₂) For all $t \geq 0$, $h(t)$ is a closed linear operator from $\mathbb{D}(h)$ to $\mathbb{H}$ and $h(t) \in L_b(\mathbb{V}, \mathbb{H})$. For any $y \in \mathbb{V}$ the map $t \rightarrow h(t)y$ is bounded, differentiable and the derivative $t \rightarrow h'(t)y$ is bounded and uniformly continuous on $\mathbb{R}^+$.

(H₃) The resolvent operator $\mathcal{R}(t)$, $t \geq 0$ is continuous in operator norm topology, and there exists a constant $M > 0$ such that $\|\mathcal{R}(t)\| \leq M$.

(H₄) The nonlinear function $G : I \times \mathcal{PC}([0, T], \mathbb{H}) \rightarrow \mathbb{H}$ satisfies

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- (1) For each $y \in \mathcal{PC}([0, T], \mathbb{H})$, $G(., y)$ is measurable and for any $t \in I$, $G(t, .)$ is continuous.
- (2) For some positive number $r > 0$, there exists a constant $k_1 > 0$, function $\omega \in L^1(I, \mathbb{R}^+)$ and a continuous non-decreasing function $\pi : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ such that

$$\mathbb{E}\|G(t, y)\|^2 \leq \omega(t)\pi_\varphi(\mathbb{E}\|y\|^2), \quad \liminf_{r \rightarrow +\infty} \frac{\pi_\varphi(r)}{r} = k_1 < +\infty.$$

- (3) There exists a positive constant k_2 such that for any bounded set $M \subset \mathbb{H}$

$$\alpha(G(t, M)) \leq k_2 \alpha(M).$$

(H₅) The function $F : [0, T] \rightarrow \mathcal{L}_2^0(\mathbb{V}, \mathbb{H})$, satisfying the following condition $\int_0^t \|F(s)\|_{\mathcal{L}_2^0}^2 ds < \infty$.

(H₆) The nonlocal function $\varphi : \mathcal{PC}([0, T], \mathbb{H}) \rightarrow \mathbb{H}$ is continuous and compact, and there exists a constant $C_\varphi > 0$, such that $\mathbb{E}\|\varphi(y)\|^2 \leq C_\varphi$.

(H₇) The impulsive function $g_k : (t_k, s_k] \times \mathbb{H} \rightarrow \mathbb{H}$ is continuous and compact, and there exist constants $C_{g_k} > 0$, $k = 1, 2, \dots, N$, such that $\mathbb{E}\|g_k(t, y)\|^2 \leq C_{g_k} \mathbb{E}\|y\|^2$.

where we have used the notation $C_g = \max_{k=1,2,\dots,N} C_{g_k}$, $\eta = \max_{k=1,2,\dots,N} \|\omega\|_{L^1[s_k, t_{k+1}]}$.

Theorem 37. Suppose that (H₁)-(H₇) are satisfied, then the problem (102) has at least one mild solution provided that

$$L := \max_{0 \leq k \leq N} \left\{ 3M^2 \left(C_g + (t_{k+1} - s_k) \eta k_1 \right) \right\} < 1.$$

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Unicité et Stabilité dans un problème inverse pour un système couplé ondes-ondes

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Résumé: Cet exposé concerne le problème inverse de la récupération des coefficients potentiels inconnus pour un système couplé ondes-ondes dans un domaine borné de $\Omega \subset \mathbb{R}^n$, avec une donnée de Dirichlet non homogène à partir d'une mesure aux limites de Neumann dépendant du temps. on présente des résultats d'unicité et de stabilité de Lipschitz pour ce problème inverse. L'idée principale pour obtenir ces résultats est basée essentiellement sur une inégalité d'observabilité et une estimation de Carleman.

Mots clés: Problèmes inverses, Système couplé ondes-ondes, Inégalités de Carleman, Inégalité d'observabilité, Unicité, stabilité.

2020 Mathematics Subject Classification: 35R30, 35Q40, 49K20.

Introduction

Tout au long de cette étude, nous utilisons les notations suivantes :

$$Q = \Omega \times [0, T], \Gamma = \partial\Omega = \overline{\Gamma_0 \cup \Gamma_1}, \Gamma_0 \cap \Gamma_1 = \emptyset, \Sigma = \Gamma \times [0, T], \Sigma_1 = \Gamma_1 \times [0, T].$$

On considère le système couplé ondes-ondes suivant

$$w_{tt}(x, t) - \Delta w(x, t) = q(x)z(x, t) \quad \text{dans } Q, \quad (105a)$$

$$z_{tt}(x, t) - \Delta z(x, t) = p(x)w(x, t) \quad \text{dans } Q, \quad (105b)$$

$$w(x, \frac{T}{2}) = w_0(x), w_t(x, \frac{T}{2}) = w_1(x) \quad \text{dans } \Omega, \quad (105c)$$

$$z(x, \frac{T}{2}) = z_0(x), z_t(x, \frac{T}{2}) = z_1(x) \quad \text{dans } \Omega, \quad (105d)$$

$$w(x, t) = g_1(x, t), z(x, t) = g_2(x, t) \quad \text{sur } \Sigma. \quad (105e)$$

Nous considérons le problème inverse de la détermination des coefficients q , et p pour le système couplé (105a)-(105e) à partir d'une observation dépendante du temps des données de Neumann $\frac{\partial w}{\partial \nu}, \frac{\partial z}{\partial \nu}$ sur la frontière. Cela signifie que nous prouverons l'unicité et la stabilité du problème inverse non linéaire caractérisé par l'application non linéaire

$$\{q, p\}|_{\Omega} \rightarrow \left\{ \frac{\partial w}{\partial \nu}, \frac{\partial z}{\partial \nu} \right\} \Big|_{\Sigma}.$$

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Dans cette étude, nous traitons deux types de problèmes inverses :

Problème inverse non linéaire. Nous répondrons plus précisément aux questions suivantes :

Unicité dans le problème inverse non linéaire ?

Stabilité dans le problème inverse non linéaire ?

Problème inverse linéaire. Après avoir soustrait le $\{w, z\}$ -système (105a)-(105e) à coefficients (q_1, p_1) du $\{w, z\}$ -système (105a)-(105e) à coefficients (q_2, p_2) , on obtient le système suivant

$$u_{tt}(x, t) - \Delta u(x, t) = q(x)v(x, t) + f(x)R_1(x, t) \quad \text{dans } Q, \quad (106a)$$

$$v_{tt}(x, t) - \Delta v(x, t) = p(x)u(x, t) + g(x)R_2(x, t) \quad \text{dans } Q, \quad (106b)$$

$$u(., \frac{T}{2}) = 0, \quad u_t(., \frac{T}{2}) = 0 \quad \text{dans } \Omega, \quad (106c)$$

$$v(., \frac{T}{2}) = 0, \quad v_t(., \frac{T}{2}) = 0 \quad \text{dans } \Omega, \quad (106d)$$

$$u(x, t) = 0, \quad v(x, t) = 0 \quad \text{sur } \Sigma. \quad (106e)$$

Le $\{u, v\}$ -système a l'avantage sur le $\{w, z\}$ -système de (105a)-(105e) que l'application

$$\{f, g\}|_{\Omega} \rightarrow \left\{ \frac{\partial u}{\partial \nu}, \frac{\partial v}{\partial \nu} \right\} \Big|_{\Sigma}$$

est linéaire. De même, dans le problème (106a)-(106e), nous répondrons plus précisément aux questions suivantes :

Unicité dans le problème inverse linéaire ?

Stabilité dans le problème inverse linéaire ?

Résultats principaux

Hypothèses H. Il existe une fonction strictement convexe (à valeurs réelles) non négative $d : \overline{\Omega} \rightarrow \mathbb{R}_+$ de classe $C^3(\overline{\Omega})$, telle que

(H_{ii}) La matrice Hessienne (symétrique) H_d de $d(\cdot)$ est strictement définie positive sur $\overline{\Omega}$, il existe une constante $\rho > 0$, telle que pour tout $x \in \overline{\Omega}$

$$H_d(x) = J_h(x) \geq \rho I. \quad (107)$$

(H_{iii}) $d(\cdot)$ n'a pas de point critique sur $\overline{\Omega}$

$$\inf_{x \in \Omega} |h(x)| = \inf_{x \in \Omega} |\nabla d(x)| = s > 0. \quad (108)$$

(H_{iv})

$$\frac{\partial d}{\partial \nu} \Big|_{\Gamma_0} = \nabla d \cdot \nu = h \cdot \nu \leq 0, \quad \text{où } h = \nabla d. \quad (109)$$

Theorem 38. (Unicité du problème inverse linéaire). Nous supposons que les hypothèses (H_{ii}), (H_{iii}), et (H_{iv}), sont vérifiées. Dans le $\{u, v\}$ -système (106a)-(106e), nous supposons

$$T > T_0 \equiv 2 \sqrt{\max_{x \in \overline{\Omega}} d(x)}, \quad (110)$$

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$$\begin{aligned} q, p \in L^\infty(\Omega), \quad f, g \in L^2(\Omega), \\ R_i, R_{it}, R_{itt} \in L^\infty(Q), \quad R_{ix_j} \left(\cdot, \frac{T}{2} \right) \in L^\infty(\Omega) \quad i = 1, 2, \quad 1 \leq j \leq n, \end{aligned} \quad (111)$$

$$\left| R_1 \left(x, \frac{T}{2} \right) \right| \geq r_1 > 0, \quad \left| R_2 \left(x, \frac{T}{2} \right) \right| \geq r_2 > 0, \quad x \in \overline{\Omega}, \quad (112)$$

avec r_1, r_2 sont des constantes positives. Alors l'implication suivante est elle vraie

$$\left\{ \begin{array}{l} \frac{\partial u(f,g)}{\partial v} \Big|_{\Sigma_1} = 0 \\ \frac{\partial v(f,g)}{\partial v} \Big|_{\Sigma_1} = 0 \end{array} \right\} \implies \left\{ \begin{array}{l} f = 0 \\ g = 0 \end{array} \right. \quad \text{sur } \Omega.$$

Theorem 39. (Unicité du problème inverse non linéaire). Nous supposons que les hypothèses (H_{ii}) , (H_{iii}) , et (H_{iv}) sont vérifiées. Soit T comme dans (110). Soient $\{w(q_1, p_1), z(q_1, p_1)\}$, et $\{w(q_2, p_2), z(q_2, p_2)\}$ les solutions du $\{w, z\}$ -système (105a)-(105e) correspondantes aux coefficients (q_1, p_1) , (q_2, p_2) respectivement, supposons en plus

$$q_1, p_1, q_2, p_2 \in L^\infty(\Omega), \quad (113)$$

$$w, z, w_t, z_t, w_{tt}, z_{tt} \in C([0, T], H_0^1(\Omega)) \cap C^1([0, T], L^2(\Omega)), \quad (114)$$

$$w_{0x_j}, z_{0x_j} \in L^\infty(\Omega), \quad 1 \leq j \leq n, \quad (115)$$

$$|w_0(x)| \geq w_0 > 0, \quad |z_0(x)| \geq z_0 > 0, \quad x \in \overline{\Omega}. \quad (116)$$

Alors l'implication suivante est elle vraie

$$\left\{ \begin{array}{l} \frac{\partial w(q_1, p_1)}{\partial v} \Big|_{\Sigma_1} = \frac{\partial w(q_2, p_2)}{\partial v} \Big|_{\Sigma_1} \\ \frac{\partial z(q_1, p_1)}{\partial v} \Big|_{\Sigma_1} = \frac{\partial z(q_2, p_2)}{\partial v} \Big|_{\Sigma_1} \end{array} \right\} \implies \left\{ \begin{array}{l} q_1 = q_2 \\ p_1 = p_2 \end{array} \right. \quad \text{sur } \Omega.$$

Theorem 40. (Stabilité du problème inverse linéaire). Considérons le $\{u, v\}$ -système (106a)-(106e). On suppose que les hypothèses (H_{ii}) , (H_{iii}) , et (H_{iv}) , (110), (111), et (112) sont vérifiées. Alors il existe une constante $C = C(\Omega, T, \Gamma_1, \varphi, q, p, R_1, R_2) > 0$, c'est-à-dire en fonction des données du problème (106a)-(106e), mais pas sur les coefficients inconnus f et g , telle que

$$\|f\|_{L^2(\Omega)} + \|g\|_{L^2(\Omega)} \leq C \left\{ \left\| \frac{\partial u(f,g)}{\partial v} \right\|_{H^1(0,T, L^2(\Gamma_1))} + \left\| \frac{\partial v(f,g)}{\partial v} \right\|_{H^1(0,T, L^2(\Gamma_1))} \right\} \quad (117)$$

pour tous $f, g \in L^2(\Omega)$.

Theorem 41. (Stabilité du problème inverse non linéaire). Nous supposons que les hypothèses (H_{ii}) , (H_{iii}) , et (H_{iv}) sont vérifiées. Considérons le $\{w, z\}$ -système (105a)-(105e) sur $[0, T]$, avec $T > T_0$ comme dans (110), une fois avec des coefficients potentiels $(q_1, p_1) \in L^\infty(\Omega) \times L^\infty(\Omega)$, et une fois avec les coefficients potentiels $(q_2, p_2) \in L^\infty(\Omega) \times L^\infty(\Omega)$, et soient $\{w(q_1, p_1), z(q_1, p_1)\}$, $\{w(q_2, p_2), z(q_2, p_2)\}$ les solutions correspondantes. Alors il existe une constante $C = C(\Omega, T, \Gamma_1, \varphi, M, w_0, w_1, z_0, z_1, g_1, g_2) > 0$, telle que

$$\begin{aligned} & \|q_1 - q_2\|_{L^2(\Omega)} + \|p_1 - p_2\|_{L^2(\Omega)} \\ & \leq C \left\{ \left\| \frac{\partial w(q_1, p_1)}{\partial v} - \frac{\partial w(q_2, p_2)}{\partial v} \right\|_{H^1(0,T, L^2(\Gamma_1))} + \left\| \frac{\partial z(q_1, p_1)}{\partial v} - \frac{\partial z(q_2, p_2)}{\partial v} \right\|_{H^1(0,T, L^2(\Gamma_1))} \right\}. \end{aligned} \quad (118)$$

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Extended Jacobi elliptic function expansion method to solve the time- fractional PHI - four equation

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Abstract: In this paper, the time fractional $(1 + 1)$ - dimensional The e PHI-four equation, which reads[1]

$$\frac{\partial^\alpha}{\partial t^\alpha} \frac{\partial^\alpha u(x, t)}{\partial t^\alpha} - \frac{\partial^2 u(x, t)}{\partial x^2} + \lambda^2 u(x, t) + \mu u^3(x, t) = 0, \quad (119)$$

is solved analytically, where $0 < \alpha \leq 1$ and $\frac{\partial^\alpha}{\partial t^\alpha}(\cdot)$ in the Sense of the new conformable derivative. The exact solutions for the (TFPHI-F) have been presented using the Jacobi Elliptic Function Rational Expansion Method combined with the fractional complex transformation. The variety of traveling wave solutions indicates that this powerful method is effective. With the help of **Mathematica 11** software package, we managed to solve difficult nonlinear algebraic systems and plotting surfaces and Contour Plots of some particular solutions.

Keywords: The PHI - four equation ,conformable derivative, fractional complex transformation, exact solution.

2020 Mathematics Subject Classification: Primary 26A33, 35C07, 35C08.

Introduction

In recent years, searching exact traveling wave solutions to nonlinear evolution equations (NLEEs) has become a remarkable topic of research. In this work, we obtain exact traveling wave solutions of significant NLEEs, namely, the PHI-four equation involving parameters by using extended Jacobi elliptic function expansion method[3].

Preliminaries

New Fractional Derivative

Definition 18. [2] Let $f : [0, +\infty) \rightarrow \mathbb{R}$ be a function. α^{th} order "conformable fractional derivative" of f is defined by

$$D_\alpha(f)(t) = \lim_{\epsilon \rightarrow 0} \frac{f(te^{\epsilon t^{-\alpha}}) - f(t)}{\epsilon} \quad (120)$$

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for all $t > 0, \alpha \in (0, 1)$. If f is α -differentiable in some $(0, a), a > 0$ and $\lim_{t \rightarrow 0^+} f^{(\alpha)}(t)$ exists then define $f^{(\alpha)}(0) = \lim_{t \rightarrow 0^+} f^{(\alpha)}(t)$.

The Extended Jacobi Elliptic Function Expansion Method

The general time-fractional nonlinear evolution equation, say in tow independent variables x and t , is given by Consider a given time-fractional nonlinear PDE, say in tow variables x and t

$$P(u, D_t^\alpha u, D_{tt}^{2\alpha} u, u_x, D_t^\alpha u_x, u_{xx} \dots) = 0, \quad (121)$$

Where $0 < \alpha \leq 1$, $u = u(x, t)$ is an unknown function and P is a polynomial of u and its partial fractional derivatives. To construct the traveling wave solution of (121) by the extended Jacobi elliptic function expansion method [4], **we follow the following steps:**

Step 1: To obtain exact traveling wave solution, the following fractional complex transformation has been applied

$$\xi = kx - \omega \frac{t^\alpha}{\alpha}, \quad u(x, t) = U(\xi) \quad (122)$$

Where k, ω are constants to be discussed latter. Then, the Eq. (121) is reduced to the following nonlinear ordinary differential equation

$$H(U, \omega U', \omega^2 U'', kU', k\omega U', k^2 U'', \dots) = 0, \quad (123)$$

Step 2: The key step of the new method is to suppose that Eq. (126) has the formal solution

$$U(\xi) = a_0 + \sum_{i=1}^N a_i z(\xi) + \sum_{i=1}^N b_i z(\xi)^{-i} \quad (124)$$

where $z(\xi) \in \{\text{sn}(\xi|m), \text{cn}(\xi|m), \text{dn}(\xi|m), \text{sc}(\xi|m), \text{sd}(\xi|m), \text{cs}(\xi|m)\}$.

Here, a_0, a_i and b_i ($i = 1, 2, \dots$) are constants which to be determined later

Step 3: The degree N of the power series (124) is determined by considering the homogeneous balance between the nonlinear term in Eq. (123) and the highest-order derivative. We define the degree of $U(\xi)$ is $D[U(\xi)] = N$, for example,

$$D \left[U^p(\xi) \left(\frac{\partial^\beta U(\xi)}{\partial \xi^\beta} \right)^q \right] = N_p + q(\beta + N) \quad (125)$$

Step 4: Substituting Eq. (124) using (122) into Eq.(123). Then collecting the coefficients of like powers of $z(\xi)^i$ where $i = 0, 1, \dots$. By setting them to zero, we get a system of algebraic equations, and Solving the over-determined system of nonlinear algebraic equations by use of **Mathematica software package**, we would end up with explicit expressions fo $a_0, a_i, b_i, k, \omega (i = 1, 2, \dots)$

Step 5: Finally, Substitute the values of $a_0, a_i, b_i, k, \omega (i=1, 2, \dots)$ into (124), we obtain the exact traveling wave solutions of the fractional nonlinear evolution equation (123).

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Main results

Application of the method to TFPHI-four

By using (122), we obtain the following ODE

$$(\omega^2 - k^2)U'' + \lambda^2 U + \mu U^3 = 0 \quad (126)$$

Now, considering the homogeneous balance between the highest order linear term U'' and non-linear term of the highest order U^3 in equation (126), we obtain $3N = N + 2$, so $N = 1$ we can choose the following form solution of (126)

$$U(\xi) = a_0 + a_1 z(\xi) + b_1 \frac{1}{z(\xi)} \quad (127)$$

where $z(\xi) \in \{\text{sn}(\xi|m), \text{cn}(\xi|m), \text{dn}(\xi|m), \text{sc}(\xi|m), \text{sd}(\xi|m), \text{cs}(\xi|m)\}$.

Substituting Eq. (127) using (122) into Eq.(126). Then collecting the coefficients of like powers of $z(\xi)^i$ where $i = 0, 1, \dots$. By setting them to zero, we get a system of algebraic equations, and Solving the over-determined system of nonlinear algebraic equations by use of **Mathematica software package**, we would end up with explicit expressions for a_0, a_1, b_1, k, ω

Finally, Substitute the values of a_0, a_1, b_1, k, ω into (127), we obtain the exact traveling wave solutions of the fractional nonlinear evolution equation (126).

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Applying Darbo's Fixed Point Criterion to Analyze Stability and Solutions in Hadamard Nonlinear Variable Order Problems: An Ulam-Hyers-Rassias Approach

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Abstract: The existence aspects along with the stability of solutions to a Hadamard variable order fractional boundary problem (VOFBVP) are investigated in this research study. Our results are obtained via generalized intervals and piece-wise constant functions and the relevant Green function, by converting the existing Hadamard VOFBVP to an equivalent standard Hadamard FBVP of the fractional constant order. Further, Darbo's fixed point criterion along with Kuratowski's measure of noncompactness (KMNCS) are used in this direction. As well as, the Ulam-Hyers-Rassias stability (UHRS) of the proposed Hadamard VOFBVP is established. A numerical example is presented to express our results' validity.

Keywords: Variable order; Darbo criterion; Green function; Ulam-Hyers-Rassias stability.

2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

Introduction

Fractional calculus is fundamentally established by having arbitrary numbers in the order of derivation operators instead of natural numbers. This idea is considered preliminary and simple. However, it involves remarkable effects and outcomes which describe some physical processes, dynamics, mathematical modelings, control theory, bioengineering, and biomedical applications and etc [1, 2].

In this paper, motivated by other related works in this regard, we investigate the solutions' existence of the Hadamard nonlinear VOFBVP as follows:

$${}^{\mathcal{H}}\mathcal{D}_{1+}^{\Psi(t)}r(t) + m_1(t, r(t)) = 0, \quad t \in \mathcal{U} := [1, \mathcal{T}], \quad (128)$$

via boundary conditions $r(1) = r(\mathcal{T}) = 0$, where $1 < \mathcal{T} < +\infty$, $1 < \Psi(t) \leq 2$, $m_1 : \mathcal{U} \times \mathcal{S} \rightarrow \mathcal{S}$ is a continuous function ($\mathcal{S}$ is a real (or complex) space) and ${}^{\mathcal{H}}\mathcal{D}_{1+}^{\Psi(t)}$ specifies the Hadamard derivative of variable order $\psi(t)$.

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Preliminaries

The symbol $\mathcal{C}(\mathcal{U}, \mathcal{S})$ denotes a set that contains all continuous functions $\aleph : \mathcal{U} \rightarrow \mathcal{S}$. It is a Banach space by defining

$$\|\aleph\| = \sup\{|\aleph(t)| : t \in \mathcal{U}\}, \quad \mathcal{U} := [1, T].$$

Definition 19. [3, 4] For $1 \leq a_1 < a_2 < +\infty$, we consider the mappings $\psi(t) : [a_1, a_2] \rightarrow (0, +\infty)$ and $q(t) : [a_1, a_2] \rightarrow (n-1, n)$. The Hadamard $(\Psi(t))^{th}$ -variable order integral of h_1 is

$$\mathcal{H}\mathcal{I}_{a_1^+}^{\Psi(t)} h_1(t) = \frac{1}{\Gamma(\Psi(t))} \int_{a_1}^t \left(\log \frac{t}{s}\right)^{\Psi(t)-1} \frac{h_1(s)}{s} ds, \quad t > a_1, \quad (129)$$

and the Hadamard $(q(t))^{th}$ -variable order derivative of h_1 is

$$(\mathcal{H}\mathcal{D}_{a_1^+}^{q(t)} h_1)(t) = \frac{1}{\Gamma(n-q(t))} \left(t \frac{d}{dt}\right)^n \int_{a_1}^t \left(\log \frac{t}{s}\right)^{n-q(t)-1} \frac{h_1(s)}{s} ds, \quad t > a_1. \quad (130)$$

Obviously, in case of $\Psi(t)$ and $q(t)$ are constant, then both above Hadamard variable order operators are in coincidence with the usual Hadamard constant order operators (Refer to [1, 3, 4]).

Existence of solutions

To achieve the main purpose of this section, some assumptions are proposed as:

(H1) Consider $\mathcal{F} = \{\mathcal{U}_1 := [1, T_1], \mathcal{U}_2 := (T_1, T_2], \mathcal{U}_3 := (T_2, T_3], \dots, \mathcal{U}_n := (T_{n-1}, T]\}$ as a partition for the interval $\mathcal{U}$, and $\Psi(t) : \mathcal{U} \rightarrow (1, 2]$ as a piecewise constant function w.r.t $\mathcal{F}$, i.e.,

$$\Psi(t) = \sum_{j=1}^n \Psi_j \mathcal{J}_j(t) = \begin{cases} \Psi_1, & \text{if } t \in \mathcal{U}_1, \quad 1 < \Psi_1 \leq 2, \\ \Psi_2, & \text{if } t \in \mathcal{U}_2, \quad 1 < \Psi_2 \leq 2, \\ \cdot & \\ \cdot & \\ \cdot & \\ \Psi_n, & \text{if } t \in \mathcal{U}_n, \quad 1 < \Psi_n \leq 2, \end{cases}$$

in which $\mathcal{J}_j$ interprets the indicator of $\mathcal{U}_j := (T_{j-1}, T_j]$, $j \in \mathbb{N}_1^n$, so that $T_0 = 1$ and $T_n = T$, and

$$\mathcal{J}_j(t) = \begin{cases} 1, & \text{for } t \in \mathcal{U}_j, \\ 0, & \text{for elsewhere.} \end{cases}$$

(H2) Let $(\log t)^\beta m_1 : \mathcal{U} \times \mathcal{S} \rightarrow \mathcal{S}$ be continuous, $(0 \leq \beta \leq 1)$, and $\exists K > 0$, such that $(\log t)^\beta \|m_1(t, r) - m_1(t, \bar{r})\| \leq K \|r - \bar{r}\|$, for any $r, \bar{r} \in \mathcal{S}$ and $t \in \mathcal{U}$.

First, we analyze the Hadamard VOFBVP defined in (128). In the light of (130), the Hadamard VOFBVP (128) can be rewritten by

$$\frac{1}{\Gamma(2-\Psi(t))} \left(t \frac{d}{dt}\right)^2 \int_1^t \left(\log \frac{t}{s}\right)^{1-\Psi(t)} \frac{r(s)}{s} ds + m_1(t, r(t)) = 0, \quad t \in \mathcal{U}. \quad (131)$$

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From (H1), the equation (131) on the interval $\mathcal{U}_j, j \in \mathbb{N}_1^n$, can be expressed as:

$$\begin{aligned} & \left(t \frac{d}{dt}\right)^2 \left(\frac{1}{\Gamma(2-\Psi_j)} \int_1^{\mathcal{T}_j} \left(\log \frac{t}{s}\right)^{1-\Psi_j} \frac{r(s)}{s} ds + \dots \right. \\ & \left. + \frac{1}{\Gamma(2-\Psi_j)} \int_{\mathcal{T}_{j-1}}^t \left(\log \frac{t}{s}\right)^{1-\Psi_j} \frac{r(s)}{s} ds \right) + m_1(t, r(t)) = 0, \quad t \in \mathcal{U}_j. \end{aligned} \quad (132)$$

Definition 20. The Hadamard VOFBVP (128) admits a solution like functions $r_j, j = 1, 2, \dots, n$, if $r_j \in \mathcal{C}([1, \mathcal{T}_j], \mathcal{S})$ satisfies the equation (132) and $r_j(1) = 0 = r_j(\mathcal{T}_j)$.

From above, the Hadamard VOFBVP (128) written in (131) can be given as (132) on $\mathcal{U}_j, j \in \mathbb{N}_1^n$. For $1 \leq t \leq \mathcal{T}_{j-1}$, put $r(t) \equiv 0$. Then (132) is formulated by

$$D_{\mathcal{T}_{j-1}^+}^{\Psi_j} r(t) + m_1(t, r(t)) = 0, \quad t \in \mathcal{U}_j.$$

In this case, we follow our study by considering the standard Hadamard constant-order FBVP (COFBVP) as follows:

$$\begin{cases} \mathcal{H} D_{\mathcal{T}_{j-1}^+}^{\Psi_j} r(t) + m_1(t, r(t)), & t \in \mathcal{U}_j \\ r(\mathcal{T}_{j-1}) = 0, r(\mathcal{T}_j) = 0. \end{cases} \quad (133)$$

The fundamental part of our analysis regarding solutions of the Hadamard COFBVP (133) is discussed below.

Lemma 12. A function $r \in \mathcal{E}_j$ is a solution of the Hadamard COFBVP (133) iff r fulfills the integral equation

$$r(t) = \int_{\mathcal{T}_{j-1}}^{\mathcal{T}_j} \frac{1}{s} \mathbb{G}_j(t, s) m_1(s, r(s)) ds, \quad t \in \mathcal{U}_j, \quad (134)$$

where $\mathbb{G}_j(t, s)$ stands for the Green function formulated by:

$$\mathbb{G}_j(t, s) = \frac{1}{\Gamma(\Psi_j)} \begin{cases} \left(\left(\log \frac{\mathcal{T}_j}{\mathcal{T}_{j-1}} \right)^{1-\Psi_j} \left[\left(\log \frac{t}{\mathcal{T}_{j-1}} \right) \left(\log \frac{\mathcal{T}_j}{s} \right) \right]^{\Psi_j-1} - \left(\log \frac{t}{s} \right)^{\Psi_j-1}, & \mathcal{T}_{j-1} \leq s \leq t \leq \mathcal{T}_j, \\ \left(\log \frac{\mathcal{T}_j}{\mathcal{T}_{j-1}} \right)^{1-\Psi_j} \left[\left(\log \frac{t}{\mathcal{T}_{j-1}} \right) \left(\log \frac{\mathcal{T}_j}{s} \right) \right]^{\Psi_j-1}, & \mathcal{T}_{j-1} \leq t \leq s \leq \mathcal{T}_j, \end{cases} \quad (135)$$

where $j \in \mathbb{N}_1^n$.

Theorem 42. Suppose that both (H1) and (H2) hold, and

$$\frac{K \left((\log \mathcal{T}_j)^{1-\beta} - (\log \mathcal{T}_{j-1})^{1-\beta} \right) \left(\log \mathcal{T}_j - \log \mathcal{T}_{j-1} \right)^{\Psi_j-1}}{4^{\Psi_j-1} (1-\beta) \Gamma(\Psi_j)} < 1. \quad (136)$$

Then, there is a solution to the Hadamard VOFBVP (128) on $\mathcal{U}$.

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UHRS stability

Definition 21. ([?]) Let $\rho \in \mathcal{C}(\mathcal{U}, \mathcal{S})$. Then, the Hadamard VOFBVP (128) is Ulam-Hyers-Rassias stable (UHRS) w.r.t ρ if $\exists c_m > 0$, so that $\forall \epsilon > 0$ and for every solution $\varsigma \in \mathcal{C}(\mathcal{U}, \mathcal{S})$ of

$$\|\mathcal{H}\mathcal{D}_{1+}^{\Psi(t)}\varsigma(t) - (-m_1(t, \varsigma(t)))\| \leq \epsilon\rho(t), \quad t \in \mathcal{U}, \quad (137)$$

$\exists$ a solution $r \in \mathcal{C}(\mathcal{U}, \mathcal{S})$ of (128) with

$$\|\varsigma(t) - r(t)\| \leq c_m\epsilon\rho(t).$$

Theorem 43. Let both (H1) and (H2) along with (136) hold. Also,

(H3) Let $\rho \in \mathcal{C}(\mathcal{U}_j, \mathcal{S})$ is a increasing function and $\exists \lambda_\rho > 0$ provided

$$\mathcal{H}\mathcal{I}_{\mathcal{T}_{j-1}+}^{\Psi_j}\rho(t) \leq \lambda_{\rho(t)}\rho(t), \quad \text{for any } t \in \mathcal{U}_j.$$

Then, the Hadamard VOFBVP (128) is UHRS stable w.r.t the function ρ .

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Uniform stabilisation of a thermoelastic system with static Wentzell conditions

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Abstract: In this worke, we consider a thermoelastic system with static Wentzell boundary conditions. We show the well-posedness, using the semigroup theory, and establish an explicit decay rate result, using the multiplier method. More precisely, we prove an exponential decay result under the influence of competing both boundary damping bu' and localised interior damping $l(x)u'$. Our method consists in establishing a crucial fundamental identity and exploiting it to derive the decay rates.

Keywords: Thermoelasticity; exponential stabilisation; Wentzell conditions; boundary feed-backs; localised damping

Introduction

Let Ω be a bounded domain of $\mathbb{R}^3$ with a boundary $\Gamma = \partial\Omega$ of class C^3 . In this work, we will study the uniform stabilisation for the following thermoelastic system with static Wentzell boundary conditions:

$$\begin{cases} u'' - \operatorname{div} \sigma(u) + \xi \nabla \theta + l(x)u' = 0 & \text{in } Q = \Omega \times]0, \infty[, \\ \theta' - \Delta \theta + \beta \operatorname{div} u' = 0 & \text{in } Q, \\ \theta = 0, & \text{in } \Sigma = \Gamma \times]0, \infty[, \\ \sigma_s(u) - \operatorname{div}_T \sigma_T^0(u) + au_T + bu' = 0, & \text{on } \Sigma \\ \sigma_v(u) - \sigma_T^0(u) : \partial_T v + au_v + bu' = 0, & \text{on } \Sigma \\ u(\cdot, 0) = u_0, u'(\cdot, 0) = u_1, \theta(\cdot, 0) = \theta_0, & \text{in } \Omega, \end{cases} \quad (138)$$

where $u(x, t) \in \mathbb{R}^3$ and $\theta = \theta(x, t)$ denote the displacement and temperature deviation, respectively, at $x = (x_1, x_2, x_3) \in \Omega$ and t is the time variable. We denote by the derivative with respect to the time variable. $\sigma(u) = (\sigma_{ij}(u))_{i,j=1}^3$ is the stress tensor given by $\sigma(u) = 2\alpha \varepsilon(u) + \lambda \operatorname{div}(u)I_3$, where $\lambda, \alpha > 0$ are the Lamé coefficients, $\xi, \beta > 0$ the coupling parameters, I_3 is the identity matrix of $\mathbb{R}^3$ and $\varepsilon(u) = \frac{1}{2}(\nabla u + \overline{\nabla} u) = [\varepsilon_{ij}(u)]_{i,j=1}^3$ is a 3×3 symmetric matrix. Here and in what follows we shall use the summation convention for repeated indices. Moreover, let $x \in \Gamma$ we denote by $\pi(x)$ the projection from $\mathbb{R}^3$ on the tangent plane $T_x(\Gamma)$ and for a given vector field u we have, for any point x of Γ , $u(x) = u_T(x) + u_v(x)v(x)$, where $u_T(x) = \pi(x)(u(x))$ is the tangential component of u , $u_v(x) = u(x) \cdot v(x)$ and v represents the unit outward normal field to Γ . $\sigma_s(u) = 2\alpha \varepsilon_s(u)$, $\sigma_v(u) = (\lambda + 2\alpha)\varepsilon_v(u_v) + \lambda \operatorname{tr}_2(\varepsilon_T(u))$, and $\sigma_T^0(u) : \partial_T v = \operatorname{tr}(\sigma_T^0(u) \cdot \partial_T v)$ with :

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$$\sigma_T^0(u) = 2\alpha\varepsilon_T^0(u) + \frac{2\lambda\alpha}{\lambda + 2\alpha} \text{tr}(\varepsilon_T^0(u))I_2, \quad (139)$$

where $\varepsilon_T^0(u) = \varepsilon_T(u) = \frac{1}{2}(\pi\partial_T u_t \pi + \overline{\pi\partial_T u_t \pi}) + u_\nu(\partial_T \nu)$, $\varepsilon_s(u) = \frac{1}{2}(\partial_\nu u_T - (\partial_T \nu)u_T + \overline{\partial_T u_\nu})$ and $\varepsilon_\nu(u) = \partial_\nu u_\nu$. whereas $\partial_T \nu$ denote the curvature operator on Γ , ∂_T (respectively ∂_ν) denote the tangential (respectively normal) derivative, I_2 is the identity of the tangent plane and ' tr ' means the trace of a matrix. Furthermore, $\text{div}_T \sigma_T^0(u)$ is the tangential divergence of the endomorphism field $\sigma_T^0(u)$, $a = a(x)$ and $b = b(x)$ are two nonnegative functions belong to $C^1(\Gamma)$ and $l(x) \in L^\infty(\Omega)$ is a nonnegative coefficient of the damping term such that

$$l(x) \geq l_0 \geq 0 \text{ over } \vartheta \subset \Omega, \quad (140)$$

where ϑ is a neighbourhood of the whole boundary

$$\Gamma = \{x \in \Gamma : m(x) \cdot \nu(x) > 0\}. \quad (141)$$

Main results

Theorem 44. Assume that the hypothesis (140) and (141) are in place. Let (u, θ) be the weak solution of the coupled system (138) then there exist two positive constants M and ω such that

$$\mathcal{E}(t) \leq \mathcal{E}(0)e^{1-\omega t} \quad (142)$$

In order to simplify the notations, we shall write (sometimes) m_ν, q_ν, u_ν , instead of $m \cdot \nu, q \cdot \nu, u \cdot \nu$, etc.

This theorem give us the exponential stablition of the problem (138).

First step we calculate the functional energy of the problem (138) then we estimate It,

$$2 \int_0^T \mathcal{E}(t)dt \leq K_1 \mathcal{E}(0) + \int_0^T \mathcal{E}(t)dt + K_2 \int_{\Sigma_T} \sigma_T^0(u) : \varepsilon_T^0(u) + a|u|^2 d\Gamma dt$$

where K_1 and K_2 are positive constants such that $k_1 = C_1 + C_2 + C_9 + C_{10} + C_{11}$ and $k_2 = C_5 + C_6 + C_{10} + \max(C_4 + C_7, C_8 + C_9)$. Then,

$$(1 - \eta k_2) \int_0^T \mathcal{E}(t)dt \leq \left(K_1 + \frac{\tilde{C}k_2}{\eta} \right) \mathcal{E}(0) \quad (143)$$

Choosing δ so that $0 < \eta < \frac{1}{k_2}$ and letting $T \rightarrow +\infty$, by a wellknown argument (see for instance [komornik, Theorem 8.1]), we conclude that

$$\mathcal{E}(t) \leq \mathcal{E}(0)e^{1-\omega t} \quad (144)$$

where $\omega = \frac{\eta(1-\eta k_2)}{k_1 \eta + \tilde{C}k_2}$, is a positive constant depending on $\text{meas}(\Omega), \text{meas}(\Gamma), \alpha, \lambda, \beta, \xi, R_0, \eta, \delta, \rho, \kappa, d, \varepsilon, \|l(x)\|_{L^\infty}, a_0$ and b_0 .

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Well-posedness and exponential stability of thermoelastic shear model

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Abstract: In this presentation, we consider a thermoelastic shear beam model system with one dissipative mechanism. We prove a well posedness result by the use of Faedo–Galerkin method and an exponential stability by the multiplier method.

Keywords: Shear beam model, well–posedness, exponential stability, Faedo–Galerkin method.

2020 Mathematics Subject Classification: 35B40; 65N12; 74F05; 74K99.

Introduction

In this work we consider the system

$$\begin{cases} \rho \varphi_{tt} - \kappa(\varphi_x + \psi)_x + \mu \theta_x = 0, & \text{in } (0, L) \times (0, \infty), \\ -b \psi_{xx} + \kappa(\varphi_x + \psi) = 0, & \text{in } (0, L) \times (0, \infty), \\ c \theta_t - \delta \theta_{xx} + \mu \varphi_{xt} = 0, & \text{in } (0, L) \times (0, \infty), \end{cases} \quad (145)$$

where φ is the transversal displacement, ψ is the rotation angle of the filament and θ is the difference of the temperature.

In addition, we supplement the system (145) with the following initial and boundary conditions

$$\varphi(x, 0) = \varphi_0(x), \varphi_t(x, 0) = \varphi_1(x), \psi(x, 0) = \psi_0(x), \theta(x, 0) = \theta_0(x), \quad x \in [0, L] \quad (146)$$

$$\varphi(0, t) = \varphi(L, t) = \psi(0, t) = \psi(L, t) = \theta_x(0, t) = \theta_x(L, t) = 0, \quad t \in (0, +\infty). \quad (147)$$

The energy of the system (145) is defined by

$$E(t) := \frac{\rho}{2} \int_0^L |\varphi_t|^2 dx + \frac{b}{2} \int_0^L |\psi_x|^2 dx + \frac{c}{2} \int_0^L |\theta|^2 dx + \frac{\kappa}{2} \int_0^L |\varphi_x + \psi|^2 dx. \quad (148)$$

We have

$$\frac{d}{dt} E(t) = -\delta \int_0^L |\theta_x|^2 dx \leq 0. \quad (149)$$

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Well-Posedness

We introduce the phase space:

$$\mathcal{H} := H_0^1(0, L) \times L^2(0, L) \times H_0^1(0, L) \times H_*^1(0, L) \quad (150)$$

where

$$H_*^1(0, L) := \left\{ \phi \in H^1(0, L), \int_0^L \phi(x) dx = 0 \right\}. \quad (151)$$

Definition 22. Let $(\varphi_0, \varphi_1, \psi_0, \theta_0) \in \mathcal{H}$. A weak solution of (145)–(147) is a function $U = (\varphi, \psi, \theta) \in \mathcal{H}$ which satisfies:

$$\left\{ \begin{array}{l} \rho \frac{d}{dt} \int_0^L \varphi_t u dx + \kappa \int_0^L (\varphi_x + \psi) u_x dx + \mu \int_0^L \theta_x u dx = 0, \quad \forall u \in H_0^1(0, L), \\ b \int_0^L \psi_x w dx + \kappa \int_0^L (\varphi_x + \psi) w dx = 0, \quad \forall w \in H_0^1(0, L), \\ c \frac{d}{dt} \int_0^L \theta v dx + \delta \int_0^L \theta_x v_x dx + \mu \int_0^L \varphi_t v_x dx = 0, \quad \forall v \in H_*^1(0, L) \end{array} \right. \quad (152)$$

for a.e. $t \in (0, T)$ and

$$\varphi(0) = \varphi_0, \varphi_t(0) = \varphi_1, \psi(0) = \psi_0, \theta(0) = \theta_0.$$

The well-posedness result reads as follows:

Theorem 45. For any initial data $(\varphi_0, \varphi_1, \psi_0, \theta_0) \in \mathcal{H}$ and any $T > 0$, the problem (145)–(147) has a weak solution (φ, ψ, θ) satisfying

$$\begin{aligned} \varphi &\in L^\infty(0, T; H_0^1(0, L)), \varphi_t \in L^\infty(0, T; L^2(0, L)), \\ \psi &\in L^\infty(0, T; H_0^1(0, L)), \theta \in L^\infty(0, T; H_*^1(0, L)) \cap L^2(0, T; H_*^1(0, L)). \end{aligned} \quad (153)$$

Proof The proof to this result is done by the use of Feado–Galerkin method through four steps. ■

Exponential Stability

Theorem 46. There exist two positive constants $A, \gamma > 0$, such that the energy $E(t)$ defined by (148) satisfies along the solution (φ, ψ, θ) the estimate

$$E(t) \leq AE(0)e^{-\gamma t}, \quad \forall t \geq 0.$$

Proof The proof of Theorem 46 will be given by the use of the multiplier method, in fact we construct a Lyapunov functional $\mathcal{L}(t; x)$ which is positive definite, equivalent to the energy $E(t)$ and $\mathcal{L}_t(t; x)$ is negative definite. We write $\mathcal{L}$ as a combination of functionals $\{E(t), \mathcal{F}_1(t); \mathcal{F}_2(t)\}$ where each derivative of $E(t), \mathcal{F}_1(t); \mathcal{F}_2(t)$ satisfies an estimate of a term from the energy with negative sign. We obtain, $\mathcal{L}'(t) \leq -\gamma \mathcal{L}(t)$. Thus, an integration we respect to t and the equivalence between $\mathcal{L}$ and $E(t)$ lead to the exponential stability of $E(t)$. ■

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STABILITY OF WAVE EQUATION WITH VARIABLE DELAY TERM

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Abstract: In this work we consider a evolution problem with memory in a bounded domain and varying delay term. Under suitable assumptions on the weight of the damping and the weight of the delay, we show the exponential stability of the solution by introducing a suitable Lyapunov functional. Our main result in this work is to study the general decay result.

Keywords: Wave equation, Evolution problem, Memory, Exponential stability.

2020 Mathematics Subject Classification: 35B37, 35L55, 74D05.

Introduction

We consider the evolution problem with a varying delay term,

$$\left\{ \begin{array}{ll} u_{tt}(x, t) - a\Delta u(x, t) - \Delta u_{tt}(x, t) + \int_0^t g(t-s)\Delta u(s)ds \\ \quad \mu_1 u_t(x, t) + \mu_2 u_t(x, t - \tau(t)) = k_1 |u_t|^{q-1} u_t, & (x, t) \in \Omega \times (0, +\infty), \\ u(x, t) = 0, & (x, t) \in \Gamma_0 \times (0, +\infty) \\ u(x, 0) = u_0(x), u_t(x, 0) = u_1(x), & x \in \Omega, \\ u(x, t - \tau(t)) = f_0(x, t - \tau(t)) & (x, t) \in \Omega \times (0, +\infty), \end{array} \right.$$

where k_1, a, μ_1 are positive constants, and μ_2 is a real number $\tau(t) > 0$ is the decay function.

Main results

stability of solutions of mathematical problems always attracts the attention of researchers. This work considered a evolution problem with memory and varying delay term, in which the exponential decay of solutions was established by introducing a suitable Lyapunov functional. We propose the study of stability of solutions.

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Controllability of Mild Solutions to Impulsive Functional Evolution Equations

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Abstract: This study aims to establish the controllability of first-order impulsive functional evolution equations within a separable Banach space, employing Schauder's fixed point theorem. We investigate controllability under local conditions and explore the effects of impulses on system evolution. Leveraging the properties of separable Banach spaces, we seek to elucidate control mechanisms in such contexts. Our analysis contributes to the theoretical foundations of controllability theory, emphasizing the significance of understanding and manipulating impulsive systems within functional evolution frameworks.

Keywords: Impulsive functional differential equations, mild solutions, controllability, fixed point, semigroups, separable Banach space.

2020 Mathematics Subject Classification: Primary 18B40, 93B05, 47H10.

Introduction

Recent advances, particularly in fixed-moment equations, have focused on impulsive equations, which are critical in scientific dynamics. Bainov and Simeonov [1], Benchohra et al. [2], and Lakshmikantham et al. [3] explain these developments. In this paper, we provide sufficient conditions to assure the controllability of mild solutions on the semiinfinite interval $J = [0, +\infty)$ for first order impulsive functional evolution equations of the type

$$y'(t) - Ay(t) = f(t, y_t) + BU(t), \quad \text{a.e. } t \in J := [0, +\infty), \quad t \neq t_k, \quad k \in \mathbb{N}^*; \quad (154)$$

$$\Delta y|_{t=t_k} = I_k(y(t_k^-)), \quad k \in \mathbb{N}^*; \quad (155)$$

$$y(t) = \varphi(t), \quad t \in H := [-r, 0], \quad \text{where } 0 < r < +\infty; \quad (156)$$

Where $f : J \times D \rightarrow E$ is a given function, $\varphi \in D$, $I_k \in C(E, E)$, $t_k < t_{k+1}$, $\forall k \in \mathbb{N}^*$, $A : D(A) \subset E \rightarrow E$ is the infinitesimal generator of a C_0 -semigroup $T(t)$, $t \geq 0$, and E is a real separable Banach space with norm $|\cdot|$, the control function $U(\cdot)$ is given in $L^2(J, E)$, the Banach space of admissible control function and B is a bounded linear operator from E into E . We denote by y_t the element of D defined by: $y_t(\theta) = y(t+\theta)$, $\theta \in [-r, 0]$, here $y_t(\cdot)$ represents the history of the state from $t-r$ up to the present time t . Such that $D = \{\psi : [-r, 0] \rightarrow E : \psi|_{I_t} \text{ is continuous and } \psi(t_k^-), \psi(t_k^+) \text{ exist with } \psi(t_k^-) = \psi(s) \text{ and } I_t = [-r, +\infty) \setminus \{t_k, k \in \mathbb{N}^*\}\}$.

Abada et al. [1] provided conditions for controllability within bounded intervals for certain impulsive functional differential equations with finite delay. These conditions apply to separable Banach spaces and incorporate both local and nonlocal aspects, building upon the work of Burton and Kirk.

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Main results

Definition 23. (Mild solution) A function $y \in BPC([-r, +\infty), E)$ is said to be a mild solution of problem (154) – (156) if $y(t) = \varphi(t)$, $t \in [-r, 0]$ and for each $t \in J$, we have

$$y(t) = T(t)\varphi(0) + \int_0^t T(t-s)f(s, y_s)ds + \int_0^t T(t-s)BU(s)ds + \sum_{0 < t_k < t} T(t-t_k)I_k(y(t_k^-)).$$

We will need to introduce the following one which is assumed hereafter:

(H1) $A : D(A) \rightarrow E$ is the infinitesimal generator of a strongly continuous semi-group $T(t)$, $t \in J$ which is compact for $t > 0$ in the Banach space E . Let $\|T(t)\|_{B(E)} \leq M$ for all $t \geq 0$ such that $M > 0$;

(H2) The function $f : J \times D \rightarrow E$ is caratheodory;

(H3) There exists a continuous function $k : J \rightarrow [0, +\infty)$ such that:

$$|f(t, u) - f(t, v)| \leq k(t)\|u - v\|, \quad t \in J, \quad u, v \in D, \quad f(t, 0) = 0$$

$$\text{and } k^* := \sup_{t \in J} \int_0^t k(s)ds;$$

(H4) There exist constants $d_k > 0$, $k \in \mathbb{N}^*$ with $\sum_{k \in \mathbb{N}^*} d_k = d < \infty$ such that for each $x, y \in E$, we have:

$$|I_k(y) - I_k(x)|_E \leq d_k|y - x|_E \text{ and } I_k(0) = 0;$$

(H5) For each $b \in \mathbb{N}$, the linear operator $W : L^2([0, b], E) \rightarrow E$ is defined by

$$WU = \int_0^b T(b-s)BU(s)ds,$$

has a pseudo-invertible operator $\widetilde{W}^{-1}$ which takes values in $L^2([0, b], E)/\text{Ker}W$ and there exists positive constants $\widetilde{M}$ and $\widetilde{M}_1$ such that :

$$\|B\| \leq \widetilde{M} \text{ and } \|\widetilde{W}^{-1}\| \leq \widetilde{M}_1.$$

Theorem 47. Assume that (H1)–(H5) hold. If $M[k^* + d + \widetilde{M}\widetilde{M}_1b(1 + Mk^* + Md)] < 1$, then the problem (154) – (156) is controllable on $[-r, +\infty)$.

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Smoothness of solutions of differential equations of constant strength in Roumieu spaces

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Abstract: We show in this work that every solution of hypoelliptic differential equations with constant strength with coefficients in Roumieu spaces is in some Roumieu space.

Keywords: Differential operators of constant strength, hypoelliptic operators, iterates of operators, Roumieu spaces.

2020 Mathematics Subject Classification: Primary 35H10, 35A18, 35H30.

Introduction

A general class of hypoelliptic differential operators with variable coefficients is the class of differential operators of constant strength. Hörmander in [2], has given a basic result of hypoellipticity of these operators. Using this result and the theorem of Newberger-Zielezny [3] concerning the problem of iterates of differential operators with constant coefficients, Shafii-Mousavi and Zielezny [4] have obtained a result of Gevrey regularity of solutions of linear differential equations of constant strength with coefficients in Gevrey classes. In the case of differential operators with constant coefficients, various results of Gevrey and Roumieu hypoellipticity have obtained with help of iterates of operators or more generally of iterates of systems of operators.

Our purpose in this paper is to give an extension of the result of Hörmander [2] in the classes of ultradifferentiable functions (or classes of Roumieu) to linear differential equations of constant strength with coefficients in Roumieu spaces.

For the definition of Roumieu spaces we will consider sequences of positive real numbers (M_p) satisfying the following conditions:

logarithmic convexity :

$$M_0 = 1 \text{ and } M_p^2 \leq M_{p-1}M_{p+1}, \forall p \in \mathbb{N}^*, \quad (157)$$

stability under derivation and multiplication:

$$\exists H > 0 : C_p^j M_{p-j} M_j \leq M_p \leq H^p M_{p-j} M_j, \forall p \in \mathbb{N}, j \leq p. \quad (158)$$

Example 4. The sequence $M_p = p!^s$, $s \geq 1$, called Gevrey sequence of order s , satisfies the conditions (157) – (158).

For more details on the spaces of ultra-differentiable functions and ultra-distributions see [?].

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Definition 24. Let Ω be an open subset of $\mathbb{R}^n$, $Q(D) = \sum_{|\alpha| \leq m} a_\alpha D^\alpha$ a linear differential operator with constant coefficients of order m . We call Roumieu vector (or vector of type M_p) of the operator $Q(D)$ in Ω , any function $u \in C^\infty(\Omega)$ such that

$$\forall H \text{ compact of } \Omega, \exists C > 0, \forall l \in \mathbb{Z}_+ : \|Q^l(D)u\|_{L^2(H)} \leq C^{l+1} M_{lm}$$

The space of Roumieu vectors of $Q(D)$ in Ω is denoted $R_M(\Omega, Q)$.

Main results

Roumieu regularity of hypoelliptic operators with constant coefficient

The first result of this paper is the Roumieu regularity of hypoelliptic operators with constant coefficients the following theorem. But first let us show the following estimate of higher order of hypoelliptic operators with constant coefficients.

For simplify the notations set $d = \frac{\mu}{\nu}$ and $\gamma = dm\mu$ where $\mu, \nu \in \mathbb{N}^*$.

Proposition 4. Let ω be a bounded open subset of $\mathbb{R}^n$ and let Q be a d -hypoelliptic operator of order m , then

$$\exists C > 0, \forall k \geq 0, \forall \alpha \in \mathbb{Z}_+^n, |\alpha| \leq km\nu, \forall \delta > 0, \forall u \in C^\infty(\omega),$$

$$\|D^\alpha u\|_{L^2(\omega_\delta)} \leq C^k \sum_{i=0}^k \binom{k}{i} \left(\frac{k}{\delta}\right)^{(k-i)dm} \|Q^i u\|_{L^2(\omega)} \quad (159)$$

Theorem 48. Let Ω be an open subset of $\mathbb{R}^n$ and Q a d -hypoelliptic differential operator with constant coefficients, and let (M_p) be a sequence satisfying (157) – (158), and such that

$$(p!)^d \subset M_p, \quad (160)$$

then

$$R_M(\Omega, Q) \subset R_{M^d}(\Omega).$$

Roumieu regularity of hypoelliptic operators with constant strength

Before establish the regularity of solutions of differential equations associated with differential operators of constant strength, we will recall some definitions related to this question and the fundamental result due to Hörmander [2] of regularity of these operators.

Definition 25. A differential operator $P(x, D)$ with variable coefficients in Ω is said of constant strength in Ω , if for arbitrary fixed $x, y \in \Omega$, the differential operators $P(x, D)$ and $P(y, D)$ with constant coefficients are equally strong, that means

$$\forall x, y \in \Omega, \exists C_{x,y} > 0 : \widetilde{P}(x, \xi) \leq C_{x,y} \widetilde{P}(y, \xi), \quad \forall \xi \in \mathbb{R}^n,$$

$$\text{where } \widetilde{P}^2(\xi) = \sum_\alpha |P^{(\alpha)}(\xi)|^2.$$

The following theorem is the basic result of hypoellipticity of differential operators of constant strength, see Hörmander [2, Theorem 7.3.1 and Theorem 7.4.1].

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Theorem 49. Let $P(x, D)$ be a differential operator with C^∞ coefficients and of constant strength in a neighborhood of $x_0 \in \mathbb{R}^n$. Then there exists a sufficiently small open neighborhood Ω of x_0 , and a linear mapping E of $\mathcal{E}'(\mathbb{R}^n)$ into $\mathcal{E}'(\mathbb{R}^n)$ with the following properties:

$$P(x, D)Eu = u \text{ in } \Omega \text{ if } u \in \mathcal{E}'(\mathbb{R}^n),$$

$$EP(x, D)v = v \text{ in } \Omega \text{ if } v \in \mathcal{E}'(\Omega), \quad (161)$$

$$\|Eu\|_{p, \bar{P}_0 h} \leq C_h \|u\|_{p, h} \text{ if } u \in \mathcal{E}'(\Omega) \cap \mathcal{B}_{p, h}, \quad h \in \mathcal{H}, \quad (162)$$

where $P_0(D) = P(x_0, D)$ and C_h is independent of u .

If in addition $P_0(D)$ is hypoelliptic then $P(x, D)$ is so also.

The second result of this paper the following theorem :

Theorem 50. Let $P(x, D)$ be a differential operator of constant strength in a neighborhood of x_0 , and let (M_p) a sequence satisfying the conditions (157) – (158). Then there exists a open neighborhood Ω_0 of x_0 such that, if the coefficients of $P(x, D)$ are in $R_M(\Omega_0)$ and the operator $P_0(D) = P(x_0, D)$ is d -hypoelliptic and the sequence (M_p) satisfyies the condition (160), we have

$$\forall u \in \mathcal{D}'(\Omega_0), P(x, D)u \in R_M(\Omega_0) \Rightarrow u \in R_{M^d}(\Omega_0).$$

Corollary 2. Let Ω be an open subset of $\mathbb{R}^n$, $P(x, D)$ a differential operator of constant strength in Ω with coefficients in $R_M(\Omega)$, and let (M_p) be a sequence satisfying the conditions (157) – (158). If for some $x_0 \in \Omega$ the operator $P_0(D) = P(x_0, D)$ is d -hypoelliptic and the sequence (M_p) satisfyies the condition (160), then

$$\forall u \in \mathcal{D}'(\Omega), P(x, D)u \in R_M(\Omega) \Rightarrow u \in R_{M^d}(\Omega).$$

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The fractional $p(.,.)$ -Neumann boundary conditions for the nonlocal $p(.,.)$ -Laplacian operator

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Abstract: In this paper, we deal with the fractional $p(.,.)$ -Laplacian problem with nonlocal Neumann boundary conditions

$$\begin{cases} (-\Delta)_{p(.,.)}^s u + |u|^{\bar{p}(x)-2} u = \lambda V_1(x) |u|^{q(x)-2} u, & \text{in } \Omega, \\ \mathcal{N}_{s,p(.,.)} u = \mu V_2(x) |u|^{r(x)-2} u, & \text{on } \mathbb{R}^N \setminus \Omega, \end{cases}$$

here, $\lambda, \mu > 0$ are parameters and V_1, V_2 are two nonnegative weighted functions. The domain $\Omega \subset \mathbb{R}^N$ ($N \geq 1$) is smooth and bounded, $\bar{p}(x) = p(x, x)$, q, r are continuous bounded functions. Applying the Ekeland principle and the variational method, under appropriate assumptions, we show that the above problem has nontrivial weak solutions.

Keywords: Fractional $p(.,.)$ -Laplacian, $p(.,.)$ -Neumann boundary condition, Ekeland variational principle.

2020 Mathematics Subject Classification: 35D05, 35D30, 35J58, 35J60, 35J65.

Introduction

Operators involving variable growth are extensively studied due to the precision in modeling of various phenomenon where the property of the subject under consideration depends on the point of the observation, for example in image restoration, electrorheological fluid and in non-Newtonian processes etc, (for more details and applications, see [1, 2, 1] and the references therein.)

In present work our objective is to study the nonlocal fractional $p(.,.)$ -Laplacian problem under $p(.,.)$ -Neumann boundary conditions

$$\begin{cases} (-\Delta)_{p(.,.)}^s u + |u|^{\bar{p}(x)-2} u = \lambda V_1(x) |u|^{q(x)-2} u, & \text{in } \Omega, \\ \mathcal{N}_{s,p(.,.)} u = \mu V_2(x) |u|^{r(x)-2} u, & \text{on } \mathbb{R}^N \setminus \Omega, \end{cases} \quad (163)$$

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where Ω be a smooth bounded domain of $\mathbb{R}^N$, $N \geq 1, s \in (0, 1), p : \mathbb{R}^{2N} \rightarrow (1, +\infty)$ is a symmetric, continuous function bounded away from 1, λ, μ are positive parameters, $\bar{p}(\cdot) = p(\cdot, \cdot)$, q, r are given continuous functions in Ω satisfying the conditions introduced in the next section, $V_1, V_2 \in C(\bar{\Omega})$ are two nonnegative weighted functions and $(-\Delta)_{p(\cdot, \cdot)}^s$ is the fractional $p(\cdot, \cdot)$ -Laplacian operator defined as follows

$$(-\Delta)_{p(\cdot, \cdot)}^s u(x) := p.v. \int_{\mathbb{R}^N} \frac{|u(x) - u(y)|^{p(x, y)-2} (u(x) - u(y))}{|x - y|^{N+s \cdot p(x, y)}} dy, \quad x \in \Omega,$$

and

$$\mathcal{N}_{s, p(\cdot, \cdot)} u(x) := \int_{\Omega} \frac{|u(x) - u(y)|^{p(x, y)-2} (u(x) - u(y))}{|x - y|^{N+s \cdot p(x, y)}} dy, \quad x \in \mathbb{R}^N \setminus \bar{\Omega}, \quad (164)$$

is the nonlocal normal $p(\cdot, \cdot)$ -derivative, or $p(\cdot, \cdot)$ -Neumann boundary condition and describes the natural Neumann boundary condition in presence of the fractional $p(\cdot, \cdot)$ -Laplacian.

Main results

Theorem 51. .

Assuming that assumptions (S), (H) and (V) holds. In the case when $q^- < p^-$, there exists $\beta^* > 0$ such that for all $\lambda, \mu \in (0, \beta^*)$, our problem has a non trivial weak solution.

The second result is obtained, using variational method

Theorem 52. .

Suppose that assumptions (S), (H) and (V) are fulfilled. Then, for any $\lambda > 0$ our problem has a non trivial weak solution provided that, $\max(q^+, r^+) < p^-$.

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On swelling porous-elastic soils system with thermal effect and delay: Well-posedness and Stability results

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Abstract: In this paper, we consider a swelling porous thermoelastic system with the presence of a temperature effect and delay terms. Our work studies the case $(\mu_1 = |\mu_2|)$, which gives a new contribution to this coupling to the theory associated with asymptotic behaviors of swelling porous elastic soils. We will established the well-posedness and the general decay result is proved by the energy method.

Keywords: Swelling porous system, General decay, Temperature damping, Delay term.

2020 Mathematics Subject Classification: Primary 35B40, 35L70, 93D15.

Introduction

The goal of this paper is the thermal effects and the delay terms in case of equality $(\mu_1 = |\mu_2|)$. So we assume that the heat capacity $c_0 > 0$, and we assume that the thermal conductivity is existent $\kappa \neq 0$.

We consider the following system,

$$\begin{cases} \rho_u u_{tt} - a_1 u_{xx} - a_2 \phi_{xx} + \mu_1 u_t + \mu_2 u_t(x, t - \tau) = 0, \\ \rho_\phi \phi_{tt} - a_3 \phi_{xx} - a_2 u_{xx} - \gamma \theta_x = 0, \\ c_0 \theta_t - k_0 \theta_{xx} - \gamma \phi_{tx} = 0, \end{cases}$$

where

$$(x, t) \in (0, 1) \times (0, \infty),$$

with the initial and boundary conditions

$$\begin{aligned} u(x, 0) &= u_0(x), u_t(x, 0) = u_1(x), \theta(x, 0) = \theta_0(x) \\ \phi(x, 0) &= \phi_0(x), \phi_t(x, 0) = \phi_1(x), x \in (0, 1), \\ u_t(x, t - \tau) &= f_0(x, t - \tau), (x, t) \in (0, 1) \times (0, \tau), \\ u(0, t) &= u(1, t) = \phi(0, t) = \phi(1, t) = 0, \\ \theta(0, t) &= \theta(1, t) = 0, t \geq 0. \end{aligned}$$

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Well-posedness

Exponential decay

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Well-posedness and energy decay of solutions to the swelling porous elastic soils with fluid saturation and internal control with fractional derivative

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Abstract: In this work, we study the well-posedness and polynomial stability to the one-dimensional system in the linear isothermal theory of swelling porous elastic soils subject with internal damping of fractional derivative type. We prove well-posedness by using the semi-group theory. Also we establish an optimal decay result by frequency domain method and Borichev Tomilov theorem.

Keywords: Semigroup Theory; Elastic Porous Media Swelling; Polynomial Stability.

Introduction

In this work we consider the swelling problem in porous elastic soils with fluid saturation given by

$$\begin{cases} \rho_z z_{tt} - a_1 z_{xx} - a_2 u_{xx} + \gamma \partial_t^{\alpha, \eta} z(x, t) = 0, & x \in (0, 1), \quad t \geq 0, \\ \rho_u u_{tt} - a_3 u_{xx} - a_2 z_{xx} = 0, & x \in (0, 1), \quad t \geq 0, \\ z(x, 0) = z_0(x), \quad z_t(x, 0) = z_1(x), \\ u(x, 0) = u_0(x), \quad u_t(x, 0) = u_1(x), \\ z(0, t) = z(1, t) = 0, \quad u(0, t) = u(1, t) = 0, \quad \text{for } t \geq 0, \end{cases} \quad (165)$$

where the dependent variables $z = z(x, t)$ and $u = u(x, t)$ represent the displacement of the fluid and elastic solid material, respectively. The constants ρ_z and ρ_u are the densities of each constituent, a_1, a_3 are positive constants and $a_2 \neq 0$, satisfying the relation $a_2^2 < a_1 a_3$. γ_1, γ_2 are positive constants and the notation $\partial_t^{\alpha, \eta}$ stands for the generalized Caputo's fractional derivative of order α , $0 < \alpha < 1$, with respect to the time variable. It is defined as follows

$$\partial_t^{\alpha, \eta} w(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} e^{-\eta(t-s)} \frac{dw}{ds}(s) ds, \quad \eta \geq 0.$$

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In [5], B. Mbodje investigate the asymptotic behavior of solutions of the wave equation with a boundary damping of the fractional derivative type, that is,

$$(PBF) \quad \begin{cases} u_{tt}(x, t) - u_{xx}(x, t) = 0 & \text{in } (0, 1) \times (0, +\infty), \\ u(0, t) = 0 & \text{on } (0, +\infty), \\ u_x(1, t) + \gamma \partial_t^{\alpha, \eta} u_t(1, t) = 0 & \text{on } (0, +\infty), \\ u(x, 0) = u_0(x), u_t(x, 0) = u_1(x) & \text{on } (0, 1). \end{cases}$$

Using energy methods, he proves strong asymptotic stability under the condition $\eta = 0$ and a polynomial type decay rate $E(t) \leq C/t$ if $\eta \neq 0$.

Very recently, in [1], Ammari et al., studied the wave equation with internal fractional damping. The system considered is as follows:

$$\begin{cases} u_{tt}(x, t) - \Delta u(x, t) + \gamma \partial_t^{\alpha, \eta} u(x, t) = 0 & \text{in } \Omega \times (0, +\infty), \\ u(x, t) = 0 & \text{on } \partial\Omega \times (0, +\infty), \\ u(x, 0) = u_0(x), \quad u_t(x, 0) = u_1(x) & \text{on } \Omega. \end{cases}$$

The authors proved that the energy decays polynomially as $t^{-2/(1-\alpha)}$.

The main result of this paper is to obtain a precise and optimal energy decay estimate for an internal fractional damping acting only on the first equation of the system.

Main results

Our first result states that the energy is not conserving.

Lemma 13. *For every solution (z, u) be a solution of the system (165), the total energy $\mathcal{E} : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is defined at time t by*

$$\begin{aligned} \mathcal{E}(t) := & \frac{\rho_z}{2} \int_0^L |z_t|^2 dx + \frac{\rho_u}{2} \int_0^L |u_t|^2 dx + \frac{1}{2} \left(a_1 - a_2^2/a_3 \right) \int_0^L |z_x|^2 dx \\ & + \frac{1}{2} \int_0^L \left| \frac{a_2}{\sqrt{a_3}} z_x + \sqrt{a_3} u_x \right|^2 dx + \frac{\zeta}{2} \int_0^L \int_{\mathbb{R}} |\varphi(x, \xi, t)|^2 d\xi dx, \quad \forall t \geq 0, \end{aligned}$$

and satisfies

$$\frac{d}{dt} \mathcal{E}(t) = -\gamma \int_0^1 \partial_t^{\alpha, \eta} z(x, t) \partial_t z(x, t) dx.$$

Our main result is the following.

Theorem 53. *The semigroup $S_{\mathcal{A}}(t)_{t \geq 0}$ is polynomially stable and*

$$\mathcal{E}(t) = \|S_{\mathcal{A}}(t)\Psi_0\|_{\mathcal{H}}^2 \leq \frac{1}{t^{\frac{2}{1-\alpha}}} \|\Psi_0\|_{D(\mathcal{A})}^2. \quad (166)$$

Moreover, the rate of energy decay $t^{2/(1-\alpha)}$ is optimal for general initial data in $D(\mathcal{A})$.

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Fixed Point Theorems in a Cone of Banach Space for a new type of Contractive Mappings under an Abstract Measure of Non-Compactness and Applications

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Abstract: Under an abstract measure of non compactness, we introduce a new class of $\psi\phi\mu$ -contractive mappings for which we compute its index under Leary-Schauder, Furi-Pera and interior boundary conditions. Some consequences of this new result are given, the fixed point theorem for the sum of two operator, In the end, we give an application for integral equations.

Keywords: Fixed point index, fixed point, Leary-Schauder, Furi-Pera and interior conditions, abstract measure of non compactness, contractive mappings,

Introduction

Let E be a real Banach space, and K be a cone in E endowed with a partial order $\leq$ such that $x \leq y$ iff $y - x \in K$, and let G be a bounded open subset of K . Consider $f : \overline{G} \rightarrow K$ a $\mu\psi\phi$ - mapping satisfying:

$$\psi(\mu(f(B))) \leq \phi(\mu(B)), \quad \text{for all } B \subset \mathcal{B}(E),$$

where $\mathcal{B}(E) = \mathcal{B}$ denotes the set of all bounded subset of E , μ is an abstract measure of non-compactness, (*MNC forshort*) defined on E . $\psi : [0, +\infty) \rightarrow [0, +\infty)$ is a nondecreasing function such that $\psi(0) = 0$ and $\phi : [0, +\infty) \rightarrow [0, +\infty)$ is a semi-continuous function. In this work, we extend the notion of the fixed point index, developed in [1], and denoted by $i(f, G, K)$ to the $\mu\psi\phi$ -mapping f defined by the abstract *MNC* μ . Then, we will use it to establish some new fixed point theorems. In addition to compression and expansion of a cone principle, we prove that any $\mu\psi\phi$ -strict set contractive mapping defined on a convex set and satisfying the Furi-Pera's condition, or defined on a Strict star shaped set and satisfying the interior condition has a fixed point. As an application of our results, we present at the end of the chapter existence results for Urysohn integral equation.

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Main results

Fixed point index for positive $\mu\psi\phi$ - set contractive mappings

Theorem 54. Let $f : \overline{G} \rightarrow X$ be a $\mu\psi\phi$ -set contractive mapping which has no fixed point in ∂G . Then the fixed point index $i(f, G, X)$ is well define and satisfies the following properties

1. **Solutions property** If $i(f, G, X) \neq 0$, then f has at least one fixed point in G .
2. **Normality** $i(f, G, X) = 1$, whenever $fx = x_0 \in G$.
3. **Excision** If U is an open subset of G such that f has not a fixed point in $G \setminus U$, then we have $i(f, G, X) = i(f, U, X)$.
4. **Additivity** $i(f, G, X) = i(f, G_1, X) + i(f, G_2, X)$ where G_i are disjoint open subsets of G and f does not have a fixed point in $G \setminus G_1 \cup G_2$.
5. **Homotopy invariance** Let $I = [0, 1]$ and let $H : [0, 1] \times \overline{G} \rightarrow X$ be a continuous function and assume that H is a $\mu\psi\phi$ -set contractive mapping, that is

$$\psi\left[\mu\left(H([0, 1] \times B)\right)\right] \leq \phi(\mu(B)), \quad (167)$$

for all B is a bounded set of G , then if $H(t, x) \neq x$ for all $t \in [0, 1]$ and $x \in \partial G$ we have that the fixed point index $i(H(t, \cdot), G, X)$ is independent of t .

6. **Permanence** If X' is a convex subset of X with $f(G) \subset X'$, then $i(f, G, X) = i(f, G \cap X', X')$.

Lemma 14. Let $a \in G$. Assume that

$$(LS) \quad \forall x \in \partial G \text{ and } \lambda \geq 1, \quad fx \neq \lambda x + (\lambda - 1)a \quad (168)$$

then, $i(f, \overline{G}, K) = 1$.

Lemma 15. If the mapping f satisfies the following condition

1. there exists $e > 0_{\mathbb{E}}$ such that

$$x \neq fx + te \text{ for all } t \geq 0 \text{ and all } x \in \partial G. \quad (169)$$

Then, $i(f, G, k) = 0$.

Fixed point theorems for positive $\mu\psi\phi$ -set contractive mappings

Theorem 55. Let G_1, G_2 be two bounded and open subsets in $\mathbb{E}$ such that $0 \in G_1$ and $\overline{G_1} \subset G_2$, and let $f : \overline{G_2} \rightarrow K$ be a $\mu\psi\phi$ -set contractive mapping. If one of the hypotheses

$$(H_1) \quad fx \not\leq x, \forall x \in K \cap \partial G_1 \text{ and } fx \not\leq x, \forall x \in K \cap \partial G_2,$$

$$(H_2) \quad fx \not\leq x, \forall x \in K \cap \partial G_1 \text{ and } fx \not\geq x, \forall x \in K \cap \partial G_2,$$

holds, then f has at least one fixed point in $K \cap (\overline{G_2} \setminus G_1)$.

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Theorem 56. Let $f : \overline{G} \rightarrow \mathbb{E}$ be an $\psi\phi\mu$ -set contractive mapping, where G be an open bounded convex subset of $\mathbb{E}$ and $a \in G$. If f satisfies the Furi – Pera's condition at a , then f has a fixed point.

You can see the definition of Furi – Pera's condition in [1].

Theorem 57. et $f : \overline{G} \rightarrow \mathbb{E}$ is an $\psi\phi\mu$ -set contractive mapping, where G be an open bounded subset of $\mathbb{E}$ and let $a \in G$. If G is strictly star shaped at a and f satisfies the interior condition at a , then f has a fixed point.

You can see the definition interior condition in [3].

Existence result for Uryshon integral equation

Using our fixed point theorems we will prove an existence result of following integral equation

$$u(t) = f(t, u(t)) - \int_0^1 \gamma(t, s)g(s, u(s))ds \quad (170)$$

where $\gamma : [0, 1]^2 \rightarrow [0, +\infty[$ and $f, g : [0, 1] \times [0, +\infty[\rightarrow [0, +\infty[$ are continuous functions.

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White noise driven higher order nonlinear dispersive equation

Aissa Boukarou

Abstract:

The current work is devoted to the Cauchy problem for the white noise higher order nonlinear dispersive equation with positive dispersion. We obtain the local well-posedness for the initial data $u_0(x) \in L^2(\Omega, H^s)$, $s > \frac{5}{8}$, and the globally well-posedness for the initial data $u_0(x) \in L^2(\mathbb{R})$. The key ingredients that we used in this work are some bilinear estimates in $X_{s,b}$ spaces.

Keywords: Higher order nonlinear dispersive equation, White noise driven, Bilinear estimates
2020 Mathematics Subject Classification: Primary 35E15, 35Q53, 35B65, 35G25

Introduction

In this work we consider a higher order nonlinear dispersive equation as follows

$$\begin{cases} \partial_t u + \lambda_1 \partial_x^7 u + \lambda_2 \partial_x^5 u + \lambda_3 \partial_x^3 u + \lambda_4 \partial_x u + u \partial_x u = 0, & x \in \mathbb{R}, t \geq 0 \\ u(x, 0) = u_0(x), \end{cases} \quad (171)$$

where $\lambda_1 \neq 0, \lambda_2, \lambda_3, \lambda_4$ are real numbers.

We consider the following white noise higher order nonlinear dispersive equation with positive dispersion

$$\begin{cases} du(t) = (\lambda_1 \partial_x^7 u + \lambda_2 \partial_x^5 u + \lambda_3 \partial_x^3 u + \lambda_4 \partial_x u + u \partial_x u) dt + \Phi dw(t), & x \in \mathbb{R}, t \geq 0 \\ u(x, 0) = u_0(x), \end{cases} \quad (172)$$

Main results

Theorem 58. Assume that $\lambda_1 \lambda_2 < 0$ and $\lambda_3 > 0$, let $\delta > 0$ and $s > -5/8$. Then for any $u_0 \in H^s$, there exists $T > 0$ and a unique solution u of (171) on the time interval $(0, T)$ such that

$$u \in C([0, T], H^s).$$

Moreover the solution depends continuously on the data u_0 . Here we have

$$T = \frac{c_0}{(1 + \|u_0\|_{H^s})^\beta}, \quad (173)$$

for some constants $c_0 > 0$ and $\beta > 1$ depending only on s . Furthermore, the solution u satisfies the bound

$$\|u\|_{X_{s,b}} \leq 2C \|u_0\|_{H^s} \quad 1/2 < b < 1, \quad (174)$$

with a constant $C > 0$ depending only on s and b .

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Thus, this result shows that for local-in-time the radius of analyticity remains constant.

The main results of this work are as follows

Theorem 59. *Let $u_0 \in L^2(\Omega, H^s)$ with $s > -5/8$. Assume that $\lambda_1 \lambda_2 < 0$ and $\lambda_3 > 0$, $s > -5/8$. Then for any $u_0 \in L^2(\Omega, H^s)$, there exists $T_\omega > 0$, $\omega \in \Omega$ and a unique solution u of (172) on the time interval $(0, T_\omega)$ such that*

$$u \in C([0, T_\omega], H^s).$$

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Dynamic analysis of a population model with double Allee effect

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Abstract: The aim of this work is to qualitatively analyze a discrete-time predator-prey model with a double Allee effect on prey populations. This model is characterized by a double Allee effect on prey populations. The conditions for the existence and stability of fixed points were studied. A bifurcation and Neimark analysis were performed at the positive fixed point. This analysis is based on using the central manifold theorem of fixed point theory, while choosing an Allee effect parameter as the bifurcation parameter

Keywords: Discrete time model, Allee effect, stability, bifurcation

2020 Mathematics Subject Classification: Primary 37N25 , 34C23, 34D35 .

Introduction

Predator-prey models have received increasing attention in recent years, particularly with the work of Lotka-Volterra. Several researchers have modified the Lotka-Volterra system by adding other elements to make the model more realistic [1, 3, 4, 5]. One important phenomenon that has attracted the attention of many researchers and that can be used in predator-prey models is the Allee effect [2]. Broadly speaking, the Allee effect is a biological phenomenon characterised by a positive correlation between the density of a population and its growth rate per individual at low densities, which can lead to critical population thresholds. The main causes of this effect are difficulties in finding mates, lack of inbreeding, social dysfunction in small populations, predator avoidance and exploitation of food resources. Several experimental studies have demonstrated the existence of this effect in different natural species, including plants, insects, birds and mammals.

In this paper, we analyse the dynamic behaviour of a model in which the prey population is influenced by the double Allee effect by modifying the logistic growth rate of the prey population according to the following functional form

$$f(x) = rx \left(1 - \frac{x}{k}\right) \left(\frac{x - \theta}{x + \mu}\right).$$

Consider the discrete time predator-prey model

$$\begin{cases} x_{n+1} = x_n \exp \left[r \left(1 - \frac{x_n}{k}\right) \left(\frac{x_n - \theta}{x_n + \mu}\right) - b y_n \right], \\ y_{n+1} = y_n \exp (v x_n - d). \end{cases} \quad (175)$$

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Where x_n and y_n denote the number of prey and predators, respectively, at time n ; $k > 0$ is the carrying capacity of the prey; $r > 0$ is the intrinsic growth rate of the prey population; c is the conversion rate of prey into a new predator, and the per capita mortality rate of predators is denoted by d . $\theta > 0$ is the Allee effect parameter, and μ quantifies the strength of the Allee effect, $v = bc$. Of course, the initial values $x(0) = x_0$ and $y(0) = y_0$ are assumed to be non-negative real numbers.

The model in equation 175 reduces, in the absence of the predator, to the one-dimensional Ricker model with double Allee effects. The model has three fixed points $E_0 = (0, 0)$, $E_1 = (k, 0)$, $E_2 = (\theta, 0)$ and one positive fixed point $E_3 = (x, \frac{r}{bk})$.

Flip bifurcation

In this section, we will give the condition for the existence of the flip bifurcation at the positive fixed point E_3 .

Theorem 60. Assume that $L_2 \neq 0$, then the system 175 undergoes flip bifurcation at the unique point E_3 , when parameter θ varies in small neighborhood of θ_1 . Furthermore, if $L_2 > 0$, then the period-two orbits that bifurcate from positive fixed point E_3 are stable, and if $L_2 < 0$, then these orbits are unstable.

Where $L_2 = \left(\frac{1}{6} \frac{\partial^3 F}{\partial u^3} + \left(\frac{1}{2} \frac{\partial^2 F}{\partial u^2} \right)^2 \right)_{(0,0)}$ and F the map restricted to the center manifold $W^c(0, 0)$.

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Modeling and Analysis of Coupled Bending and Torsional Vibrations in Rotor Systems: A Simplified Dynamic Model and Runge-Kutta Method Approach

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Abstract:

The nonlinear coupling between bending and torsional vibrations has a significant impact on the dynamic behavior of rotor systems. This coupling arises from mass imbalance terms, and the torsional and lateral movements experience external excitations. However, modeling these dynamic behaviors is challenging, primarily due to the nonlinearity of their equations of motion. In our study, we developed a simplified dynamic model for a rotor-bearing system with two discs. To derive the governing equations, we employed the Lagrange approach, considering three degrees of freedom, including gravity. These equations were then solved using the Runge-Kutta method implemented in the MATLAB environment. To assess the effectiveness of our developed dynamic model, we conducted simulation examples using MATLAB. We specifically focused on detecting the lateral frequency components in lateral vibration spectra and vice versa. Our results demonstrate that the developed dynamic model accurately simulates the behavior of the rotor system. Furthermore, we found that the Runge-Kutta method is a reliable and efficient numerical technique for addressing the nonlinearity present in the equations of motion for rotor systems.

Keywords: Rotor dynamics, coupled torsional-lateral vibrations, rotor system, imbalance, external excitations.

Introduction

The study of vibrations in rotor systems is of paramount importance in many industrial applications, such as turbines, engines, and rotating machinery in general. These vibrations can affect equipment performance, durability, and even safety. Thus, understanding the phenomena of torsional-lateral rotor vibrations is essential for designing reliable and efficient systems.

In this context, several researchers have contributed to developing models and analysis methods to understand these complex vibrations. Among these contributions, Tondel [1] developed a theory on torsional-lateral vibration of rotor systems, laying the groundwork for many subsequent research endeavors. Mohieldin and Khalif [2] proposed an elastodynamic model using the finite element method to study bending and torsional motions of rotating shafts, providing a precise approach to analyze the dynamic response of such systems. Al-Bdoor [3] presented a coupled

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bending-torsion vibration model of an unbalanced rotor, using the dynamic Lagrange method to derive the motion equations, revealing nonlinear phenomena such as inertial coupling and nonlinear interaction between rotor bending and torsion vibrations. Cohen and Porat [4] examined an unbalanced rotor model driven by a flexible shaft, highlighting stability aspects during torsional-lateral resonance, a crucial aspect for the design and operation of rotating machinery. Belmans [5] studied the influence of torsional vibrations in induction motor drive systems, while Perera [6] conducted a comprehensive study of coupled torsional-lateral vibrations of unbalanced rotors through analytical and experimental analyses, thereby contributing to validating theoretical models and improving our understanding of vibrational phenomena in these systems. Finally, Shen et al. [7] presented a recent study considering gyroscopic effects, gravity, and two types of imbalance (static and dynamic) in the rotor system, subjecting torsional and lateral motions to excitations.

The objective of this study is to examine the dynamics of rigid rotors in steady-state conditions. To achieve this, a model of a rigid rotor with two discs was employed to simulate their behavior when supported by a pair of rigid bearings, subjected to drive torque, with imbalance and gravity effects, without considering gyroscopic effects. The analysis focused on the frequency response of lateral and torsional vibrations for various rotational speeds and torsional excitations, while maintaining constant rotational speed (steady-state). The obtained results were used to establish spectra for each case, identifying coupled torsional-lateral vibration frequencies. Subsequently, these results were analyzed and interpreted.

System Description

In this study, we examine a system consisting of a rotating shaft carrying two unbalanced disks (see Figure 1). The coordinate system used is illustrated in Figure 1. The coordinate O represents the axis of rotation of the shaft, O_1 is the axis of rotation of the first disk, and O_2 is the center of mass of the disk. The coordinates of O_1 are given by (x_i, y_i) , and the coordinates of O_2 are given by (x_{mi}, y_{mi}) . The rotation angle of the center of mass of the disk is represented by ϕ_i . The variables m_i , e_i , and I_i respectively represent the masses, eccentricities, and polar moments of inertia of the disks, with the index $(i = 1, 2)$ corresponding to the disk number.

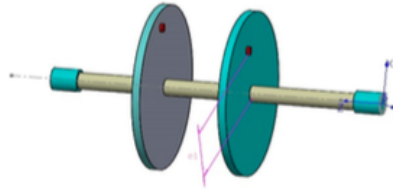


Figure 2: A shaft carrying two disks. .

In deriving the equations of motion, we focus on the torque applied directly to the rotors while also including the effect of gravitational force. We neglect the mass of the shaft compared to that of the disks.

$$\begin{cases} \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{x}_i} \right) - \frac{\partial T}{\partial x_i} + \frac{\partial D}{\partial \dot{x}_i} + \frac{\partial U}{\partial x_i} = F_{xi} \\ \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{y}_i} \right) - \frac{\partial T}{\partial y_i} + \frac{\partial D}{\partial \dot{y}_i} + \frac{\partial U}{\partial y_i} = F_{yi} \\ \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\phi}_i} \right) - \frac{\partial T}{\partial \phi_i} + \frac{\partial D}{\partial \dot{\phi}_i} + \frac{\partial U}{\partial \phi_i} = T_i \end{cases} \quad (176)$$

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Where T , U , and D are the kinetic, potential, and dissipation energies of the system, and (F_{xi}, F_{yi}, T_i) are the generalized forces and moments.

For the model given in Figure (2), the expressions for the kinetic, potential, and dissipation energies of the system are :

$$\begin{cases} T = \frac{1}{2}m_1(\dot{x}_{m1}^2 + \dot{y}_{m1}^2) + \frac{1}{2}I_1\dot{\phi}_1^2 + \frac{1}{2}m_2(\dot{x}_{m2}^2 + \dot{y}_{m2}^2) + \frac{1}{2}I_2\dot{\phi}_2^2 \\ U = \frac{1}{2}F_{x1}x_1 + \frac{1}{2}F_{y1}y_1 + m_1gy_{m1} + \frac{1}{2}F_{x2}x_2 + \frac{1}{2}F_{y2}y_2 + m_2gy_{m2} + \frac{1}{2}k_T(\phi_2 - \phi_1)^2 \\ D = \frac{1}{2}(c_x\dot{x}_1^2 + c_y\dot{y}_1^2) + \frac{1}{2}c_T\dot{\phi}_1^2 + \frac{1}{2}(c_x\dot{x}_2^2 + c_y\dot{y}_2^2) + \frac{1}{2}c_T\dot{\phi}_2^2 \end{cases} \quad (177)$$

Where F_{x1} and F_{y1} are the forces in the x and y directions at the position of disk 1, and similarly, F_{x2} and F_{y2} are the forces at the position of disk 2. g represents the acceleration due to gravity.

The expressions for F_{x1} , F_{y1} , F_{x2} , and F_{y2} are given by:

$$\begin{cases} F_{x1} = k_{11}x_1 + k_{12}x_2 \\ F_{y1} = k_{11}y_1 + k_{12}y_2 \end{cases} \quad \begin{cases} F_{x2} = k_{21}x_1 + k_{22}x_2 \\ F_{y2} = k_{21}y_1 + k_{22}y_2 \end{cases} \quad (178)$$

Where k_{11} , k_{12} , k_{21} , k_{22} , c_x , c_y , c_T are the stiffness and damping coefficients of the shaft at the corresponding disk positions.

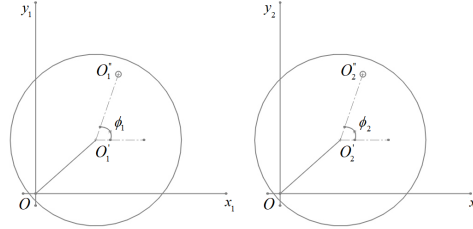


Figure 3: Cartesian coordinates of the two-disk rotor system.

To locate the position of the unbalance mass m , let's examine the diagram of the disk shown in Figure (2).

$$\begin{cases} x_{m1} = x_1 + e_1 m_1 \cos \phi_1 \\ y_{m1} = y_1 + e_1 m_1 \sin \phi_1 \end{cases} \quad et \quad \begin{cases} x_{m2} = x_2 + e_2 m_2 \cos \phi_2 \\ y_{m2} = y_2 + e_2 m_2 \sin \phi_2 \end{cases} \quad (179)$$

Taking the first derivative with respect to time of equation (179).

$$\begin{cases} \dot{x}_{m1} = \dot{x}_1 - e_1 m_1 \dot{\phi}_1 \sin \phi_1 \\ \dot{y}_{m1} = \dot{y}_1 + e_1 m_1 \dot{\phi}_1 \cos \phi_1 \end{cases} \quad et \quad \begin{cases} \dot{x}_{m2} = \dot{x}_2 - e_2 m_2 \dot{\phi}_2 \sin \phi_2 \\ \dot{y}_{m2} = \dot{y}_2 + e_2 m_2 \dot{\phi}_2 \cos \phi_2 \end{cases} \quad (180)$$

By substituting equations (178, 179, 180) into (177), the Lagrange equation (176) is applied for the generalized coordinates x , y , and ϕ . Solving with respect to x , y , and ϕ , a set of 6 equations is obtained (see "Appendix" for the corresponding equations).

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$$\begin{cases} m_1 \ddot{x}_1 + c_x \dot{x}_1 + k_{11} x_1 + \frac{1}{2}(k_{12} + k_{21})x_2 = m_1 e_1 (\ddot{\phi}_1 \sin \phi_1 + \dot{\phi}_1^2 \cos \phi_1) \\ m_1 \ddot{y}_1 + c_y \dot{y}_1 + k_{11} y_1 + \frac{1}{2}(k_{12} + k_{21})y_2 = m_1 e_1 (-\ddot{\phi}_1 \cos \phi_1 + \dot{\phi}_1^2 \sin \phi_1) - m_1 g y_1 \\ m_2 \ddot{x}_2 + c_x \dot{x}_2 + k_{22} x_2 + \frac{1}{2}(k_{12} + k_{21})x_1 = m_2 e_2 (\ddot{\phi}_2 \sin \phi_2 + \dot{\phi}_2^2 \cos \phi_2) \\ m_2 \ddot{y}_2 + c_y \dot{y}_2 + k_{22} y_2 + \frac{1}{2}(k_{12} + k_{21})y_1 = m_2 e_2 (-\ddot{\phi}_2 \cos \phi_2 + \dot{\phi}_2^2 \sin \phi_2) - m_2 g y_2 \\ (m_1 e_1^2 + I_1) \ddot{\phi}_1 + c_T \dot{\phi}_1 + k_T \phi_1 - k_T \phi_2 = -m_1 e_1 ((g + \ddot{y}_1) \cos \phi_1 + \ddot{x}_1 \sin \phi_1) \\ (m_2 e_2^2 + I_2) \ddot{\phi}_2 + c_T \dot{\phi}_2 + k_T \phi_2 - k_T \phi_1 = -m_2 e_2 ((g + \ddot{y}_2) \cos \phi_2 + \ddot{x}_2 \sin \phi_2) \end{cases} \quad (181)$$

The matrix form of the six equations of motion is presented (The matrices $[M]_{cte}$, $[M]_{var}$, $[K]$, $[C]$, and $\{F\}$ are detailed in the appendix).

$$[M]_{cte} = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & (m_1 e_1^2 + I_1) & 0 & 0 & 0 \\ 0 & 0 & 0 & m_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & (m_2 e_2^2 + I_2) \end{bmatrix} \quad (182)$$

$$[M]_{var} = \begin{bmatrix} 0 & 0 & -m_1 e_1 \sin \phi_1 & 0 & 0 & 0 \\ 0 & 0 & m_1 e_1 \cos \phi_1 & 0 & 0 & 0 \\ -m_1 e_1 \sin \phi_1 & m_1 e_1 \cos \phi_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -m_2 e_2 \sin \phi_2 \\ 0 & 0 & 0 & 0 & 0 & m_2 e_2 \cos \phi_2 \\ 0 & 0 & 0 & -m_2 e_2 \sin \phi_2 & m_2 e_2 \cos \phi_2 & 0 \end{bmatrix} \quad (183)$$

$$[C] = \begin{bmatrix} c_x & 0 & 0 & 0 & 0 & 0 \\ 0 & c_y & 0 & 0 & 0 & 0 \\ 0 & 0 & c_T & 0 & 0 & 0 \\ 0 & 0 & 0 & c_x & 0 & 0 \\ 0 & 0 & 0 & 0 & c_y & 0 \\ 0 & 0 & 0 & 0 & 0 & c_T \end{bmatrix} \quad (184)$$

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$$[K] = \begin{bmatrix} k_{11} & 0 & 0 & \frac{1}{2}(k_{12} + k_{21}) & 0 & 0 \\ 0 & k_{11} & 0 & 0 & \frac{1}{2}(k_{12} + k_{21}) & 0 \\ 0 & 0 & k_T & 0 & 0 & -k_T \\ \frac{1}{2}(k_{12} + k_{21}) & 0 & 0 & k_{22} & 0 & 0 \\ 0 & \frac{1}{2}(k_{12} + k_{21}) & 0 & 0 & k_{22} & 0 \\ 0 & 0 & -k_T & 0 & 0 & k_T \end{bmatrix} \quad (185)$$

$$\{F\} = \begin{pmatrix} e_1 m_1 \dot{\phi}_1^2 \cos \phi_1 \\ e_1 m_1 \dot{\phi}_1^2 \sin \phi_1 - m_1 g \\ T_1 - m_1 e_1 g \cos \phi_1 \\ e_2 m_2 \dot{\phi}_2^2 \cos \phi_2 \\ e_2 m_2 \dot{\phi}_2^2 \sin \phi_2 - m_2 g \\ T_2 - m_2 e_2 g \cos \phi_2 \end{pmatrix} \quad (186)$$

Where the general form is.

$$([M]_{cte} + [M]_{var})\{\ddot{e}\} + [C]\{\dot{e}\} + [K]\{e\} = \{F\} \quad (187)$$

Now, the equation of motion can be written in the general combined form as follows:

$$[M]\{\ddot{e}\} + [C]\{\dot{e}\} + [K]\{e\} = \{F\} \quad (188)$$

In the given context, the notations used are as follows

1. $[M] = ([M]_{cte} + [M]_{var})$: Mass matrix.
2. $[C]$: Damping matrix.
3. $[K]$: Stiffness matrix.
4. $\{F\}$: The vector of external forces (including unbalanced force, gravitational force, and external torque).
5. $\{e\}$: The vector of nodal coordinates for the entire system.

By introducing the identity:

$$[M]\{\dot{e}\} - [M]\{\dot{e}\} = 0 \quad (189)$$

And by using the state vector, we obtain a first-order system equivalent to the second-order system (188), as follows:

$$\begin{bmatrix} [M] & 0 \\ 0 & [M] \end{bmatrix} \frac{d}{dt} \begin{pmatrix} e \\ \dot{e} \end{pmatrix} + \begin{bmatrix} 0 & -[M] \\ [K] & [C] \end{bmatrix} \begin{pmatrix} e \\ \dot{e} \end{pmatrix} = \begin{pmatrix} 0 \\ \{F\}(\dot{\phi}, \phi, t) \end{pmatrix} \quad (190)$$

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by multiplying each of the equations (189) and (190) on the left with $(-[M]^{-1})$, we can obtain a second form of the system:

$$\frac{d}{dt} \begin{Bmatrix} \{e\} \\ \{\dot{e}\} \end{Bmatrix} = \begin{bmatrix} [0] & [I] \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix} \begin{Bmatrix} \{e\} \\ \{\dot{e}\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ -[M]^{-1}\{F\} \end{Bmatrix} \quad (191)$$

The equation (191) can be written in the form:

$$\frac{d}{dt} \{Q\} = [A]\{Q\} + \{B\} \quad (192)$$

Where:

$$A = \begin{bmatrix} [0] & [I] \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix} B = \begin{Bmatrix} \{0\} \\ -[M]^{-1}\{F\} \end{Bmatrix} Q = \begin{Bmatrix} \{e\} \\ \{\dot{e}\} \end{Bmatrix} \quad (193)$$

Numerical example and discussion:

The six governing equations mentioned earlier are reformulated into twelve first-order differential equations in the form $\dot{Q} = f(Q)$. Subsequently, we utilize the fourth-order Runge-Kutta method to integrate this set of equations. The code is written in MATLAB. This numerical approach allows us to solve the differential equations and obtain corresponding numerical solutions for the variables Q . The system parameters used in this study are listed in Table (1).

Note that we assume both rotor disks are identical, i.e., $I_1 = I_2$ and $m_1 = m_2$.

Steady-State Analysis:

Steady-state analysis refers to the study of a system's behavior when conditions have stabilized and do not vary over time. In this analysis, we focus on the steady solutions of the equations that represent the system. For a steady-state analysis, we consider the case of a constant rotor speed ($\dot{\phi} = \Omega$). An essential aspect of this analysis is the identification of critical speeds, which represent the resonance frequencies of the system. These critical speeds are determined by the stiffness and mass characteristics of the system, and they play a crucial role in preventing excessive vibration issues. Using the values mentioned in Table (1), the function "eig" is used to calculate the critical speeds as follows:

$$[V, D] = \text{eig}([K], [M]_{cte}) \begin{cases} w_{cr(i)} = \text{diag}(\text{sqrt}(D))(\text{rad/s}) \\ f_{cr1} = w_{cr(i)} \cdot \frac{1}{2\pi} (\text{Hz}) \\ \Omega_{cr1} = w_{cr(i)} \cdot \frac{60}{2\pi} (\text{tr/min}) \end{cases} \quad (194)$$

The first two lateral critical speeds and the first critical torsional speed obtained from (194).

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Symbol	Parameters	Value	Units
L	Shaft length	1.00	m
r_i	Inner diameter of the shaft	0.00	m
r_e	Outer diameter of the shaft	0.154	m
I_a^d	Diametral moment of inertia of the shaft	2.7609×10^{-5}	$Kg.m^2$
k_{11}, k_{22}	Lateral stiffness (direct)	1.25264×10^9	$N.m^{-1}$
k_{12}, k_{21}	Lateral stiffness (crossed)	-1.0481×10^9	$N.m^{-1}$
k_T	Torsional stiffness	3.9757×10^6	$N.m^{-1}$
C	Internal damping coefficient of the shaft	5000.0	$N.m^{-1}$
E	Elastic modulus	2.10^{11}	$N.m^{-2}$
G	Shear modulus	72.10^9	$N.m^{-2}$
ρ	Mass per unit volume	7680	$Kg.m^{-3}$
		Disk 1	Disk 2
L_1	Distance from the first bearing support	0.3	m
L_2	Distance from the first bearing support		0.7 m
m	Mass of the disk	138.723	138.723 Kg
I_D^p	Polar moment of inertia of the disk	10.2544	10.2544 $Kg.m^2$
I_D^d	Diametral moment of inertia of the disk	0.0172	0.0172 $Kg.m^2$
h	Disk thickness	0.038	0.038 m
R_i	Inner diameter of the disk	0.154	0.154 m
R_e	Outer diameter of the disk	0.769	0.769 m
e	Rotor disk eccentricity	0.0038	0.0038 m

Table 1: Shaft-Disk Data

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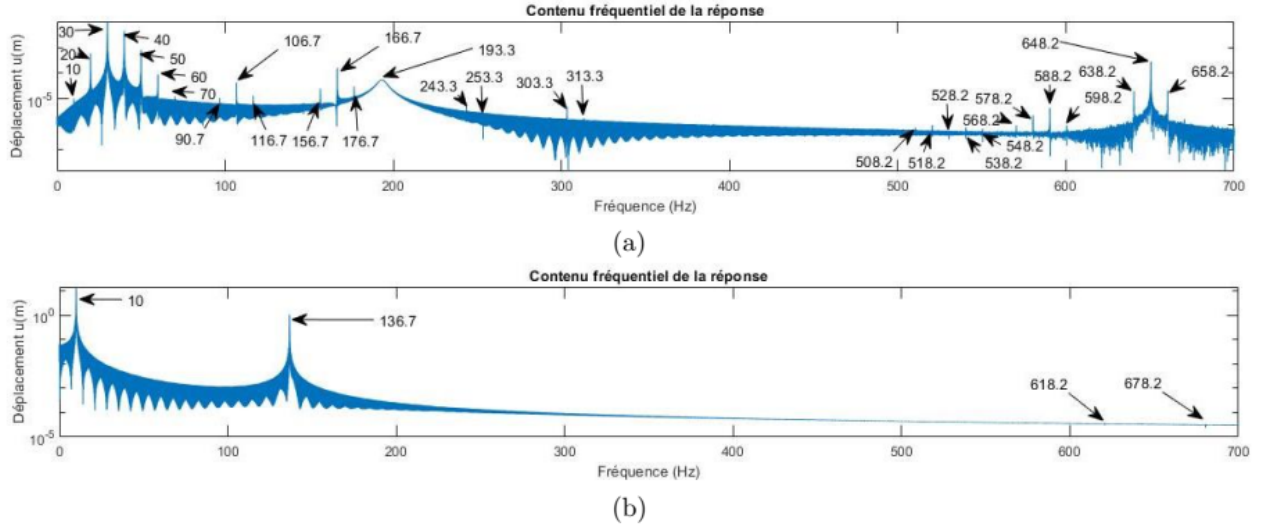


Figure 4: Spectrum plots when $f = 30(\text{Hz})$: (a) Lateral motion; (b) Torsional motion.

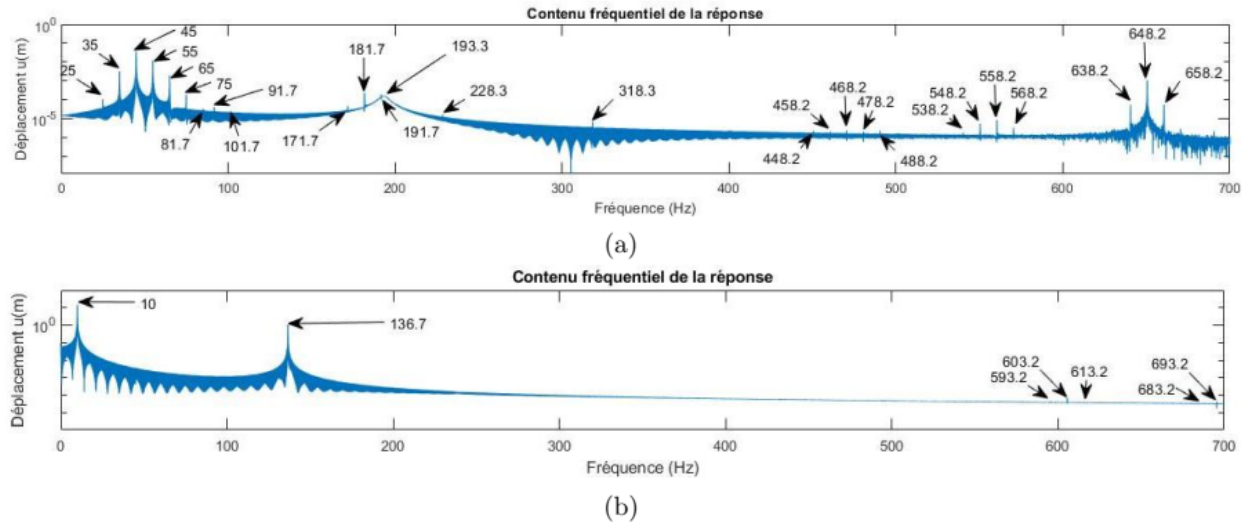


Figure 5: Spectrum plots when $f = 45(\text{Hz})$: (a) Lateral motion; (b) Torsional motion.

$$\begin{cases} w_{cr1} = 1214.3(\text{rad/s}) \\ w_{cr2} = 4072.5(\text{rad/s}) \\ w_{tors} = 8588.0(\text{rad/s}) \end{cases} \begin{cases} f_{cr1} = 193.2569(\text{Hz}) \\ f_{cr2} = 648.1562(\text{Hz}) \\ f_{tors} = 136.6792(\text{Hz}) \end{cases} \begin{cases} \Omega_{cr1} = 11595.0(\text{tr/min}) \\ \Omega_{cr2} = 38889.0(\text{tr/min}) \\ \Omega_{tors} = 8201.0(\text{tr/min}) \end{cases} \quad (195)$$

In order to explore the coupling phenomenon in steady-state, an external sinusoidal excitation $T = T_0 \cos(f_T \cdot 2\pi \cdot t)$ is applied to the first disk, with $f_T = 10\text{Hz}$ as the excitation frequency and T_0 as the torque amplitude in $(N.m)$. The shaft rotation speed is f . The first two critical frequencies obtained from the relation (194) are $f_{cr1} = 193.2569(\text{Hz})$ and $f_{cr2} = 648.1562(\text{Hz})$, and the first critical torsional frequency is given by $f_{tors} = 136.6792(\text{Hz})$.

The objective is to study how the torsional excitation affects the lateral vibration response of an unbalanced rotor. The vibrational responses for three different shaft rotation frequencies, $f = 30\text{Hz}$, and $f = 45\text{Hz}$, are presented in Figures (3.a, 3.b, 4.a, 4.b).

When an external torque is applied to the torsional motion of the rotor system, it leads to lateral

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vibrations of the rotor with multiple frequency components.

In all the graphs, we can observe the presence of the shaft rotation frequency (f) in the lateral frequency spectrum. This indicates that the shaft rotation frequency is clearly detectable in the lateral frequency spectrum. Additionally, the existence of the torsional excitation frequency (f_T) can be noticed in the lateral frequency spectrum, resulting from the coupling between torsional and flexural motion. The appearance of sum and difference frequencies ($f \pm n \cdot f_T$) can be observed in Figure (3.a) around the shaft rotation frequency [$(f - 2 \cdot f_T)=10\text{Hz}$, $(f - f_T)=20\text{Hz}$, $(f)=30\text{Hz}$, $(f + f_T)=40\text{Hz}$, $(f + 2 \cdot f_T)=50\text{Hz}$, $(f + 3 \cdot f_T)=60\text{Hz}$]. This indicates that the rotor undergoes lateral movement in different directions and at different frequencies. This phenomenon is due to the coupling between torsional and lateral vibration modes. Furthermore, the appearance of sum and difference frequencies ($f_{tors} \pm f \pm n \cdot f_T$) can also be noticed around the first torsional critical frequency [$f_{tors}-f-f_T=96.7\text{Hz}$, $(f_{tors}-f)=106.7\text{Hz}$, $(f_{tors}-f+f_T)=116.7\text{Hz}$, $(f_{tors}+f-f_T)=156.7\text{Hz}$, $(f_{tors}+f)=166.7\text{Hz}$, $(f_{tors}+f+f_T)=176.7\text{Hz}$], resulting from the coupling between torsional and flexural motion.

Additionally, one can observe the appearance of sum and difference frequencies ($f_{cr1} \pm n \cdot f_T$), ($f_{cr2} \pm n \cdot f_T$), and ($f_{cr1} \pm 2 \cdot n \cdot f \pm n \cdot f_T$), ($f_{cr2} \pm 2 \cdot n \cdot f \pm n \cdot f_T$) in the lateral frequency spectrum around the first and second critical frequencies. For the second critical frequency [$(f_{cr2} - f_T)=638.2\text{Hz}$, $f_{cr2}=648.2\text{Hz}$, $(f_{cr2} + f_T)=658.2\text{Hz}$, $((f_{cr2} - 2 \cdot f) - f_T)=578.2\text{Hz}$, $(f_{cr2} - 2 \cdot f)=588.2\text{Hz}$, $((f_{cr2} - 2 \cdot f) + f_T)=598.2\text{Hz}$, and $((f_{cr2} - 4 \cdot f) - f_T)=518.2\text{Hz}$, $(f_{cr2} - 4 \cdot f)=528.2\text{Hz}$, $((f_{cr2} - 4 \cdot f) + f_T)=538.2\text{Hz}$], and for the first critical frequency [$(f_{cr1} + 2 \cdot f - f_T)=303.3\text{Hz}$], the sum and difference frequencies around the first critical frequencies are identical to those around the second critical frequencies. The differences between these two are mainly attributed to the presence of certain amplitude components that are too small to be clearly visualized.

The torsional motion in Figure (3.b) is relatively simple, with four vibrations generated with frequencies [$f_T=10\text{Hz}$, $f_{tors}=136.7\text{Hz}$, $(f_{cr2} - f)=618.2\text{Hz}$, $(f_{cr2} + f)=678.2\text{Hz}$].

The frequencies present in Figures (4.a) are identical to those present in Figure (3.a). The differences between these two figures are primarily due to the shaft rotation frequency (f). Due to this lateral and torsional coupling mechanism, the torsional excitation affects the lateral vibration response in unbalanced rotor systems.

Conclusion:

The study of the effect of torsional excitation on the lateral vibratory response of an unbalanced rotor has revealed crucial insights into the dynamics of this complex mechanical system. By analyzing vibratory responses for different shaft rotation frequencies, we have observed that torsional excitation induces significant coupling between torsional and lateral vibration modes. This complex interaction manifests in the presence of multiple frequency components in the lateral frequency spectrum, with sum and difference frequencies derived from the first and second critical frequencies as well as the torsional excitation.

The findings of this study underscore the importance of considering torsional excitation in the design and analysis of unbalanced rotor systems. The coupling phenomenon between vibration modes can lead to nonlinear and unpredictable vibratory responses, necessitating a careful approach to prevent excessive vibration problems and ensure system stability and reliability.

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The Blow-Up Existence of Solution to Caputo–Katugampola Fractional Partial Differential Equation with Fractional Laplace

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Abstract: In the present paper, the semilinear time-space fractional diffusion equation, where the time derivative is in the Caputo–Katugampola sense and the spatial derivative is the fractional Laplacian, necessary or sufficient conditions of blow-up in a finite time are established.

Keywords: Caputo–Katugampola derivative, Fractional Laplacian, Blow-up, Global weak solution.

2020 Mathematics Subject Classification: 35R11 , 35B44, 35A01

Introduction

The main goal of this work is to discuss the global non-existence of solutions for the semilinear time-space fractional diffusion equation

$$\begin{cases} {}_C D_{0,t}^{\alpha,\rho} u + (-\Delta)^{\beta/2} u = h(x,t)|u|^p, (x,t) \in \mathbb{R}^N \times (0,T), \\ u(x,0) = u_0(x) \geq 0, x \in \mathbb{R}^N, \end{cases} \quad (196)$$

where $\alpha \in (0,1), \rho > 0, \beta \in (1,2), {}_C D_{0,t}^{\alpha,\rho}$ denotes the α -th-order Caputo–Katugampola derivative operator and $(-\Delta)^{\beta/2}$ represent the fractional Laplace operator.

The α -th-order Caputo–Katugampola derivative of a function $f(t)$ is defined by [1],

$${}_C D_{0,t}^{\alpha,\rho} f(t) = I_{0,t}^{1-\alpha,\rho} \gamma f(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \left(\frac{t^\rho - u^\rho}{\rho} \right)^{-\alpha} f'(u) du, t \in (0,T],$$

where $\gamma = t^{1-\rho} \frac{d}{dt}$ and $\Gamma(\cdot)$ is the Euler Gamma function.

Main results

Concerning the function $h(x,y)$ we require the condition (H) :

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$$h(x, t) \geq C_h |x|^\sigma t^s \quad \text{for } x \in \mathbb{R}^N, t > 0, C_h > 0.$$

The assumptions on σ and s will be determined through the convergence of certain integrals in the proof.

Theorem 61. *Let $N \geq 1$ and $p > 1$. Assume that (H) is satisfied. If*

$$1 < p \leq p_c = 1 + \frac{\alpha\rho(\beta + \sigma) + \beta s + \beta(1 - \rho)}{\alpha\rho N + \beta\rho(1 - \alpha)},$$

then the weak solution of problem (196) blows-up in a finite time.

Theorem 62. *Suppose that problem (196) has a blows-up in a finite time weak solution. Then, there is a positive constant K such that*

$$\liminf_{|x| \rightarrow \infty} \left[u_0(x) |x|^{\alpha\rho(p'-1)} h(x)^{1-p'} \right] \leq K,$$

where $\frac{1}{p} + \frac{1}{p'} = 1$.

Remark 3. *When $\rho = 1$, we recover the case studied by M. Kirane et al.[3].*

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On the study the existence and asymptotic behavior of solution for a viscoelastic wave equations with logarithmic nonlinearity

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Abstract: In this work, we introduce the viscoelastic wave equation

$$u_{tt} - \Delta(au_t + u) + \int_0^t h(t-s)\Delta u(s)ds + b|u_t|^{m(x)-2}u_t = u(\ln|u|^{r(x)} - 1).$$

with logarithmic nonlinear and a nonlinear feedback having a variable exponent $m(\cdot)$. Firstly, Solutions are established both locally and globally through the application of the Galerkin and energy methods . Thereafter, we investigate a general decay result.

Keywords:Faedo-Galerkin, variable exponent, local solution, Lyapunov functional

2020 Mathematics Subject Classification: 35B40, 35L70..

Introduction

Within this presentation, we discuss the existence(local, global) and stability behavior of solutions of the initial value problem

$$\begin{cases} u_{tt} - \Delta(au_t + u) + u + \int_0^t h(t-s)\Delta u(s)ds + b|u_t|^{m(x)-2}u_t \\ \hspace{15em} = (\ln|u|^{r(x)} - 1)u & \text{in } \mathcal{O} \times (0, \infty) \\ u(x, t) = 0, & \text{in } \Gamma \times (0, \infty) \\ u(x, 0) = u_0(x), u_t(x, 0) = u_1(x), & \text{in } \mathcal{O}. \end{cases} \quad (197)$$

where $\mathcal{O}$ is a bounded domain in $\mathbb{R}^n, n \in \mathbb{N}$, with a smooth boundary $\partial\mathcal{O} = \Gamma$, the constants a, b are positives, $h(\cdot)$ is a nonegative non-increasing function defined on $\mathbb{R}_+$ ang (u_0, u_1) are the initial data belonging to a suitable function space. The exponents $r(\cdot), m(\cdot)$ is given measurable functions on $\mathcal{O}$ satisfying

$$r(x) = \frac{1}{(1 + |x|)^\beta}, \quad (198)$$

with $\beta > 0$, and

$$2 \leq m_1 \leq m(x) \leq m_2 < \infty,$$

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with

$$m_1 = \operatorname{ess\,inf}_{x \in \mathcal{O}} m(x), \quad m_2 = \operatorname{ess\,sup}_{x \in \mathcal{O}} m(x),$$

we also assume that $m(\cdot)$ be log-continuous in $\mathcal{O}$ such that

$$|m(x_1) - m(x_2)| \leq \frac{A}{|\log(|x_1 - x_2|)|}, \quad \forall x_1, x_2 \in \bar{\mathcal{O}}, \text{ such that } |x_1 - x_2| < \delta, \quad (199)$$

for any $x_1, x_2 \in \mathcal{O}$, $A > 0$ and $0 < \delta < 1$ (log-Hölder condition).

The equation in (197) is a class of typical semilinear wave equations with strong damping and logarithmic nonlinearity, which often appear in the viscoelastic mechanics, quantum mechanics theory etc. Problem (197) describes several interesting phenomena in mathematical physics for example in the case $n = 1$ or 2 this type of equations describe the vibrations of a homogeneous viscous thread and the longitudinal vibrations of a homogeneous viscous bar, respectively.

Main results

We use the Faedo-Galerkin criterion to confirm the existence of the local and global solution to the wave equation (197). Then main result is in this theorem:

Theorem 63. *Let $(u_0, u_1) \in H_0^1(\mathcal{O}) \times L^2(\mathcal{O})$ and $\varepsilon \in (0, 2p - 1)$ and $t_0 > 0$. Suppose that (H1)-(H3) verified. Then, for β in (198) so large, there is a constant $M_1 \geq 0$ such that the solution of (197) satisfies*

$$E(t) \leq M_1 \left(1 + \int_{t_0}^t \zeta^{2p-1+\varepsilon}(s) ds \right)^{\frac{-1}{2p-2+\varepsilon}}, \quad \text{for any } t \geq t_0.$$

Additionally, if there is $\varepsilon_1 \in (0, 2p - 1)$ and $t_0 > 0$ such that

$$\int_{t_0}^{\infty} \left(1 + \int_{t_0}^t \zeta^{2p-1+\varepsilon_1}(s) ds \right)^{\frac{-1}{2p-2+\varepsilon_1}} dt < \infty,$$

then, $\forall \varepsilon \in (0, p)$ and $t_0 > 0$, there exists a constant $M_2 \geq 0$ such that the solution of (197) verifies

$$E(t) \leq M_2 \left(1 + \int_{t_0}^t \zeta^{p+\varepsilon}(s) ds \right)^{\frac{-1}{p-1+\varepsilon}}, \quad \forall t \geq t_0.$$

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Etude d'un problème de Transmission dans un Domaine non Borné, cadre holdérien

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 Spécialité : Analyse fonctionnelle

Abstract: On se propose d'étudier le problème abstrait du second ordre suivant

$$(p) \begin{cases} u''(x) + Au(x) = g(x), & x \in]-\infty, 0[\cup]0, \delta[, \\ u'(\delta) = f_+, \text{ et } u(-\infty) = f_-, \\ u(0_-) = u(0_+) \text{ et } \mu_- u'(0_-) = \mu_+ u(0_+). \end{cases}$$

posée dans un espace de Banach E , avec

- A est un opérateurs linéaires fermés à valeur dans E .
- f_-, f_+ sont deux éléments de E .
- μ_-, μ_+ sont deux constantes réelles positives non nulles et distinctes exprimant la conductivité des deux milieux.

L'étude se fait sous l'unique l'hypothèse d'ellipticité sur l'opérateur linéaire fermé A de domaine $D(A) \subset E$ i.e.,

$$(H) \begin{cases} \rho(A) \supset [0, +\infty[, \text{ où } \rho(A) \text{ est l'ensemble résolvant de } A \\ \exists C > 0 \text{ tel que } \forall \lambda \in [0, +\infty[: \|(A - \lambda I)^{-1}\| \leq \frac{1}{1 + \lambda} \end{cases}$$

dans le cadre continue, plus précisément lorsque le second membre $g = (g_-, g_+)$ est holdérienne

$$g_- \in BUC^{2\alpha}(-\infty, 0; E), \quad g_+ \in C^{2\alpha}(0, \delta; E)$$

d'exposant $2\alpha \in]0, 1[$.

L'objectif est de montrer l'existence, l'unicité et la régularité maximale de la solution stricte c'est à dire les restrictions (u_+, u_-) de la solution u sur les deux intervalles $[0, \delta]$ et $]-\infty, 0]$ vérifient

$$\begin{cases} u_+ \in C^2([0, \delta]; E) \cap C([0, \delta]; D(A)), \\ u_- \in BUC^2(]-\infty, 0]; E) \cap BUC(]-\infty, 0]; D(A)). \end{cases}$$

en plus $u_+, Au_+ \in C^{2\alpha}(0, \delta; E)$ et $u_-, Au_- \in BUC^{2\alpha}(-\infty, 0; E)$.

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Résultats obtenus

Theorem 64 (Solution Stricte). *Supposons (H). Soient*

$$g_+ \in C^{2\alpha}([0, \delta]; E), g_- \in BUC^{2\alpha}([-\infty, 0]; E) \text{ avec } 0 < 2\alpha < 1$$

Si $f_+ \in D\left((-A)^{\frac{1}{2}}\right)$ et $f_- \in D(A)$. Alors le problème (P_0) admet une unique solution stricte si et seulement si $g_+(0) - g_-(0) \in \overline{D(A)}$, $Af_-, (-A)^{\frac{1}{2}}f_+ \in \overline{D(A)}$.

Theorem 65 (Régularité). *Supposons (H_1) . Soient*

$$g_+ \in C^{2\alpha}([0, \delta]; E), g_- \in BUC^{2\alpha}([-\infty, 0]; E)$$

avec $0 < 2\alpha < 1$ et $f_- \in D(A)$, $f_+ \in D\left((-A)^{\frac{1}{2}}\right)$. Alors

$$u_+'' , Au_+ \in C^{2\alpha}(0, \delta; E); u_+'' , Au_+ \in BUC^{2\alpha}(-\infty, 0; E)$$

si seulement si $g_+(0) - g_-(0), Af_-, (-A)^{\frac{1}{2}}f_+ \in D_A(\alpha, +\infty)$ où

$$D_A(\alpha, \infty) = \left\{ x \in E, \|x\|_\alpha = \sup_{t>0} \|t^{1-\alpha} Ae^{At}x\| < \infty \right\}.$$

Les techniques employées ici reposant sur la théorie de semi-groupes (voir par exemple [4] pour cette théorie) et le travail de Sinestrari [3].

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Numerical solution of nonlinear Volterra integral equations using wavelets method

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Abstract: he wavelets method with orthogonal polynomials is used for solving nonlinear Volterra integral equations.

This method reduces the integral equations to a system of algebraic equations, which can be solved by using an appropriate method. This method is used to solve some nonlinear Volterra integral equations.

Numerical experiments show that the proposed technique is accurate and efficient.

Keywords: wavelets method, orthogonal polynomials, nonlinear Volterra integral equations.

2020 Mathematics Subject Classification: 65T60, 65R20, 45D05.

Introduction

wavelet and polynomials wavelets

One of the key advantages of wavelet methods is their ability to handle a wide range of equations, including ordinary differential equations (ODEs), partial differential equations (PDEs), integral equations, and integral-differential equations. Moreover, wavelet methods can accommodate problems defined on irregular domains, non-uniform grids, or in higher dimensions, making them suitable for a diverse array of applications in computational mathematics and scientific computing.

Wavelet is a family of functions generated from the dilation and translation a function called mother wavelet. We have the following family of continuous wavelets, when the translation parameter a and the dilation parameter b vary continuously [2, 4]

$$\psi_{a,b}(x) = |a|^{-\frac{1}{2}} \psi\left(\frac{x-b}{a}\right), \quad a, b \in \mathbb{R}, a \neq 0.$$

If we put $a = 2^{-k}$ and $b = 2^{-k}n$, we get this forms of family:

$$\psi_{k,n}(x) = 2^{-\frac{k}{2}} \psi(2^k x - n),$$

where $\psi_{k,n}(x)$ forms a wavelet basis for $L^2(\mathbb{R})$. We can define polynomials wavelets $\psi_{n,m}(x)$ on the

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interval $[0;1)$ as follows:

$$\psi_{n,m}(x) = \begin{cases} \frac{2^{-\frac{k}{2}}}{\|P_m(x)\|_{L^2}} P_m(2^k x - n), & \frac{n}{2^k} \leq x < \frac{n+1}{2^k}, \\ 0, & \text{otherwise,} \end{cases}$$

where k is assumed to be any positive integer, $n = 0, 1, 2, \dots, 2^k - 1$ and the coefficient $\|P_m(x)\|_{L^2}$ is for normality and $m = 0, 1, \dots, M - 1$. Here $P_m(x)$ is a polynomials of degree m we can choice any polynomials like Legendre, Chebyshev, Hermit... for define polynomials wavelets. This family can approximate function from $L^2(\mathbb{R})$, which led to the propose method for solve nonlinear Volterra integral equations second kind as:

$$u(x) = f(x) + \lambda \int_0^x k(x, t, u(t)) dt \quad (200)$$

where $u(x)$ the unknown function. By converting the Eq (200) to nonlinear system algebraic as following:

$$U^T - S^T = F^T \quad (201)$$

where U is the unknown $2^k M$ vector and $u(x) \simeq U^T \psi(x)$, $\psi(x)$ are $2^k M$ vector of element $\psi_{nm}(x)$.

Main results

The present work have lead to highlight the following results:

1. A novel proposed method for solving integral equations.
2. The obtained results show that the maximum absolute error decreases with the increasing values of k and m using this method for different examples.

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ERROR ESTIMATES FOR FINITE ELEMENT APPROXIMATIONS OF A 2D-VISCOELASTIC WAVE EQUATION WITH DYNAMIC BOUNDARY CONDITIONS

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Abstract: In this paper, we study the 2D linear wave equation with dynamical control on the boundary. We show and analyze typical semi-discrete systems as well as an implicit fully discrete scheme. For both semi discrete and fully discrete methods, optimal a priori error estimates are demonstrated. Finally, some numerical findings and a priori error estimate are derived.

Keywords: Error estimates, Finite element method, Viscoelastic, Wave equation, Dynamic boundary conditions.

2020 Mathematics Subject Classification:

Introduction

In this paper, we consider the following viscoelastic wave equations with strong damping and dynamic boundary conditions:

$$\begin{cases} u_{tt}(x, t) - \Delta u(x, t) - \Delta u_t(x, t) - \Delta u_{tt}(x, t) = 0, & \text{in } Q = \Omega \times [0, T], \\ u(x, t) = 0, & \text{on } \Sigma_0 = \Gamma_0 \times [0, T], \\ \frac{\partial u}{\partial n}(x, t) + \xi(x, t) = 0, & \text{on } \Sigma_1 = \Gamma_1 \times [0, T], \\ \xi_{tt}(x, t) + \xi(x, t) = u_t(x, t), & \text{on } \Sigma_1 = \Gamma_1 \times [0, T]. \end{cases} \quad (202)$$

With the following initial conditions

$$\begin{cases} u(x, 0) = u_0(x), & u_t(x, 0) = u_1(x), & x \in \Omega, \\ \xi(x, 0) = \xi_0(x), & \xi_t(x, 0) = \xi_1(x), & x \in \Gamma_1. \end{cases} \quad (203)$$

Where $\xi(x, t)$ designates the dynamical control. Ω is a bounded open set in $\mathbb{R}^2$ with boundary Γ of class C^2 and $\Gamma = \Gamma_0 \cup \Gamma_1$ with $\Gamma_0 \neq \emptyset$ and $\bar{\Gamma}_0 \cap \bar{\Gamma}_1 = \emptyset$.

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$\frac{\partial}{\partial n}$ denotes the unit outer normal derivative.

The viscoelastic equation explain how waves travel through materials that have both elastic and viscous characteristics. Understanding the evolution of waves in materials that can simultaneously dissipate energy and deform is made possible by the use of viscoelastic wave equations, for the physical aspects, see Love [6].

The non-ordinary boundary conditions defined on (202) are the primary source of difficulty for problem (202). Boundary conditions of this kind are referred to as dynamic boundary conditions, moreover these conditions have numerous practical uses in addition to being significant from a theoretical one.

For example, in one space dimension, the problem (202) can modelize the dynamic evolution of a viscoelastic rod that is fixed at one end and has a tip mass attached to its free end.

The dynamic boundary conditions, which emerge when we analyse the transverse motion of an extensible membrane whose boundary may be affected by the vibrations only in an area, describe Newton's law for the attached mass (see [3], [4] and [5] for more details).

The study of different distributed systems with dynamical boundary controls, from a theoretical point of view, was done by many authors see for example ([7], and [8]).

The numerical study of a wave equation with dynamical boundary control is new. Very little attention has been paid to this field. We mention for example ([9] and [10]). It is demonstrated that the semi-discrete energy decays exponentially, uniformly with respect to the mesh parameter. The system also converges, but no error limitations are given.

Motivated by the previous works, in the present paper we investigate problem (202) in which we generalize the results obtained in [4] by supposing new boundary conditions and add a viscosity term and strong damping.

Main results

Denote by $|\cdot|$, $|\cdot|_1$, $(\cdot, \cdot)$ and $(\cdot, \cdot)_1$ the norms and scalar products in $L^2(\Omega)$ and $L^2(\Gamma_1)$ respectively.

Existence and Uniqueness

Theorem 66. Assume that $u_0, u_1 \in H_{\Gamma_0}^1(\Omega) \cap H^2(\Omega)$, $\xi_0, \xi_1 \in L^2(\Gamma_1)$ and $\frac{\partial u_0}{\partial n}(x) = -\xi_0(x)$, $\frac{\partial u_1}{\partial n}(x) = -\xi_1(x)$ for all $x \in \Gamma_1$.

Then for each $T > 0$, problem (202) admits a unique solution (u, ξ) satisfying the following conditions:

1. $u \in L^\infty(0, T, H_{\Gamma_0}^1(\Omega))$,
2. $u_t \in L^\infty(0, T, L^2(\Omega)) \cap L^\infty(0, T, H_{\Gamma_0}^1(\Omega))$,
3. $u_{tt} \in L^\infty(0, T, L^2(\Omega)) \cap L^\infty(0, T, H_{\Gamma_0}^1(\Omega))$,
4. $\xi, \xi_{tt} \in L^\infty(0, T, L^2(\Gamma_1))$.

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Semi-Discrete Approximation

Theorem 67. Assume that the assumption of Theorem 66 holds with $\xi_0, \xi_1 \in H_{\Gamma_0}^2(\Omega)$ and $\xi \in H^1(0, T, H_{\Gamma_0}^2(\Omega))$, then we have the following estimate

$$|u_t - u_t^h|^2 + |\nabla u - \nabla u^h|^2 + |\nabla u_t - \nabla u_t^h|^2 + |\xi - \xi^h|_1^2 + |\xi_t - \xi_t^h|_1^2 \leq Ch^2,$$

holds for $0 \leq t \leq T$.

Fully-Discrete Approximation

Let us denote $\hat{u}^{n+1} = \frac{u^{n+1} - u^n}{\Delta t}$, $\hat{\xi}^{n+1} = \frac{\xi^{n+1} - \xi^n}{\Delta t}$, $\hat{u}(t_n) = u_t(t_n)$ and $\hat{\xi}(t_n) = \xi_t(t_n)$.

Theorem 68. Assume that the assumption of theorem 66 holds, then if $u \in H^3(0, T, L^2(\Omega)) \cap W^{2,\infty}(0, T, H^3(\Omega))$ and $\xi \in H^2(0, T, H^2(\Omega)) \cap W^{1,\infty}(0, T, H^2(\Omega))$ we have

$$|u_t(t_n) - \hat{u}^n|^2 + |\nabla u(t_n) - \nabla u^n|^2 + |\nabla u_t(t_n) - \nabla \hat{u}^n|^2 + |\xi(t_n) - \xi^n|_1^2 + |\xi_t(t_n) - \hat{\xi}^n|_1^2 \leq C(h^2 + (\Delta t)^2).$$

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Solving Fuzzy System of Volterra Integral Equations by using Fuzzy Kamal Transform

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Abstract:

In this survey, we use the Fuzzy Kamal Transform to solve a system of fuzzy volterra integral equation of the seconf kind. Some application are given in this paper.

Keywords: fuzzy volterra intergral equation, system, Fuzzy kamal transform, Convolution theorem.

2020 Mathematics Subject Classification: 03E72, 26A42, 45J05

Introduction

The subject of integral equations is one of the most useful mathematical tools in both pure and applied mathematics. It has enormous applications in many physical problems. Basic concept of fuzzy was first introduced by Professor Zadeh in 1965 after his publications on fuzzy set theory [1]. The topics of fuzzy integral equations have attracted the attention of many researchers in recent years, particularly, in fuzzy control, fuzzy finance and economics systems. The fuzzy systems of integral equations have attracted increasing attention, with regard to fuzzy control, have been developed mathematical models used in many problems of physics, biology, chemistry, engineering, and in other fields depend on an integral equation [4]. Existence and uniqueness of solutions of fuzzy volterra integral equations have been considered in [2]. Alternative methods have been introduced by many authors to solve fuzzy volterra integral equations and systems of fuzzy volterra integral equations. In this work we use fuzzy kamal tranform to solve fuzzy system of fuzzy volterra integral equation.

The parametric form of the FSVIDE-2 is given by for all $i, j = 1, \dots, n$:

$$\begin{cases} \underline{f}_i(t) = \underline{g}_i(t) + \sum_{j=1}^n \lambda_{ij} \int_0^t \underline{k}_{ij}(t-x) \underline{f}_j(x) dx \\ \overline{f}_i(t) = \overline{g}_i(t) + \sum_{j=1}^n \lambda_{ij} \int_0^t \overline{k}_{ij}(t-x) \overline{f}_j(x) dx \end{cases}$$

Fuzzy Kamal transform method

Definition 26. [3]

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- The fuzzy Kamal transform of a fuzzy valued function $f(t)$ denoted $G(v)$ is defined by :

$$G(v) = \mathcal{K}(f(t)) = \int_0^\infty e^{-\frac{t}{v}} f(t) dt$$

For all $t \geq 0$ suppose that $f(t)$ is piecewise continuous and of exponential order, then the fuzzy Kamal transform of f exists .

The lower and upper fuzzy kamal transform of f are defined by

$$G(v, \alpha) = \mathcal{K}(f(t, \alpha)) = [\mathcal{K}(\underline{f}(t, \alpha)), \mathcal{K}(\overline{f}(t, \alpha))].$$

- If $\mathcal{K}(f(t)) = G(v)$ then $f(t)$ is defined as the fuzzy inverse Kamal transform of $G(v)$

$$f(t) = \mathcal{K}^{-1}(G(v))$$

Main results

Fuzzy Kamal Transform method for solving FSVIE-2

Taking Fuzzy Kamal transform operator on system of FSVIE-2 and using convolution theorem of fuzzy Kamal transform, we have

$$\left\{ \begin{array}{l} \mathcal{K}(f_i(t)) = \mathcal{K}(g_i(t)) + \sum_{j=1}^n \lambda_{1j} \mathcal{K}(k_{ij}(t) * f_j(t)) \end{array} \right.$$

The parametric form of this fuzzy system as:

$$\left\{ \begin{array}{l} \mathcal{K}(\underline{f}_i(t, \alpha)) = \mathcal{K}(\underline{g}_i(t, \alpha)) + \sum_{j=1}^n \lambda_{1j} \mathcal{K}(\underline{k}_{ij}(t, \alpha) \mathcal{K}(\underline{f}_j(t, \alpha))) \\ \mathcal{K}(\overline{f}_i(t, \alpha)) = \mathcal{K}(\overline{g}_i(t, \alpha)) + \sum_{j=1}^n \lambda_{1j} \mathcal{K}(\overline{k}_{ij}(t, \alpha) \mathcal{K}(\overline{f}_j(t, \alpha))) \end{array} \right.$$

For solving this system ,we should discuss $\mathcal{K}(\underline{k}_{ij}(t, \alpha)) \mathcal{K}(\underline{f}_j(t, \alpha))$ and $\overline{\mathcal{K}(\underline{k}_{ij}(t, \alpha)) \mathcal{K}(\underline{f}_j(t, \alpha))}$ for $i=1,2,...,n$ and $j=1,2,...,n$. For example using case (01) if $\underline{k}_{ij}(t, \alpha)$ and $\underline{f}_i(t, \alpha)$ are positive, So by using the method Cramer for solving systems and fuzzy inverse Kamal transforms we get :

$$\underline{f}_{-1}(t, \alpha) = \mathcal{K}^{-1} \left(\frac{\begin{vmatrix} \mathcal{K}(\underline{g}_1(t, \alpha)) & -\lambda_{12}\mathcal{K}(\underline{k}_{1n}(t, \alpha)) & \dots & -\lambda_{1n}\mathcal{K}(\underline{k}_{1m}(t, \alpha)) \\ \mathcal{K}(\underline{g}_2(t, \alpha)) & (1 - \lambda_{22})\mathcal{K}(\underline{k}_{22}(t, \alpha)) & \dots & -\lambda_{2n}\mathcal{K}(\underline{k}_{2n}(t, \alpha)) \\ \vdots & & & \vdots \\ \mathcal{K}(\underline{g}_n(t, \alpha)) & -\lambda_{n2}\mathcal{K}(\underline{k}_{n2}(t, \alpha)) & \dots & (1 - \lambda_{nn})\mathcal{K}(\underline{k}_{nn}(t, \alpha)) \end{vmatrix}}{\begin{vmatrix} (1 - \lambda_{11})\mathcal{K}(\underline{k}_{11}(t, \alpha)) & -\lambda_{12}\mathcal{K}(\underline{k}_{12}(t, \alpha)) & \dots & \lambda_{1n}\mathcal{K}(\underline{k}_{1n}(t, \alpha)) \\ -\lambda_{21}\mathcal{K}(\underline{k}_{21}(t, \alpha)) & (1 - \lambda_{22})\mathcal{K}(\underline{k}_{22}(t, \alpha)) & \dots & -\lambda_{2n}\mathcal{K}(\underline{k}_{2n}(t, \alpha)) \\ \vdots & & & \vdots \\ -\lambda_{n1}\mathcal{K}(\underline{k}_{n1}(t, \alpha)) & -\lambda_{n2}\mathcal{K}(\underline{k}_{n2}(t, \alpha)) & \dots & (1 - \lambda_{nn})\mathcal{K}(\underline{k}_{nn}(t, \alpha)) \end{vmatrix}} \right)$$

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$$\underline{f}_{-n}(t, \alpha) = \mathcal{K}^{-1} \left(\frac{\begin{vmatrix} (1 - \lambda_{11})\mathcal{K}(\underline{k}_{11}(t, \alpha)) & -\lambda_{12}\mathcal{K}(\underline{k}_{12}(t, \alpha)) & \dots & \mathcal{K}(\underline{g}_1(t, \alpha)) \\ -\lambda_{21}\mathcal{K}(\underline{k}_{21}(t, \alpha)) & (1 - \lambda_{22})\mathcal{K}(\underline{k}_{22}(t, \alpha)) & \dots & \mathcal{K}(\underline{g}_1(t, \alpha)) \\ \vdots & & & \vdots \\ -\lambda_{n1}\mathcal{K}(\underline{k}_{n1}(t, \alpha)) & -\lambda_{n2}\mathcal{K}(\underline{k}_{n2}(t, \alpha)) & \dots & \mathcal{K}(\underline{g}_1(t, \alpha)) \end{vmatrix}}{\begin{vmatrix} (1 - \lambda_{11})\mathcal{K}(\underline{k}_{11}(t, \alpha)) & -\lambda_{12}\mathcal{K}(\underline{k}_{12}(t, \alpha)) & \dots & \lambda_{1n}\mathcal{K}(\underline{k}_{1n}(t, \alpha)) \\ -\lambda_{21}\mathcal{K}(\underline{k}_{21}(t, \alpha)) & (1 - \lambda_{22})\mathcal{K}(\underline{k}_{22}(t, \alpha)) & \dots & -\lambda_{2n}\mathcal{K}(\underline{k}_{2n}(t, \alpha)) \\ \vdots & & & \vdots \\ -\lambda_{n1}\mathcal{K}(\underline{k}_{n1}(t, \alpha)) & -\lambda_{n2}\mathcal{K}(\underline{k}_{n2}(t, \alpha)) & \dots & (1 - \lambda_{nn})\mathcal{K}(\underline{k}_{nn}(t, \alpha)) \end{vmatrix}} \right)$$

With the same analysis we obtain $\bar{f}_1(t, \alpha)$, $\bar{f}_2(t, \alpha)$, ..., and $\bar{f}_n(t, \alpha)$

Example:

Consider the following two dimensional fuzzy Volterra integral equation

$$\begin{cases} f_1(t) = (2\alpha, 2\alpha^2)t^2 + \int_0^t (t-x)f_1(x)dx + \int_0^t (t-x)f_2(x)dx \\ f_2(t) = (\alpha, \alpha^2)(-2t^2) - \int_0^t (t-x)f_1(x)dx - \int_0^t (t-x)f_2(x)dx \end{cases}$$

By using the method of fuzzy kamal transform we obtain

$$\begin{cases} \mathcal{K}(\underline{f}_1(t, \alpha)) = 4\alpha v^3, & \mathcal{K}(\bar{f}_1(t, \alpha)) = 4\alpha^2 v^3 \\ \mathcal{K}(\underline{f}_2(t, \alpha)) = -4\alpha v^3, & \mathcal{K}(\bar{f}_2(t, \alpha)) = -4\alpha^2 v^3 \end{cases}$$

Operating fuzzy inverse Kamal transform we have :

$$\begin{cases} \underline{f}_1(t, \alpha) = 2\alpha t^2, & \bar{f}_1(t, \alpha) = 2\alpha^2 t^2 \\ \underline{f}_2(t, \alpha) = -2\alpha t^2, & \bar{f}_2(t, \alpha) = -2\alpha^2 t^2 \end{cases}$$

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Results Of Existence And Approximate Solutions For Volterra Fredholm Integro-Differential Equations

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Abstract: Using the Krasnoselskii and Banach fixed point theorems, we prove in this work the existence and uniqueness of a fractional Volterra Fredholm integro-differential equation involving Caputo's fractional derivative. Moreover, we use the modified Adomian decomposition approach to examine the convergence of the approximated solutions to the same problem. An example is given to substantiate the usefulness of the obtained results.

Keywords: Krasnoselskii's fixed point theorem, Adomian decomposition method, Volterra Fredholm integro-differential equation, Caputo fractional derivative.

2020 Mathematics Subject Classification: Primary 49M27, 45J05, 26A33.

Introduction

In the recent years, the integro fractional differential equation arises in a lot of phenomena of various areas of science such as physics, engineering, chemistry, rheology, dynamical processes in self-similar, porous structures, electrochemistry of corrosion and viscoelasticity, etc. For more details see ([20], [6], [30], [8], [2]) and the references cited therein.

Some results on the existence of solutions of integro fractional differential equations have been discussed by many mathematicians using the fixed point theorem. Moreover, many researches occur about the approximate solution of this type of equations by the Adomian decomposition method (ADM) that has been proposed by the American mathematician George Adomian [1] during the second half of the 20th century. The Adomian method is based on the use of polynomials that bear his name, this technique has the advantage of elegance and especially ease of use. The solution is given in the form of a series of which each term is easily computable using the Adomian polynomials adapted to nonlinearity.

In this paper we investigate the existence and uniqueness of solutions using the Krasnoselskii and Banach fixed point theorems and we apply the (MADM) for the following Caputo fractional Volterra Fredholm integrodifferential equation

$${}^C D^\alpha (u(x) - h(u(x))) = g(x) + a(x)u(x) + \int_0^x K_1(x,t)F_1(u(t))dt + \int_0^1 K_2(x,t)F_2(u(t))dt, \quad (204)$$

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with the initial condition

$$u(0) = u_0,$$

where ${}^C D^\alpha$ is Caputo's fractional derivative, $0 < \alpha < 1$, $u : [0, 1] \rightarrow \mathbb{R}$ is the continuous function which has to be determined, $g : [0, 1] \rightarrow \mathbb{R}$, $h : \mathbb{R} \rightarrow \mathbb{R}$ is Lipschitz continuous function, $K_i : [0, 1] \times [0, 1] \rightarrow \mathbb{R}$, $i = 1, 2$ are continuous functions, $F_i : \mathbb{R} \rightarrow \mathbb{R}$, $i = 1, 2$ are Lipschitz continuous functions.

We introduce the following assumptions

(H1) There exist two real constants $L_{F_1}, L_{F_2} > 0$, such that, for any $u_1, u_2 \in C([0, 1], \mathbb{R})$

$$|F_1(u_1(x)) - F_1(u_2(x))| \leq L_{F_1} |u_1 - u_2|.$$

and

$$|F_2(u_1(x)) - F_2(u_2(x))| \leq L_{F_2} |u_1 - u_2|.$$

(H2) There exists areal constant $0 < L_h < 1$ such that, for any $u_1, u_2 \in C([0, 1], \mathbb{R})$

$$|h(u_1(x)) - h(u_2(x))| \leq L_h |u_1 - u_2|.$$

(H3) There exist two positive constants K_1^* and K_2^* such that

$$K_1^* = \sup_{x \in [0, 1]} \int_0^x |K_1(x, t)| dt < \infty, K_2^* = \sup_{x \in [0, 1]} \int_0^x |K_2(x, t)| dt < \infty.$$

(H4) The two functions $a, g : [0, 1] \rightarrow \mathbb{R}$ are continuous.

Main results

We give the existence results based on the Krasnoselskii fixed point theorem.

Theorem 69. Assume that (H1) – (H4) are satisfied. If

$$\rho = L_h + \frac{\|a\|_\infty + L_{F_1} K_1^* + L_{F_2} K_2^*}{\Gamma(\alpha + 1)} < 1, \quad (205)$$

then the problem (204) has at least one solution on $[0, 1]$.

Now, we give the existence and uniqueness results based on the Banach fixed point theorem.

Theorem 70. Assume that (H1) – (H4) and (205) hold. Then, the problem (204) has a unique solution.

After that, we use the modified Adomian decomposition method (MADM) to solve the problem (204) which has the advantage to accelerate the convergence towards the exact solution if compared with the standard Adomian decomposition method (ADM). This last allows us to solve by a direct scheme the considered problem where the solution is represented in the form of series given as follows:

$$u = \sum_{n=0}^{\infty} u_n,$$

and the nonlinear functions h, F_1 and F_2 are decomposed as

$$F_1 = \sum_{n=0}^{\infty} A_n, F_2 = \sum_{n=0}^{\infty} B_n, h = \sum_{n=0}^{\infty} D_n,$$

where A_n, B_n, D_n represent Adomian polynomials

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Theorem 71. Assume that (H1) – (H4) and (205) hold.

If the series solution $u(x) = \sum_{i=0}^{\infty} u_i(x)$ and $\|u\|_{\infty} < \infty$ is convergent, then it converges to the exact solution of the fractional differential equation (204)

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Elliptic problem with nonlocal boundary conditions in L^p spaces

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Abstract: This work is devoted to the study of General Problem of elliptic type in the framework of UMD Banach spaces. Here, we obtain some results about existence, uniqueness and regularity of the solution. We define two types of solutions strict and semi-strict solutions and we give necessary and sufficient conditions on the data to obtain these results.

Keywords: Nonlocal boundary conditions, analytic semigroups, bounded imaginary powers of operators, UMD spaces, interpolation spaces, Dore-Venni operators sums theory.

2020 Mathematics Subject Classification: Primary 34G10, 35J15, 35J25.

Introduction

In this work, we study a complete abstract second order differential equation of elliptic type with non local boundary conditions in the framework of L^p spaces.

More precisely we consider the following abstract second order differential equation

$$-u''(x) + (L - M)u'(x) + LM u(x) = f(x), \text{ a.e. } x \in]0, 1[\quad (206)$$

with the following nonlocal boundary conditions:

$$\begin{cases} u(0) = u_0, \\ u(1) - u(x_0) = u_{1,x_0}, \end{cases} \quad (207)$$

where $f \in L^p(0, 1; X)$, $1 < p < +\infty$, X is an UMD Banach space, u_0 and u_{1,x_0} are elements of X , $x_0 \in [0, 1[$. L and M are closed linear operators in X with domains $\mathcal{D}(L)$ and $\mathcal{D}(M)$, respectively.

Our main goal is to give (under some hypotheses) necessary and sufficient conditions on the data to obtain some results about existence, uniqueness and regularity of the solution of problem (206)-(207).

The authors B. Hamdi et al. [1] has studied the particular case $L = M$ of problem (206)-(207), and they proved that this problem is well posed.

We define two types of solutions :

- A semi-strict solution, i.e. u verify (206)-(207) and:

$$u \in W^{2,p}(0, 1 - \varepsilon; X) \cap L^p(0, 1 - \varepsilon; \mathcal{D}(LM)) \text{ and } u' \in L^p(0, 1 - \varepsilon; \mathcal{D}(L - M)) \cap L^p(0, 1; X).$$

- A strict solution, i.e. u verify (206)-(207) and:

$$u \in W^{2,p}(0, 1; X) \cap L^p(0, 1; \mathcal{D}(LM)) \text{ and } u' \in L^p(0, 1; \mathcal{D}(L - M)).$$

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The method used is based on finding a formula to represent the solution in each case using the semigroups and fractional powers of the operators. Then an analysis of this representation is made to find regularity results of the solution using the interpolation spaces and the Dore-Venni operators sums theory.

Main results

We suppose that

$$\begin{cases} \mathcal{D}(L) = \mathcal{D}(M) \\ \mathcal{D}(ML) = \mathcal{D}(LM) \\ ML = LM \end{cases} \quad (208)$$

The operators L and M generate an analytic semigroups, noted $\{e^{xL}\}_{x \geq 0}$ and $\{e^{xM}\}_{x \geq 0}$. (209)

The operator $(L + M)^{-1}$ is bounded. (210)

Under hypotheses (209) and (210), the solution of our problem is of the form

$$u(x) = e^{xL}(u_0 - J_0) + e^{(1-x)M}v + I_x + J_x + R(x),$$

where

$$\begin{cases} v = -I_1 + u_{1,x_0} + e^{x_0L}(u_0 - J_0) + I_{x_0} + J_{x_0}, \\ I_x = -(L + M)^{-1} \int_0^x e^{(x-s)L} f(s) ds, \\ J_x = -(L + M)^{-1} \int_x^1 e^{(s-x)M} f(s) ds, \end{cases}$$

and R is the regular part of the solution.

Theorem 72. *Under the hypotheses (208) – (210), the problem (206) – (207) has a unique strict (resp. semi-strict) solution if and only if*

$$\begin{cases} u_0 \in \mathcal{D}(LM, X)_{\frac{1}{2p}, p}, \\ u_{1,x_0} + e^{x_0L}(u_0 - J_0) + I_{x_0} + J_{x_0} \in \mathcal{D}(LM, X)_{\frac{1}{2p}, p} \text{ (resp. } \in \mathcal{D}(LM, X)_{\frac{1}{2p} + \frac{1}{2}, p}). \end{cases} \quad (211)$$

Where $(\mathcal{D}(LM, X))_{\frac{1}{2p}, p}$ is an interpolation space between $\mathcal{D}(LM)$ and X , see for example [2].

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Numerical method for solving nonlinear singular pseudoparabolic equations with nonlocal mixed conditions in Reproducing Kernel Hilbert Space Method

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Abstract: Numerical methods play an important role in modern mathematic, In this work, we will examine the process of finding an approximate solution to the Solving nonlinear singular pseudoparabolic equations with nonlocal mixed conditions using a new numerical technique called the reproducing kernel Hilbert space method. We will delve into the various steps of this approach and exemple illustrations to demonstrate its effectiveness.

Keywords: nonlinear singular, pseudoparabolic equations , nonlocal mixed conditions , Reproducing Kernel Hilbert Space , approximate solution

Introduction

Nonlocal mixed problems, which involve conditions that extend beyond a single point, are commonly found in different branches of engineering. These problems arise in many areas of physics and technology, particularly in partial differential equations with integral conditions. One type of nonlocal problem is represented by the pseudoparabolic equation with nonlocal mixed conditions.

This equation is used to model various physical processes, including long dispersive waves, heat conduction with different temperatures, and population aggregation .

This work investigates the approximate solution of the following nonlinear singular pseudoparabolic equations with nonlocal mixed conditions using New numerical method [...]. This method is called reproducing kernel Hilbert space method (RKHM).

Reproducing kernel Hilbert space method is very powerful method. There are many advantages of this method. We can quickly acquire approximate solutions to the problems.

The application of reproducing kernel theory extends across various domains, including numerical analysis, differential equations, probability, and statistics, offering significant utility. Recent literature has delved into the utilization of the reproducing kernel Hilbert space method in tackling challenges posed by fractional differential equations and nonlinear partial differential equations.

The work is organized as follows some properties of reproducing kernel spaces. , the exact and

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approximate solution of the problem . exemple The RKHSM systematically produces analytic and approximate solutions in the form of series. The convergence analysis . prove that the approximate solution converges to the exact solution uniformly. Some numerical experiments are illustrated and Some conclusions.

the example that we used in this work was taken from an article [2] ,This example was previously applied to the method of Reproducing kernel Hilbert Space by the authors Wei Jiang and Minggen Cui.

Main results

Theorem 73. • *the approximate solution $\mathfrak{U}_n(x, t)$ uniformly converge to exact solution $\mathfrak{U}(x, t)$*

- *the derivatives $\partial_{x,t}^{i+j} \mathfrak{U}_n(x, t), i = 0, 1, 2, j = 0, 1$ uniformly converge to $\partial_{x,t}^{i+j} \mathfrak{U}(x, t), i = 0, 1, 2, j = 0, 1$*

where

$$\mathfrak{U}_n(x, t) = \sum_{i=1}^n \wp_{ik} F(x_k, t_k) \bar{\psi}_i(x, t).$$

and

$$\mathfrak{U}(x, t) = \sum_{i=1}^{\infty} \wp_{ik} F(x_k, t_k) \bar{\psi}_i(x, t).$$

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An Approximate Solution of Hyperbolic Partial Differential Equation

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Abstract: The numerical investigations on two dimensional hyperbolic equation have drawn considerable interest in different aspects.

Galerkin method, is considered for approximating solutions of linear and nonlinear hyperbolic problems. Bounds reducing the estimation of the error to questions in approximation theory are derived for the several methods studied.

In this work, the Galerkin method is applied to construct the numerical solution of time dependent hyperbolic partial differential equation with variable coefficient.

This study shows that the Galerkin method is a very effective and powerful tool in solving such problems numerically. Some numerical experiments examples are made.

Keywords: Numerical solution, hyperbolic equation, Galerkin method, weak solution

2020 Mathematics Subject Classification: 65L20, 65L12, 37N30.

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A Numerical Technique for Solving Nonlinear Fractional Integro-Differential Equations

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Abstract: In this work, we aim to apply an efficient numerical technique based on the Adomian decomposition method (ADM) to solve a class of nonlinear fractional integro-differential equations, in particular nonlinear fractional Volterra integro-differential equations. The fractional derivative is considered in the Caputo sense. The approximate analytical solution and some numerical examples show the accuracy and efficiency of the proposed method, which is very simple and precise.

Keywords: Volterra integro-differential equation, Caputo fractional derivative, Adomian decomposition method, Numerical solution

2020 Mathematics Subject Classification: 34A08, 26A33, 34K28, 35C10.

Introduction

The fractional integro-differential equations have attracted much more interest of mathematicians and physicists which provides an efficiency for the description of many practical dynamical arising in engineering and scientific disciplines such as, physics, biology, electrochemistry, chemistry, economy, electromagnetic, control theory and viscoelasticity. Recently, various numerical and analytical methods have been applied for the approximate solutions of fractional integro-differential equations [2, 3, 5, 6],

The main objective of this work is to present the ADM to construct the approximate solutions of the following nonlinear fractional Volterra integro-differential equation

$$D^\alpha u(x) = f(x) + \int_0^x G(u(x), u'(x)) dx,$$

with the initial condition

$$u(0) = u_0,$$

where D^α is the Caputo fractional derivative operator of order α with $0 < \alpha \leq 1$, $u : I = [0, 1] \rightarrow \mathbb{R}$ is the continuous function which has to be determined, $f : I \rightarrow \mathbb{R}$ and $G : \mathbb{R} \rightarrow \mathbb{R}$ is nonlinear function of $u(x)$ and $u'(x)$.

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Basic definitions

In this section, we give some definitions and properties of the fractional calculus and fractional differential transform method

Definition 27. [5] The Riemann-Liouville fractional integral operator $I^\alpha u(x)$ of order $\alpha > 0$ is defined as

$$I^\alpha u(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x - \xi)^{\alpha-1} u(\xi) d\xi,$$

where $\Gamma(\cdot)$ is the well-known gamma function.

Definition 28. [5] The Caputo fractional derivative $D^\alpha u(x)$ of order $n - 1 < \alpha \leq n$, $n \in \mathbb{N}$ is defined as

$$\begin{aligned} D^\alpha u(x) &= I^{n-\alpha} D^n u(x) \\ &= \frac{1}{\Gamma(n-\alpha)} \int_0^x (x - \xi)^{n-\alpha-1} u^{(n)}(\xi) d\xi. \end{aligned}$$

For the Riemann-Liouville fractional integral and Caputo fractional derivative, we have the following relation

$$I^\alpha D^\alpha u(x) = u(x) - \sum_{k=0}^{n-1} u^{(k)}(0^+) \frac{x^k}{k!}.$$

Main results

In this section, we present the ADM for solving nonlinear fractional Volterra integro-differential equation

Theorem 74. Consider the following nonlinear fractional Volterra integro-differential equation

$$D^\alpha u(x) = f(x) + \int_0^x G(u(x), u'(x)) dx, 0 < \alpha \leq 1, \quad (212)$$

with the initial condition

$$u(0) = u_0. \quad (213)$$

The ADM gives the solution of equations (212) and (213) in the form of infinite series which converges rapidly to the exact solution as follows

$$u(x) = \sum_{k=0}^{\infty} u_k(x),$$

where

$$u_n(x) = u_0 + I^\alpha(f(x, t)) + I^\alpha \left(\int_0^x A_{n-1}(x) dx \right),$$

and A_n are Adomian polynomials [7] of the nonlinear terms and it can be calculated by formula given below

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[G \left(\sum_{i=0}^{\infty} \lambda^i u_i \right) \right]_{\lambda=0}, n = 0, 1, 2, \dots$$

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Nonlinear Schrödinger equation on a star-shaped network

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Abstract: In this paper, we prove the exponential stability of the solution of the nonlinear dissipative Schrödinger equation posed on a star-shaped network $\mathcal{R}$, where the damping is localized on one branch at the infinity and the initial data is assumed to be in $L^2(\mathcal{R})$. First, we use the fixed point argument and Strichartz estimates on a star-shaped network to obtain results of local and global well posedness. Then we prove that, the energy of the solutions of the dissipative system decay exponentially to zero when the time tends to infinity. The proof is based on smoothing properties for Schrödinger equation, on the unique continuation and on the semigroup properties.

Keywords: Exponential stability, dissipative Schrödinger equation, star-shaped network

2020 Mathematics Subject Classification: Primary 2010, 35L05, 34K35.

Introduction

We study the stabilization problem of the nonlinear Schrödinger equation on a star-shaped network as in the following figure for $N = 3$.

More precisely, we consider the following initial and boundary-value problem:

$$\begin{cases} i \partial_t u_1 + \partial_x^2 u_1 + \lambda u_1 |u_1|^{\alpha-1} + i a(x) u_1 = 0 \text{ for } x > 0, t > 0, \\ i \partial_t u_i + \partial_x^2 u_i + \lambda u_i |u_i|^{\alpha-1} = 0 \text{ for } x > 0, t > 0, 2 \leq i \leq N, \\ \sum_{i=1}^N \partial_x u_i(t, 0) = 0, t > 0, \\ u_i(t, 0) = u_j(t, 0), \forall t > 0, 1 \leq i, j \leq N, \\ u_i(0, x) = \varphi_i(x), x > 0, 1 \leq i \leq N, \end{cases} \quad (214)$$

where $N \in \mathbb{N}^*, N \geq 3$, $\lambda \in \mathbb{R}^*$ and $\alpha \in \{3, 5\}$ will be treated in this paper. The presence of the damping term in (214) is responsible for the localized mechanism of dissipation of the system since the function $a = a(x)$ is assumed to be in $L^\infty(\mathbb{R}_+)$, almost everywhere non-negative function, and to satisfy for some $R > 0$, and $\alpha_0 > 0$,

$$a(x) \geq \alpha_0 > 0 \quad \text{for } |x| > R. \quad (215)$$

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This means that the damping term $a(x)u_1$ is effective at infinity, i.e., in the exterior of the ball of radius R .

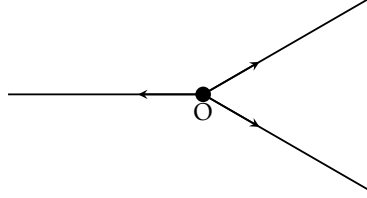


Figure 6: Star-Shaped Network for $N = 3$

The considered graph consists of a finite number of edges of infinite length attached to a common vertex, each of them being identified with a copy of the positive semi-axis. The coupling between the edges is through a boundary condition at the vertex which are given by:

$$u_1(0) = u_2(0) = \dots = u_N(0), (\text{continuity condition}), \quad (216)$$

$$\text{and } \sum_{i=1}^N \partial_x u_i(0) = 0 \text{ (Kirchhoff condition)}. \quad (217)$$

Let $\mathcal{R}_i, i = 1, 2, \dots, N$ be N disjoint sets identified with $(0, +\infty)$. We set $\mathcal{R} := \bigcup_{i=1}^N \overline{\mathcal{R}_k}$. Define the Hilbert space $\mathcal{H} = \bigoplus_{i=1}^N L^2(\mathcal{R}_k) = L^2(\mathcal{R})$ with inner product $((u_k), (v_k))_{\mathcal{H}} = \sum_{i=1}^N (u_k, v_k)_{L^2(\mathcal{R}_k)}$. The system (214) displays L^2 -energy defined by $E_0(u(t)) := \|u\|_{L^2(\mathcal{R})}^2$.

It is easy to check that the L^2 -energy of u satisfies the following identity,

$$\begin{aligned} E_0(u(t)) &:= \sum_{i=1}^N E_0(u_i(t)) = -2 \int_0^t \int_{\mathbb{R}_+} a(x) |u_1(s, x)|^2 dx ds + \sum_{i=1}^N \|\varphi_i\|_{L^2(\mathbb{R}_+)}^2 \\ &= -2 \int_0^t \int_{\mathbb{R}_+} a(x) |u_1(s, x)|^2 dx ds + E_0(u(0)), \forall t \geq 0, \end{aligned} \quad (218)$$

where $u = (u_1, \dots, u_N)$ and $E_0(u_i(t))$ is the energy of $u_i, 1 \leq i \leq N$ defined by

$$E_0(u_i(t)) := \int_0^{+\infty} |u_i(t, x)|^2 dx.$$

As a consequence $E_0(u(t))$ is decreasing in time and the system (214) is therefore dissipative.

Main results

First, we establish local well-posedness, then, extending it to all times.

Proposition 5. *Given $\varphi = (\varphi_1, \dots, \varphi_N) \in L^2(\mathcal{R})$ and $\alpha \in (1, 5)$, then there exist $T = T(\|\varphi\|_{L^2}, \|a\|_{L^\infty}, \alpha, \lambda) > 0$ and a unique solution $u = (u_1, \dots, u_N)$ of system (214) in the time interval $[0, T]$ which depends continuously on the initial data with*

$$u_i \in \mathcal{C}([0, T]; L^2(\mathcal{R}_i)) \cap L^r((0, T); L^{\alpha+1}(\mathcal{R}_i)), 1 \leq i \leq N, \quad (219)$$

where $r = \frac{4(\alpha+1)}{\alpha-1}$.

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Next, we show how to extend the local well posedness result to the critical case $\alpha = 5$.

Proposition 6. *If $\alpha = 5$, then for each $\varphi \in L^2(\mathcal{R})$ there exist $T = T(\varphi, \|a\|_{L^\infty}, \alpha, \lambda) > 0$ and a unique solution $u = (u_1, \dots, u_N)$ of system (214) in the time interval $[0, T]$ which depends continuously on the initial data with*

$$u_i \in C([0, T]; L^2(\mathcal{R}_i)) \cap L^6((0, T); L^6(\mathcal{R}_i)), 1 \leq i \leq N. \quad (220)$$

Our major concern will be to prove the exponential decay of the global L^2 -energy at infinity. More precisely we have the following theorem:

Theorem 75. *Consider $\alpha = 3$, a function a satisfying assumption (215) and the initial data φ in $L^2(\mathcal{R})$. For any solution u of the system (214), there exist $c > 0$ and $\omega > 0$ such that:*

$$E_0(u(t)) \leq c e^{-\omega t}, \text{ for all } t \geq 0. \quad (221)$$

Note that this result remains true provided the initial data satisfies $\|\varphi\|_{L^2(\mathcal{R})} \ll 1$, if one considers the case $\alpha = 5$.

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Solving Nonlocal Initial-Boundary Value Problems for Hyperbolic Integro-Differential Equations by applying a new numerical method

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Abstract: In this work, we will examine how to find the approximate solution to Nonlocal Initial-Boundary Value Problems for Hyperbolic Integro-Differential Equations by applying a new numerical method (reproducing kernel Hilbert space method). This research analyzes how well the new method converges. Specific assumptions are made to establish a theoretical foundation for the method. Additionally, the accuracy of the numerical approach within a reproducing kernel Hilbert space is evaluated by estimating the errors. In addition to giving examples to show how accurate this method is.

Keywords: convergence and error analysis,hyperbolic integro-differential,nonlocal Initial-Boundary, reproducing kernel Hilbert space.

2020 Mathematics Subject Classification: Primary .

Introduction

PDEs express the relationships between various quantities and their rates of change with respect to multiple independent variables. These variables could be space, time, temperature, pressure, or any other relevant physical or abstract dimension. The equations themselves are written in a concise and expressive language using partial derivatives, which represent the in

infinitesimal change of a function with respect to one variable while holding all others constant. Solving these equations unveils the hidden patterns and behaviors of complex systems, allowing us to predict, analyze, and even control them. Hyperbolic Integro-Differential Equations are a type of such equations,the solution of this equations is very important and several methods of solution have been found,including analytical and numerical),in this work, we will address a new numerical method(reproducing kernel Hilbert space method). The concept emerged in a paper by Zaremba (1908)[1], who used it to explore boundary value problems in harmonic functions, and Bergman [2] (1930s-40s) played a central role in developing the theory, de

fining kernels for harmonic and analytic functions with applications in partial differential equations. The pivotal theorem Mercer's [3] (1909) demonstrated how positive de

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nite kernels can be expressed as in

nite sums of eigenfunctions, providing a deeper understanding of their structure. Aronszajns work solidi

ed the theoretical framework of reproducing kernel Hilbert spaces (RKHS), establishing connec-
tions to various mathematical

elds.

Main result

Theorem 1: f $u(x, t) \in W_{(3,1)}$, then there exists a constant $M > 0$, such that

$$|\partial_x^m \partial_t^p u(x, t)| \leq M \|u(x, t)\|_{W_{(3,1)}}, m = 0, 1, 2, p = 0, 1.$$

Theorem 2: Let $(x_i, t_i)_{i=1}^\infty = 1$ be a dense subset in D , then

$$\partial_x^m \partial_t^p u_n(x, t), m = 0, 1, 2, p = 0, 1$$

converge uniformly to

$$\partial_x^m \partial_t^p u(x, t)$$

.

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Some results for new contraction based on $\mathcal{C}_g$ simulation functions via w -distance with applications.

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Abstract: Based on the notion of class of $\mathcal{C}_g$ simulation functions, we launched the concept of (ρ, ϑ) - $\mathcal{Z}_g$ -contraction on the setting of w -distance mappings. We employed our new concept to formulate and prove several fixed point results over the complete metric spaces. To illustrate our results, some examples and applications to integral equations and fractional differential equations are provided throughout the paper to illustrate the usability of the obtained results. The results presented in our manuscript generalize many of the results known in the literature.

Keywords: w -distance mappings; $\mathcal{C}_g$ simulation functions; (ρ, ϑ) - $\mathcal{Z}_g$ -contraction; fixed point.

2020 Mathematics Subject Classification: Primary 45G10, 46N20, 54H25.

Introduction

In 1922, Banach [2] proved the Banach's contraction principle which is one of the main pre-eminent tests for the existence and uniqueness of the solution of many problems in the field of nonlinear analysis. This principle has been improved, extended and generalized by many authors in several ways and in various spaces over the years.

Later on, Kada et al.[2] employed the notion of metric spaces to introduce the concept of w -distance mappings and used it to establish some fixed point results. Subsequently, number of authors studied fixed point theory for such class of mappings. Recently, with the aim of generalizing the concept of simulation functions as introduced by Khojasteh et al.[3], Liu et al.[12] introduced the notion of $\mathcal{C}_g$ simulation functions and demonstrated the existence and uniqueness of results within complete metric spaces.

In this work, we defined the concept of (ρ, ϑ) - $\mathcal{Z}_g$ -contraction and established some fixed point results for such mappings in complete metric spaces.

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First, we present the notion of w -distance mapping on a metric space.

Definition 29. [2] On a metric space $(\mathcal{E}, d)$, a function $\rho : \mathcal{E} \times \mathcal{E} \rightarrow \mathbb{R}^+$ is said to be w -distance function if the assumptions below hold:

$$(\rho_1) \quad \rho(\omega, \varpi) \leq \rho(\omega, \nu) + \rho(\nu, \varpi) \quad \forall \omega, \varpi, \nu \in \mathcal{E};$$

$$(\rho_2) \quad \text{For any } \omega \in \mathcal{E}, \text{ the function } \rho(\omega, \cdot) : \mathcal{E} \rightarrow \mathbb{R}^+ \text{ is a lower semi-continuous;}$$

$$(\rho_3) \quad \text{for each } v > 0, \text{ there is } \sigma > 0 \text{ such that, for all } \omega, \varpi, \nu \in \mathcal{E}$$

$$\rho(\omega, \varpi) \leq \sigma \text{ and } \rho(\omega, \nu) \leq \sigma \text{ imply } d(\varpi, \nu) \leq v.$$

For properties and examples related to w -distance, see: [5, 6, 7].

Ansari [8] in 2014 introduced the notion of C -class functions. After that, Liu et al.[12] generalized the class of simulation function by employing the notion of C -class functions as follows:

Definition 30. [12] We say that a mapping $\mathcal{G} : \mathbb{R}^+ \times \mathbb{R}^+ \rightarrow \mathbb{R}$ has the property $\mathcal{C}_g$, if there is $\mathcal{C}_g \geq 0$ such that:

$$\mathcal{C}(i) \quad \mathcal{G}(\eta, \lambda) > \mathcal{C}_g \Rightarrow \eta > \lambda;$$

$$\mathcal{C}(ii) \quad \mathcal{G}(\eta, \eta) \leq \mathcal{C}_g, \text{ for each } \eta \in \mathbb{R}^+.$$

Definition 31. [12] The function $\xi : \mathbb{R}^+ \times \mathbb{R}^+ \rightarrow \mathbb{R}$ is called a $\mathcal{C}_g$ simulation function if the assertions below hold:

$$\xi(i) \quad \xi(\lambda, \eta) < \mathcal{G}(\eta, \lambda), \text{ for every } \eta, \lambda > 0, \text{ where } \mathcal{G} \in \mathcal{C} \text{ which has the property } \mathcal{C}_g;$$

$$\xi(ii) \quad \text{if } \{\eta_t\}, \{\lambda_t\} \text{ are two sequences in } \mathbb{R}^+ \text{ with } \lim_{t \rightarrow \infty} \lambda_t = \lim_{n \rightarrow \infty} \eta_t > 0 \text{ and } \lambda_t < \eta_t \text{ for each } t \in \mathbb{N}, \\ \text{then } \limsup_{t \rightarrow \infty} \xi(\lambda_t, \eta_t) < \mathcal{C}_g.$$

Let $\mathcal{Z}_g$ denotes the set of all $\mathcal{C}_g$ -simulation functions.

Main results

In our work, we utilized the notions of $\mathcal{C}_g$ simulation functions and w -distance mappings to construct new fixed point results in metric spaces. First, we introduced the concept of (ρ, ϑ) - $\mathcal{Z}_g$ -contraction as follows.

Definition 32. On a metric space $(\mathcal{E}, d)$, we let ρ be a w -distance on $\mathcal{E}$. The self mapping $\mathcal{F} : \mathcal{E} \rightarrow \mathcal{E}$ is called (ρ, ϑ) - $\mathcal{Z}_g$ -contraction, if there exist $\vartheta \in \Psi$ and $\xi \in \mathcal{Z}_g$ such that:

$$\xi(\vartheta(\rho(\mathcal{F}\omega, \mathcal{F}\nu)), \vartheta(\rho(\omega, \nu))) \geq \mathcal{C}_g \quad (222)$$

for every $\omega, \nu \in \mathcal{E}$ with $\rho(\omega, \nu) > 0$. (Ψ denotes the set of all altering distance functions given by Khan et al.[9])

Below is the main result we obtained through our work.

Theorem 76. On a complete metric space $(\mathcal{E}, d)$, we let ρ be a w -distance on $\mathcal{E}$. Suppose $\mathcal{F} : \mathcal{E} \rightarrow \mathcal{E}$ is a (ρ, ϑ) - $\mathcal{Z}_g$ -contraction. Moreover, assume that

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(C1) there exists $\omega_0 \in \mathcal{E}$ such that $\rho(\omega_0, \mathcal{F}\omega_0) > 0$,

(C2) if $\omega, \bar{\omega} \in \mathcal{E}$ such that $\rho(\omega, \bar{\omega}) > 0$, then $\rho(\mathcal{F}\omega, \mathcal{F}\bar{\omega}) > 0$.

(C3) One of the following conditions holds

(i) $\mathcal{F}$ is continuous or

(ii) if $\omega \in \mathcal{E}$ with $\mathcal{F}\omega \neq \omega$, then

$$\inf\{\rho(\bar{\omega}, \omega) + \rho(\mathcal{F}\bar{\omega}, \omega) : \bar{\omega} \in \mathcal{E}\} > 0. \quad (223)$$

Then, $\mathcal{F}$ has a unique fixed point in $\mathcal{E}$.

In addition, we provided some corollaries and some concrete examples to show the usability of the obtained results. Furthermore, we applied Theorem 76 to study the existence of a unique solution of nonlinear fractional differential equations, nonlinear integral equations and spring-mass system problems.

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Analysis on some retarded Volterra-Fredholm type integral inequalities of several variables and applications

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Abstract: In this paper, we generalize some known integral inequalities to more general situations. We establish bounds on the unknown function of some integral inequalities of Volterra-Fredholm type of several variables. Our results can be used as useful tools in the study of the properties of the solutions of certain multidimensional integral and differential equations of Volterra-Fredholm type. An application is given to illustrate the usefulness of our results.

Keywords: Integral inequality; Properties of solutions; Volterra-Fredholm type integral inequalities; Several variables.

2020 Mathematics Subject Classification: 26D15; 45B05; 45D05.

Introduction

The Gronwall–Bellman inequality and its various generalizations are crucial in the discussion of the existence, uniqueness, continuation, boundedness, oscillation and stability and other qualitative properties of solutions of differential and integral equations. Recently, Many authors improved the Gronwall-Bellman inequality to more general forms [2, 3]. In the past few decades many such new interesting retarded integral inequalities of Volterra-Fredholm type have been established [4, 8, 12].

Pachpatte in [9] has established the following useful linear Volterra-Fredholm type integral inequality

$$u(t) \leq c + \int_{\alpha}^t a(t,s)u(s)ds + \int_{\alpha}^{\beta} b(t,s)u(s)ds, \quad (224)$$

Recently, A. Boudeliou [5] discovered the retarded Volterra-Fredholm type integral inequality in n variables

$$\begin{aligned} u(x) \leq & u_0 + \int_{\gamma(x^0)}^{\gamma(x)} a(x,s)\omega(u(s)) \left[u(s) + \int_{\gamma(x^0)}^s b(s,\tau)\omega(u(\tau))d\tau \right] ds \\ & + \int_{\gamma(x^0)}^{\gamma(T)} a(x,s)\omega(u(s)) \left[u(s) + \int_{\gamma(x^0)}^s b(s,\tau)\omega(u(\tau))d\tau \right] ds. \end{aligned} \quad (225)$$

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However, in certain situations, such as some classes of delay integral equations of Volterra-Fredholm type, it is desirable to find some new inequalities, in order to achieve a diversity of desired goals. In this paper, we discuss a class of multidimensional retarded integral inequalities of Volterra-Fredholm type. Finally, we give an application of our results.

Main results

In what follows, $I = [x^0, T] = I_1 \times I_2 \times \dots \times I_n$ where $I_i = [x_i^0, T_i], i = 1, \dots, n$, and $x^0 = (x_1^0, \dots, x_n^0), T = (T_1, \dots, T_n) \in \mathbb{R}^n, \Delta = \{(x, s) \in I^2 : x^0 \leq s \leq x \leq T\}$. We also adopt the notation $x = (x_1, x_2, \dots, x_n) = (x_1, x^1)$, where $x^1 = (x_2, \dots, x_n), (x^0)^1 = (x_2^0, \dots, x_n^0)$, and

- $D_i = \frac{\partial}{\partial x_i}, i = 1, \dots, n$,
- $dx^1 = dx_n \dots dx_2$,
- $\int_{\alpha(x^0)}^{\alpha(x)} \dots ds = \int_{\alpha(x_1^0)}^{\alpha_1(x_1)} \dots \int_{\alpha(x_n^0)}^{\alpha_n(x_n)} \dots ds_n \dots ds_1$.

Theorem 77. Let $u(x) \in C(I, \mathbb{R}_+), f(x, s), \gamma_1(x, s), \gamma_2(x, s) \in C(\Delta, \mathbb{R}_+)$ and $f(x, s), \gamma_1(x, s), \gamma_2(x, s)$ are nondecreasing in x for each $s \in I, \alpha(x) = (\alpha_1(x_1), \dots, \alpha_n(x_n)) \in C^1(I, I)$, where $\alpha_i(x_i) \in C^1(I_i, I_i)$ be nondecreasing functions on I_i , with $\alpha_i(x_i) \leq x_i, i = 1, \dots, n$. And let $\psi, \omega \in C(\mathbb{R}_+, \mathbb{R}_+)$ be nondecreasing with $\{\psi, \omega\}(u) > 0$, for $u > 0$, and $\lim_{u \rightarrow +\infty} \psi(u) = +\infty$, and $u_0 > 0$ be a constant. If $u(x)$ satisfies

$$\begin{aligned} \psi(u(x)) \leq & u_0 + \int_{\alpha(x^0)}^{\alpha(x)} \gamma_1(x, s) \left[f(x, s) \omega(u(s)) + \int_{\alpha(x^0)}^s \gamma_2(s, \tau) \omega(u(\tau)) d\tau \right] ds \\ & + \int_{\alpha(x^0)}^{\alpha(T)} \gamma_1(x, s) \left[f(x, s) \omega(u(s)) + \int_{\alpha(x^0)}^s \gamma_2(s, \tau) \omega(u(\tau)) d\tau \right] ds, \end{aligned} \quad (226)$$

for $x \in I$, then

$$\begin{aligned} u(x) \leq & \psi^{-1} \left\{ F_1^{-1} \left(F_1 \left[H_1^{-1} \left(\int_{\alpha(x^0)}^{\alpha(T)} \gamma_1(x, s) \left[f(x, s) + \int_{\alpha(x^0)}^s \gamma_2(s, \tau) d\tau \right] ds \right) \right] \right. \right. \\ & \left. \left. + \int_{\alpha(x^0)}^{\alpha(x)} \gamma_1(x, s) \left[f(x, s) + \int_{\alpha(x^0)}^s \gamma_2(s, \tau) d\tau \right] ds \right) \right\}, \end{aligned} \quad (227)$$

for $x \in I$, where

$$F_1(v) = \int_{v_0}^v \frac{ds}{\omega(\psi^{-1}(s))}, v \geq v_0 > 0, F_1(+\infty) = \int_{v_0}^{+\infty} \frac{ds}{\omega(\psi^{-1}(s))} = +\infty, \quad (228)$$

and

$$H_1(t) = F_1(2t - u_0) - F_1(t), \quad (229)$$

is increasing for $t \geq u_0$.

Remark 4. For $\gamma_1 = 1, \gamma_2 = 0, \alpha(x) = x, \psi(u) = \omega(u) = u$ and for x^1 fixed, (226) reduces to (224). Further, for $\gamma_1 = 1, \gamma_2 = 0, \psi(u) = \omega(u) = u$ and $x^2 = (x_3, \dots, x_n)$ fixed, then Theorem 77 reduces to Theorem 2.1 in [11].

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Remark 5. For $\psi(u) = u$, $\gamma_1(x, s) = \gamma_1(s)$, $\gamma_2(s, \tau) = \gamma_2(\tau)$, $f(x, s) = f(s)$ and x^1 fixed, then Theorem 77 reduces to Theorem 2.1 in [8]. Further, for $\psi(u) = u$ and x^2 fixed, theorem 77 reduces to theorem 3.1 in [8].

Corollary 3. Let functions $u(x)$, $f(x)$, $\gamma_1(x)$, $\gamma_2(x)$, $\alpha(x)$, ψ , ω , F_1 , F_1^{-1} , and u_0 are as in theorem 77. If $u(x)$ satisfies (226) and

$$\bar{H}_1(t) = F_1(2t - u_0) - F_1(t) - \int_{\alpha(x^0)}^{\alpha(T)} \gamma_1(X, s) \left[f(X, s) + \int_{\alpha(x^0)}^s \sigma_2(s, \tau) d\tau \right] ds,$$

is increasing and $\bar{H}_1(t) = 0$ has a solution c_1 for $t \geq u_0$. Then

$$u(x) \leq \psi^{-1} \left\{ F_1^{-1} \left(F_1(c_1) + \int_{\alpha(x^0)}^{\alpha(x)} \gamma_1(x, s) \left[f(x, s) + \int_{\alpha(x^0)}^s \sigma_2(s, \tau) d\tau \right] ds \right) \right\},$$

for $x \in I$.

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On the numerical solution of first order partial differential equation on half-line

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Abstract: This paper proposed a numerical method for solving first order partial differential equations on semi-infinite domains. The method of solution is based on the transformation of the original problem by means of a suitable mapping and one use the classical Jacobi polynomials collocation method to solve the mapped partial differential equation. This is by using the properties of Jacobi polynomials with the vec-operation and Kronecker product to reduce the partial differential equation to a system of linear algebraic equations with unknown Jacobi coefficients. Finally, some numerical examples are presented to illustrate the efficiency of the proposed method compared with other approaches.

Keywords: Partial differential equation, A semi-infinite domains, Jacobi polynomials.

Introduction

Partial differential equations (**PDEs**) on semi-infinite domains are encountered as model in many fields of science and engineering such as the earth quake and underwater acoustic problems. In this work, we consider the **PDEs** on semi-infinite domains of the form

$$\partial_t u(x, t) = a_1 \partial_x u(x, t) + a_2 u(x, t) + K(x, t), \quad x, t \in [0, \infty), \quad (230)$$

with the following condutions

$$u(0, t) = g_1(t), \quad u(x, 0) = g_2(x), \quad x, t \in [0, \infty), \quad (231)$$

where g_1, g_2, K are given sufficiently smooth functions and a_1, a_2 are constants whereas u is unknown function to be determined.

Main results

Example 5. Consider the following equation

$$\partial_t u(x, t) = \partial_x u(x, t) + u(x, t) - \sqrt{2}e^{-\sqrt{2}t-x}, \quad x, t \in [0, \infty), \quad (232)$$

subject to the conditions

$$u(0, t) = e^{-\sqrt{2}t}, \quad u(x, 0) = e^{-x}, \quad t \in [0, \infty). \quad (233)$$

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The exact solution

$$u(x, t) = e^{-\sqrt{2}t-x}, \quad x, t \in [0, \infty). \quad (234)$$

If applying a technique described in the previous section, for different degrees N and M with different scaling factors β_1 and β_2 , the following results are obtained

Table 2: Comparison of the L_∞ errors for [2], **EMJPs** and **AMJPs** methods to solve the Example 5 with different factors $\alpha_1 = \alpha_2, \beta_1 = \beta_2 = 1$ $s = 7$.

	AMJPs		EMJPs		[2]	
N	$\alpha_1 = 1/2$	$\alpha_1 = 0$	$\alpha_1 = 1/2$	$\alpha_1 = 0$	$\alpha_1 = 1/2$	$\alpha_1 = 0$
M	$\beta_1 = 1/2$	$\beta_1 = 0$	$\beta_1 = 1/2$	$\beta_1 = 0$	$\beta_1 = 1/2$	$\beta_1 = 0$
4	9.92e-02	9.05e-02	3.16e-01	2.64e-01	9.33e-01	-
6	9.47e-03	7.25e-03	3.38e-02	2.66e-02	8.37e-02	7.47e-02
8	1.93e-04	2.23e-05	8.75e-04	7.51e-04	1.12e-02	1.07e-02
10	4.86e-06	5.19e-09	4.40e-07	1.16e-06	1.41e-03	1.72e-03
12	2.28e-08	1.41e-10	1.58e-09	2.07e-09	2.38e-04	2.36e-04
14	5.79e-12	1.94e-12	3.31e-11	4.09e-11	3.50e-05	3.38e-05
16	2.22e-13	2.58e-14	1.48e-12	4.15e-12	7.09e-06	7.53e-06

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The approximate solution of stochastic differential equation (SDE) by an algebraic system of a numerical method

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Abstract: The definition of block pulse functions (fb) and the definition of second kind Chebyshev wavelets (SCWs) have been used. The approximate of exact solution by (fb) and the approximate of exact solution by (SCWs) have been studied. Relationship matrix between block function and (SCWs) have been involved, The stochastic operational matrix method for solving stochastic Itô–Volterra have been found. The theoremes and the remarks have been used. The algebraic system of a numerical method for the approximation of exact solution have been found, Converge of exact solution to approximate solution has been studied.

Keywords: Equation, stochastic operational matrix, approximation, Brownian motion, numerical method of second kind Chebyshev, algebraic system.

2020 Mathematics Subject Classification: Primary 60G65, 60J70, 60J65 .

Introduction

There are many equations that no one has found its exact solution, but these equations are very important in scientific problems, we have to find a rough solution to solve these equations.

Consider the stochastic differential equation for $t \in [0, 1)$:[3]

$$V(t) = U(t) + \int_0^t K_0(s, t) V(s) ds + \sum_{i=1}^n \int_0^t K_i(s, t) V(r) dB_i(s), i = 1, 2, \dots, n, \quad (235)$$

where $B(t)$ is standard Brownian motion and $\int_0^t K(s, t) V(s) dB_i(s)$ is Itô integral .

We use an algebraic system to solve the equation (1), this algebraic system resulted in many matrices.

In this paper, We study numerical solutions by using block pulse functions (fb) and second kind Chebyshev wavelets (SCWs) in order to find the algebraic system and approaching the exact solution of the equation (1) .

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Definitions, Theoremes and Remarks

Definition of block pulse functions (fb) .[3] page 292

The approximation of function $f(t)$ by (fb):[3].

The integration of (fb) for s [3]. .

The integration of (fb) for $B(s)$ [3].

Definition of second kind Chebyshev wavelets (SCWs) $W(t)$ [3].

Approximation of fonction by using (SCWs): [3].

L'approximation of function $z(s, t)$ by using $W(t)$.[3].

The relationship matrix between (fb) and (SCWs). [4]

Theoreme1[4].

Theoreme2[4].

Remark 1 [4].

Remark 2 [4].

The approximation solution of stochastic differential equation (1) by an algebraic system [3]

For approach the solution of equation (1) by (SCWs) $W(t)$ we use the definitions, the Theoremes and the remarks.

yielding the algebraic system ,

$$V^T - \widehat{Y}_0^T - \sum_{i=1}^n \widehat{Y}_i^T = U^T. \quad (236)$$

Theoreme[3] converge.

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Etude des solutions d'équations intégrales Hammerstein et Hammerstein-Volterra

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Résumé: Dans ce travail, on démontre l'existence des solutions dans l'espace $L^p([a, b])$ des équations intégrales non-linéaires de types Hammerstein, et l'existence et l'unicité de la solution continue des équations intégrales non-linéaire de Hammerstein-Volterra en utilisant des théorèmes du point fixe.

Mots-clés: Equation intégrale, non-linéaire, Hammerstein, Volterra.

Mathematics subject classification (2020): 45G10, 47H30, 47N20.

Introduction

La théorie des équations intégrales intervient dans plusieurs domaines des mathématiques, beaucoup de problèmes dans le domaine des équations différentielles ordinaires et partielles, la physique mathématique, les problèmes de contacts et de l'astrophysique peut être formulée comme une équation intégrale. Ainsi, la théorie de l'équations intégrales a été un domaine de recherche actif dans la mathématiques appliquées et la physique mathématique.

L'importance des équations intégrales dans toutes les branches de la science et l'ingénierie nous amène à étudier certaines de ces équations et les réaliser numériquement.

Principaux résultats

Dans ce travail nous étudions l'existence des solutions d'équation intégrale non linéaire de type Hammerstein dans $L^p([a, b])$,

$$\varphi(t) = g(t) + \int_a^b K(t, s) f(s, \varphi(s)) ds, \quad -\infty < a \leq t \leq b < +\infty. \quad (237)$$

Un intérêt particulier est consacré au cas où le noyau $K(.,.)$ satisfait à la condition

$$K(t, s) = 0, \quad \text{pour } a \leq t \leq s \leq b,$$

dans ce cas, l'équation (237) est réduite à l'équation de Hammerstein-Volterra suivante

$$\varphi(t) = g(t) + \int_a^t K(t, s) f(s, \varphi(s)) ds, \quad -\infty < a \leq t \leq b < +\infty. \quad (238)$$

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Nous avons étudié quelques résultats d'existence et l'unicité de la solution de (238) dans $C([a, b])$.

On considère l'opérateur T défini par:

$$T\varphi(t) = g(t) + \int_a^b K(t, s) f(s, \varphi(s)) ds, \quad \forall t \in [a, b]. \quad (239)$$

On applique la théorie du point fixe sur l'opérateur T sous des conditions données sur T . A fin de prouver que l'équation admet une solution, il faut montrer que l'opérateur T admet un point fixe.

Dans le cas particulier où $X = C([a, b])$, le théorème d'Arzela-Ascoli est généralement utilisé pour prouve la compacité de T . En utilisant certaines conditions sur les fonctions $g(\cdot)$ et $f(\cdot, \cdot)$ en combinant le théorème d'Arzelz-Ascoli avec un résultat densité L^p , nous prouvons la compacité de l'opérateur T dans $L^p([a, b])$, $p \geq 1$.

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Rational Chebyshev collocation method for the numerical solution of Fredholm integral equations on the whole line

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Abstract: A spectral solution method is proposed to solve a class of linear integral equations on the real line. The method is developed by means of mapped Chebyshev rational functions. A special quadrature rule based on mapped Chebyshev-Gauss points and weights is then utilised to evaluate the infinite integrals appeared in the scheme. Thus, the solution of the problem reduces to the solution of a simple system of algebraic equations. Convergence and error analysis are discussed and numerical examples illustrate the efficiency of the method.

Keywords: Fredholm integral equations, real line, mapped Chebyshev rational functions, error analysis.

Introduction

Fredholm integral equations in unbounded domain has enormous applications and having many important uses in engineering and sciences. For example, they appear naturally in radiative transfer theory, kinetic theory of gases, the theory of neutron transport and the traffic theory. In comparison with the abundant research for integral equations in finite domain, a few numerical approaches have been investigated for infinite domain and specially for the real line (See [2, 3, 3]). In the present work we propose an efficient numerical method for the Fredholm integral equation defined on the real-line by:

$$u(x) = f(x) + \int_{-\infty}^{\infty} k(x, t)u(t)dt, \quad x \in \mathbb{R}, \quad (240)$$

where $f(x)$ and $k(x, t)$ are given smooth functions and $u(x)$ is unknown function.

The Chebyshev collocation method is presented for the solution of the equation previous in which the integral part is replaced by their operational matrix representations with collocation points apply the Chebyshev spectral collocation method to solve the resulting equation. The rate of convergence and error analysis are provided. Moreover, some numerical examples are carried out to verify the spectral accuracy of the proposed method.

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Main results

Solution method

The function u can be accurately approximated by

$$U(x) = \sum_{j=0}^N c_j T_j(x), \quad j = 0, 1, \dots, N. \quad (241)$$

where $T_j(x)$ are the Chebyshev polynomials. To define the equation (240) in the finite interval $[-1, 1]$ we need to the following variable change

$$x = \varphi_s(y) = \frac{sy}{\sqrt{1-y^2}}, \quad y = \theta_s(x) = \frac{sx}{\sqrt{1+x^2}}.$$

It is clear that

$$\frac{dx}{dy} = \frac{s}{(1-y^2)^{3/2}} \quad x \in \mathbb{R}, y \in I,$$

The the equation 240 can defined by

$$U(y) = F(y) + \int_{-1}^1 K(y, z) U(z) dz \quad (242)$$

where

$$U(y) = u(\theta_s(y)), F(y) = f(\theta_s(y)) \\ K(y, z) U(z) = k(\theta_s(y), \theta_s(z)) U(\theta_s(z)).$$

To obtain such a solution, we need to substitute $\{\zeta_i\}_{i=0}^N$ the Chebyshev points into the integral equation (242) so that the latter becomes

$$U(\zeta_i) = F(\zeta_i) + \int_{-1}^1 K(\zeta_i, z) U(z) dz \quad i = 0 \dots N \quad (243)$$

The main difficulty in obtaining high order accuracy is to compute the integral term. To overcome this difficulty, we need to use the point Gauss quadrature formula relative to the Chebyshev weights $\{w_j\}_{j=0}^N$. Then the equation (243) can be written as follows:

$$U(\zeta_i) = F(\zeta_i) + \sum_{j=0}^N K(\zeta_i, \zeta_j) U(\zeta_j) w_j, \quad i = 0 \dots N \quad (244)$$

Then, the discrete spectral rational Chebyshev-collocation method for solving Eq. (244) leads to the following linear system:

$$(T - sKDT)U = F$$

where $\mathbf{T}, \mathbf{K}, \mathbf{D}$ and $\mathbf{F}$ are given by:

$$\mathbf{F} = F(\zeta_i^N) \quad , \quad \mathbf{K} = K(\zeta_i^N, \zeta_j^N), \\ \mathbf{T} = U(\zeta_i^N) \quad , \quad \mathbf{D} = \text{diag}(w_j). \quad (245)$$

which can be easily solved by using a standard linear algebra routine.

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Convergence analysis

Theorem 78. *Let u be the exact solution to Eq. (240) and the approximate solution u_s^N obtained by using the spectral-collocation scheme in the previous section. If $u \in H_s^m(\mathbb{R}_+)$, then for $m \geq 1$,*

$$\|u - u_s^N\|_\infty = \mathcal{O}(N^{1/2-m}). \quad (246)$$

Numerical experiments

Consider the following example of Fredholm integral equations (240) with different kernel then we calculate the maximum absolute errors obtained by using MCRFs-collocation method described above for different degree N and various values of s -parameter on the interval $[-50, 50]$.

Example 6.

$$f(x) = e^{-x^2} - \operatorname{erf}(1)\pi e/(x^2 + 1); \quad k(x, t) = 1/(1 + x^2)(1 + t^2); \quad u(x) = e^{-x^2}.$$

N	$s = 1.5$	$s = 2$	$s = 4$
20	8.49e-06	2.62e-05	6.54e-04
40	4.82e-07	8.54e-08	2.39e-08
60	5.96e-09	3.38e-10	1.03e-12
80	1.17e-10	1.81e-12	8.22e-14
100	2.78e-12	2.58e-14	1.80e-14

Table 3: A comparison of the errors of Example 6.

Conclusion

In this study, an efficient approach based on rational Chebyshev functions basis is described for the numerical solution of Fredholm integral equations on the real-line. The spectral accuracy associated with L_∞ norm error estimates is demonstrated theoretically. Numerical examples are given to support the theoretical results. The results show that the rational Chebyshev approach is very accurate. Moreover, the proposed method can be further extended to solving similar types of equations as well.

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Numerical method for state-dependent delay integral equations

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Abstract: In this work, we will establish the existence and the uniqueness of the solution of Differential and integral equations with state-dependent delays. We will also enhance the theoretical study with a numerical investigation, relying on modern numerical techniques and sophisticated software. Additionally, we will present the obtained results in this context.

Keywords: Delay integral equation, State-dependent or Self-referred integral equation, Leray-Schauder theorem, Picard iteration method

2020 Mathematics Subject Classification: Primary: Partial differential equations and functional analysis.

Introduction

Differential delay equations constitute a very important field of study for modeling hereditary phenomena encountered in physics, biology, chemistry, economics, ecology, etc. Despite the fact that in most models, the delay is estimated non-indicative and ignored to simplify the study, it has been proven that in many cases, the delay plays a dominant role in several domains, and models with delay provide more accurate and realistic results than their counterparts without delay.

Differential equations with state-dependent delays attract the interest of specialists as they largely arise from application models, such as the two-body problem in classical electrodynamics, position control, mechanical models, transmission of infectious diseases, population models, dynamics of economic systems ([?],[?]), etc. As a special type of state-dependent delay differential equations,

$$\begin{cases} \frac{du(t)}{dt} = g(t) + \lambda f(t)u(\varphi(t)), & t \in I. \\ u(a) = u_a, \end{cases}$$

Iterative differential equations have distinctive characteristics and have been studied in recent years, including smoothing, equivariance, analyticity, monotonicity, convexity, as well as numerical solution. In the theory of differential equations, one of the fundamental and important problems is the initial value problem, and there are many existence results, on special iterative differential equations. In 1984, Eder [?] proved the existence of the solution.

According to Volterra's works, integral equations are considered a suitable pattern for modeling such problems. In our work, we will present some numerical methods and the results obtained

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for this purpose.

We followed the following outline in order to provide a good study for solving this equation.

The relationship between a delay differential equation (DDE) and a delay integral equation (DIE)

([?])([?])

The compactness of the integral operator.

[?]

Existence and uniqueness of solution to the equation.

[?]

Some numerical methods for solving equations

[?]

the trapezoidal method

[?]

The successive method (Picard's Method of Successive Approximations)

[?]

Conclusion

Processing delay differential equations has garnered significant attention across various scientific disciplines. We selected a model from among these equations and transformed it into an integral equation. Subsequently, we conducted an analytical study to prove the existence and uniqueness of solutions. Following this, we pursued numerical methods, where we observed that the Picard iterative method is the most effective under certain conditions. Therefore, the field remains open for further research and the development of software and methodologies. This contribution sheds light on a particular type of these equations.

Vibration control of a viscoelastic flexible marine riser

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Abstract: In this work, we investigate asymptotic behavior of solution of the flexible marine riser problem with vessel dynamics when a pipeline is made by the viscoelastic materials (see [1, 2]). Under a suitable control applied at the top boundary of the riser, we establish different types of decay rates for a large class of relaxation functions. In particular, exponentially and polynomially (or power type) decaying functions are included in this class. Our method is based on the multiplier technique.

Keywords: Stability, vibration control, arbitrary decay, viscoelasticity

2020 Mathematics Subject Classification: 35L20; 93D15; 93D20.

Introduction

In this work, we consider a flexible marine riser with vessel dynamics when a pipeline is made by the viscoelastic material. Namely

$$\rho w_{tt}(x, t) + EI w_{xxxx}(x, t) - EI \int_0^t h(t-s) w_{xxx}(s) ds - T w_{xx}(x, t) = 0, \quad (x, t) \in (0, L) \times [0, \infty) \quad (247)$$

with the boundary conditions and the initial data

$$\left\{ \begin{array}{l} w(0, t) = w_x(0, t) = w_{xx}(L, t) = 0, \quad t \geq 0, \\ -EI w_{xxx}(L, t) + EI \int_0^t h(t-s) w_{xxx}(L, s) ds + T w_x(L, t) \\ = u(t) - d_s w_t(L, t) - M_s w_{tt}(L, t), \quad t \geq 0 \\ w(x, 0) = w_0(x), \quad w_t(x, 0) = w_1(x), \quad x \in [0, L]. \end{array} \right. \quad (248)$$

Here $w(x, t)$, $w(L, t)$ and $w_t(L, t)$ are the displacement of the riser at the position x for the time t , the position and velocity of the vessel, respectively. The positive constants ρ , L , EI and T represents, the uniform mass per unit length, the length, the bending stiffness and the tension of the riser respectively. The coefficients M_s and d_s denotes the mass of the surface vessel and the vessel damping, and $u(t)$ is the top boundary control of the riser. The integral term in the Eq. 247 represents the viscoelastic damping term.

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Main results

The main objective of this work, is to prove different types of decay rates of the system (247)–(248) under a suitable control acting on top of the riser. The efficiency of the proposed boundary control is confirmed by some numerical simulations.

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Probability statistics, Modeling and Control

Statistical control of manufacturing processes. Application to the company Sarl el Wifak (Setif)

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Abstract: The Statistical Process Control (SPC) is the set of methods and preventive actions that aim to bring the process to the required level of quality. To this end, the performance of the production processes are evaluated, monitored and adjusted statistically to avoid excesses and maintain product compliance. The SPC was used to supervise the wafer manufacturing process in a private company. The positional and dispersion parameters of the studied population were reported as plots on control charts. Their easy interpretation makes it an effective visual tool to supervise the evolution of the process.

Keywords: Statistical Process Control, Control Card, capability.

2020 Mathematics Subject Classification: 62.

Introduction

In recent years, the industry has faced increasingly fierce competition. The internationalization of competition and the race for development have pushed companies to seek assets that will allow them to win the game. The quest for quality then became a key point in the competitiveness of companies. This research does not date from today. Indeed, quality has always been an important objective since man has been making objects. However, the new context of global competition revives this quest for quality by requiring more formalism in its approach. So quality has always been a qualifying criterion. And to better appreciate it, you have to know how to measure it while including the processing capacity of the available data. For this, it is necessary to use tools and methods that allow the realization of such a task. Statistical quality control [1], or more specifically statistical process control, proves to be a privileged means that allows this essential approach to obtain better quality.

Statistical Process Control (SPC)

It is a real information system applied to manufacturing processes either directly (control of its parameters) or indirectly (control of product characteristics). Statistical process control SPC [2]

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is a method of monitoring a process to identify specific causes of variation and signal the need to take corrective action. The implementation of the SPC allows, through the knowledge of the variations of the machines and processes:

To raise the level of product quality.

Increase machine performance and stability.

To refine the settings, reduce the cost of non-quality (less rejects, less downtime, less control).

To better predict future production by ensuring greater safety and better reliability of the operation of the process.

To improve productivity.

The basic concepts on which the application of SPC is based are:

Variability monitoring and control using control charts.

Assessing process capability by calculating capabilities.

Application of SPC to the wafer manufacturing process

We now place ourselves in a real situation, the implementation of this method in the wafer manufacturing workshops : Sarl El Wifak. The wafer manufacturing process is described in detail in its marketing authorization file [1]. The workshop of the company el wifak respects this process and almost continuously produces flawless wafers. Thus, despite compliance with the file and despite the know-how of the company and like any company, production is sometimes affected, let us give for example the three problems that we have identified :

1) Just after baking the sheets (see Manufacturing Process page 27 in [3]) and cooling them, we noticed that as these sheets are on their way to the spreader, some of them are cracked and must therefore be removed before passing through the spreader.

2) After these plates are spread and cooled, they pass through the cutter, which cuts them into wafers. At this level too, it is found that there are also some badly cut wafers and therefore must be removed.

3) The packs of wafers weigh much more than the weight mentioned on the packaging.

The plates used to obtain the wafers and even the wafers which are cracked or broken must be discarded from production. For this, a sorting person has to be put in place : the plates and wafers are then removed manually. This extra step in the manufacturing process increases the cost of production, due to the labor required, yield may be slightly affected, not to mention that any defective wafer plates are scrapped. The purpose of our work within the company el Wifak is the application of SPC to the manufacturing process of wafers and the establishment of control charts in order to determine if the manufacturing process is stable and possibly provide information concerning this process to try to find the origin of some problems and possibly improve the performance of the company. Recall that the company El Wifak is very well known for the quality of its products nationwide. We wanted to intervene at the level of several points of the manufacturing process. Unfortunately, we were only authorized to do so at the level of two points which are:

After cutting the plates into wafers .

Weighing boxes of wafers at the end of the production cycle.

In order to carry out this work, we decided to apply the SPC to the manufacturing process at the level of these two points; we then suggested to use two types of control charts: the first is the control chart with attributes of type np [4] and the second is the measurement chart or Shewhart chart [5].

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Main results

The results obtained [3] show that the company's wafer manufacturing process is capable but not centered, we can see it at the level of the interpretation of the np card where it was asserted that the process is under control and mastered, likewise the average graph is generally normal, on the other hand the range graph reflects an abnormality in its graphics card. The spans of two samples are outside the control limits: A too large span, produces wafer bundles weighing more than necessary. In this case, it is necessary to find the cause of this problem. The value 0.95 of the real capability obtained on the basis of the upper limit of tolerance that we have proposed indicates a non-centered process. This process is pulled towards the upper limit of tolerance, therefore probably a problem of adjustment or a problem of raw material such as the cream used to spread the wafers which sometimes can be denser than normal or even a failure of the measuring device.

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Predictive Density Order Selection of Mixture Autoregressive Models

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Abstract: This paper deals with the problem of joint determination of the number of regimes and autoregressive orders of the Mixture Autoregressive MAR models. More specifically, we propose a variant of the Predictive Density Criterion (PDC) for the purpose of MAR model selection. The efficiency of this variant is evaluated by means of Monte Carlo experiments. Our results indicate that (PDC) serves as an effective tool for selecting the correct order and the number of regimes.

Keywords: Mixture AR model, Bayesian approach, model selection problem, Predictive density criterion.

2020 Mathematics Subject Classification:

Introduction

The mixture of statistical models have been studied long time ago, but the adoption of the mixture philosophy in nonlinear time series models has a relatively short history. Wong and Li (2000) introduced a model called mixture autoregressive MAR model. This mixture of models makes possible a time series to have a different dynamic according to the regimes in which it is located. It is based on the idea that a relationship can be non-linear over the global period considered, but linear by sub-periods. Indeed, the MAR model is a finite mixture of autoregressive models where parameters at each time point depend on a latent transition process which is independent and identically distributed i.i.d.

It is well known that one of the major problems in the analysis of the MAR models is to specify the number of models and the appropriate autoregressive order of each component. Many approaches to this problem can be used. Indeed, Wong and Li (2000) have adapted the Bayesian Information Criterion (BIC) (Schwarz, 1978) for the MAR setting. They suggested, as the goodness-of-fit term, the maximized log-likelihood which is computed from the conditional probability density of the model rather than one defined from the complete-data log-likelihood. In the same work, they have pointed out that the (AIC) is not appropriate for selecting the orders in the MAR framework. Furthermore, the predictive density function of each observation of the data has been proven useful to solve the order selection problem for some classes of time series models. Indeed, Djuric and Kay (1992) proposed the Predictive Density Criterion (PDC) for the order selection problem of the linear autoregressive AR model. Bentarzi and al. (2008) extended the (PDC) to solve the order selection problem for periodic AR models. Since the self-exciting threshold autoregressive SETAR model is piecewise linear autoregressive, Hamdi and Khalfi (2016) extended the (PDC)

for the problem of joint determination of the delay parameter and autoregressive orders. The demonstrated effectiveness of the (PDC) criterion as a model selection tool in AR, PAR and SETAR models motivates the exploration of an extended variant for MAR modeling. More specifically, we propose a variant of (PDC) for the problem of joint determination of the number of regimes and autoregressive orders of MAR models.

The article is organized as follows. Section 2 is dedicated to our contribution. Using a non-informative prior density, we determine an explicit expression of the predictive density function which allows us to derive our criterion. The performance of the proposed criterion is discussed through a simulation study in the Section 3.

Predictive Density Criterion for MAR Model

Consider the following Mixture Autoregressive model

$$f(y_t|F_{t-1}) = \sum_{k=1}^K \frac{\alpha_k}{\sigma_k} f^{(k)}\left(\frac{y_t - \varphi_{k,0} - \sum_{j=1}^{p_k} \varphi_{k,j} y_{t-j}}{\sigma_k}\right), \quad t \in \mathbb{Z} \quad (249)$$

We denoted this model by $MAR(K; p_1, p_2, \dots, p_K)$, where $f^{(k)}$ and $f(\cdot|F_{t-1})$ are, respectively, a probability density with zero mean and unit variance and the conditional probability density of y_t given the past information. F_t is the information set up time t . The σ_k are positive scalars, $\varphi_{k,i}$ $i = 0, 1, \dots, p_k$ the autoregressive parameters and $\sum_{k=1}^K \alpha_k = 1$, $\alpha_k > 0, k = 1, \dots, K$.

Let $Z_t = \{Z_t : t \in \mathbb{Z}\}$, is a sequence of independent and identically distributed (i.i.d.) K -dimensional vectors with component $Z_{t,k}$ equal to 1 if the observation y_t comes from the k -th component of the conditional density which is supposed strictly positive on $\mathbb{R}$, and 0 otherwise. The (PDC) criterion proposed by Djuric et Kay (1992) for selecting the optimal order of the linear AR model, is given as follows:

$$PDC(p) = - \sum_{t=2}^n \log f(y_t|y_{1,t-1}, p), \quad p \in \{0, \dots, P\} \quad (250)$$

The optimal order estimation is given by:

$$p_0 = \arg \left(\min_p \left(- \sum_{t=2}^n \log f(y_t | y_{1,t-1}, p) \right) \right) \quad (251)$$

To derive the (PDC) version for the model selection of MAR, we must to give explicitly the expression of predictive density function $f(y_t|y_{1,t-1}, p)$. We consider the following assumptions, under we derive our extend predictive density criterion.

- A1. The autoregressive parameters $\varphi_{k,i}$ $i = 0, 1, \dots, p_k, k = 1, \dots, K$ satisfy the necessary and sufficient condition for the MAR process to be stationary.
- A2. The probability distribution of the parameters vector is given by non-informative prior density.
- A3. The distribution of the probability density function $\alpha_k > 0, k = 1, \dots, K$ is supposed to be Dirichlet distribution.

Simulation study

In this section, we are interested, on one hand, in studying, via Monte Carlo experiments, the performance of the PDC order selection criterion and, on the other hand, in comparing the performance of the PDC to the performances of some well-known classical criteria.

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Novel results on the stabilization of stochastic systems

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Abstract: We study systems in Hilbert space that are subjected to stochastic structured bounded perturbations. These systems may represent partial differential equations with stochastic uncertainties and boundary control. Our objective is to investigate the issue of maximizing the stability margin through state feedback. Presuming the semigroup to be analytic, we prove the existence of suboptimal controllers.

Keywords: Stochastic stability, robust stabilization, state feedback, Analytic semigroup

2020 Mathematics Subject Classification: Primary 93D05, 93D15, 93D21.

Introduction

In recent times, researchers from the mathematical and engineering communities have paid close attention to the system robustness analysis that occurs in a wide range of applications. Stability radii have been applied and examined for many different types of systems in a great deal of research articles.

The maximization of the stability radius has been investigated in [3]. In their study, the authors assumed that the input operator is bounded. In practice, it is important to study this problem in the case where the input operator is unbounded, because it covers the case of partial differential equations with boundary and pointwise control.

In this work, we investigate the maximization of the stability radius by state feedback. Assuming that the semigroup is analytic, we establish some conditions for the existence of suboptimal controllers. The maximum stability margin is obtained by using operator Riccati equations.

Main results

Let $H, Z, U_i, Y_i, i \in \overline{N}$, real separable Hilbert spaces. Consider the controlled stochastic system

$$\begin{cases} dx(t) = Ax(t)dt + \sum_{i=1}^N D_i \Delta_i (E_i x(t)) dw_i(t) + Bu(t)dt, & t \in \mathbb{R}^+, \\ x(0) = x_0 \in H, \end{cases} \quad (252)$$

where A is the infinitesimal generator of an exponentially stable analytic semigroup $(S(t))_{t \geq 0}$ on H . B is a linear operator from Z to H , (B is generally unbounded as an operator from Z to H), such that $(-A)^{-\eta} B \in L(Z, H)$ for some fixed η , $0 \leq \eta < \frac{1}{2}$.

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For $i \in \overline{N}$, D_i is a linear operator from U_i to H , E_i is a linear operator from H to Y_i and $\Delta_i \in Lip(Y_i, U_i)$.

$(w_i(t))_{t \in \mathbb{R}_+}$, $i \in \overline{N}$, are independent real Wiener processes on a probability space $(\Omega, \mathcal{F}, \mu)$ relative to an increasing family $(\mathcal{F}_t)_{t \in \mathbb{R}_+}$ of σ -algebras $(\mathcal{F}_t)_{t \in \mathbb{R}_+} \subset \mathcal{F}$. u takes its values in the real separable Hilbert space Z .

We assume that (A, B) is stabilizable. Our aim is to identify the highest stability radii that linear state feedback $u = Fx$ can achieve.

Let

$$\overline{\mathcal{F}} = \left\{ \begin{array}{l} F \in L(H, Z); A + BF \text{ is the infinitesimal generator} \\ \text{of an exponentially stable semigroup} \end{array} \right\}$$

and define

$$\overline{r^w}(A; (D, E)) = \sup \{ r^w(A + BF; (D, E)); F \in \overline{\mathcal{F}} \}.$$

Our first main result is stated as follows.

Theorem 79. Let $F \in \overline{\mathcal{F}}$. Assume that there exist $\sigma > 0$ and $\varepsilon > 0$ such that the Lyapunov inequality

$$2 \langle P(A + BF)x, x \rangle + \varepsilon^{-2} \langle B^*Px, B^*Px \rangle + \langle E(\alpha)x, E(\alpha)x \rangle \leq 0, \quad x \in D(A + BF) \quad (253)$$

has a solution $P_0 \in L^+(H)$ which satisfies the condition

$$I - \sigma^2(\theta_j/\alpha_j^2)D_j^*P_0D_j \geq 0, \quad j \in \overline{N} \quad (254)$$

Then the Riccati equation

$$2 \langle Ax, Px \rangle + \langle E(\alpha)x, E(\alpha)x \rangle - \varepsilon^{-2} \langle B^*Px, B^*Px \rangle = 0, \quad x \in D(A) \quad (255)$$

has a solution $P \in L^+(H)$ satisfying

$$I - \sigma^2(\theta_j/\alpha_j^2)D_j^*PD_j \geq 0, \quad j \in \overline{N}, \quad F_\varepsilon = -\varepsilon^{-2}B^*P \in \overline{\mathcal{F}}$$

and

$$r^w(A - \varepsilon^{-2}BB^*P; (D_i, E_i)_{i \in \overline{N}}) \geq \sigma.$$

Now we establish conditions for the existence of suboptimal controllers.

Proposition 7. Let $\sigma, \varepsilon > 0$. Assume that the Riccati equation (255) has a solution $P \in L^+(H)$ such that $I - \sigma^2(\theta_j/\alpha_j^2)D_j^*PD_j \geq 0$, $j \in \overline{N}$. Then

$$F_\varepsilon \in \overline{\mathcal{F}} \text{ and } \sigma \leq r^w(A + BF_\varepsilon; (D_i, E_i)_{i \in \overline{N}}).$$

As a result of Proposition 7, we use the Riccati equation (255) to determine the maximum achievable stability radius.

Corollary 4. We have

$$\overline{r^w}(A; (D_i, E_i)_{i \in \overline{N}}) = \sup \left\{ \begin{array}{l} \sigma > 0; \text{ there exist } \varepsilon > 0 \text{ such that (255) has a solution} \\ P \in L^+(H) \text{ with } I - \sigma^2(\theta_j/\alpha_j^2)D_j^*PD_j \geq 0, \quad j \in \overline{N} \end{array} \right\}.$$

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Stability analysis of the ABC Algorithm : A Lyapunov based Approach

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Abstract: The Artificial Bee Colony (ABC) is one of the most prominent natural inspired algorithms introduced in 2005 by Karaboga. Like for other meta-heuristics, their parameters setting is critical to the algorithm progress and its convergence. In this paper we suggest the ABC to be described as a stochastic discrete-time state-space system, then we investigate its stability analysis using Lyapunov stability theory. Based on the derived stability conditions, new ranges of the parameter Φ are recommended.

Keywords: Artificial bee colony , Stability analysis, Parameter setting, Swarm intelligence.

2020 Mathematics Subject Classification: 49M05, 68W40, 68W25.

Introduction

The field of global optimization has witnessed an explosion of the so-called meta-heuristic algorithms that were successfully applied to various problems, such as traffic and transportation problems, imagery and many other range of problems. Though these algorithms do not guarantee the optimal solution, they generally have a tendency to find a good solution in a reasonable laps of time. The main problem with these algorithms is a determination of their parameters, where a good performance of an algorithm depends essentially on its fine-tuning.

The Artificial Bee Colony (ABC) originally developed by Karaboga in 2005 [1] has become a widely used optimization technique. It is a multi-agent algorithm that imitates the bees' foraging behavior to find the best nectar source. In ABC, bees get divided into groups sharing the tasks of exploration and exploitation of the search area. Each bee i is represented, at step t , by a vector of D (D being the search space dimension) decision variables $x_i(t)$. These positions $x_i(t)$ are initialized randomly. Then, they evolve in different ways depending on the group to which they belong. The first group is composed of N employed bees. Each particle in this group looks within its neighborhood for a new candidate solution $v_i(t)$ as follows

$$\mathbf{v}_i = \mathbf{x}_i + \Phi_{1,i} \odot (\mathbf{x}_i - \mathbf{x}_k), \quad 1 \leq i \leq N, \quad (256)$$

where k is a randomly selected index in $\{1, \dots, N\}$ different from i , and $\Phi_{1,i}$ is a uniform random vector, and ' $\odot$ ' denotes the Hadamard product, i.e., the element wise product. If the new position has better quality than the current solution, then $\mathbf{v}_i$ replaces $\mathbf{x}_i$. If the quality of the solution is not

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improved within a predefined number of iterations, called the abandonment limit, the employed bee position is randomly regenerated. The second group is composed of M onlooker bees that wait for the information gathered by the employed bees. The onlooker bees choose the employed to be followed and update their positions as follows:

$$\mathbf{v}_i = \mathbf{x}_i + \Phi_{2,i} \odot (\mathbf{x}_i - \mathbf{x}_k), \quad N+1 \leq i \leq N+M, \quad (257)$$

where $\Phi_{2,i}$ is a random uniform vector and k is a random integer between 1 and N . The choice of the index k is proportional to the probability P_k . The new position $\mathbf{v}_i$ replaces the previous one $\mathbf{x}_i$ only if it presents enhanced quality.

Note that the components of the random vectors $\Phi_{1,i}$ and $\Phi_{2,i}$ are commonly generated in the interval $[-1, 1]$ [1, 2].

Given ABC's success, a few amount of theoretical work has been performed on the stochastic search algorithm to try to predict and understand its underlying behavior. At our knowledge, only two works have been carried out to analyze the ABC stability analysis. Bansal et al. focused on the ABC parameterization using the Von Neumann stability criteria [2]. Based on this criteria and the conducted experiments, the best range for the Φ parameter is $[-1, 1]$. In a recent study [3] a matrix-iterative model for the ABC is introduced, taking into account the coupling within bees. Under the quasi-deterministic assumption, the necessary conditions for absolute and stochastic stability were investigated, and new ranges for setting the uniform distributions of the ABC algorithm were reported. In this work we modeled the equations describing the ABC algorithm as a stochastic discrete-time system. The behavior of this system depends on its dynamic and preservation stability conditions. Sufficient conditions for stability are conducted based on the Lyapunov stability theory, and then new area of the location of the parameter Φ are recommended.

Main results

In this section, we present our model for the dynamics of ABC. Then, we derive the stochastic stability criteria.

Model

Without loss of generalities, we can consider each direction independently, given that directions are decoupled (Eqs. 256 and 257). Thus, the analysis reduces to the study of scalars x_i instead of the vectors $\mathbf{x}_i$. We denote the groups of employed and onlooker bees at each step t respectively:

$$\mathbf{x}_E^{(t)} = \left(x_1^{(t)}, \dots, x_N^{(t)} \right)^\top,$$

and

$$\mathbf{x}_O^{(t)} = \left(x_{N+1}^{(t)}, \dots, x_{N+M}^{(t)} \right)^\top,$$

where ' $\top$ ' indicates the transpose of a vector or a matrix. Therefore, the algorithm can be presented in the state-space form:

$$\begin{pmatrix} \tilde{\mathbf{x}}_E \\ \tilde{\mathbf{x}}_O \end{pmatrix}^{(t+1)} = \begin{pmatrix} 1 + \Phi_{1,i} & \mathbf{0} \\ 0 & 1 + \Phi_{2,i} \end{pmatrix}^{(t)} \begin{pmatrix} \tilde{\mathbf{x}}_E \\ \tilde{\mathbf{x}}_O \end{pmatrix}^{(t)}. \quad (258)$$

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Let $\Phi_{1,i} \sim \mathcal{U}_{[a,b]}$ and $\Phi_{2,i} \sim \mathcal{U}_{[c,d]}$, such that $b > a$ and $d > c$. The Lyapunov stability theory and Sylvester's criterion applied to the stochastic dynamic of ABC determine the condition for asymptotic stability and convergence to the equilibrium point. Our objective is to derive conditions on parameters a , b , c , and d to ensure the stochastic stability. The main result of this work is established in the following theorem.

Analysis results

Theorem 80. Let $\Phi_{1,i} \sim \mathcal{U}_{[a,b]}$ and $\Phi_{2,i} \sim \mathcal{U}_{[c,d]}$, such that $b > a$ and $d > c$. If the following conditions :

i.

$$-3 < a, c < 1$$

ii.

$$\begin{aligned} -\frac{a}{2} - \frac{3}{2} - \frac{\sqrt{-3a^2 - 6a + 9}}{2} < b < -\frac{a}{2} - \frac{3}{2} + \frac{\sqrt{-3a^2 - 6a + 9}}{2}, \\ -\frac{c}{2} - \frac{3}{2} - \frac{\sqrt{-3c^2 - 6c + 9}}{2} < d < -\frac{c}{2} - \frac{3}{2} + \frac{\sqrt{-3c^2 - 6c + 9}}{2}, \end{aligned}$$

hold, then the proposed ABC algorithm is stochastically stable.

Figure 7: Stability regions

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Euler schemes for solving stochastic delay differential equations

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Abstract: Mathematical models play a major role across many fields, and have contributed to our understanding of the dynamic behaviour of biological systems. Various models based on ordinary differential equations have been used to describe the behaviour of biological systems. However, by combining features of the classical approaches, there is a potential to extending such models. This work investigates numerically stochastic delay differential equations. The proposed approach is based on some strategies in the delay Euler scheme. Some numerical experiments are presented to demonstrate the applicability and effectiveness of the suggested technique.

Keywords: Delay differential equation, Euler scheme, numerical solution, Brownian motion.
2020 Mathematics Subject Classification: Primary 34Kxx, 34k50, 37A50, 60-xx.

Introduction

A stochastic differential equation (SDE) is a mathematical model that describes the evolution of a system subject to random noise or uncertainty. Unlike ordinary differential equations, which only involve deterministic processes, SDEs incorporate stochastic or random elements, making them suitable for modeling systems subject to random fluctuations or external influences.

In many applications, it is assumed that the future state of a process does not depend on past states and is determined by the present. If the system is assumed to be governed by an equation that involves the state and the rate of change of the state, ordinary or partial differential equations are usually considered. However, in other situations, it appears that these assumptions are only a first approximation of the actual situation and that a more realistic model would include some of the past states of the system. Similarly, in some cases where dependence on past system states is not imagined.

In mathematics, delay differential equations (DDEs) are a type of differential equations where the derivative of the unknown function at a certain time is given in terms of the function's values at previous times. Delay equations are used to model phenomena where there is a temporal mixture between the system's action and its response to that action. We consider the problem of

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the numerical solution of stochastic delay differential equations of Ito form:

$$\begin{cases} dX(t) = f(X(t), X(t-\tau))dt + g(X(t), X(t-\tau))dW(t), & t \in [0, T] \\ X(t) = \psi(t) & \text{for } t \in [-\tau, 0], \end{cases} \quad (259)$$

where $X(t)$ is the state variable at time t , $f(t, X(t), X(t-\tau))$ is the drift term, $g(t, X(t), X(t-\tau))$ is the diffusion term, $W(t)$ is the Wiener process, and τ is the delay time. The delay term makes the numerical solution more challenging since it involves the state variable at a previous time.

The theorem presented in reference [3] guarantees the existence and uniqueness of the exact solution of the SDDE (259).

Main results

This work aims to identify an approximate solution to the equation (259) using a numerical technique based on the Euler method. It is widely adapted for numerically solving stochastic differential equations (SDEs) and stochastic delay differential equations (SDDEs). The Euler-Maruyama scheme is based on dividing the time domain into discrete intervals to approximate the solution at the end of each interval by adding the deterministic and stochastic terms. This method is derived from the Taylor expansion of the solution and is a first-order weak scheme that converges in the mean-square sense. We have calculated the solution to within an acceptable level of accuracy by using the formula provided by Euler. :

$$X(t_{n+1}) = X(t_n) + f(t_n, X(t_n), X(t_n - \tau))\Delta t + g(t_n, X(t_n), X(t_n - \tau))\Delta W(t_n),$$

$X(t_n)$ is the numerical solution at time t_n , and $\Delta W(t_n)$ is the Wiener increment over the time interval $[t_n, t_{n+1}]$. To obtain the numerical solution, we start from the initial condition $X(t_0) = X_0$ and compute $X(t_{n+1})$ recursively using the above formula.

To confirm the applicability and effectiveness of the proposed technique, we employ various numerical simulations.

Example 7. Consider the SDDE

$$\begin{cases} dX(t) = (aX(t) + bX(t-\tau))dt + (\beta_1 + \beta_2 X(t) + \beta_3 X(t-\tau))dW(t), & t \in [0, T] \\ \psi(t) = 1 + t & \text{for } t \in [-1, 0], \end{cases}$$

where the exact solution on $t \in [0, 1]$ is given by

$$X(t) = e^{at} \left(1 + \frac{b}{a^2} \right) - \frac{b}{a} t - \frac{b}{a^2} + \beta_1 e^{at} \int_0^t e^{-as} dW(s).$$

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Semiparametric von Mises kernel circular density estimator

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Abstract:

In this paper, we apply the semi-parametric approach based on von Mises kernel is developed. Some properties of the semi-parametric HG and JSH von Mises circular kernel density estimators are provided. The popular cross validation technique is adapted for bandwidth selection. The performances of the proposed von Mises estimators are illustrated by a simulation studies and real applications for circular data.

Keywords: Bandwidth; Bayesian approach; Circular density; Cross validation; MBC techniques; Markov chain Monte Carlo (MCMC); von Mises kernel.

2020 Mathematics Subject Classification: 62G05, 62G99

Introduction

The aim of this article is threefold. First, we propose the semi-parametric circular kernel estimator, as well as its statistical properties. Secondly, we construct a new semi-parametric circular kernel estimator, using the JSH technique of [6] in order to accelerate the convergence of the bias. Statistical properties of this new estimator will also be studied. As a third objective, an application on simulated data and real data will be carried out to compare the performance of the developed estimators.

Let Θ be a circular random variable with unknown circular density function f on $[0, 2\pi[$. Consider the following identity

$$f(\theta) = g(\theta) \left\{ \frac{f(\theta)}{g(\theta)} \right\} := g(\theta)r(\theta), \quad (260)$$

where $g(\theta)$ is a starting (initial) model; see below discussion on choosing g and $r(\theta)$ is a correction factor. The semi-parametric circular kernel approach consists of estimating g by a parametric or

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semi-parametric circular model and estimating f in r by the non-parametric approach using the circular kernel method.

Let $\Theta_1, \dots, \Theta_n$ be a n -random sample of a bounded continuous circular density f . Depending on the identity (260), we investigate the semi-parametric estimator of the following form:

$$\tilde{f}(\theta) := g(\theta)\hat{r}(\theta) := g(\theta) \left\{ \frac{1}{n} \sum_{i=1}^n \frac{K_\nu(\theta - \Theta_i)}{g(\Theta_i)} \right\}, \quad (261)$$

where g is the starting model which can be chosen as a uniform circular distribution, a known (fixed) circular parametric model, an unknown circular parametric model or even a circular semi-parametric model, $K_\nu(\cdot)$ is a circular kernel and ν is the concentration parameter. Depending on the choice of g , several estimators can result from the estimator (261):

- If g is a uniform circular distribution; i.e., $g(\theta) = 1/2\pi$, then the estimator (261) becomes the standard circular non-parametric estimator, studied by [1], [2] and [3];
- If g is a standard circular non-parametric estimator, the estimator (261) becomes the non-parametric JLN estimator, studied by [4];
- If g is an unknown circular parametric model, then the estimator (261) is the semi-parametric HG estimator with circular kernel.
- If g is a circular semi-parametric model, then the estimator given by (261) is the semi-parametric JSH estimator with circular kernel.

Semi-parametric HG estimator with circular kernel of the density function

The semi-parametric circular estimator HG of f is defined by:

$$\widehat{f}_\nu^{HG}(\theta) = g(\theta; \widehat{\vartheta}) \left\{ \frac{1}{n} \sum_{i=1}^n \frac{K_\nu(\theta - \Theta_i)}{g(\Theta_i; \widehat{\vartheta})} \right\}. \quad (262)$$

where $\widehat{\vartheta}$ is the maximum likelihood estimator of $\vartheta \in \mathcal{V} \subseteq \mathbb{R}^p$ of the starting model g ; it can be chosen as a von Mises model with unknown parameters $\vartheta = (\mu, k)$.

Using the circular von Mises kernel, the estimator (262) is written as follows:

$$\widehat{f}_\nu^{HG-VM}(\theta) = g(\theta; \widehat{\vartheta}) \left\{ \frac{1}{n} \sum_{i=1}^n \frac{\frac{1}{2\pi I_0(\nu)} \exp\{\nu \cos(\theta - \Theta_i)\}}{g(\Theta_i; \widehat{\vartheta})} \right\}. \quad (263)$$

JSH semi-parametric estimator with circular kernel of the density function

The circular semi-parametric JSH estimator of f is given as follows:

$$\widehat{f}_\nu^{JSH}(\theta) = \widehat{f}_\nu^{HG}(\theta) \left\{ \frac{1}{n} \sum_{i=1}^n \frac{K_\nu(\Theta_i - \Theta_j)}{\widehat{f}_\nu^{HG}(\theta)} \right\}. \quad (264)$$

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where $\widehat{f}_v^{HG}(\cdot)$ is the circular HG estimator given by the equation (262).

Using the von Mises circular kernel, the estimator (264) is written as follows:

$$\widehat{f}_v^{JSH-VM}(\theta) = \widehat{f}_v^{HG-VM}(\theta) \left\{ \frac{1}{n} \sum_{i=1}^n \frac{\frac{1}{2\pi I_0(v)} \exp\{v \cos(\theta - \Theta_i)\}}{\widehat{f}_v^{HG-VM}(\theta)} \right\}. \quad (265)$$

Main results

The following result presents the asymptotic bias and variance of the semiparametric estimator

Theorem 81. Let $\widehat{f}_v^{HG-VM}$ be the HG-VM estimator defined by (263). Let $g_0(\cdot) = g(\cdot, \vartheta_0)$ be the best start approaching the unknown function f in the sense of the Kullback-Leibler distance, and let $r(\cdot) = f/g_0$ a twice continuously differentiable weight function. When $v \rightarrow \infty$ and $v^{1/2}/n \rightarrow 0$ when $n \rightarrow \infty$, then:

(i) the bias of the HG-VM estimator is given by:

$$\text{bias}[\widehat{f}_v^{HG-VM}(\theta)] = \frac{1}{4v} g_0(\theta) r''(\theta) + o(v^{-1}),$$

(ii) the variance of the HG-VM estimator is given by:

$$\mathbb{V}[\widehat{f}_v^{HG-VM}(\theta)] = \frac{v^{1/2}}{2n\sqrt{\pi}} f(\theta) + o\left(\frac{v^{1/2}}{n}\right).$$

Theorem 82. Let $\widehat{f}_v^{JSH-VM}$ be the JSH-VM estimator defined by (265).

Let $g_0(\cdot) = g(\cdot, \vartheta_0)$ be the best start approaching the unknown function f in the sense of the Kullback-Leibler distance, and let $r(\cdot) = f/g_0$ a function four times continuously differentiable. When $v \rightarrow \infty$ and $v^{1/2}/n \rightarrow 0$ when $n \rightarrow \infty$, then:

(i) the bias of the JSH-VM estimator is given by:

$$\text{bias}[\widehat{f}_v^{JSH-VM}(\theta)] = -f(\theta)q(\theta, r) \frac{1}{v^2} + o\left(\frac{1}{v^2}\right),$$

$$\text{where } q(\theta, r) = \left(\frac{r''(\theta)}{4r(\theta)} \right)'',$$

(ii) the variance of the JSH-VM estimator is given by:

$$\mathbb{V}[\widehat{f}_v^{JSH-VM}(\theta)] = \frac{v^{1/2}}{2n\sqrt{\pi}} f(\theta) + o\left(\frac{v^{1/2}}{n}\right).$$

we examine and compare the performance of the JHS-VM, HG-VM, JLN-VM estimators and the standard von Mises estimator. We simulate data following the 20 circular models already; see [5]

For each model, 500 replications of sample size $n = 10, 50, 100$ and 200 are generated. We use the JHS-VM, HG-VM, JLN-VM estimators and the standard von Mises estimator to estimate the density of the data generated from the models considered.

In the case of the semi-parametric estimators HG-VM and JHS-VM, we considered the von Mises model with parameters μ and k as a starting model. The parameters μ and k are estimated using the maximum likelihood method.

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We examine the performance of the estimators via the integrated squared error (ISE) and the integrated squared bias (ISB) given respectively by

$$\text{ISE} := \int_0^{2\pi} [\widehat{f}_v(\theta) - f(\theta)]^2 d\theta \quad (266)$$

And

$$\text{ISB} := \int_0^{2\pi} [\mathbb{E}\{\widehat{f}_v(\theta)\} - f(\theta)]^2 d\theta, \quad (267)$$

where $\widehat{f}_v$ is the JHS-VM, HG-VM, JLN-VM estimator or the standard von Mises estimator.

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Solving stochastic integral equation using Jacobi approximation

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Abstract: A proposed accurate computational method, utilizing Jacobi approximation and the Jacobi-Gauss collocation method, is recommended for solving a set of nonlinear stochastic Itô-Volterra integral equations (SIVIEs). Due to the lack of exact solutions for these equations, there's considerable interest in finding precise approximate solutions. In this approach, Jacobi polynomials are used for approximation, allowing the conversion of the original system of equations into linear and nonlinear systems of algebraic equations.

Keywords: Stochastic integral equation, Jacobi polynomials, Lagrange interpolation, Gauss-quadrature, Collocation method.

2020 Mathematics Subject Classification: 60H10, 60H15 · 65C30 · 37N30 · 60G07.

Introduction

One of the most important theory in the applied mathematics is integral equations. As we all know that many problem in life can be modeled into integral equation which is linear or nonlinear, deterministic or stochastic. We can found this theory in many discipline such that Engineering, Finance, Biology, Graphic representation etc

Formulation of problem

On a filtrated probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, P)$, we will focus on solving the nonlinear stochastic integral equation

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$$X(t) = p(t) + \int_0^t W_1(t,s)\psi_1(X(s))ds + \int_0^t W_2(t,s)\psi_2(X(s))dB(s), \quad t \in [0, T]. \quad (268)$$

where the kernels W_1 , W_2 and ψ_1 , ψ_2 are a smooth functions on a filtrated space $(\Omega, \mathcal{F}, \mathcal{F}_t, P)$, and $\{B(t), t \geq 0\}$ is the Brownian process $\mathcal{F}_t$ -measurable. $X(t)$ is the unknown process that we need to find, and $p(t)$ is a random function.

Main results

In this paper, we introduce a novel algorithm for addressing these problems through the utilization of the collocation method and Jacobi approximations. Employing this approach, the problem is transformed into an algebraic system amenable to solution via an iterative method. We confirm the convergence of our method and demonstrate its efficiency and accuracy through numerical examples

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On the existence of solutions of SDE

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Abstract: In this work, we give a new statement for the solutions of stochastic differential equations where the coefficients are sufficiently smooth, the Picard approximation converges in the $\mathbb{C}^\infty$ sense, which means that the solution of the stochastic differential equation is almost certainly of class $\mathbb{C}^\infty$ in the initial value x . This work is inspired from the work given in [1].

Introduction

In the real world, random phenomena can no be ignored, such as electronic circuit, portfolio theory, option pricing in financial economics, and other aspects are widely used. Therefore, a lot of scholars have tried to use various methods for the investigation of the theoretical analysis of stochastic integral equations. In this work, we considered the following SDE

$$dY(t) = K_1(t, s)\phi_1(Y(s))ds + K_2(t, s)\phi_2(Y(s))dB_t, t \in [0, T] \quad (269)$$

$$Y(0) = y$$

The equation (269) is equivalent to integral equation

$$Y(t, y) = y + \int_0^t K_1(t, s)\phi_1(Y(s, x))ds + \int_0^t K_2(t, s)\phi_2(Y(s, x))dB_s,$$

where $K_1(t, s)$, $\phi_1(s)$, $K_2(t, s)$ and $\phi_2(s)$ are known functions and $Y(t)$ is unknown process defined on the same probability space (Ω, F, P) , $B(t)$ is the Brownian motion or Wiener process.

The use of nonlinear stochastic Itô-volterra integral equations with m -dimensional Brownian motion process covers various fields of science and engineering, including finance biology, physics, and more scientific fields.

Defining $Y_0(t) = y$, the Picard iterations are subsequently established for $n = 1, 2, \dots$

$$Y_n(t, x) = y + \int_0^t K_1(t, s)\phi(Y_{n-1}(s, y))ds + \int_0^t K_2(t, s)\phi(Y_{n-1}(s, y))dB_s$$

for all $t \in [0, T]$. Let $D^m Y$ denote the m -order derivatives of Y with respect to y . We can easily verify that $D^m(Y_{n+1})(t, y)$ for any integer $m \geq 0$ is continuous because Y_0 is continuous in (t, y) . If we could prove Y_n converges in $\mathbb{C}^\infty(\mathbb{R})$ then the solution Y of equation is $\mathbb{C}^\infty$ in $y \in \mathbb{R}^d$.

Let (Ω, F, P) be a filtered Space and $M_{2,T}$ denote set of stochastic process such that :

$$E\left(\sup_{0 \leq t \leq T} |X_t|^2\right) < \infty.$$

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We define the norme on $M_{2,T}$ by

$$\|X\|_{2,T}^2 = E(\sup_{0 \leq t \leq T} |X_t|^2).$$

The following theorem indicate tat under some assumptions the solutions $Y(\Omega, t, y)$ is of classe $\mathbb{C}^\infty$ with respect to y .

Theorem 83. *Let $X = X(t, x, \omega)$ $t \in [0, T]$ be the solution of Eq (269). Under assumption, there is a modification of $X(t, x, \omega)$ denoted by $X(t, x, \omega)$ again, so that the mapping $x \in \mathbb{R}^d \mapsto X(t, x, \omega) \in \mathbb{R}$ is $\mathbb{C}^\infty$ for each fixed t , almost surely ω .*

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Modeling economic environment monitoring to enhance economic tasks using deep learning with economical education on various sources

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Abstract: This research study proposed a new model which consists of modeling, designing and implementing a CNN-LSTM-based deep learning model for monitoring economic environment to enhance economic tasks using economical education from various sources through safe information. LSTM is used because of its ability to learn long-term dependencies, in taking the concatenated function of commercial, marketing or financial data as input, while integrating encapsulations, using objects connected, smart data and multiple social media sources to retrieving relevant safe information by combining multiple search results. It provides some good ideas that is extended to improve corporate marketing, business strategies, and financial times series forecasting. It is shown that the proposed model learns to both localize and recognize different aspects of marketing, business, and finance. The model evaluates the challenging task of detecting fraud in business and finance and shows that it is more accurate than the state-of-the-art of other neural networks and uses fewer parameters and less computation.

Keywords: CNN-LSTM-based multimodal deep learning model, Economic entities, Multiple social media sources, Education sustainability, Modeling, Objects connected, Smart Data

Introduction

Various information on business, marketing, and finance circulates in daily life on various sources namely connected objects, smart data, and multiple social media sources with generally qualitative and significative differences in obtained information through advances in IoT to get a comprehensive view of the ongoing event. Managers are faced with a huge retrieval of information shared on the web with huge awareness: it is a great challenge.

Several artificial intelligence systems are designed for monitoring the networking environment to facilitate managers identifying and filtering relevant useful shared content. Some work is focused on using just social networks as an information source while others are concurrently dedicated to warning, multiple sources, and education.

Modeling and designing economic environment monitoring to enhance economic tasks using deep learning with economical education is a great challenge.

Research on collecting content using deep learning is a problem of sequential learning. Therefore,

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a new model is proposed for improving economic tasks based on a CNN-LSTM used to learn long-term dependencies. This new approach makes it possible to integrate artificial intelligence technologies, deep learning, social media and IoT rechnologies for managing economic tasks using safe information.

Built on prior work that forms the background for this approach, this new model combines representation training with warning and economical education while integrating encapsulations from connected objects, smart data, and multiple social media sources to capture relevant and safe content while combining various search results, providing the best ideas for its extending to enhance economic tasks.

This research study consists of monitoring the economic environment to identify relevant content of an eventual upcoming economic crisis event. Once content is retrieved and cleaned of non-informative information, it is used to update content (warning or economical education) for citizens and managers to make quick and effective decisions to help people in need and avoid economic crises. Therefore, the study provides not only a solution to the challenge but also achieves promising results.

Results

This research study consists of monitoring the economic environment to identify relevant safe content to enhance economic tasks. Once content is retrieved and cleaned of non-informative information, it is used to update content (warning or economical education) for managers to make quick and effective decisions to enhance economic tasks.

Much research exists for Financial Time Series Forecasts. Research for Fraud Detection is common. On the other hand, there are only two studies for the Market Price and only one for Placing products in a supermarket.

Twitter is the first social network to be used in applications to exchange news via tweets. The NN-based model follows it to enhance warning that uses Twitter & Facebook.

The FFNN-based ALE follows this to enhance warning and education from the Web: it is the first to use multiple social media sources with all the Web. There are some other works namely LSTM, RNN, and Deep CNN-LSTM-based model to improve warning, awareness, and education in crisis event from all the Web.

There is the first model to use smart data & all the Web: it is the RNN-based ALE improving awareness.

With advances in IoT, other applications are used based on sensor technology. Thus, the study was able to integrate the deep learning concept.

The new proposed model consists of modeling, designing, and implementing deep learning model to enhance economic tasks through economical education while using safe information. There are other models to enhance preventing fake news, namely the improvements in commerce and business, financial time series forecasting, fraud detection and marketing with deep learning while avoiding false information, followed by Deep learning-based managing disaster with smart education in environmental sustainability while avoiding false information, followed by Smart education with deep learning and social media for managing disaster in the pursuit of environmental sustainability while avoiding false information, followed by Context-based detecting false information, and Detecting fake information using deep CNN-BLSTM-hybrid model through context social: all these models use multiple social media sources (all the Web).

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The new CNN-LSTM-based deep learning model is presented here. The LSTM layer is powerful in handling temporal correlation. Its extension has convolutional structures in both input-state and state-to-state transitions, which will solve this problem. By stacking multiple CNN-LSTM layers and building a coding prediction structure, the research study created a network model for these space-time sequence prediction problems. The economic crisis forecasting goal uses previously observed social networking to prevent an economic event in a local region, such as Algiers, London, or Paris. From the automated learning perspective, this is a problem of predicting space-time sequences.

Suppose the research study has a dynamic system represented by an $(M \times N)$ grid with M rows and N columns. In each cell of the grid, there are P measures (word, bias) varying in time. At any time, the observation can be represented by a tensor X belonging to $R^{P \times M \times N}$, with R denoting the domain of observed traits. With recording periodically observations, the study will have a sequence of tensors $X_1, X_2, \dots, X_t$. Spatio-time sequence prediction predicts the most probable sequence of length K , given previous J observations (including the current sequence) formulated.

Observing at each timestamp is a 2D map. In dividing this map into non-tiled, non-overlapping patches and visualizing pixels inside a patch as its measurements, the problem is naturally a spatio-time sequence prediction. This spatio-time sequence prediction problem is different from that of one-step time series prediction because this prediction target contains both spatial and temporal structures.

CNNs use very little pre-processing: they learn the filters, hand-engineered in conventional algorithms.

The objective is to maximize the size K of E_K set.

The study presents the experiments carried out to compare the performance of this proposed deep learning model, tested with the dataset, introduced in the following subsection, which was preprocessed.

The Root Mean Squared Error (RMSE), the mean absolute error (MAE), the mean square error (MSE) and R-squared (R^2) are the measures used to assess model performance across all experiments. Since the F score is derived from recall and precision, the study also shows these two measures for reference. The results are presented, discussed, and analyzed in the following sections.

An excellent monitoring economic environment is needed to collect messages from an upcoming economic crisis event. To verify the performance of the proposed model, the study applied three evaluation indices, including the mean squared error (RMSE), the mean absolute error (MAE), and goodness of fit (R-Square) as the loss function for model training.

The study divided the data sets into a training set and a verification set. The learning set is applied to train different deep-learning models while updating the weights and biases of the neural cell. Then the verification set checks the skill of these models.

LSTM is an important part of the CNN-LSTM framework and provides vector characteristics based on historical information.

The research study checked the effectiveness of the proposed CNN-LSTM-based deep learning model against the benchmarks: the RNN and LSTM prediction methods are the widely used deep learning models.

On the Hyper-order of Analytic Solutions of Linear Differential Equations near a Finite Singular Point

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Abstract: In this work, we study the hyper-order of analytic solutions of linear differential equations with analytic coefficients having the same order near a finite singular point. We improve previous results given by Cherief and Hamouda [3]. We also consider the nonhomogeneous linear differential equations.

Keywords: Linear differential equations, hyper-order, a finite singular point, Nevanlinna theory.

2020 Mathematics Subject Classification: Primary 34M10, 30D35.

Introduction

Throughout this study, we assume that the reader is familiar with the fundamental results and the standard notations of the Nevanlinna value distribution theory of meromorphic function in the complex plane $\mathbb{C}$ and in the unit disc $D = \{z \in \mathbb{C} : |z| < 1\}$ (see [1, 4, 6]). We denote respectively the order and the hyper-order of growth of a meromorphic function f , by $\sigma(f)$ and $\sigma_2(f)$.

Many authors as in ([3, 4, 2]) have studied the growth of solutions of linear differential equations near a finite singular point. They have investigated different forms of linear differential equations with analytic coefficients near a finite singular point by using adapted notions and properties of Nevanlinna theory. In this research, we continue this investigation near a finite singular point.

First, we recall the appropriate definitions. Set $\overline{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$ and suppose that f is meromorphic in $\overline{\mathbb{C}} \setminus \{z_0\}$, where $z_0 \in \mathbb{C}$. Define the counting function near z_0 by

$$N_{z_0}(r, f) = - \int_{\infty}^r \frac{n(t, f) - n(\infty, f)}{t} dt - n(\infty, f) \log r,$$

where $n(t, f)$ counts the number of poles of f in the region $\{z \in \mathbb{C} : t \leq |z_0 - z| \} \cup \{\infty\}$, each pole according to its multiplicity, and the proximity function by

$$m_{z_0}(r, f) = \frac{1}{2\pi} \int_0^{2\pi} \log^+ |f(z_0 - re^{i\phi})| d\phi.$$

The characteristic function of f is defined in the usual manner by

$$T_{z_0}(r, f) = m_{z_0}(r, f) + N_{z_0}(r, f).$$

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In addition, the order of a meromorphic function f near z_0 is defined by

$$\sigma_T(f, z_0) = \limsup_{r \rightarrow 0} \frac{\log^+ T_{z_0}(r, f)}{-\log r}.$$

For an analytic function f in $\overline{\mathbb{C}} \setminus \{z_0\}$, we have also the definition

$$\sigma_M(f, z_0) = \limsup_{r \rightarrow 0} \frac{\log^+ \log^+ M_{z_0}(r, f)}{-\log r}.$$

where $M_{z_0}(r, f) = \max_{|z_0 - z| = r} |f(z)|$.

When the order is infinite, we introduce the notion of hyper-order near z_0 that is defined as follows :

$$\begin{aligned} \sigma_{2,T}(f, z_0) &= \limsup_{r \rightarrow 0} \frac{\log^+ \log^+ T_{z_0}(r, f)}{-\log r}, \\ \sigma_{2,M}(f, z_0) &= \limsup_{r \rightarrow 0} \frac{\log^+ \log^+ \log^+ M_{z_0}(r, f)}{-\log r}. \end{aligned}$$

Main results

Recently, Cherief and Hamouda [3] proved the following two results :

Theorem 84 ([3]). *Let $n \in \mathbb{N} \setminus \{0\}$, $k \geq 2$ be an integer and $A_j(z)$ ($j = 0, \dots, k-1$) be analytic functions in $\overline{\mathbb{C}} \setminus \{z_0\}$, such that $\sigma(A_j, z_0) < n$ and $A_0(z) \not\equiv 0$. If a_j ($j = 0, \dots, k-1$) are distinct complex numbers, then every solution $f \not\equiv 0$ of the differential equation*

$$f^{(k)} + A_{k-1}(z) \exp \left\{ \frac{a_{k-1}}{(z_0 - z)^n} \right\} f^{(k-1)} + \dots + A_0(z) \exp \left\{ \frac{a_0}{(z_0 - z)^n} \right\} f = 0 \quad (270)$$

that is analytic in $\overline{\mathbb{C}} \setminus \{z_0\}$, satisfies $\sigma(f, z_0) = +\infty$.

Theorem 85 ([3]). *Let $n \in \mathbb{N} \setminus \{0\}$, $k \geq 2$ be an integer and $A_j(z)$ ($j = 0, \dots, k-1$) be analytic functions in $\overline{\mathbb{C}} \setminus \{z_0\}$, such that $\sigma(A_j, z_0) < n$ and $A_0(z) \not\equiv 0$. Let a_j ($j = 0, \dots, k-1$) be complex constants. Suppose that there exist nonzero numbers a_s and a_l , such that $0 < s < l \leq k-1$, $a_s = |a_s|e^{i\theta_s}$, $a_l = |a_l|e^{i\theta_l}$, $\theta_s, \theta_l \in [0, 2\pi)$, $\theta_s \neq \theta_l$. $A_s A_l \not\equiv 0$ and for $j \neq s, l$, a_j satisfies either $a_j = d_j a_s$ ($0 < d_j < 1$) or $a_j = d_j a_l$ ($0 < d_j < 1$). Then every solution $f \not\equiv 0$ of equation (270) that is analytic in $\overline{\mathbb{C}} \setminus \{z_0\}$, satisfies $\sigma(f, z_0) = +\infty$.*

We continue to consider these above theorems and investigate the hyper-order of analytic solutions of equation (270). We also consider the nonhomogeneous equation.

Theorem 86. *Let $n \in \mathbb{N} \setminus \{0\}$, $k \geq 2$ be an integer and $A_j(z), a_j$ ($j = 0, \dots, k-1$) satisfy the additional hypotheses of Theorem 2.1. Then every solution f of equation (270) that is analytic in $\overline{\mathbb{C}} \setminus \{z_0\}$ satisfies $\sigma_2(f, z_0) = n$, where z_0 is an essential singular point for f .*

Example 8. *Consider the differential equation*

$$f''' + \frac{3}{z} \left(2 + \frac{1}{z}\right) f'' - \frac{1}{z^4} \exp \left\{ \frac{2}{z} \right\} f' - \frac{2}{z^4} \left(3 + \frac{3}{z} + \frac{1}{z^2}\right) \exp \left\{ \frac{1}{z} \right\} f = 0. \quad (271)$$

Obviously, the conditions of Theorem 1.4 are satisfied. Hence every solution f of equation (271) that is analytic in $\overline{\mathbb{C}} \setminus \{0\}$ satisfies $\sigma_2(f, 0) = 1$, where 0 is an essential singular point for f .

Remark that the function $f(z) = \exp\{\exp(\frac{1}{z})\}$ is a solution of equation (271) that is analytic in $\overline{\mathbb{C}} \setminus \{0\}$ with $\sigma_2(f, 0) = 1$.

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Theorem 87. Let $n \in \mathbb{N} \setminus \{0\}$, $k \geq 2$ be an integer and $A_j(z), a_j (j = 0, \dots, k-1)$ satisfy the additional hypotheses of Theorem 2.2. Then every solution f of equation (270) that is analytic in $\overline{\mathbb{C}} \setminus \{z_0\}$, satisfies $\sigma_2(f, z_0) = n$, where z_0 is an essential singular point for f .

Theorem 88. Let $n \in \mathbb{N} \setminus \{0\}$, $k \geq 2$ be an integer and $A_j(z), a_j (j = 0, \dots, k-1)$ satisfy hypotheses of Theorem 2.1 or those of Theorem 2.2. Let $F \not\equiv 0$ be analytic function in $\overline{\mathbb{C}} \setminus \{z_0\}$ of order $\sigma = \sigma(F, z_0) < n$. Then every solution f of equation

$$f^{(k)} + A_{k-1}(z) \exp \left\{ \frac{a_{k-1}}{(z_0 - z)^n} \right\} f^{(k-1)} + \dots + A_0(z) \exp \left\{ \frac{a_0}{(z_0 - z)^n} \right\} f = F \quad (272)$$

that is analytic in $\overline{\mathbb{C}} \setminus \{z_0\}$ satisfies $\sigma(f, z_0) = +\infty$ and $\sigma_2(f, z_0) = n$, where z_0 is an essential singular point for f , with at most one exceptional analytic solution f_0 of finite order in $\overline{\mathbb{C}} \setminus \{z_0\}$.

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Boundary stabilization for weakly coupled degenerate wave equations under fractional damping

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Abstract: In this paper, we consider the well-posedness and stability of a one-dimensional system of degenerate wave equations coupled via zero order terms with one boundary fractional damping acting on one end only. We prove optimal polynomial energy decay rate of order $1/t^{(3-\tau)}$. The method is based on the frequency domain approach combined with multiplier technique .

Keywords: System of coupled degenerate wave equations, Fractional boundary damping, Strong asymptotic stability, Bessel functions, Optimal polynomial decay.

2020 Mathematics Subject Classification: 35B40, 35L80, 74D05, 93D15.

Introduction

In this communication , we investigate the existence and energy decay rate of a system of coupled degenerate wave equations with only one fractional boundary damping. This system defined on $(0, 1) \times (0, +\infty)$ takes the following form

$$(P) \quad \left\{ \begin{array}{ll} u_{tt}(x, t) - (a(x)u_x)_x(x, t) + \alpha v = 0 & \text{in } (0, 1) \times (0, +\infty), \\ v_{tt}(x, t) - (a(x)v_x)_x(x, t) + \alpha u = 0 & \text{in } (0, 1) \times (0, +\infty), \\ \left\{ \begin{array}{ll} u(0, t) = 0 & \text{if } 0 \leq m_a < 1 \\ (au_x)(0, t) = 0 & \text{if } 1 \leq m_a < 2 \end{array} \right. & \text{in } (0, +\infty), \\ \left\{ \begin{array}{ll} v(0, t) = 0 & \text{if } 0 \leq m_a < 1 \\ (av_x)(0, t) = 0 & \text{if } 1 \leq m_a < 2 \end{array} \right. & \text{in } (0, +\infty), \\ v(1, t) = 0 & \text{for } t \in (0, +\infty), \\ \beta u(1, t) + (au_x)(1, t) = -\rho \partial_t^{\tau, \omega} u(1, t) & \text{in } (0, +\infty), \\ u(x, 0) = u_0(x), u_t(x, 0) = u_1(x), v(x, 0) = v_0(x), v_t(x, 0) = v_1(x) & \text{for } x \in (0, 1), \end{array} \right.$$

where $a \in C([0, 1]) \cap C^1(]0, 1])$ is positive on $]0, 1]$ but vanishes at zero, α denote the coupling parameter, which is assumed to be real and small enough, $\beta \geq 0$ and $\rho > 0$. The notation $\partial_t^{\tau, \omega}$

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stands for the generalized Caputo's fractional derivative of order τ , ($0 < \tau < 1$), with respect to the time variable (see [11]). It is defined as follows

$$\partial_t^{\tau,\omega} g(t) = \begin{cases} g_t & \text{for } \tau = 1, \omega \geq 0, \\ \frac{1}{\Gamma(1-\tau)} \int_0^t (t-s)^{-\tau} e^{-\omega(t-s)} \frac{dg}{ds}(s) ds & \text{for } 0 < \tau < 1, \omega \geq 0. \end{cases}$$

The initial data (u_0, u_1, v_0, v_1) belong to a suitable function space.

Main results

We state a well-posedness result for problem (P1) by means of the semigroup theory . Next, we show the lack of exponential stability by spectral analysis and we obtain an optimal polynomial energy decay rate using multipliers method , frequency domain approach and Borichev-Tomilov theorem.

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Enhancing Image Quality Using Nonlinear Differential Equation-Based Filters

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Abstract: In the realm of image processing, the journey of an image from acquisition to interpretation is riddled with challenges, with noise posing as a primary obstacle. Whether it arises during acquisition, transmission, or storage, noise takes on multiple forms, from random pixel variations to electronic interference, diminishing image clarity and hindering interpretation. To combat this, filtering techniques come into play, aiming to suppress noise while safeguarding crucial image features for accurate analysis. Among these techniques, nonlinear differential equation-based filters have risen to prominence for their efficacy in enhancing image quality.

Keywords: Nonlinear Differential Equation-Based Filters, Image Enhancement, Nonlinear filters, Anisotropic diffusion model, Total Variation (TV) filter.

2020 Mathematics Subject Classification: 58J90.

Introduction

In the realm of image processing, the journey of an image from acquisition to interpretation is fraught with challenges. One of the primary obstacles encountered during this journey is noise, which can corrupt images during acquisition, transmission, and storage. This noise manifests in various forms, such as random variations in pixel values, interference from electronic devices, or imperfections in the imaging sensors. As a result, the images may suffer from reduced clarity, making interpretation and reading difficult. To mitigate the impact of noise and enhance image quality, various filtering techniques are employed. These filters aim to suppress noise while preserving essential image features to ensure accurate interpretation and analysis. Among the diverse array of filtering methods available, nonlinear differential equation-based filters have gained prominence for their effectiveness in improving image quality.

Understanding Image Noise

Before delving into the realm of filtering techniques, it is crucial to understand the nature of image noise and its implications. Image noise refers to random variations in pixel values that obscure the true underlying information contained within an image. This noise can arise from a multitude of sources, including environmental factors, equipment limitations, and transmission errors.

One of the primary challenges posed by image noise is its detrimental effect on image quality.

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Noise can introduce artifacts and distortions, making it difficult to discern relevant details within the image. Additionally, noise can degrade the performance of image analysis algorithms, leading to erroneous conclusions and interpretations.

The Role of Filtering in Image Enhancement

To address the adverse effects of noise on image quality, filtering techniques are employed to remove or suppress unwanted noise while preserving the essential features of the image. Filtering operates by applying mathematical operations to the pixel values of the image, with the goal of smoothing out noise without significantly altering the underlying content.

Various types of filters exist, ranging from simple linear filters to more complex nonlinear filters. Linear filters, such as the Gaussian filter and the mean filter, are widely used for their simplicity and computational efficiency. However, these filters may not always be effective in scenarios where noise exhibits nonlinear characteristics or when preserving fine image details is paramount. Nonlinear filters offer an alternative approach to noise reduction, leveraging more sophisticated mathematical models to better adapt to the complex nature of image noise. Among these nonlinear filters, those based on differential equations have shown promise in effectively suppressing noise while preserving image details. By modeling the image as a dynamic system governed by differential equations, these filters can achieve superior noise reduction performance compared to their linear counterparts.

Nonlinear Differential Equation-Based Filters

One class of nonlinear filters that has garnered significant attention is filters based on differential equations. These filters operate by formulating the process of image denoising as a dynamic system governed by nonlinear partial differential equations (PDEs). By solving these PDEs iteratively, the filters are able to remove noise while preserving important image features. One notable example of a nonlinear differential equation-based filter is the Total Variation (TV) filter. The TV filter is based on the principle of minimizing the total variation of the image, which represents the sum of the absolute differences between neighboring pixel values. By penalizing rapid intensity variations, the TV filter effectively smooths out noise while preserving edges and fine details in the image.

Another popular nonlinear filter is the Perona-Malik (PM) filter, which is based on an anisotropic diffusion model inspired by heat diffusion processes. The PM filter employs a nonlinear diffusion equation to selectively smooth regions of low contrast while preserving edges and boundaries. This adaptive smoothing behavior makes the PM filter particularly well-suited for denoising images with complex textures and structures.

Advantages and Challenges of Nonlinear Differential Equation-Based Filters

Nonlinear differential equation-based filters offer several advantages over traditional linear filters in terms of noise reduction and image preservation. By capturing the nonlinear relationships inherent in image data, these filters can achieve superior denoising performance while preserving important image features such as edges, textures, and structures. Additionally, the adaptability of these filters allows them to effectively handle a wide range of noise types and intensities, making them versatile tools for image enhancement.

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However, despite their effectiveness, nonlinear differential equation-based filters are not without their challenges. One of the primary concerns is the computational complexity associated with solving nonlinear PDEs, which can result in increased processing times, especially for large-scale images or real-time applications. Additionally, the selection of appropriate parameters and regularization terms for the filters can be nontrivial and may require careful tuning to achieve optimal results.

Conclusion

In conclusion, image noise poses a significant challenge in the acquisition, transmission, and storage of images, leading to degradation in image quality and hindering accurate interpretation and analysis. To address this challenge, various filtering techniques are employed to suppress noise while preserving essential image features. Among these techniques, nonlinear differential equation-based filters have emerged as powerful tools for enhancing image quality.

By formulating the process of image denoising as a dynamic system governed by nonlinear partial differential equations, these filters are able to effectively remove noise while preserving important image details. Examples such as the Total Variation and Perona-Malik filters demonstrate the efficacy of this approach in achieving superior denoising performance.

While nonlinear differential equation-based filters offer significant advantages in terms of noise reduction and image preservation, they also present challenges such as computational complexity and parameter selection. Despite these challenges, the continued research and development of these filters hold promise for further advancements in image enhancement and analysis.

Therefore, the utilization of nonlinear differential equation-based filters represents a significant step forward in the quest to improve image quality and enable more accurate interpretation and analysis in various fields ranging from medical imaging to remote sensing and beyond.

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Event-driven model reference tracking control of switched systems in networked environments

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Abstract: This paper considers the model reference state tracking control problem for a class of networked switched systems under an event-triggering data transmission scheme. Firstly, a novel tracking delay model is developed, incorporating network-induced delays along with the proposed triggering scheme. Some linear matrix inequality (LMI)-based conditions are derived for the co-design problem of the tracking controller and the triggering matrix using the average dwell time (ADT) switching method and an appropriate multiple Lyapunov–Krasovskii function (MLKF).

Keywords: networked switched systems, Event-triggered control, tracking control, linear matrix inequality.

2020 Mathematics Subject Classification: Systems theory; control, Networked control.

Introduction

Networked switched systems (NSS) constitute a class of systems where the plant is a switched system (SS) [1], with control tasks transmitted over a communication network, known as networked control systems (NCS) [2]. NSS have been attracting research interest due to their prevalence in practical applications [3, 4], offering various advantages, such as fast switching and low maintenance cost. However, controlling NSS inevitably introduces transmission delays and packet loss, which can impact system performance.

Most research employs a time-triggered control scheme, potentially leading to a waste of the network energy because of the important amount of unnecessary transmitted data. An alternative approach, the event-triggering control (ETC) scheme, has been developed to select only significant sampled packets [5]. Thus, the control strategy based on this approach can reduce communication resources while ensuring the system's performance. Over the last decade, many scholars have explored the application of event-triggered schemes in NSS [6, 7].

In recent years, tracking control has been widely adopted in the control area, including flight control [8]. The main objective of model reference tracking control is to drive the controlled plant to track a prescribed reference model under a well-designed controller. Several valuable results concerning the event-triggered tracking problem have been obtained for SS and NCS [9, 10].

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For NSS, it is more challenging to investigate the effects of the hybridization of SS and NCS on tracking performance. It is well known that designing event-triggered tracking control laws for NSS is more difficult compared to stabilization [11].

Main results

Consider the switched system defined by the following model:

$$\dot{x}(t) = A_i x(t) + B_i u(t) + D_i \omega(t) \quad (273)$$

The main goal is to design an event-based controller (276) to track the following reference model:

$$\dot{x}_r(t) = A_r x_r(t) + B_r r(t) \quad (274)$$

where $x(t) \in \mathbb{R}^n$ is the state, $x_r(t) \in \mathbb{R}^n$ is the model reference state, $u(t) \in \mathbb{R}^m$ is the control input, $r(t) \in \mathbb{R}^m$ is the bounded reference input, and $w(t) \in \mathbb{R}^m$ is the external disturbance. The function $\sigma : [t_0, \infty[\rightarrow \mathbb{I} = \{1, 2, \dots, N\}$ is the switching signal with N being the number of subsystems. For $\sigma(t) = i \in \mathbb{I}$; A_i, B_i, A_r, B_r , and D_i are constant matrices of appropriate dimensions.

By defining $\bar{x}(t) = x(t) - x_r(t)$ as the tracking error, $\bar{x}(t)$ is sampled periodically as $\bar{x}(vh), v \in \mathbb{N}$ with $h > 0$ being the sampling period. The triggered data $\bar{x}(t_k h)$ are transmitted under the event-triggered scheme (??) over the network subject to delays $d_k < d_M$ at time instants $t_k h, k \in \mathbb{N}, \{t_0, t_1, t_2, \dots, t_k\} \subseteq \mathbb{N}$.

$$t_{k+1} h = t_k h + \min_{v \in \mathbb{N}} \left\{ v h \mid e_k^T(t) e_k(t) \geq \bar{x}^T(t_k h + v h) \Omega_i \bar{x}(t_k h + v h) \right\} \quad (275)$$

where $\Omega_i > 0$ are the matrices threshold to be designed, $e_k(t) = \bar{x}(t_k h) - \bar{x}(t_k h + v h)$ is the measurement error between the current sampled data $\bar{x}(t_k h + v h)$ and the last transmitted data $\bar{x}(t_k h), t \in [t_k h + d_k, t_{k+1} h + d_{k+1}[$. An event-based controller is proposed to control the switched system (273) where K_i is the feedback gain of appropriate dimension:

$$u(t) = K_i \bar{x}(t_k h) \quad (276)$$

By combining equations (273)-(276), we get the following system dynamics:

$$\begin{cases} \dot{\bar{x}}(t) = A_i \bar{x}(t) + B_i K_i \bar{x}(t - d(t)) + B_i K_i e_k(t) + \epsilon(t) \\ \epsilon(t) = (A_i - A_r) x_r(t) - B_r r(t) + D_i \omega \\ x(s) = \phi(s), \quad s \in [-d_M, 0] \end{cases} \quad (277)$$

where $\phi(s)$ is a continuous initial function on $[-d_M, 0]$.

Theorem 89. Consider the closed-loop networked switched system (277), for given positive parameters d_M, h, λ , and $\mu \geq 1$, if for any consecutive $i, j \in \mathbb{I}, i \neq j$, there exist symmetric positive-definite matrices

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$X_i > 0, Z_i > 0, W_i > 0$, and matrices Y_i of appropriate dimensions such that:

$$\Theta_i = \begin{bmatrix} A_i X_i + X_i A_i^T + \lambda X_i & B_i Y_i & 0 & B_i Y_i & X_i & \sqrt{d_M} X_i & X_i \\ \star & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & -d_M Z_i & 0 & 0 & 0 & 0 \\ \star & \star & \star & -2X_i + I & 0 & 0 & 0 \\ \star & \star & \star & \star & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & -Z_i & 0 \\ \star & \star & \star & \star & \star & \star & -W_i \end{bmatrix} < 0, \quad (278)$$

$$X_j \leq \mu X_i, \quad Z_j \leq \mu Z_i \quad (279)$$

then the switched delayed tracking error system (277) is exponentially stable under the event-triggered controller (276), and the time-dependent switching rule satisfying the ADT condition:

$$\tau_a > \frac{\ln \mu}{\lambda} \quad (280)$$

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New adaptative projection method for generalized variational inequalities problem

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Abstract: We propose a new projection method (extragradient method) for generalized variational inequalities problem. Our method is well-defined and is proven to be globally convergent to a solution of the problem under the assumptions that the multivalued mapping is continuous and pseudomonotone (where the condition is strictly weaker than the pseudomonotonicity). To support our results, we provide some numerical tests and a comparison has been carried out with recent works.

Keywords : Projection method, Generalized variational inequalities problem, multivalued mapping, Pseudomonotone mapping.

2020 Mathematics Subject Classification: 90C25, 90C33, 65K05.

Introduction

Let C be a nonempty closed and convex set in $\mathbb{R}^n$ and $F : \mathbb{R}^n \mapsto 2^{\mathbb{R}^n}$ be a set valued mapping with nonempty compact and convex values. We consider the following generalized variational inequality problems abbreviated $GVIP(F, C)$ which consists to find a vector $\bar{x} \in C$ and a vector $t \in F(\bar{x})$ such that:

$$\langle t, x - \bar{x} \rangle \geq 0, \text{ for all } x \in C, \quad (1.1)$$

where $\langle \cdot, \cdot \rangle$ and $\|\cdot\|$ denote the usual inner product and norm respectively in $\mathbb{R}^n$.

Throughout this work, we denote the solutions set of $GVIP(F, C)$ by T .

If F is single-valued, then $GVIP(F, C)$ reduces to the variational inequality (in short $VIP(F, C)$), i.e., find $\bar{x} \in C$ such that:

$$\langle F(\bar{x}), x - \bar{x} \rangle \geq 0, \text{ for all } x \in C. \quad (1.2)$$

In this work, we will focus the topic of projection methods. Thus, we should recall some development process of this type of methods where the simplest version among these methods for solving $VIP(F, C)$ is based on its equivalent fixed point reformulation $\bar{x} = P_C(\bar{x} - \alpha F(\bar{x}))$. In order to get the convergence, this first projection method requires very strong conditions, i.e., the strong monotonicity and Lipschitz continuity of the operator F . In 1976, Korpelevich constructed a double projection method known by extragradient method such that the strong monotonicity of F is relaxed to its monotonicity. Then in 1997, Iusem and Svaiter introduced a variant of Korpelevich's

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method by using Armijo linear search procedure such that the Lipschitz continuity of F is further relaxed to the continuity. Later, some studies were completed, in particular, Solodov and Svaiter, Wang & al and consequently, many developments were brought to improve the algorithmic behavior of this type of methods. In the same form of these last algorithms which require only two projections onto C at each iteration and under the same convergence hypotheses of continuity and pseudomonotony of F , Grar and Benterki proposed an algorithm with a new displacement stepsize which must satisfy a certain condition, this last ensures a faster convergence towards a solution, and our preliminary and comparative computational experience using this algorithm on many classes of $VIP(F, C)$ were very encouraging.

If F is maximal monotone operator, Iusem and Pere suggested an extragradient method to solve $GVIP(F, C)$. In 2009, Li and He devised a projection method to solve $GVIP(F, C)$, where for taking an element in $F(x)$, they have to solve another single-valued variational inequality. Therefore the price at each iteration is somewhat expensive. To overcome this difficulty, Fang and He modified the algorithms and it is arbitrary to take an element in $F(x)$ where the convergence is proved under the same assumptions.

We suggest in this work a new projection method for $GVIP(F, C)$ and under very mild assumptions, we prove that the sequence generated by the proposed algorithm is globally convergent to a solution of the problem.

Main results

Based on the framework of last projection methods, we have proposed a new method to solve the generalized variational inequalities. Under very weak conditions of the underling operator, we have established the theoretical results to prove the global convergence of the associated algorithm. Further more, we have also presented some preliminary numerical tests which show and confirm the efficiency of our algorithm for this class. As we can see from the obtained results, there is a significant reduction in iterations number and in computation time comparing with some recents algorithms. This expresses the relevant role played by the imposed condition in the numerical performance of our algorithm.

As future perspective, we will try to propose new procedures to determine the stepsize λ_k in order improve more the convergence of the proposed algorithm.

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A novel topological methodology for probing the existence of solutions in nonlinear fractional impulsive wave equations

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Abstract:

This study delves into a category of ψ Caputo fractional impulsive wave equations and advances upon prior findings. Employing a novel topological method, it establishes the existence of classical solutions characterized by complex arguments arising from impulsive disturbances. Notably, the scarcity of results pertaining to these impulsive equations underscores the significance of this contribution.

Keywords: ψ -Caputo fractional derivative type; sum of operators; classical solution ; boundary value problem; fixed point theorems.

Analytical solution of time - fractional Navier – Stokes equation by use of HPLTM

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Abstract: We introduce an analytical method, namely the Homotopy perturbation Laplace transforms method (HPLTM), which is a combination of the Homotopy perturbation Laplace transforms method (HPLTM). A new application of the HPLTM is presented for the solution of the fractional order fractional Navier – Stokes equation. The accuracy and efficiency of the proposed method is verified through with exact solutions extracted. However, the plan has proven to be a very dependable, strong and efficient method for resolving a variety of issues in the sciences and engineering. Besides, its strength is that it requires less calculation in contrast to the other techniques. It will be covered in more detail in future applied work in the fields of fluid mechanics.

Keywords: Fractional Navier–Stokes equation; Riemann–Liouville fractional integral; HPLTM.

Main results

Applications : time-fractional Navier–Stokes equations If the value of P is known, then all the values of $g_1 = -\frac{1}{P} \frac{\partial P}{\partial x}$, $g_2 = \frac{1}{P} \frac{\partial P}{\partial y}$ can be determined. Application 1 From Eq. (28), 2-dimensional NS equation of fractional order with $g_1 = -g_2 = g$ may be written as:

$$\begin{cases} D_t^\alpha U(x, y, t) + U \frac{\partial U}{\partial x} + V \frac{\partial P}{\partial y} = g + \rho_0 \left(\frac{\partial^2 U}{\partial x^2} + V \frac{\partial^2 P}{\partial y^2} \right), \\ D_t^\alpha V(x, y, t) + U \frac{\partial U}{\partial x} + V \frac{\partial P}{\partial y} = -g + \rho_0 \left(\frac{\partial^2 U}{\partial x^2} + V \frac{\partial^2 P}{\partial y^2} \right), \\ \frac{\partial U}{\partial x} + \frac{\partial P}{\partial y} = 0, \end{cases} \quad \Omega \times (0, T), 0 < \alpha \leq 1 \quad (29)$$

with IC: $U(x, y, 0) = -\cos(x + y)$, $V(x, y, 0) = \cos(x + y)$.

Thus, the solutions $U(x, y, t)$ and $V(x, y, t)$ are written in the form of:

$$\begin{cases} U(x, y, t) = U_0(x, y, t) + U_1(x, y, t) + U_2(x, y, t) + U_3(x, y, t) + \dots \\ \quad = -\cos(x + y)e^{-2\rho_0 t^\alpha} \\ V(x, y, t) = V_0(x, y, t) + V_1(x, y, t) + V_2(x, y, t) + V_3(x, y, t) + \dots \\ \quad = \cos(x + y)e^{-2\rho_0 t^\alpha} - g \frac{t^\alpha}{\Gamma(\alpha+1)}. \end{cases} \quad (47)$$

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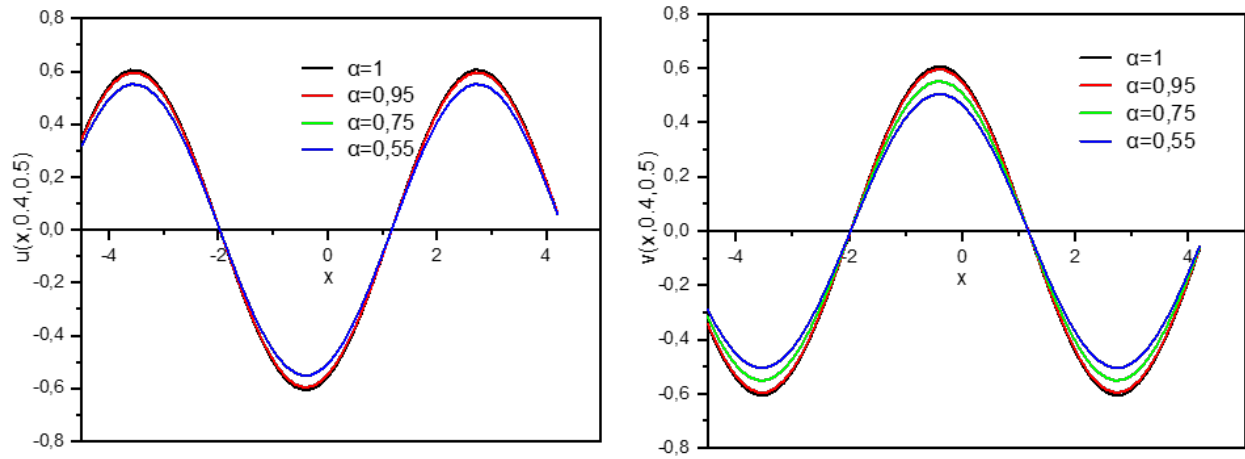


Figure 8: Plots solution application 1 using Eqs. (47) With $\rho_0 = 0.5, \alpha_0 = 0.14, y = 0.4$ and $x = -5 : 0.1 : 5, t = 0.5, g = 0, \alpha = 1, 0.95, 0.75, 0.55$

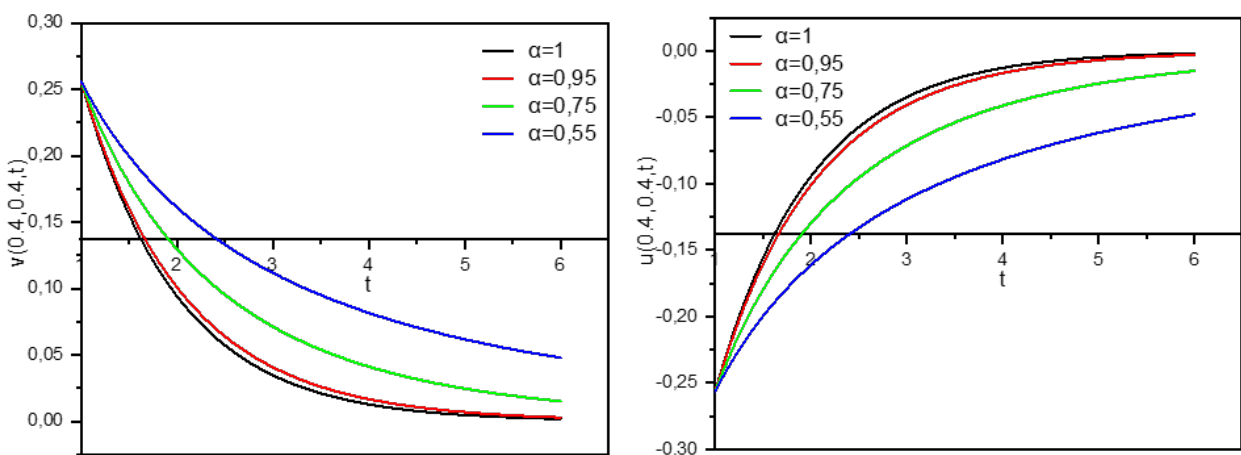


Figure 9: Plots solution application 1 using Eqs. (47) With $\rho_0 = 0.5, \alpha_0 = 0.14, x = y = 0.4$ and $t = 1 : 0.1 : 6, g = 0, \alpha = 1, 0.95, 0.75, 0.55$

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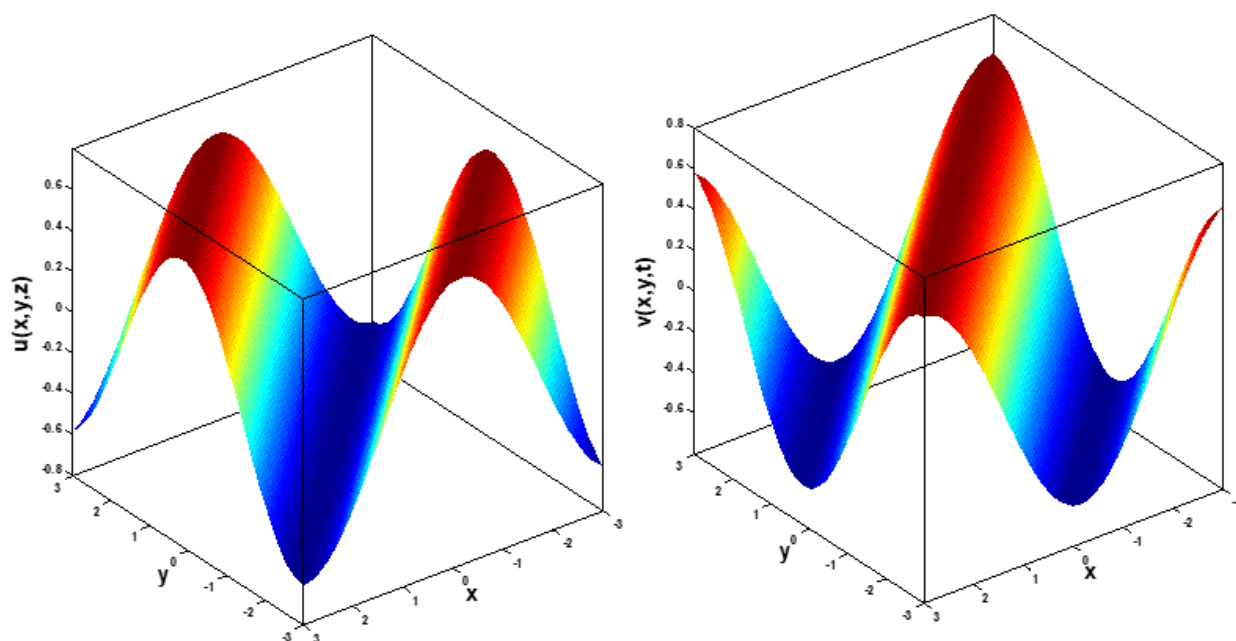


Figure 10: Plots solution application 1 using Eqs. (47) With $\rho_0 = 0.5, \alpha_0 = 0.14, x = y = -3 : 0.1 : 3, t = 0.5, g = 0, \alpha = 1$

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Optimal Control for Marguerre-von Kármán Equations

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Abstract: The classical Marguerre-von kármán equations are two-dimensional equations for a nonlinearly elastic shallow shell subjected to boundary conditions analogous to those of a von kármán plate. They were initially proposed by Marguerre and von kármán and Tsien , and later justified by Ciarlet and Paumier by means of a formal asymptotic analysis. In this paper we prove the existence of an optimal control for the cost. Then, by proving the *Frèchet* differentiability of the state we establish necessary optimality condition for the optimal control corresponding to the cost. Further, we study the uniqueness of he optimal controls, refering our study to I.Bock and B.Rao.

Keywords:Non-linear elasticity; stationary Marguerre-von kármán's equations; optimal control.

2020 Mathematics Subject Classification: Primary 47H10; 54H25; 54C30; 47H09.

Introduction

Let $\omega \in \mathbb{R}^2$ be a bounded domain with smooth boundary $\partial\omega$. We consider the Von Kármán equations for a thin elastic plate by means the right hand side of the equilibrium equation is considered, where the second side of the first equation depends to $\lambda\Delta$ operator.

In this paper, our system may be described as follows: Let $\omega \subset \mathbb{R}^2$ be a bounded domain with smooth boundary $\partial\omega$. We consider the Marguerre-Von Kármán of the shalow shell model with Dirichlet boundary conditions

$$\begin{cases} \Delta^2 \zeta = [\varphi, \zeta] + [\varphi, \theta] + f & x \in \omega \\ \Delta^2 \varphi = -[\zeta, \zeta] - 2[\zeta, \theta] & x \in \omega \end{cases} \quad (281)$$

where

$$\begin{cases} \zeta = \frac{\partial \zeta}{\partial \nu} = 0 & x \in \partial\omega \\ \varphi = \varphi_0, \quad \frac{\partial \varphi}{\partial \nu} = \varphi_1 & x \in \partial\omega \end{cases} \quad (282)$$

Next, we use the reduced cubic form for the Marguerre-von Kármán equations, following the recent definitions with an unknown changes $\tilde{\zeta} = u = \zeta + \theta$ and $\tilde{F} = F + \theta$. Finally, we define the energy given by Ciarlet $J(v) = \mathcal{J}(u(v)) + j(v)$ to study the optimal control. Moreover, in this paper, we discuss the existence and uniqueness of optimal control.

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We show *Frèchet* differentiability of $\mathcal{J}$, j and of the state $x(\cdot)$. As a consequence, we prove uniqueness of optimal control, and necessary condition of optimality. This is another novelty of the paper.

Reduced Cubic Canonical Problem

Definition 33. The cubic operator is C . First, we let the bilinear and linear continuous operators are respectively B , and L_2, L_1

$$B : H^2(\omega) \times H^2(\omega) \longrightarrow H_0^2(\omega)$$

The canonical Marguerre-von Kármán equations are reduced to a cubic operator equation, such that the pair $(\tilde{\zeta}, \lambda) \in (H_0^2(\omega), \mathbb{R})$ satisfies the continuous operator equation:

$$\tilde{\zeta} - \lambda L_2(\tilde{\zeta}) - L_1(\tilde{\zeta}) + C(\tilde{\zeta}) - \tilde{F} = 0 \quad (283)$$

B be a bilinear operator, unique solution of $\Delta^2 B(\xi, \eta) = [\xi, \eta]$. Then define the operator $C : H_0^2(\omega) \longrightarrow H_0^2(\omega)$, written by $C(\eta) = B(B(\eta, \eta), \eta)$, and we define the linear operators $L_2, L_1 : H_0^2(\omega) \longrightarrow H_0^2(\omega)$ by letting $\lambda L_2(\eta) = B(\theta_0, \eta)$, and $L_1(\eta) = B(\chi, \eta) = B(B(\theta, \eta), \theta)$. Let $\chi \in H^2(\omega)$ is the unique solution of the boundary value problem

$$\Delta^2 \chi = 0 \text{ and } \chi = \varphi_0, \frac{\partial \chi}{\partial \nu} = \varphi_1 \text{ in } \partial\omega, \text{ Where } \chi = B(\theta, \theta), \text{ Therefore, } \varphi = \chi - [\tilde{\zeta}, \tilde{\zeta}].$$

The Variational Problem equation

Definition 34. The canonical Marguerre-von Kármán equations are reduced to a cubic operator equation, such that the pair $(\tilde{\zeta}, \lambda) \in (H_0^2(\omega), \mathbb{R})$ satisfies the variational equation:

$$\left\{ \begin{array}{ll} \text{Find } (\tilde{\zeta}, \lambda) \in (H_0^2(\omega), \mathbb{R}) & \text{such that,} \\ (\tilde{\zeta} - \lambda L_2(\tilde{\zeta}) - L_1(\tilde{\zeta}) + C(\tilde{\zeta}) - \tilde{F}, \eta)_{\Delta} = 0 & \forall \eta \in H_0^2(\omega) \end{array} \right. \quad (284)$$

Then $\forall \eta \in H_0^2(\omega)$, $\tilde{\zeta}$ it is a weak solution of problem (281)

Eigen theory for operators: L_1 and L_2

Theorem 90. The linear operators L_2 and L_1 are compact, self-adjoint and positive definite, then L_2 and L_1 have an infinite number of distinct positive eigenvalues, such that

$$\begin{aligned} 0 < \lambda_1 < \lambda_2 < \lambda_3 < \dots < \lambda_k < \dots \rightarrow +\infty \text{ for } L_2, \\ 0 < \alpha_1 < \alpha_2 < \alpha_3 < \dots < \alpha_k < \dots \rightarrow +\infty \text{ for } L_1. \end{aligned}$$

where

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$$\lambda_1 = \inf_{u \in H_0^2(\omega), \|u\|_{2,\omega} \neq 0} \frac{\|u\|_{2,\omega}^2}{(L_2 u, u)_\Delta}, \mu_1 = \frac{1}{\lambda_1} = \sup_{u \in H_0^2(\omega), \|u\|_{2,\omega} = 1} (L_2 u, u)_\Delta,$$

$$\text{then } \alpha_1 = \inf_{u \in H_0^2(\omega), \|u\|_{2,\omega} \neq 0} \frac{\|u\|_{2,\omega}^2}{(L_1 u, u)_\Delta}, \beta_1 = \frac{1}{\alpha_1} = \sup_{u \in H_0^2(\omega), \|u\|_{2,\omega} = 1} (L_1 u, u)_\Delta$$

Existence and Uniqueness of a Solution of the State Problem

Theorem 91. *Let*

$$\|Mv + \theta\|_{2,\omega} < \|B\|^{-1}, \quad (285)$$

$$\lambda < \lambda^*(\|Mv + \theta\|_{2,\omega}). \quad (286)$$

Where $\lambda^*(\|Mv + \theta\|_{2,\omega}) = \frac{\lambda_1}{\alpha_1} + \lambda_1(1 - \|B\|^{\frac{2}{3}} \|Mv + \theta\|_{2,\omega}^{\frac{2}{3}})$, Then there exists a unique solution $u(v) \in H_0^2(\omega)$ of the equation (283). Moreover $u(v)$ satisfies the estimate

$$\|u(v)\|_{2,\omega} \leq \tilde{K}_1(\lambda)^{-1} \|Mv + \theta\|_{2,\omega}, \quad (287)$$

where

$$\tilde{K}_1(\lambda) = 1 - \left(\frac{\lambda}{\lambda_1} + \frac{1}{\alpha_1} \right) \quad (288)$$

The Cost function and the minimization problem

Let the functional

$$\forall v \in H_0^2(\omega), \mathcal{I}(v) := \mathcal{J}(u(v)) + j(v), \quad (289)$$

$$\mathcal{I}(u) := \frac{1}{4}(C(u), u)_\Delta + \frac{1}{2}((I - \lambda L_2 - L_1)u, u)_\Delta + (Mv + \theta, u)_\Delta.$$

Theorem 92. *solving the reduced Marguerre-Von Karman equation is equivalent to finding all the stationnary points of the functional $\mathcal{I}$, that is, those ζ satisfy*

$$\zeta \in H_0^2(\omega) \text{ and } \mathcal{I}'(\zeta) = 0$$

Then, there exists at least one minimizer for the energy functional $\mathcal{I}$:

$$\zeta \in H_0^2(\omega) \text{ and } \mathcal{I}(\zeta) = \inf_{u \in H_0^2(\omega)} \mathcal{I}(u) \quad (290)$$

Existence of the optimal control

Assuming that $G(v) := \|v + \Delta^2 \theta\|_{0,\omega} - \frac{\alpha}{c} \|B\|^{-1} \tilde{K}_1(\lambda)^{\frac{3}{2}}, \alpha \in]0, 1[.$

Let us define the admissible set of controls

$$\mathcal{U}_{ad} = \left\{ v \in L^2(\omega) : G(v) \leq 0 \right\} = G^{-1}(]-\infty, 0]) ,$$

$$, \text{ when } \lambda < \lambda_1(1 - \frac{1}{\alpha_1}).$$

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Where $\mathcal{U}_{ad}$ is a closed and convex set.

Theorem 93. Let $\mathcal{J} : H_0^2(\omega) \longrightarrow \mathbb{R}$, $j : L^2(\omega) \longrightarrow \mathbb{R}$ be weakly lower semicontinuous. Then there exists a solution $u \in \mathcal{U}_{ad}$ of problem

$$\mathcal{I}(u) = \min_{v \in \mathcal{U}_{ad}} \mathcal{I}(v) \quad (291)$$

Uniqueness of the Control

Assuming that $\mathcal{J}$ and j are Fréchet differentiable and satisfying conditions

$$\langle \mathcal{J}'(u_1) - \mathcal{J}'(u_2), u_1 - u_2 \rangle_* \geq m \|u_1 - u_2\|_{2,\omega}, \forall m > 0, \forall u_1, u_2 \in H_0^2(\omega). \quad (292)$$

$$\langle j'(v_1) - j'(v_2), v_1 - v_2 \rangle_0 \geq N \|v_1 - v_2\|_{0,\omega}, \forall N > 0, \forall v_1, v_2 \in L^2(\omega). \quad (293)$$

If $v \in \mathcal{U}_{ad}$ is a solution of problem (291), then $\langle \mathcal{I}'(v), w - v \rangle_0 \geq 0, \forall w \in \mathcal{U}_{ad}$.

$$\|\mathcal{J}'(x)\|_* \leq a_0 \|x\|_{2,\omega} + a_1, \forall u \in H_0^2(\omega); a_0, a_1 > 0, \quad (294)$$

where $\|\cdot\|_* = \|\cdot\|_{\mathcal{L}(H_0^2(\omega), H_0^2(\omega))}$,

Theorem 94. Let the functionals $\mathcal{J}$, j be lower bounded, weakly lower semi-continuous with Fréchet derivatives satisfying the conditions (292), (293) and (294), if

$$\left[-m + (a_0 \alpha \|B\|^{-1} \tilde{k}_1(\lambda)^{\frac{1}{2}} + a_1) \cdot \frac{6\alpha}{1-\alpha^2} \tilde{k}_1(\lambda)^{-\frac{1}{2}} \|B\| \right] \cdot c^2 [\tilde{K}_1(\lambda)(1-\alpha^2)]^{-2} < N$$

where $\tilde{k}_1(\lambda) = 1 - (\frac{\lambda}{\lambda_1} + \frac{1}{\alpha_1})$, then there exists a unique solution u of the problem (290).

Necessary Condition of Optimality

$$\langle p, v - u \rangle + \langle R_2 j'(u), v - u \rangle \geq 0, \forall v \in \mathcal{U}_{ad}, \quad (295)$$

$$[I - \lambda L_2 - L_1 + C'(x(u))]p = R_1 \mathcal{J}'(x(u)), \quad (296)$$

$$x(u) - \lambda L_2 x(u) - L_1 x(u) + C(x(u)) = Mu + \theta, \quad (297)$$

The system (295), (296) and (297) is the optimality system of the problem (291). The problem (296) is the adjoint problem to the problem (297)

Conclusion

This study is concerned with optimal control problem for Marguerre-von Kármán equations. First, we establish the existence and uniqueness of non trivial solutions. Then we define the admissible set of control to study the existence and by the Fréchet differentiability of the state we

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get the uniqueness of the control. At the end by Riesz theorem, and the adjoint problem we establish the system of optimality to define the necessary condition of optimality. Note that, in the case $\theta \equiv 0$ in $\bar{\omega}$, we recover the optimal control problem for von Kármán equations.

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Solving Nonlinear Volterra Integral Equation Using Lucas Polynomials

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Abstract: For this work, the main idea is to approximate the solution of the Nonlinear Volterra integral equation (NLVIE for short) of the second kind numerically, Lucas Polynomials (LP) with different degrees were used for this purpose. First we derive the operational matrices of integration for Lucas polynomials, these matrices combined with collocation method, the NLVIE reduces to a system of algebraic equations with unknown LP coefficients. A comparison between numerical results obtained by this method with the exact solutions for giving examples is done in order to show the efficiency of the current method.

Keywords: Nonlinear Volterra Integral Equation, Lucas Polynomials, Error Analysis

2020 Mathematics Subject Classification: 45G10, 11B39, 65G20.

Introduction

Nonlinear integral equation is an important branch in mathematics, it arises in many applied areas which include engineering problems, such as mechanics, physics, astronomy, biology, economics, potential theory and electrostatics. The general form of these equation is giving by

$$\varphi(s) - \int_{\Omega} k(s, t, \varphi(t)) dt = f(s), \quad s, t \in \Omega \quad (298)$$

where the functions $f(s)$, and $k(s, t; \varphi)$ are given and continuous functions in Ω and $D(k) = \Omega \times \Omega \times I$, ($I \in \mathbb{R}$), respectively, the function $\varphi(t)$ is to be determined as continuous function $\Omega \rightarrow I$.

The existence of the solution can be guaranteed under the following conditions

- $k(s, t, \varphi(t)) : \Omega \times \Omega \times I \rightarrow \mathbb{R}$ is continuous and bounded in this domain
- $k(s, t; \varphi) : \Omega \times \Omega \times I \rightarrow \mathbb{R}$ is Lipschitzian for the third variable. In other words there exists $L > 0$ such that

$$|k(s, t, \varphi_1) - k(s, t, \varphi_2)| \leq L |\varphi_1 - \varphi_2| \quad s, t \in \Omega, \quad \varphi_1, \varphi_2 \in I.$$

In general, solving this kind of equations is very hard and sometimes not possible so many different methods are used to approximate solution of the nonlinear integral equations numerically. In this paper, we are interested to approximate the solution of the NLVE where the upper bound of the region for the integral part is variable using Lucas collocation method

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Main results

The proposed method gives a good approximation to the solution of the Volterra integral equation of the second kind according to the comparison between the obtained results and the exact solutions of some given examples
furthermore, the absolute error decreases when the degree of the polynomials increases

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Wellposedness and general stability result for MGT problem in unbounded domains

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Abstract: In this presentation, we are interested in a viscoelastic Moore–Gibson–Thompson equation with a type-II memory term and a relaxation function satisfying $g(t) \leq -\eta(t)g(t)$. By constructing appropriate Lyapunov functionals in the Fourier space, we establish a general decay estimate of the solution under the condition $\left(\beta - \frac{\gamma}{\alpha} - \frac{\rho}{2}\right) > 0$. We then give the decay rate of the L^2 -norm of the solution. We also give two examples to illustrate our theoretical results.

Keywords: Moore–Gibson–Thompson equation, Relaxation function, General decay, Fourier space .

2020 Mathematics Subject Classification: Primary 35B37 , 35L55 , 74D05 , 93D15 , 93D20.

Introduction

In this work, we consider, in the whole space $\mathbb{R}^n$, the following Moore–Gibson–Thompson equation (MGT) with a finite type-II memory term:

$$u_{ttt}(t) + \alpha u_{tt}(t) - \beta \Delta u_t(t) - \gamma \Delta u(t) + \int_0^t g(t-s) \Delta u_t(s) ds = 0, \quad t > 0, \quad (299)$$

together with initial data

$$u(0) = u_0, \quad u_t(0) = u_1, \quad u_{tt}(0) = u_2, \quad (300)$$

where α, β, γ are positive constants and $g > 0$ is the relaxation function. The MGT equation arises from modeling high-frequency ultrasound waves as an alternative model to the well-known Kuznetsov equation, which is a classical model describing acoustic wave propagation in gases and liquids.

The mathematical community have got interested in the third-order PDE models for acoustic wave propagation as a consequence of their physical justification. It is clear and well known that the use of the Maxwell-Cattaneo law for the heat flux is responsible for the presence of the term u_{ttt} , and leads to the change from a second-order to third-order (in time) PDE, as well as from a parabolic to hyperbolic character of the linear dynamics. This justifies the appearance of a higher-order quasilinear PDE, its linearization, as well as several possible of its variants. The interest was certainly motivated at least initially by the difficulties and challenges encountered in the study and analysis of the MGT equation, differently from the linearization of the Kuznetsov and Westervelt equations, which is the well-known, parabolic-like, strongly damped wave equation. As a result of this interest, many papers related to the well-posedness and asymptotic behavior of

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the Moore–Gibson–Thompson equation, employing various types of dissipation, have appeared. Let's first cite few works treating the MGT problems in the abstract form

$$\tau u_{ttt} + \alpha u_{tt} + c^2 \mathcal{A}u + b \mathcal{A}u_t = 0, \quad (301)$$

where $\mathcal{A}$ is a self-adjoint positive operator. In this regard, Kaltenbacher et al. [5] proved that problem (301) is well-posed and its solution is exponentially stable in the subcritical case, namely, $\alpha - \frac{c^2 \tau}{b} > 0$. Whereas, the energy remains constant in the critical case $\alpha - \frac{c^2 \tau}{b} = 0$. Similar result was also obtained by Marchand et al. in [9], where they used the spectrum analysis of the operator. Furthermore, Pellicer et al. [10] completed the work of Marchand et al. [9], where they proved an optimal exponential decay rate of the solutions by introducing an explicit scalar product. Chaotic behaviour of the solutions of the equation (301) is shown in [3] when $\alpha - \frac{c^2 \tau}{b} < 0$. Pellicer and Said-Houari [11] established the well-posedness and investigated the decay rate of the solutions of (301) in the whole space $\mathbb{R}^N$ (by replacing $\mathcal{A}$ with $-\Delta u$).

Next, we discuss some results obtained for the MGT equation with memory. In this regard, Lasiecka and Wang in [6] investigated the following MGT equation

$$u_{ttt}(t) + \alpha u_{tt}(t) + \beta \mathcal{A}u_t(t) + \gamma \mathcal{A}u(t) - \int_0^t g(s) \mathcal{A}z(t-s) ds = 0, \quad (302)$$

where they took z to be one of the three cases:

$$\begin{cases} z = u & \text{(type I memory),} \\ z = u_t & \text{(type II memory),} \\ z = u + u_t & \text{(type III memory),} \end{cases} \quad (303)$$

and the function g is decaying exponentially. Precisely, they studied the effect of the memory on the decay of the energy in the subcritical case $\chi = \alpha\beta - \gamma > 0$ and showed that the damping mechanism induced by the memory term leads to an exponential decay of the solution energy. Furthermore, in [7], they generalized the first work by allowing the memory kernel to satisfy a more general decay estimate of the form

$$g'(t) + H(g(t)) \leq 0, \quad \text{for all } t > 0, \quad (304)$$

with $H(0) = 0$. In addition to some extra conditions, they showed that the energy decays at the same rate as the memory kernel does, when $\chi > 0$. Alves et al. in [1] proved existence and uniform stability results of (302), (303), replacing the finite memory by an infinite one. In [8], Liu et al. proved new general decay results for the solution in the subcritical case $\chi > 0$, by allowing a more general class of functions g than those in (304). Precisely, they allowed the kernel g to satisfy

$$g'(t) \leq -\zeta(t)H(g(t)), \quad (305)$$

for some positive functions $\zeta(t)$ and H . Their proof is based on the perturbed energy method and on some properties of the convex function. Bounadja and Said-Houari [2] investigated the existence and stability of the solution of (302) in the whole space $\mathbb{R}^n$ (with $\mathcal{A} = -\Delta$), using type I and type-III memory terms and showed that the obtained decay results in the critical case $\chi = 0$ and the subcritical case $\chi > 0$ depend on the rate of decay of the memory kernel. In particular, for the type-I memory, they proved in the critical case that the decay rate is of regularity-loss type.

Main results

Our aim in this work is to investigate (299)-(300) for relaxation functions g satisfying

$$g'(t) \leq -\eta(t)g(t) \quad \forall t \geq 0, \quad (306)$$

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and establish a general decay rate result under the condition

$$\beta - \frac{\gamma}{\alpha} - \frac{\rho}{2} > 0, \quad (307)$$

where $\rho = \int_0^\infty g(s)ds$. Then, we find the decay rate of the L^2 -norm of the vector $U = (u_{tt} + \alpha u_t, \nabla(u_t + \alpha u), \nabla u_t)$ and those of its higher-order derivatives by applying Plancherel's theorem. To the best of our knowledge, the MGT equation with a type II memory satisfying the general condition (306) has never been discussed in the whole space $\mathbb{R}^n$. Our main result then is in establishing the decay estimates of the L^2 -norm of U , as well as the L^2 -norm of its derivatives. To prove our main result, we first establish estimates on the Fourier image of U , then we apply the Plancherel identity to obtain the desired result.

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A predictive spatio-temporal model for bovine Babesiosis epidemic transmission

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Abstract: The main purpose of this talk is to analyze a new dynamical model pertaining to bovine Babesiosis transmission, and investigate its consequent morphology. We present and study various ramifications of our mathematical model for bovine Babesiosis spread, given, firstly, by a temporal system of ordinary differential equations and, finally, by a spatio-temporal system consisting of reaction-diffusion equations. Diffusion terms are incorporated into the model, using specific derivations for both infected ticks and infected bovines. Furthermore, mechanisms for the nearest neighbors' infection are integrated into the model. We determine mathematically the basic reproduction number $\mathcal{R}_0$ via the next-generation matrix. Then, we analyze the stability of the equilibria and the effects of the mobility of infectious agents, being they either ticks or bovines. Finally, model-based analytical-numerical results are obtained and displayed in graphical profiles. The results of the proposed model and the health ramifications are then raised, discussed and validated.

Keywords: Babesiosis disease, Bovine mobility, Reaction-diffusion equations, Spatio-temporal model.

2020 Mathematics Subject Classification: Journal of Theoretical Biology 480 (2019) 192–204.

Introduction

Bovine Babesiosis is caused by the sting of ticks, a kind of parasitic infection which attacks the cattle. *Babesia bovis* and *Babesia bigemia* are the most known varieties found in tropical and subtropical areas (Friedman and Yakubu, 2014). Intravascular haemolysis, coagulation disorders and release of pharmacologically active substances resulting in vascular malfunction and hypotensive shock are the major pathogenesis of this disease. High fever, depression, anorexia, hemoglobinemia and hemoglobinuria are the predominant signs (Demeke et al., 2018).

To control the transmission of Babesiosis, vaccination and antiparasitic treatments are the basic (control) strategies which have been performed (Friedman and Yakubu, 2014). But, due to some problems and side effects, some vaccines and drugs have been eliminated from these strategies (Solorio-Rivera and Rodriguez-Vivas, 1997a; 1997b). All these reasons make this disease interesting to be modeled mathematically in order to know its dynamical behavior. In fact, knowing its behavior can lead to the design of new control strategies.

In this talk, we provide and study a more accurate spatio-temporal model for the bovine Babesiosis transmission by using individual models for the infection of nearest neighbors (Postnikov and Sokolov, 2007) and model derivation (Brauer et al., 2008) of the infected bovines and infected

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ticks. We study two important factors concerning the diffusion of bovine Babesiosis: the infection of nearest neighbors effect and the mobility of the infectious individuals (bovine and tick). The corresponding ordinary differential equations describing temporal evaluation of bovines and ticks are given by k

$$\left\{ \begin{array}{l} S'_B(t) = -\beta_{TB}BS_B(t)I_T(t) + (\mu_B + \alpha_B)R_B(t) \\ E'_B(t) = \beta_{TB}BS_B(t)I_T(t) - \nu_BE_B(t) \\ I'_B(t) = \nu_BE_B(t) - \lambda_BI_B(t) \\ R'_B(t) = \lambda_BI_B(t) - (\mu_B + \alpha_B)R_B(t) \\ S'_T(t) = -\beta_{BT}BS_T(t)I_B(t) + p\mu_T(I_T(t) + E_T(t)) \\ E'_T(t) = \beta_{BT}BS_T(t)I_B(t) - (p\mu_T + \nu_T)E_T(t) \\ I'_T(t) = -p\mu_TI_T(t) + \nu_TE_T(t). \end{array} \right. \quad (308)$$

Main results

Proposition 8. *System (308) has the disease-free steady state $E_0 = (1, 0, 0, 0, 0)$ whatever the values of the parameters of the system, whereas, only if $\mathcal{R}_0 > 1$ there is a (unique) endemic steady state $E_1 = (S_B^*, E_B^*, I_B^*, E_T^*, I_T^*)$ in the interior of Ω .*

Proof We denote by $E_{i=0,1}$ the steady states points of system (??), that is, the equilibrium point where $S'_B = E'_B = I'_B = E'_T = I'_T = 0$ for all $t > t_0$. Then, the disease-free steady state $E_0 = (1, 0, 0, 0, 0)$ and the endemic point is given by

$$\begin{aligned} S_B^* &= \frac{ad^2fk + cd^2fk + acdeg + acdek + acdfk}{abceg + abdeg + acdeg + bcdeg + acdek} \\ E_B^* &= \frac{abdeg - ad^2fk}{abceg + abdeg + acdeg + bcdeg + acdek} \\ I_B^* &= \frac{abceg - acdfk}{abceg + abdeg + acdeg + bcdeg + acdek} \\ E_T^* &= \frac{abcegk - acdfk^2}{abceg^2 + abcegk + abcf gk + abdf gk + bcd f gk} \\ I_T^* &= \frac{abceg - acdfk}{abceg^2 + abcegk + abcf gk + abdf gk + bcd f gk} \end{aligned}$$

where

$$\begin{aligned} a &= \mu_B + \alpha_B, & b &= \beta_{TB}B, & c &= \nu_B, & d &= \lambda_B, \\ e &= \beta_{BT}B, & f &= p\mu_T + \nu_T, & g &= \nu_T, & k &= p\mu_T. \end{aligned} \quad (309)$$

Notice that if $\mathcal{R}_0 > 1$, then $(bge - dfk) > 0$. Therefore, since all the parameters of the equations are greater than zero,

$$S_B^*, E_B^*, I_B^*, E_T^*, I_T^* > 0$$

So, there is a (unique) endemic equilibrium point $E_1 = (S_B^*, E_B^*, I_B^*, E_T^*, I_T^*)$ in the interior of Ω .

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Now, we will demonstrate that if $\mathcal{R}_0 \leq 1$ then the unique equilibrium point in Ω is the disease free equilibrium point $E_0 = (1, 0, 0, 0, 0)$.

We shall distinguish two cases:

- $\mathcal{R}_0 < 1$, in this case: $(bge - dfk) < 0$. So it is easy to check that $E_B^*, I_B^*, E_T^*, I_T^* < 0$. Therefore, this point remains outside the region Ω .
- $\mathcal{R}_0 = 1$, In this situation because $bge - dfk = 0$, we have that $E_B^*, I_B^*, E_T^*, I_T^* = 0$. Therefore, the last four equations of the system (183) are checked automatically and, replacing the value of $E_B^*, I_B^*, E_T^*, I_T^*$ in the first equation, we found

$$S_B' = (\mu_B + \alpha_B)(1 - S_B)$$

Thus, it must be $S_B = 1$. But, if it is $S_B = 1$ we have disease-free steady state.

■

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The optimal control of a nonlinear model described by Marguerre von Karman equations

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Abstract: In this work we study the optimal control of a nonlinear model described by Marguerre von Karman equations with long memory with variable change to simplify this problem, first we prove the uniqueness and existence of optimal control for the quadratic cost. Then, by proving the strong Gâteaux differentiability of nonlinear solution mapping $u \rightarrow \xi(u)$ we establish necessary optimality condition for the optimal control corresponding to the quadratic cost.

Keywords: Marguerre von Karman system with long memory, optimal control, Gâteaux differentiability, necessary optimality condition.

Introduction

We consider the following Marguerre von Kármán system with long memory and the clamped boundary condition in the variables ζ , representing the position of the plate and the Airy's stress function ψ and θ represents the geometric shape of this shell.

$$\begin{cases} \zeta_{tt} - \Delta \zeta_{tt} + \Delta^2 \zeta + \hbar \star \Delta^2 \zeta = [\psi, \zeta + \theta] + f & \text{in } (0, T) \times \Omega, \\ \Delta^2 \psi = -[\zeta, \zeta + 2\theta] & \text{in } (0, T) \times \Omega, \\ \zeta = \partial_\nu \zeta = \psi = \partial_\nu \psi = 0 & \text{on } (0, T) \times \partial\Omega, \\ \zeta(0, x) = \zeta_0(x) \quad \zeta_t(0, x) = \zeta_1(x) & \text{in } \Omega, \end{cases} \quad (310)$$

such that :

- Ω bounded open of R^2 with smooth boundary $\partial\Omega$.
- $(\hbar \star \Delta^2)(\omega)\zeta = \int_0^t \hbar(t-\omega) \Delta^2 \zeta(\omega) d\omega$, and $\hbar \in C^1([0, T])$.
- The vector ν an external normal, $f \in L^2$.
- $[\zeta, \psi] = \frac{\partial^2 \zeta}{\partial x_1^2} \frac{\partial^2 \psi}{\partial x_2^2} + \frac{\partial^2 \psi}{\partial x_1^2} \frac{\partial^2 \zeta}{\partial x_2^2} - 2 \frac{\partial^2 \zeta}{\partial x_1 \partial x_2} \frac{\partial^2 \psi}{\partial x_1 \partial x_2}$ and call the non-linear, symmetrical Monge-Ampère bracket

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By variable change according to ([1]) we have:

$$\begin{cases} \xi_{tt} - \Delta \xi_{tt} + \Delta^2 \xi + \hbar \star \Delta^2 \xi = [\xi, \Phi] + \Delta^2 \theta + f & \text{in } (0, T) \times \Omega \\ \Delta^2 \Phi = -[\xi, \xi] + [\theta, \theta] & \text{in } (0, T) \times \Omega \\ \xi = \frac{\partial \xi}{\partial \nu} = \Phi = \frac{\partial \Phi}{\partial \nu} = 0 & \text{on } (0, T) \times \partial \Omega \\ \xi(0, x) = \xi_0(x), \quad \xi_t(0, x) = \xi_1(x) & \text{in } \Omega, \end{cases} \quad (311)$$

Firstly, we survey the well-posedness of Eq. (70) with respect to y in the Hadamard sense relying on some previous results. Especially, in order to prove the local Lipschitz continuity of the solution mapping, we employ the energy equality of Volterra-type integro-differential equation which is proved in [2].

Secondly, based on this result, we study the following optimal control problem

$$\text{Minimize} \quad J(u), \quad (312)$$

we propose the quadratic cost functional J as studied in Lions [5], which is to be minimized within U_{ad} , an admissible set of control variables in U . The problem 2 consists of two problems:

- The existence of optimal control.
- The necessary condition for the optimal control.

We need to show the existence of $u^* \in U_{ad}$ that minimizes the quadratic cost functional J . However, differently from the linear equation case, we are faced with difficulty that the weak convergence of the controlled state $\xi(u_n)$ is insufficient to cover the convergence of the nonlinear part of Eq (70). Also, this method is quite useful in proving the strong Gâteaux differentiability of the nonlinear solution mapping $u \rightarrow \xi(u)$, which is used to define the associate adjoint system. Then, we establish a necessary condition of optimality of the optimal control u^* .

Main results

The uniqueness of the solution:

Theorem 95. Assume that $(\xi_0, \xi_1) \in H_0^2 \times H_0^1$, $\hbar \in C^1([0, T])$, $f \in L^2(0, T; L^2)$, and $\theta \in H_0^2$. Then Eq. (70) has a unique weak solution ξ . Moreover, the solution mapping $p = (\xi_0^1, \xi_1^1, f_1) \rightarrow (\xi(p), \xi_t(p), \Phi(p))$ of $\wp = H_0^2 \times H_0^1 \times L^2(0, T; L^2)$ into $C([0, T]; H_0^2) \times C([0, T]; H_0^1) \times C([0, T]; W_\infty^2)$ is locally Lipschitz continuous.

Indeed, let $p_1 = (\xi_0^1, \xi_1^1, f_1) \in \wp$ and $p_2 = (\xi_0^2, \xi_1^2, f_2) \in \wp$. We prove Theorem ?? by showing the inequality:

$$\|\nabla(\xi'(p_1; t) - \xi'(p_2; t))\| + \|\Delta(\xi(p_1; t) - \xi(p_2; t))\| + \|\Phi(p_1; t) - \Phi(p_2; t)\|_{W_\infty^2} \leq C \|p_1 - p_2\|_\wp, \quad (313)$$

where $C > 0$ is a constant depending on data, and

$$\|p_1 - p_2\|_\wp = (\|\xi_0^1 - \xi_0^2\|_{H_0^2} + \|\xi_1^1 - \xi_1^2\|_{H_0^1} + \|f_1 - f_2\|_{L^2(0, T; L^2)})^{\frac{1}{2}}.$$

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The existence of the solution:

Theorem 96. Assume that the hypotheses of Theorem (2.1) and [4] are satisfied. Then there exists at least one optimal control u for the control this problem with the cost :

$$J(u) = \|Cy(u) - Y_d\|_M^2 + (Ru, u)_U, \quad \text{for } u \in U \quad (314)$$

where C is an operator called the observer, M is a Hilbert space of observation variables, $Y_d \in M$ is a desired value of $y(u)$, and $R \in L(U, U)$ is symmetric and positive.

Necessary condition of an optimal control:

Theorem 97. The optimal control u^* is characterized by the following system of equations and inequality:

$$\begin{cases} \xi_{tt}(u^*) - \Delta \xi_{tt}(u^*) + \Delta^2 \xi + \hbar \star \Delta^2 \xi(u^*) = [\xi(u^*), \Phi(u^*)] + \Delta^2 \theta + Bu^* & \text{in } (0, T) \times \Omega \\ \Delta^2 \Phi(u^*) = -[\xi(u^*), \xi(u^*)] + [\theta, \theta] & \text{in } (0, T) \times \Omega \\ \xi(u^*) = \frac{\partial \xi(u^*)}{\partial \nu} = \Phi(u^*) = \frac{\partial \Phi(u^*)}{\partial \nu} = 0 & \text{on } (0, T) \times \partial \Omega \\ \xi(0, u^*) = \xi_0(u^*), \quad \xi_t(0, u^*) = \xi_1(u^*) & \text{in } \Omega, \end{cases} \quad (315)$$

using the Gâteaux differentiable derivative of $\xi(u)$, with the direction $u - u^*$, we find:

$$\begin{cases} p_{tt}(u^*) - \Delta p_{tt}(u^*) + \Delta^2 p + \int_t^T \hbar(\sigma - t) \Delta^2 p(u^*, \sigma) d\sigma = [p(u^*), G(-[\xi(u^*), \xi(u^*)] + [\theta, \theta])] \\ + 2[\xi(u^*), -G[p(u^*), \xi(u^*)]] + \xi(u^*) - Y_d & \text{in } (0, T) \times \Omega \\ p(u^*) = \frac{\partial p(u^*)}{\partial \nu} = 0 & \text{on } (0, T) \times \partial \Omega \\ p(u^*, T) = 0, \quad p_t(u^*, T) = 0 & \text{in } \Omega, \end{cases} \quad (316)$$

Then

$$\int_0^T (p(u^*), B(u - u^*))_2 dt + (Ru^*, u - u^*)_2 \geq 0 \quad \forall u \in U_{ad}.$$

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Galerkin Method for solving Singular integro-differential equations

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Abstract: In this paper, we present a Galerkin method for solving Fredholm integro-differential equations of the second kind defined on real half line. The main idea in this method is applying mapped Legendre functions, these properties together with the Legendre-Gauss points are then utilized to reform the Fredholm integro-differential equation in semi-infinite interval into a singular equation in finite interval and to reduce it to the solution of a simple matrix equation. Some examples are given to demonstrate the validity, efficiency and applicability of the method.

Keywords: Fredholm integro-differential equations, Mapped Legendre , Galerkin method, Error estimation.

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Exploration of the Exponential Convergence Speed of the Metropolis-Hastings Algorithm

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Abstract: This work presents the exponential convergence of the Metropolis-Hastings algorithm towards the target distribution. It demonstrates that this algorithm rapidly reaches a stable state consistent with the desired distribution. This exponential convergence is essential for the efficiency and reliability of the algorithm across various domains.

Keywords: Metropolis-Hastings Algorithm , Exponential Convergence, Target Distribution , Markov Chain , Monte Carlo Method

2020 Mathematics Subject Classification: 65C40, 60J22 , 62F15.

Introduction

The Metropolis-Hastings algorithm [1, 2] is a powerful tool in statistics and machine learning, enabling exploration of complex probability distributions. Its ability to efficiently generate samples, even in high-dimensional spaces, makes it a major asset in many fields. One of its most fascinating features lies in its convergence speed, which measures its ability to reach the desired target distribution in a reasonable amount of time. In this article, we will delve deeply into the concept of exponential convergence speed of the Metropolis-Hastings algorithm.

Main results

This study aims to examine an approach for estimating the rate at which the π -reversible Markov kernel $P = P_{MH}$ converges to the distribution π .

Theorem 98. *The Markov chain $(X_n)_{n \in \mathbb{N}}$ associated with P in the Metropolis-Hastings algorithm satisfies the following properties for all n : P is irreducible and admits a reversible measure π . Therefore, the eigenvalues $\lambda_1 > \lambda_2 \cdots > \lambda_N$ of P satisfy $\lambda_1 = 1$. Additionally, for any initial distribution π_0 , we have:*

$\|\pi_n - \pi\|_{TV} \leq C e^{-\rho n}$ with $C = \frac{1}{2} \sqrt{\max_{x \in \Omega} \left(\frac{1}{\pi(x)} - 1 \right)}$. Where $\rho = \min(1 - |\lambda_2|, 1 - |\lambda_N|)$ is the spectral gap

of P , and $\pi_n = \pi_0 P^n$ represents the marginal distribution of $(X_n)_{n \in \mathbb{N}}$, and P is exponentially ergodic and converges exponentially fast towards the target distribution π .

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Proof If P is π -irreducible, π -reversible, and primitive. By the Perron-Frobenius theorem [1], P has a strictly positive real eigenvalue λ_1 , and $\lambda_1 > \lambda_2 \cdots > \lambda_N$. We need to show that these eigenvalues are contained in the interval $[-1, 1]$.

Let μ be an eigenvector of λ , then we have:

$$\begin{aligned} \left| \langle \mu | \mu P \rangle_{\frac{1}{\pi}} \right| &= \left| \int \int \mu(x) \mu(y) P(x, y) \frac{dx dy}{\pi(x)} \right| = \left| \int \int \pi(y) \frac{\mu(x)}{\pi(x)} \frac{\mu(y)}{\pi(y)} P(x, y) dx dy \right| \\ &\leq \frac{1}{2} \int \pi(x) \left(\frac{\mu(x)}{\pi(x)} \right)^2 dx + \frac{1}{2} \int \pi(y) \left(\frac{\mu(y)}{\pi(y)} \right)^2 dy \leq \int (\mu(x))^2 \frac{dx}{\pi(x)} = \langle \mu | \mu \rangle_{\frac{1}{\pi}} \end{aligned}$$

And since μ is an eigenvector of λ , we have $\left(\left\| \mu \right\|_{\frac{1}{\pi}} \right)^2 \geq |\lambda| \left(\left\| \mu \right\|_{\frac{1}{\pi}} \right)^2$, which implies $1 \geq |\lambda|$. Therefore, we have: $1 \geq \lambda_1 > \lambda_2 \cdots > \lambda_N \geq -1$ If $\lambda = 1$ then:

$$\frac{\mu(x)}{\pi(x)} \frac{\mu(y)}{\pi(y)} = \frac{1}{2} \left(\frac{\mu(x)}{\pi(x)} \right)^2 + \frac{1}{2} \left(\frac{\mu(y)}{\pi(y)} \right)^2 \Rightarrow \frac{\mu(x)}{\pi(x)} = \frac{\mu(y)}{\pi(y)}$$

That is, $\mu = c\pi$ with c being a constant.

Reciprocally, by the definition of an invariant measure, π indeed satisfies $\pi = \pi P$. Thus, we have established that the eigenvalue 1 is a simple eigenvalue which we denote as $\lambda_1 = 1$.

Now, let's show that -1 is not an eigenvalue of P . We assume by contradiction that there exists an eigenvector f for the eigenvalue -1 . Let x be a state in the state space Ω such that $f(x) > 0$ and $f(x) = \max_{y \in \Omega} |f(y)|$ (we can take $-f$ instead of f if necessary). Since P is aperiodic according to Proposition 5.1, for sufficiently large n , $P^n(x, x) > 0$ [16] (see Lemma 3.15, page 39). We have:

$$-f(x) = P^n(f)(x) = \int P^n(x, y) f(y) dy > - \int P^n(x, y) f(x) dy = -f(x)$$

This implies that $-f(x) > -f(x)$. It is a contradiction.

Finally, let's show that $\|\pi_n - \pi\|_{TV} \leq Ce^{-\rho n}$. Using the properties of the total variation norm [13], we have:

$$2\|\pi_n - \pi\|_{TV} = \|\pi_n - \pi\|_1 \leq \|\pi_n - \pi\|_2 \leq \left(\int (\pi_n(x) - \pi(x))^2 \frac{dx}{\pi(x)} \right)^{\frac{1}{2}} = \|\pi_n - \pi\|_{\mathcal{L}^2(\Omega, \frac{1}{\pi})}$$

Since P is a symmetric operator for its right action on $\mathcal{L}^2\left(\Omega, \frac{1}{\pi}\right)$, there exists an orthonormal basis $(\mu_1, \mu_2, \dots, \mu_N)$ of this space corresponding to the eigenvalues $(1, \lambda_1, \dots, \lambda_N)$. The orthonormal eigenvector μ_1 for the eigenvalue 1 is simply the invariant measure π . Let's write the decomposition of the initial distribution π_0 in the eigenbasis of P : $\pi_0 = c\pi + \sum_{i=2}^N c_i \mu_i$. For certain real coefficients c and c_i , $i \in [2, N]$, with $c_i = \langle \pi_0 | \mu_i \rangle_{\frac{1}{\pi}}$, applying P^n to the formula above

yields: $\pi_n = c\pi + \sum_{i=2}^N c_i (\lambda_i)^n \mu_i$ Since $|\lambda_i| < 1$ for $i \in [2, N]$, then $\lim_{n \rightarrow \infty} \pi_n = c\pi + \lim_{n \rightarrow \infty} \sum_{i=2}^N c_i (\lambda_i)^n \mu_i = c\pi$, which implies that $c = 1$. Therefore: $\pi_n - \pi = \sum_{i=2}^N c_i (\lambda_i)^n \mu_i$ et $\left(\|\pi_n - \pi\|_{\frac{1}{\pi}} \right)^2 = \sum_{i=2}^N c_i^2 (\lambda_i)^{2n} \leq$

$\sum_{i=2}^N c_i^2 (1 - \rho)^{2n} \leq \left(\|\pi_0 - \pi\|_{\frac{1}{\pi}} \right)^2 e^{-2n\rho}$ With $\rho = \min(1 - |\lambda_2|, 1 - |\lambda_N|)$. To conclude, we need to bound the value $\left(\|\pi_0 - \pi\|_{\frac{1}{\pi}} \right)^2$ which we denote as $F(\pi_0)$, $F(\pi_0) = \left(\|\pi_0 - \pi\|_{\frac{1}{\pi}} \right)^2 = \int (\pi_0(x) - \pi(x))^2 \frac{dx}{\pi(x)}$ The function F is quadratic and, therefore, convex over a convex part of $\mathbb{R}^N$. Hence, it reaches its maximum at an extreme point of the convex set formed by probability measures. The extreme points

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of this convex set are precisely the Dirac measures δ_{x_0} (convex combination of Dirac measures). Thus,

$$\max_{\pi_0 \text{ probability on } \Omega} \left(\|\pi_0 - \pi\|_{\frac{1}{\pi}} \right)^2 = \max_{x_0 \in \Omega} \left(\|\delta_{x_0} - \pi\|_{\frac{1}{\pi}} \right)^2$$

For fixed x_0 we have: $\left(\|\delta_{x_0} - \pi\|_{\frac{1}{\pi}} \right)^2 = \frac{(1-\pi(x_0))^2}{\pi(x_0)} + \int_{\Omega} \pi(x) \delta_{x \neq x_0} dx = \frac{1-\pi(x_0)}{\pi(x_0)} = \frac{1}{\pi(x_0)} - 1$

We conclude that $\|\pi_n - \pi\|_{TV} \leq C e^{-\rho n}$ with $C = \frac{1}{2} \sqrt{\max_{x \in \Omega} \left(\frac{1}{\pi(x)} - 1 \right)}$ ■

Conclusion

The exponential convergence of the Metropolis-Hastings algorithm towards the target distribution, based on the concept of Markov chain and utilized in the Monte Carlo method, provides a robust approach for efficient sampling, with significant implications in various domains of research and practical applications.

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On Optimal Control for Mean-Field Systems Driven by Local martingales: Necessary Conditions

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Abstract: In this work, we derive necessary conditions for optimal control of mean-field equations driven by a spatially parameterized continuous local martingale that nonlinearly depends on both the state process and its law via the expectation of some function of the state. Through illustrative examples, we employ these findings to tackle particular instances of the stochastic maximum principle documented in existing literature, while also examining a linear quadratic stochastic optimal control problem of mean-field type.

Keywords: Mean-field SDE, local martingale, stochastic maximum principle.

2020 Mathematics Subject Classification: Primary 60H10, 60G55, 93E20.

Introduction

Let $T > 0$ be a fixed time horizon and $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$ be a probability filtered space satisfying the usual conditions. Let $\mathbb{U}$ be a closed convex subset of $\mathbb{R}$. The control variable is a suitable process u such that

- (i) $u : [0, T] \times \Omega \rightarrow \mathbb{U}$ is measurable and $(\mathcal{F}_t)_{0 \leq t \leq T}$ -adapted process,
- (ii) $\mathbb{E} \left[\sup_{t \in [0, T]} |u_t|^2 \right] < \infty$.

We denote $\mathcal{U}_{ad}$, the set of all admissible controls. In this paper, for any $u \in \mathcal{U}_{ad}$, we study the optimal control problem for systems governed by controlled stochastic differential equations driven by spatial parameters local martingale of mean-field type, of the form

$$\begin{cases} dX_t = b(t, X_t, \mathbb{E}[\varphi(X_t)], u_t) dt + M(dt, X_t, \mathbb{E}[\kappa(X_t)], u_t), \\ X_0 = \xi, \quad t \in [0, T], \end{cases} \quad (317)$$

where $\{M(t, x, \tilde{x}, u); t \in [0, T]\}_{(x, \tilde{x}, u) \in \mathbb{R}^{3d}}$ is a continuous one-dimensional local martingales with joint quadratic $\langle M(t, x, \tilde{x}, u), M(t, y, \tilde{y}, u) \rangle = \int_0^t q(s, x, \tilde{x}, y, \tilde{y}) ds$ such that $q(s, x, \tilde{x}, y, \tilde{y}) = q(s, y, \tilde{y}, x, \tilde{x})$, and $b : [0, T] \times \mathbb{R}^2 \times \mathbb{U} \rightarrow \mathbb{R}$, and $\varphi, \kappa : \mathbb{R} \rightarrow \mathbb{R}$. Our goal is to minimize the following cost functional

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$$J(u) = E \left[g(X_T, \mathbb{E}[\gamma(X_T)]) + \int_0^T h(t, X_t, \mathbb{E}[\psi(X_t)], u_t) dt \right], \quad (318)$$

over the set $\mathcal{U}_{ad}$, where $h : [0, T] \times \mathbb{R}^2 \times \mathbb{U} \rightarrow \mathbb{R}$, $g : \mathbb{R}^2 \rightarrow \mathbb{R}$, and $\gamma, \psi : \mathbb{R} \rightarrow \mathbb{R}$. An admissible control $u^* \in \mathcal{U}_{ad}$ is called an optimal control, if that solves

$$J(u^*) = \inf_{u \in \mathcal{U}_{ad}} J(u) \quad (319)$$

Our primary motivation is seeing (317) as an MF-SDE driven by infinite-dimensional Brownian motion. In other words, $M(t, x, \tilde{x}, u)$ can be expressed as $M(t, x, \tilde{x}, u) = \sum_{l=1}^{\infty} \int_0^t f_l(s, x, \tilde{x}, u) dB_s^l$ where $(B^l)_{l \in \mathbb{N}}$ is a sequence of independent copies of one dimensional Brownian motions, and $(f_l)_{l \in \mathbb{N}}$ are $\mathbb{R}^d$ -valued continuous functions. This formulation naturally emerges when studying financial markets with a large number of stocks.

Given the assumption of a convex control domain, we employ the convex perturbation method for both the continuous and singular components of our control process. The proof of our result is based on the derivative of the local martingale $M(t, x, \tilde{x}, u)$ with respect to the spatial parameters x , $\tilde{x}$ and u . Moreover, the adjoint equation is a mean-field backward stochastic differential equation driven by the continuous local martingale M and contains an additional martingale orthogonal to $M(t, x, \tilde{x}, u)$ (see equation (321)). For simplification, we restrict our study to the one-dimensional case.

Main results

Stochastic maximum principle

Let $u^* \in \mathcal{U}_{ad}$ be an optimal control with associated trajectory X^* solution of the MF-SDE (317). we introduce the following perturbation u_t^θ (the so-called convex perturbation) of u_t^*

$$u_t^\theta = u_t^* + \theta(u_t - u_t^*), \quad t \in [0, T], \quad (320)$$

where $\theta > 0$ is sufficiently small and u_t an arbitrary element of $\mathcal{U}_{ad}$. Denote by X^θ the solution of corresponding to u^θ . Using the optimality of u^* , the variational inequality will be derived from the fact that

$$0 \leq J(u^\theta) - J(u^*).$$

Throughout this work, the subsequent assumptions will be in effect. **Assumption (2.1).**

(C.3) The functions φ, κ, ψ and γ are continuously differentiable. The functions b, h and g are continuously differentiable in $(x, \tilde{x}, u)$ and (x, u) , respectively.

(C.4) All the derivatives in (C.3) are continuous and bounded.

(C.5) For all $(x, \tilde{x}, u), (y, \tilde{y}, v) \in \mathbb{R}^3$, $q(t, x, \tilde{x}, u, y, \tilde{y}, v)$ belongs to $B_{ub}^{1, \delta}(\mathbb{R}^3 \times \mathbb{R}^3; \mathbb{R})$, for some $\delta \in (0, 1]$.

It follows that for $z = (x, \tilde{x}, u) \in \mathbb{R}^3$ and $\hat{z} = (y, \tilde{y}, v) \in \mathbb{R}^3$, the partial derivative $D_z D_{\hat{z}} q(t, x, \tilde{x}, u, y, \tilde{y}, v)$ is uniformly bounded in $(x, \tilde{x}, u, y, \tilde{y}, v)$.

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Now, we consider the following MF-BSDE:

$$\begin{cases} dP_t = -\left(\partial_x b(t, \Phi_t^*) P_t + \mathbb{E}[\partial_{\bar{x}} b(t, \Phi_t^*) P_t] \partial \varphi(X_t^*) + \frac{\partial}{\partial x} q(t, \Phi_t^*, \Phi_t^*) \phi_t \right. \\ \quad \left. + \mathbb{E}\left[\frac{\partial}{\partial \bar{x}} q(t, \Phi_t^*, \Phi_t^*) \phi_t\right] \partial \kappa(X_t^*) + \partial_x h(t, \Phi_t^*) + \mathbb{E}[h_{\bar{x}}(t, \Phi_t^*)] \partial \psi(X_t^*)\right) dt \\ \quad + \phi_t M(dt, \Phi_t^*) + d\mathcal{N}_t, \\ P_T = \partial_x g(X_T^*, \mathbb{E}[\gamma(X_T^*)]) + \mathbb{E}[\partial_{\bar{x}} g(X_T^*, \mathbb{E}[\gamma(X_T^*)])] \partial \gamma(X_T), \end{cases} \quad (321)$$

where for $\mathcal{L} = \varphi, \kappa, \psi, \gamma$, $\Phi_t^* = (X_t^*, \mathbb{E}[\mathcal{L}(X_t^*)], u_t^*)$, and $\mathcal{N}_t$ is a $\mathbb{R}$ -valued square integrable martingale orthogonal to $M(t, \Phi_t^*)$.

Theorem 99. *Let Assumption (2.1) holds. Let (u^*, X^*) be an optimal solution of the control problem [(317), (318), (319)]. Then there exists a unique pair $(P, \phi) \in \mathcal{L}^2([0, T]; \mathbb{R}) \times \mathcal{H}^2([0, T]; \mathbb{R})$ satisfying the adjoint equation (321) such that for all $u \in \mathcal{U}_{ad}$*

$$\partial_u H(t, X_t^*, u_t^*, P_t, \phi_t)(u_t - u_t^*) \geq 0, \text{ a.s.}, \quad (322)$$

for almost all $t \in [0, T]$, where H is the Hamiltonian defined from $[0, T] \times \mathbb{R} \times \mathbb{R} \times U \times \mathbb{R} \times \mathbb{R}$ to $\mathbb{R}$ by

$$H(t, x, u, p, \phi) = \langle p, b(t, x, \bar{x}, u) \rangle + \phi q(t, x, \bar{x}, u, x, \bar{x}, u) + h(t, x, \bar{x}, u).$$

Examples

Itô's Mean-Field SDEs: Consider the following classical controlled MF-SDE

$$\begin{cases} dX_t = b(t, X_t, \mathbb{E}[\varphi(X_t)], u_t) dt + \sigma(t, X_t, \mathbb{E}[\kappa(X_t)], u_t) dW_t, \\ X_0 = \xi, \quad t \in [0, T], \end{cases} \quad (323)$$

where $W = (W_t)_{0 \leq t \leq T}$ is an d -dimensional standard Wiener process, and the functions b and σ are defined from $[0, T] \times \mathbb{R} \times \mathbb{R} \times \mathbb{U}$ to $\mathbb{R}$ and $\mathbb{R}^{1 \times d}$. Equation (323) is equivalent to the MF-SDE (317) by setting $M(t, x, \bar{x}, u) = \sum_{k=1}^d \int_0^t \sigma^k(s, x, \bar{x}, u) dW_s^k$. Its local characteristic is $q := \sum_{k=1}^d \left(\sigma^k(t, x, \bar{x}, u)\right)^2$.

Mean-Field Linear Quadratic Control Problem: We consider the following mean-field linear quadratic stochastic control problem. Minimize

$$J(u, \eta) = \frac{1}{2} \mathbb{E} \left[D_T X_T^2 + \tilde{D}_T \mathbb{E}[X_T^2] + \int_0^T (\tilde{A}_t X_t^2 + \tilde{B}_t \mathbb{E}[X_t^2] + \tilde{C}_t u_t^2) dt \right], \quad (324)$$

subject to

$$\begin{cases} dX_t = (A_t X_t + B_t \mathbb{E}[X_t] + C_t u_t) dt + M(dt, X_t, \mathbb{E}[X_t], u_t) + F_t d\eta_t, \\ X_0 = \xi, \quad t \in [0, T], \end{cases} \quad (325)$$

where, for any $t \in [0, T]$, $\tilde{C}_t > 0$ and $\tilde{A}_t, \tilde{B}_t, D_t, \tilde{D}_t \geq 0$.

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Bivariate Poisson-XLindley Distribution and its Application in Sport

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Abstract: The Poisson-XLindley distribution (PXLD), which can be used to characterize count data that is overly dispersed or dependent, is presented in the present study. It is displayed in its bivariate form. Among the characteristics that are considered are the correlation coefficient, mean, and variance of the distribution. A comparison with various bivariate distributions is included. The goodness of fit of this new model is evaluated using data from one season in the Italian Series A and one season in the German Bundesliga using the bivariate Poisson, bivariate negative binomial, and bivariate Poisson-Lindley distributions.

Keywords: Bivariate, XLindley distribution, Poisson-XLindley distribution, Poisson distribution, Moments, Maximum likelihood estimation

2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

Introduction

In 2022, Ahsan-ul Haq et al. [1], Seghier et al. [1] introduced a new discrete distribution by compounding Poisson and XLindley distributions, named Poisson-XLindley (PXLD).

Mixing approaches were used to derive novel families of bivariate distributions. Bivariate Poisson-Lindley (BPLD), Bivariate Negative Binomial (BNBD), and Bivariate Poisson (BPD) are several examples of classes of mixed distribution that are extended from the univariate cases. For further literature, BNBD was studied by Marshall and Olkin (1985) and applied in Karlis and Ntzoufras (2003), tests for over-dispersion and independence in (BNBD) were discussed by Jung et al. (2009), Bivariate Poisson-Weighted Exponential was proposed by Zamani et al. (2014) and Bivariate Poisson Lindley was proposed by Zamani et al. (2015).

In this work, a multiplicative factor parameter is used with two Poisson XLindley margins to generate the BPXLD. The mean, variance, correlation, and joint moment-generating function of this new class of discrete distributions were given. We looked at two sports data sets to show how flexible the proposed model is compared to the BPD, BPLD, and BNBD. For the first time in the literature since the work Karlis and Ntzoufras (2003), which used the bivariate Poisson model, the novel bivariate model has been applied to sport. This paper is organized as follows: In Section 2 of the study, the new proposed model's formulation is explained along with the BPXLD's mean, variance, and correlation coefficient. Section 3 presents a moment estimation

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method for determining the unknown parameters of the proposed model. In Section 4 of the study, the bivariate Poisson, bivariate Poisson-Lindley, bivariate negative binomial, and bivariate Poisson-Lindley distributions are compared.

Bivariate Poisson-XLindley Distribution (BPXLD)

In this section, we introduce a bivariate version of the PXLD. The joint p.m.f. of BPXLD with parameters $\theta_1, \theta_2, \alpha$ is defined as:

$$P(Y_1 = y_1, Y_2 = y_2) = \frac{\theta_1^2(\theta_1^2 + 3\theta_1 + 3 + y_1)}{(1 + \theta_1)^{y_1+4}} \frac{\theta_2^2(\theta_2^2 + 3\theta_2 + 3 + y_2)}{(1 + \theta_2)^{y_2+4}} \{1 + \alpha(e^{-y_1} - c_1)(e^{-y_2} - c_2)\} \quad (1)$$

where $y_1, y_2 = 0, 1, 2, \dots; \theta_1, \theta_2 > 0$, and

$$c_t = E(e^{-Y_t}) = \frac{\theta_t^2 e^{-1}}{(1 + \theta_t)^4} \left[\frac{(1 + \theta_t)}{(\theta_t + 1 - e^{-1})^2} + \frac{(\theta_t^2 + 3\theta_t + 3)}{(\theta_t + 1 - e^{-1})} \right], \quad t = 1, 2. \quad (2)$$

The mean and variance of BPXLD $(\theta_1, \theta_2, \alpha)$ are:

$$E(Y_t) = \mu_t = \frac{\theta_t^2 + 2\theta_t + 2}{\theta_t(1 + \theta_t)^2}, \quad t = 1, 2. \quad (3)$$

$$Var(Y_t) = \frac{\theta_t^5 + 5\theta_t^4 + 11\theta_t^3 + 14\theta_t^2 + 10\theta_t + 2}{\theta_t^2(1 + \theta_t)^4}, \quad t = 1, 2. \quad (4)$$

And

$$Cov(Y_1, Y_2) = \alpha(c_{11} - \mu_1 c_1)(c_{22} - \mu_2 c_2), \quad (5)$$

where $c_{tt} = E(Y_t e^{-Y_t})$, $t = 1, 2$. differentiating m.g.f. of the PXLD with respect to z and letting $z = -1$, we have

$$\left. \frac{\partial}{\partial z} M_Y(z) \right|_{z=-1} = E(Y e^{-Y}).$$

Thus

$$c_{tt} = \frac{\theta_t^2 e^{-1}}{(1 + \theta_t)^3} \left[\frac{(1 + \theta_t + e^{-1})}{(\theta_t + 1 - e^{-1})^3} + \frac{(\theta_t^2 + 3\theta_t + 3)}{(\theta_t + 1 - e^{-1})^2} \right], \quad t = 1, 2.$$

Application

To illustrate this distribution, we choose two data sets: Set 1 displays the total goals scored by both the home and away teams in the 1991–1992 Italian Serie A; Set 2 displays the same data for the 2021–2022 German Bundesliga season. We are interested in the goals of both the home squad and the away team over a total of 306 games. see:

<https://www.the-sports.org/football-soccer-1991-1992-italian-serie-a-epr10727.html>

www.the-sports.org/football-soccer-1991-1992-italian-serie-a-epr10727.html,

<https://www.the-sports.org/football-soccer-2021-2022-german-bundesliga-epr114545.html>

www.the-sports.org/football-soccer-2021-2022-german-bundesliga-epr114545.html

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Conclusion

In this work, the bivariate Poisson-XLindley distribution was used to fit a sample of bivariate count data. The results show that, in contrast to the bivariate Poisson, bivariate negative binomial, and bivariate Poisson-Lindley distributions, the bivariate Poisson XLindley distribution corrects dependent count data that is overly distributed and positively linked. This implies that distribution might be a good substitute. The authors believe it would be interesting to use the bivariate Poisson-XLindley model to model actuarial or reliability data.

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The one-sided test and the two-sided test for the topp-leone

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Abstract: In this work, we propose two test statistics based on the partial functional mean to test the conformity of a random sample with the Topp-Leone distribution. Characterization of the distribution based on the partial functional mean has been proven. The tests are formed as the integrated deviation (or square deviation) between the sample and population partial functional means. Compared to the Kolmogorov-Smirnov (*KS*), Cramer-von Mises (*CM*), and Anderson-Darling (*AD*) tests, the proposed tests, say $\widehat{D}_{n,1}$ and $\widehat{D}_{n,2}$, generally perform better in terms of their powers. The dependence of the $\widehat{D}_{n,1}$, $\widehat{D}_{n,2}$, *KS*, *CM*, and *AD* tests on the skewness of the alternative is clear. In fact, we have found that $\widehat{D}_{n,2}$, *KS*, *CM*, and *AD* tests generally have higher powers than $\widehat{D}_{n,1}$ when testing against distributions with negative skewness and have lower powers when testing against distributions with positive skewness. We also noticed that $\widehat{D}_{n,2}$ outperforms the *KS*, *CM*, and *AD* tests when testing against negatively skewed alternatives and $\widehat{D}_{n,1}$ outperforms all of these tests when testing against positively skewed alternatives. The percentiles and powers calculations were all based on Monte Carlo simulations.

Keywords: Topp-Leone Distribution; goodness-of-fit; partial functional mean.

2020 Mathematics Subject Classification: 62F03.

Introduction

A random variable X is said to have Topp-Leone distribution of shape parameter and scale parameter if its distribution function is given by

$$F(x) = \left[1 - \left(1 - \frac{x}{\beta} \right)^2 \right]^\theta, 0 < x < \beta, \theta > 0 \quad (326)$$

and, consequently, its density is

$$f(x) = \frac{2\theta}{\beta} \left(1 - \frac{x}{\beta} \right) \left[1 - \left(1 - \frac{x}{\beta} \right)^2 \right]^{\theta-1}, 0 < x < \beta, \theta > 0 \quad (327)$$

This family of J-shaped distributions was first introduced by Topp and Leone (*TL*) [1]. No further developments in the distribution occurred until (2003) when Nadaraja and Kotz [2] drew attention

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to its suitability for application in the reliability analysis. Since then, many articles have appeared in the literature addressing different probabilistic and inferential aspects of the distribution; Van Dorp and Kotz [1] utilized the distribution to model income data. Some reliability measures of the distribution and their stochastic orderings were studied by Ghitany et al. [4]. Kotz and Seier [5] studied the kurtosis of the distribution. A two-sided generalization of the distribution was provided by Vicari et al. [4]. Zghoul [20, 3] studied order statistics and record values for samples from the TL distribution. Order statistics were also considered by Genç [4], who in [5] estimated $P(X > Y)$ when X (strength) and Y (stress) are independent random variables from (TL) distribution. Bayesian inference was carried-out by Sindhu et al. [6] and Bayoud [7]. New classes of distributions based on TL were introduced by Rezaei et al. [13]. Zghoul [30] introduced plug-in estimators for the shape parameter of TL. Al-Zahrani [8] used the Anderson-Darling goodness of fit test to model the data with the TL distribution. As far as we know, this is the only published article that deals in part with the goodness of fit (GOF) for the TL distribution.

There are good reasons to consider TL distribution for lifetime analysis. First, the closed form of its cumulative distribution function makes the distribution mathematically attractive. Second, its hazard rate function takes various shapes including the bathtub shape, which is the shape of the hazard functions of many real lifetime data. In addition, the distribution is defined on a bounded domain which is the domain of many reliability applications.

Our interest in this article is to test the null hypothesis that the distribution of a random sample is the TL distribution given in (1). Up to our knowledge, no tests have been proposed for this purpose. However, some GOF tests in the literature, could be applied to any model, particularly the empirical distribution function based tests such as the Kolmogorov Smirnov (KS), Cramer-von Mises (CM), and Anderson Darling (AD) tests. The distributions or asymptotic distributions of GOF tests depend not only on the hypothesized model, but also on the parameters, whether known or unknown. When the parameters are unknown and need to be estimated from the sample, the estimation method also affects the performance of the test. Consequently, the performance of any GOF test would be affected by one or more of these factors. For example, the Shapiro-Wilk test, which works very well as a test for normality, has comparatively poor performance when used to test for exponentiality. Since most test statistics, especially when distribution parameters are unknown, do not have a closed form, quantiles and tests power calculations must be either approximated (possibly based on the asymptotic distribution) or simulated. The latter method is generally used because it works for all sample sizes.

GOF tests are generally based on characterizations that uniquely determine the distribution. For example, the moment generating function, when exists, uniquely determines the distribution. Test statistics can therefore be constructed by measuring the difference between the empirical and theoretical moment generating functions of the assumed model.

Main results

1- $\widehat{D}_{n,2}$, on average, outperforms the other four tests for small ($n = 10$) and moderate ($n = 20$) sample sizes, while $\widehat{D}_{n,1}$ average power outperforms the other tests for $n = 50$.

2- surprising that the KS , CM and AD tests show no significant sensitivity to variations in θ , while in general $\widehat{D}_{n,1}$ indicates an increase in power and $\widehat{D}_{n,2}$ indicates a decrease in power with θ . The same is true for $n = 10$ and 50 , also tells that, on the average, KS , CM , and AD have almost the same power, $\widehat{D}_{n,2}$ outperforms all tests for values of θ less than 2.5 , and $\widehat{D}_{n,1}$ outperforms all other tests for θ of value 3.0 or more.

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3-We notice that $\widehat{D}_{n,2}$ and AD tests outperform the other tests for skewness less than -1 , and that KS , CM , and AD , roughly, perform the same for other skewness values. Although $\widehat{D}_{n,2}$ appears to be superior to other tests for negative and small positive skewness alternatives, it is nevertheless inferior to all other tests for alternatives with a skewness of values of about 0.5 or more. In the latter case, $\widehat{D}_{n,1}$ does better than any other underlined test.

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Rate of the almost sure convergence of the conditional quantile estimate based on truncated, functional and mixing data

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Abstract: In this work, we propose a local linear estimator for the conditional distribution function in the case where the real response variable is subject to left-truncation by another random variable (r.v.) and the covariate is of functional type, that is to say has values in a infinite dimensional space. In this study, the locally linear modelling was used for the functional estimation by considering an estimator adapted to a polynomial of one degree. Under standard assumptions, we get the uniform almost sure (a.s.) convergence rate of this robust estimate when the sample is an α -mixing sequence. This approach can be applied in time series analysis to the prediction problem. Then, we deduce the uniform a.s. convergence of the obtained conditional quantile estimator .

Keywords: truncation, mixing data, conditional quantile, almost sure convergence.

2020 Mathematics Subject Classification: 62G08, 62R10, 62N99, 62F12.

Introduction

The statistical problems related to the study of functional random variables have been experiencing a growing interest in the literature for many years. The development of this theme of research is ongoing and motivated by the abundance of data measured on increasingly reliable grids, this is for example the case, in meteorology, medicine, ... etc. Hence, a very widely studied problem in functional statistics is the link between two variables, the main goal of which is to predict one of the variables (the real response variable) given a new value of the other (the functional explanatory variable). One of the ways to deal with this problem is the conditional quantile which involves the conditional distribution function. Notice that the nonparametric estimation based on the local linear approach for functional independent data was, for example, studied in [3] for the conditional quantile.

However, in reality, we can perceive information gaps at different analyses and studies scales. This lack of information is an impediment during the observation process and in the results phase. Most often, these lacks are called censoring and/or truncation, which cause disturbances in the observed data. Moreover, in practice, observed data can exhibit a dependence form. A large studied example in Time Series is the case of the α -mixing dependence. We cite for example [2] for papers dealing with such functional dependent and truncation data.

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The main aim of this work is to study, under general assumptions, the uniform a.s. convergence of the local linear estimator of the conditional distribution function under a truncated model with strong mixing condition and we dedicated to study the uniform convergence of a proposed estimator of conditional quantile.

Main results

Let (X_i, Y_i) for $i = 1, \dots, N$, be N identical and independent couples distributed as (X, Y) which takes its values in $\mathcal{F} \times \mathbb{R}$, where $\mathcal{F}$ is a semi metric space endowed with a semi metric d . The unknown distribution function (d.f.) of Y is denoted by F . In the remainder of the work, we suppose that $(X_n, Y_n)_{n \geq 1}$ is strongly mixing whose coefficient $\alpha(n)$. Let T be another r.v. which has unknown d.f. G and $(T_i)_{i=1, \dots, N}$ be a sample of i.i.d. random variables that are distributed as T . T is supposed independent of (X, Y) . N is unknown but deterministic. In the left truncation model, the lifetime Y_i and the truncation r.v. T_i are both observable only when $Y_i \geq T_i$. We denote $(Y_i, T_i), i = 1, 2, \dots, n (n \leq N)$ the actual observed sample of size n .

Recall that the conditional distribution function (d.f.) $F(y|x)$ is defined, for all $x \in \mathcal{F}$, by

$$F(y|x) = E(1_{\{Y \leq y\}} | X = x)$$

and an estimator in the case of left truncated data is proposed in [1], by

$$F_n(y|x) = \frac{\sum_{i,j=1}^n W_{ij}(x) H(h_H^{-1}(y - Y_j))}{\sum_{i,j=1}^n W_{ij}(x)}, \quad \left(\frac{0}{0} := 0 \right),$$

where

$$W_{ij}(x) = \Delta_{ij}(x) G_n^{-1}(Y_i) G_n^{-1}(Y_j),$$

with

$$\Delta_{ij}(x) := \beta(X_i, x) (\beta(X_i, x) - \beta(X_j, x)) K(h_K^{-1} d(X_i, x)) K(h_K^{-1} d(X_j, x)).$$

K is a standard univariate kernel function, H is a d.f. ($H(y) = \int_{-\infty}^y H'(u) du$ where H' is a kernel of type 0) and the bandwidth $h_K := h_{K,n}$ (resp. $h_H := h_{H,n}$) is a sequence of strictly positive real numbers which plays a smoothing parameter role and $\beta(.,.)$ is a known operator from $\mathcal{F} \times \mathcal{F}$ into $\mathbb{R}$ such that, $\forall x \in \mathcal{F}$, $\beta(x, x) = 0$.

In other hand, since for any $p \in (0, 1)$, the conditional quantile of order p of r.v. Y given $X = x$ is defined by

$$Q_p(x) = F^{-1}(p|x) = \inf\{y : F(y|x) \geq p\}.$$

So a natural local linear estimator of $Q_p(x)$ is given by

$$Q_{p,n}(x) = F_n^{-1}(p|x) = \inf\{y : F_n(y|x) \geq p\}.$$

The studies of the uniform almost sure convergence of $F_n(y|x)$ and $Q_{p,n}(x)$ on some subsets $[c, d]$ of $\mathbb{R}$ and $\mathcal{S}_{\mathcal{F}}$ of $\mathcal{F}$, such that $\mathcal{S}_{\mathcal{F}} \subset \bigcup_{k=1}^{d_n} B(x_k, r_n)$, where $x_k \in \mathcal{S}_{\mathcal{F}}$ and r_n (respectively d_n) is a sequence of positive real (respectively integer) numbers, gived the following results.

Theorem 100. *Under several hypotheses, we have*

$$\sup_{x \in \mathcal{S}_{\mathcal{F}}} \sup_{y \in [c, d]} |F_n(y|x) - F(y|x)| = O(h_K^{b_1} + h_H^{b_2}) + O_{a.s.} \left(\sqrt{\frac{\ln d_n}{n \Phi(h_K)}} \right).$$

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Theorem 101. *Under standard conditions in the context, we get*

$$\sup_{x \in \mathcal{S}_{\mathcal{F}}} |Q_{p,n}(x) - Q_p(x)| = O(h_K^{b_1}) + O(h_H^{b_2}) + O_{a.s} \left(\sqrt{\frac{\ln d_n}{n\Phi(h_K)}} \right).$$

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Comparative Evaluation of Computer Experiment Designs Versus New Computer Experiment Designs with area interactions

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Abstract: In this study, we compare optimality criteria between existing computer experiment designs and our proposed computer experiment designs with area interactions. We evaluate their quality in terms of coverage of the experimental space and prediction of interactions. This comparison aims to identify the strengths and weaknesses of each approach, to guide the selection of the most suitable method. The results could provide crucial insights for researchers and practitioners in their selection of experiment designs.

Keywords: Computer Experiments Designs; Point Processes; Area-Interaction Point Processes; Voronoi Tessellations; Markov Chain Monte Carlo (MCMC) Method; Metropolis-Hastings algorithm.

2020 Mathematics Subject Classification: 05B30, 60G55, 62K99.

Introduction

Advancements in modeling and the increase in computational power have led to the development of highly complex simulators. However, these simulators can be challenging to manage due to their large size. To address this issue, approximate functions can be used as replacements for the simulator. These functions are simpler and created using approximation or interpolation methods based on computer experiment designs.

In 2008[1], Franco introduced computer experiment designs based on Strauss point processes, which involve pairwise point interaction. In 2020 [2, 1] and 2023 [4], Elmossaoui et al. proposed computer experiment designs based on marked Strauss processes and connected component point processes. This work focuses on using the area point process to simulate n experiments using Markov chain Monte Carlo (MCMC) methods and the Metropolis-Hastings algorithm (M-H algorithm).

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Main results

Algorithm for constructing computer experiments designs using Markovian area-interaction point processes

The computer experiments designs proposed in this work are generated using the following algorithm.

Algorithm

```

• Initialization step: Choose an initial configuration (a design of experiments)  $X_0 = x$  according to a given probability distribution, for example, the uniform distribution.

for  $i = 1, 2, \dots, N_{MCMC}$  do
  for each configuration  $x$  do
    - Subdivide the configuration  $x$  into neighborhood zones  $\vartheta(x_k)$  for each point  $x_k$ , where  $k \in \{1, 2, \dots, n\}$ , using the Voronoi-Dirichlet tessellations.
    - For each neighborhood  $\vartheta(x_k)$ , simulate an experiment  $y_k$  according to the proposal distribution  $q \sim U_{\vartheta(x_k)}$ . Take  $y = x_i \cup \{y_1, y_2, \dots, y_n\}$ .
    - Choose a pair of points  $\{u, v\}$  from the configuration  $y$  uniformly at random, then choose  $u$  or  $v$  with probability  $1/2$  and remove it from  $y$  (i.e.
      
$$y = \begin{cases} y / \{u\} & \text{if } u \text{ is chosen} \\ y / \{v\} & \text{if } v \text{ is chosen} \end{cases}$$

      Repeat this step  $n$  times
    end
    Calculation of the acceptance probability  $a(x, y) = \min(1, \gamma^{m(U_r(x)) - m(U_r(y))})$ .
    Take  $x = \begin{cases} y & \text{with a probability } a \\ x & \text{with a probability } 1 - a \end{cases}$ 
  end
end

```

Convergence Study

The convergence of the algorithm mentioned above is ensured by the following results:

Theorem 102. *The Markov chain $(X_N)_{N \geq 0}$ obtained from the proposed construction algorithm is geometrically ergodic, and its kernel P realizes the simulation of the Area-Interaction Point Processes with density $\pi(x) = \alpha \beta^{n(x)} \gamma^{-m(U_r(x))}$. In other words, vP^n converges to π as n approaches infinity, where v is an initial distribution, and we have: $\lim_{n \rightarrow \infty} \|vP^n - \pi\| = 0$*

Comparison of numerical results:

in this section, a comparative analysis of point distributions in computer experiment designs is conducted, based on established criteria such as minimum distance (Mindist)[12], discrepancy (Disc) [13], coverage (Cov) and the R criterion [26]. The results are based on calculating these criteria for 100 designs, with a selection including Random Designs (RD), Latin Hypercube Designs (LH), Maximin Latin Hypercube Designs (MLHS), Strauss Designs (SD) [1], Maximum Entropy

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Designs (DMAX) , Marked Strauss Designs (MSD) [2], Connected Component Designs (CCD) [4] and Proposed Designs (AID). The figures below represent the results of various criteria in the form of box plots for three dimensions.

Figure 11: Box plots of quality criteria calculated for 100 designs with 50 points in 3 dimensions.

Box plots compare AID with MSD, LH, RD, MLHS, DMAX, and SD designs based on discrepancy, distance, coverage, and ratio criteria. AID excels in distance and ratio but shows weaknesses in discrepancy.

Conclusion

The use of computer experimental designs with air interaction provides flexibility, ensuring optimal space filling and uniform point distribution. Compared to traditional statistical approaches, the combination of these designs shows promising results.

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Probabilistic and statistical process monitoring for multivariate risk analysis

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Abstract: This research is focused on dynamic system modeling and control, utilizing Dynamic Principal Component Analysis (DPCA) and probability theory for process monitoring and quality control. This combined technique detects problems by evaluating system fluctuations, as indicated by the Hotelling T square (T2) and squared prediction error (SPE). The proposed computational method uses multivariate statistical process monitoring (MSPM) and advanced machine learning to estimate the quality of Blast Furnace (BF) processes.

Keywords: Mathematical modeling, probability theory, dynamic principal component analysis, data analysis.

2020 Mathematics Subject Classification: 37A50, 60E05, 62J99.

Introduction

Many industrial operations generate large amounts of data. To diagnose and monitor a complex system, we use this information to detect and forecast the emergence of specific abnormal system behavior. Critical indicators of system failures and operating conditions encompass factors such as equipment wear or failure, extreme process faults, and contemporary automation. As a result, a crucial goal in the industry has been to develop efficient technologies for prompt fault detection and identification.

Fault detection and control charting involve comparing measured performance, such as different model parameters, with predefined performance expectations referred to as thresholds. When the deviation exceeds the control limit, a defect is signaled, or if the damage is significant, all system operation are stopped. The accuracy of fault detection from data can be enhanced by employing techniques that reduce data dimensionality, such as principal component analysis (PCA) and dynamic principal component analysis (DPCA). These strategies provide reduced dimensional representations that generalize better to new system rather than representations using the complete data dimensionality [1, 2, 3]. In this work we propose a new hybrid approach based on probability distribution and multivariate statistical process monitoring (MSPM) together with advanced machine learning to explores the simultaneous estimation of different risks associated with eight variables in nonlinear dynamic process.

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Main results

In this section, we present the main results, including the evolution of eigenvalues distribution, the principal components importance and the probability of risk occurrence based T2 and SPE.

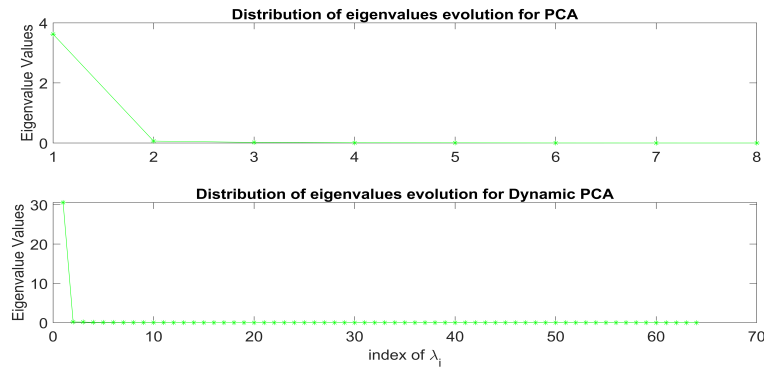


Figure 12: Distribution of eigenvalues evolution

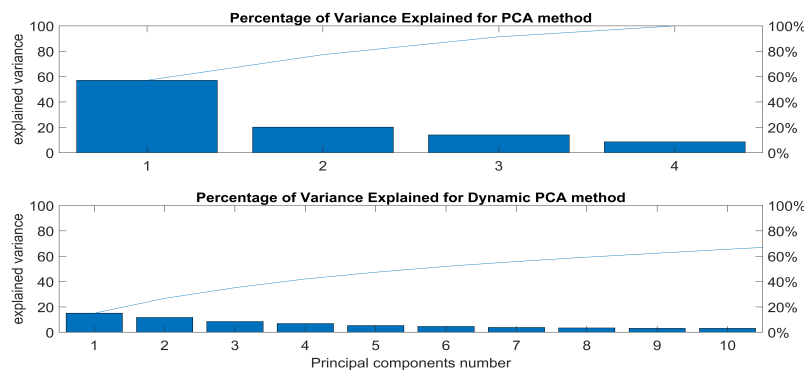


Figure 13: Principal components importance

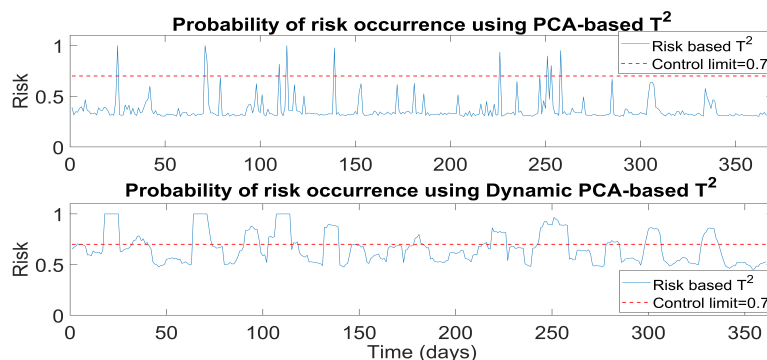


Figure 14: Probability of risk occurrence based T2

Comments: Figures 12 and 13 show a high correlation in the process for eigenvalues and PCs importance indicators. This is evident as utilizing just one PC allows us to capture over 95% of the variance, and the number of significant PCs is selected by using this different indicators. Figures 14 and 15 depict fluctuations in the T2 and SPE values over time (days), ranging from 0 to 1, with occasional spikes that exceed the control limit. These spikes indicate anomalies or events

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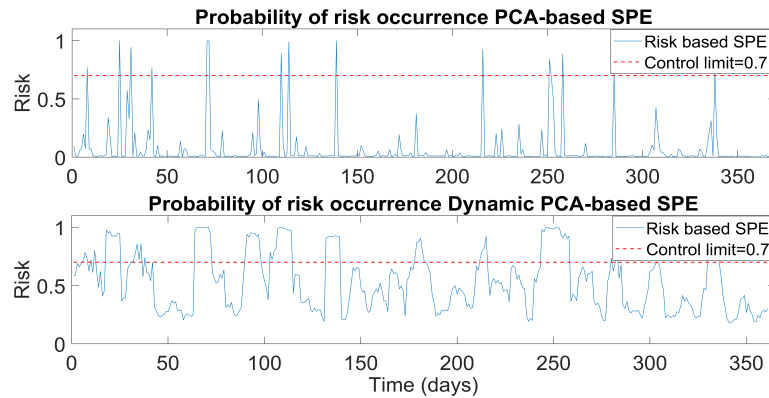


Figure 15: Probability of risk occurrence based SPE

that fall outside the expected range of the monitored process. Additionally, PCA gave lower false alarm rate than DPCA technique because there is abnormal points PCA shows them as normal.

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Calibration problem with alpha-mixing random data

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Résumé. Le but de ce travail est de proposer une méthode stochastique pour avoir des solutions approchées pour le problème de calibration linéaire. Nous établissons des inégalités exponentielles de la probabilité de la distance entre les solutions approchées et la solution exacte.

Mots-clés. Approche stochastique, Problème inverse, Problème de calibration, Donnée aléatoire...

Abstract. The aim of this article is to propose a stochastic method which gives an estimated solution for a linear calibration problem. We establish exponential inequalities for the probability of the distance between the approximate solutions and the exact one.

Keywords. Stochastic approach, inverse problem, Calibration problem, Random data ...

Introduction

Calibration, also called inverse regression, is a classical problem which appears often in a regression setup under fixed design. In simple linear regression, it is assumed that two variables x and y are linearly related with unknown intercept and slope parameters. In particular, the exogenous variable x is assumed to be precisely measurable and the endogenous variable y is assumed to be a random variable depending on x via a linear function.

In this article, we propose a stochastic method which gives an estimated solution for a calibration problem. We establish exponential inequalities for the probability of the distance between the approximate solutions and the exact solution of an unknown linear operator equation with exact second member. These inequalities will enable us to prove the almost complete convergence (a.co) with convergence rate for the sequence of approximate solutions. We finally build a confidence domain for the so-mentioned exact solution.

First section

We consider the following application

$$\begin{aligned} A : L^2[a, b] &\rightarrow L^2[a, b] \\ x &\rightarrow Ax \end{aligned}$$

where

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$$Ax(t) = \int_a^b K(t,s)x(s)ds \quad (328)$$

This integral can be written in this abstract form

$$Ax = u \quad (329)$$

A is a compact operator and self-adjoint, then we can not talk about the classical solution.

We use the works of Cuevas et al (2002), we get the coherent estimator of A ($\widehat{A}_N$).

For the second member u , by performing several experiments with dependent variables, The strong law of large numbers gives the empirical averages $\bar{u}$ as an exhaustive natural estimate.

So the problem is to solve the following equation

$$\widehat{A}_N x = \bar{u}_n \quad (330)$$

This is a typical feature of an ill-posed problem. For this, Lavrentiev et al. (1986, 2003) has replaced equation (330) by another one, precisely

$$(\widehat{A}_N + \gamma I)x = \bar{u}_n \quad (331)$$

where I is the operator identity and γ a positive real. The solution $\bar{X}_n^\gamma$ of (331) is defined by

$$\bar{X}_n^\gamma := (\widehat{A}_N + \gamma I)^{-1}(\bar{u}_n). \quad (332)$$

Second section

Results

For the model defined by (329), we assumed that the following assumptions are fulfilled :

1) let $\gamma = \gamma(N, n)$ a sequence of parameters be chosen such that, as

$n, N \rightarrow \infty$,

$\gamma \rightarrow 0$, $\frac{\log n}{\gamma \sqrt{n}} \rightarrow 0$, $\ln n \gamma^2 \rightarrow +\infty$, $\frac{\|\widehat{A}_N - A\|}{\gamma^2} \rightarrow 0$ and $S_n^{-q} \gamma^{-q} n (\ln n)^{\frac{3q}{2}-2} \rightarrow 0$.

2) The error trajectories $e_{N+1}, \dots, e_{N+n}$ are algebraically α -mixing.

Then, for all $\epsilon > 0$,

$$P\{\|\bar{X}_n^\gamma - x_{ex}\| > \epsilon\} \leq C \exp\left(-\frac{n\epsilon_0^2 \gamma^2 \sigma^2}{8} + \frac{\epsilon_0^4 \gamma^4 \sigma^4}{64}\right) + 2^q n (\ln n)^{\frac{3q}{2}-2} (\epsilon_0 \sigma)^{-q} S_n^{-q} \gamma^{-q} \quad (333)$$

with

$$\delta = 1 - \|(A + \alpha I)^{-1}\| \|\widehat{A}_N - A\|$$

$$q = \frac{(b+1)p}{b+p}, \quad p > 2 \quad \text{and} \quad b > 1$$

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and

$$s_n^{-2} = \sum_{i=1}^n \sum_{j=1}^n |cov(\|\xi_i\|, \|\xi_j\|)|$$

ie: We have prove the convergence almost completely of the probability between the approximates solutions and the exact one.

Third section

In this section, we check the validity for our result by some numerical examples. (with the commercial Package MATLAB.)

Conclusion

In this work, we have established exponential inequalities for the probability of the distance between approximate solutions and the exact one for a linear calibration problem. Some numerical examples were solved as illustrations.

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Time series forecasting using Deep Learning

Hybrid model (ARIMA-LSTM)

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Abstract: Recent research in time series forecasting using deep learning suggests that it could be a promising alternative to traditional linear methods, with mixed results when comparing ARIMA and LSTM models in terms of predictive performance. In this study, a hybrid approach combining ARIMA and LSTM models was proposed to leverage the unique strengths of each model in both linear and nonlinear modeling. Experimental results on real-world datasets show that combining the models can be an effective means to improve prediction accuracy beyond what can be achieved with either model alone.

Keywords: Time series, Deep learning, Forecasting.

2020 Mathematics Subject Classification: 62M10 , 91B84, 62P12, 62M20.

Introduction

Weather forecasting has become an important area of research in recent decades. It is always best to establish a linear relationship between the input weather data and the corresponding target data. But with the discovery of nonlinearity in the nature of meteorological data, attention has shifted to wards the nonlinear prediction of meteorological data. So it has two parts as mentioned below:

1. Rain prediction, i.e. whether it will rain or not.
2. Forecast of different meteorological factors that will help us predict rainfall. To achieve this second objective, a hybrid time series prediction model is built, which will try to learn the relationship between the weather factor and time.

Recently, the evolution of information technology has contributed to generating massive amounts of data on one hand, and preparing the necessary computing support for calculations on the other hand. This evolution has also introduced the implementation of more advanced learning models than those offered by classical statistics.

Neural networks then reached their peak and surpassed traditional models, whether for regression, classification, or more advanced problems such as multimedia data processing (image, sound)

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thanks to deep learning. The particularity of neural networks is their tendency to adapt to the features created by the data itself.

This data-oriented approach facilitates learning despite the loss of some essential features in classical statistics such as linearity, normality, homoscedasticity, or independence of observations.

In this presentation, we attempt to combine the effects of these two forms of modeling to produce a result that could surpass the individual performance of each model. We present a hybrid modeling approach for time series analysis. This approach combines Autoregressive Integrated Moving Average (ARIMA), introduced by Box and Jenkins, and artificial neural networks, specifically Long Short-Term Memory (LSTM) models. Combining the two helps us capture certain patterns that would be inaccessible to either model without the support of the other, and provide satisfactory results in time series prediction.

Main results

In this communication we presented the advantage of ARIMA-LSTM hybridization to improve the accuracy of time series forecasts. This work proposes a forecasting technique by separating a time series dataset into linear and nonlinear components. Then, autoregressive integrated moving average (ARIMA) and artificial neural network (ANN) models are used to separately recognize and predict the reconstructed detailed and approximate components, respectively. In this way, the proposed approach uses the unique strengths of ARIMA and LSTM to improve forecast accuracy. The results clearly show that the proposed method achieves the best prediction accuracies for the series studied.

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Estimation of $P(X > Y)$ for asymmetric kernels

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Abstract: This paper proposes the nonparametric kernel method for the stress strength parameter estimation. The class of asymmetric kernels based on log-symmetric distributions kernels is considered because of its several interesting properties and flexibility. Some asymptotic properties, such as bias, variance and mean squared error are established for the proposed estimator.

Keywords: Stress-Strength model; Reliability; Asymmetric kernels based on log-symmetric distributions; nonparametric estimation.

2020 Mathematics Subject Classification: 62G05, 62G07, 62G20.

Introduction

The stress strength model (SST) is used primarily in reliability engineering, but also in psychology, economics and medicine. The stress experienced by a component is often represented by the random variable Y and the strength of the component available to overcome the stress is represented by the random variable X . The system fails if $Y > X$. Model reliability is therefore defined as the probability $R = P(X > Y)$, which can be considered as a measure of component performance. Typically, R is referred to as SST reliability, as it shows the relationship between stress and component strength. To study SST models, many works make use of parametric distribution models such as weibull, exponentiated inverted weibull, generalized logistic, generalized exponential, Burr type III distributions, see, for example [1], [2] and [3]. The maximum likelihood and Bayesian approaches are the most common procedures used to estimate the reliability of these model. As good alternative to parametric approach, is the nonparametric kernel method, which does not require prior knowledge of the functional form of distributions, and is easy to implement in practice.

The aim of this paper is to estimate the reliability of the SST model, without any restrictions on density, using the asymmetric kernels based on log-symmetric distributions kernels (LS) introduced recently by Makhloufi et al [4].

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This paper is organized as follows: In section 2, we briefly review the LS kernel estimators of probability density functions (PDF). In section 3, we present the nonparametric reliability estimator of the SST model based on LS kernels (SST-LS) and develop some asymptotic properties of the proposed estimator.

A short review on PDF estimation using LS kernels

This section presents a brief recall on LS kernel estimator of the PDF. Given a random sample $X_1, \dots, X_n$ the LS kernel estimator of an unknown PDF f with non-negative support $(0, \infty)$ is given by Makhloufi et al.[4]

$$\hat{f}_{LS}(x) = \frac{1}{n} \sum_{i=1}^n \frac{1}{\sqrt{h}X_i} S\left(\frac{1}{h} \{\log(X_i) - \log(x)\}^2\right), \quad x > 0, \quad (334)$$

where $x > 0$ is the target (point where the density is estimated), $h > 0$ is a bandwidth (or smoothing parameter), and S is a generator. For more details; see Makhloufi et al [4].

The expressions of the bias and variance for $\hat{f}_{LS}$ are derived by Makhloufi et al.[4], under the following conditions

C1. The function f is twice differentiable and its second derivative is continuous and bounded,

C2. The bandwidth $h = h_n$ satisfies $\lim_{n \rightarrow \infty} h = 0$ and $\lim_{n \rightarrow \infty} nh^{3/2} = \infty$.

The asymptotic bias and variance of the estimator are given by

$$Bias\{\hat{f}_{LS}(x)\} = \frac{E(Z^2)}{2} \{xf'(x) + x^2 f''(x)\}h + o(h) \quad (335)$$

$$Var\{\hat{f}_{LS}(x)\} = \begin{cases} \frac{C_{r^2}}{nh^{1/2}} x^{-1} f(x) + o\left(\frac{1}{nh^{1/2}}\right) & si \quad x/h \rightarrow \infty \\ O\left(\frac{1}{nh^{3/2}}\right) & si \quad x/h \rightarrow q \end{cases} \quad (336)$$

where $E(Z^2)$ is the moment of order of 2 of Z with Z follows a log-symétriques distributions and $C_{r^2} = \int_{-\infty}^{+\infty} S^2(z^2)dz$ is the normalization constant for a density generator $S^2(\cdot)$ and $q > 0$ is a constant.

SST kernel estimator

The reliability R of the SST model is defined as the probability $P(X > Y)$ that a component fails if the stress Y which is applied to it exceeds the strength X and there is no failure otherwise. It is therefore a performance measurement of the component. X and Y are assumed independent random variables (r.v.) with PDFs f and g , respectively. So, R can be written as

$$R = P(X > Y) = \int_0^{+\infty} \int_0^x f(x)g(y)dydx = \int_0^{+\infty} f(x)G(x)dx,$$

where, $G(x) = \int_0^x g(y)dy$ is the cumulative distribution function (cdf) of Y calculated at x .

Let $X_1, X_2, \dots, X_n$ be a random sample on X from unknown pdf f , and let $Y_1, Y_2, \dots, Y_m$ be a random sample on Y from unknown pdf g and unknown cdf G , where n and m are the respective sizes of the two samples. The unknown pdfs f and g can be estimated by the kernel estimators given in

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(1). In addition, the unknown cdf G can be estimated by

$$\hat{G}_{LS}(x) = \int_0^x \hat{g}_{LS}(y) dy = \frac{1}{n} \sum_{j=1}^m \int_0^x K_{LS(S,y,h_m^{1/2})}(Y_j) dy$$

A natural STT-LS kernel estimator is simply given by

$$\hat{R}_{LS} = \frac{1}{nm} \sum_{i=1}^n \sum_{j=1}^m \int_0^{+\infty} \int_0^x \frac{1}{\sqrt{h_n h_m} X_i Y_j} S\left(\frac{1}{h} \{\log(X_i) - \log(x)\}^2\right) S\left(\frac{1}{h} \{\log(Y_j) - \log(y)\}^2\right) dy dx$$

Under the conditions C1, C2 given previously and the following assumptions

$$A1 \int_0^{\infty} G(x) \{x f'(x) + x^2 f''(x)\} dx < \infty$$

$$A2 \int_0^{\infty} G^2(x) x^{-1} f(x) dx < \infty$$

The bias, variance and mean square errors (MSE) of this estimator are given by

$$Bias\{\hat{R}_{LS}\} = \frac{E(Z^2)h_n}{2} \int_0^{\infty} G(x) \{x f'(x) + x^2 f''(x)\} + o(h) dx \quad (337)$$

$$Var\{\hat{R}_{LS}\} = \begin{cases} \frac{C_{r^2}}{nh^{1/2}} \int_0^{\infty} x^{-1} f(x) G^2(x) dx + o\left(\frac{1}{nh^{1/2}}\right) & \text{si } x/h \rightarrow \infty \\ O\left(\frac{1}{nh^{3/2}}\right) & \text{si } x/h \rightarrow q \end{cases} \quad (338)$$

$$\begin{aligned} MSE\{\hat{R}_{LS}\} &= Bias^2\{\hat{R}_{LS}\} + Var\{\hat{R}_{LS}\} \\ &= \frac{E^2(Z^2)h^2}{4} \left[\int_0^{+\infty} G(x) \{x f'(x) + x^2 f''(x)\} dx \right]^2 + \frac{C_{r^2}}{nh^{1/2}} \int_0^{+\infty} \frac{1}{x} f(x) G^2(x) dx \\ &\quad + o\left(h^2 + \frac{1}{nh^{1/2}}\right) \end{aligned} \quad (339)$$

By minimizing the MSE (6) on h , we obtain the optimal bandwidth

$$h_{opt} = \left[\frac{C_{r^2} \int_0^{+\infty} x^{-1} f(x) G^2(x) dx}{E^2(Z^2) \left(\int_0^{+\infty} \{x f'(x) + x^2 f''(x)\} G(x) dx \right)^2} \right]^{2/5} n^{-2/5} \quad (340)$$

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Application of the CNLRM Model with Error Correction Mechanism

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Abstract: In this work, we apply the CNLRM model to long-term relationships using an error correction mechanism generated from a steady-state process. We illustrate its application to various datasets with key tests and related relationships. The results of this study allow for the assessment of the effects of transient or permanent shocks on variables. This technique is applied to simulation data using Python software.

Keywords: stationarity, error correction model, causality, CNLRM

2020 Mathematics Subject Classification: 00A69, 11B34, 49M37, 62G05.

Introduction

The classical normal linear regression model (CNLRM) is a widely used statistical method for modeling the relationship between two variables, assuming that the relationship is linear and that errors are normally distributed with constant variance. It is a simple yet powerful method that can be used for prediction and for understanding the relationship between variables.

However, the CNLRM is not without its limitations. One of the main challenges is overfitting, where the model becomes too complex and fits to noise in the data rather than the underlying relationship between the variables. This can lead to poor predictions and misleading conclusions.

To address this challenge, an error correction mechanism can be used to control the complexity of the model and avoid overfitting. This involves adding regularization terms to the ordinary least squares (OLS) method used to estimate the model coefficients. These penalties impose constraints on the size of the coefficients, which can lead to a simpler and more accurate model.

The application of CNLRM with an error correction mechanism has many practical uses, such as in finance, economics, and engineering. For example, it can be used to model the relationship between stock prices and economic indicators, to predict sales based on marketing expenses, or to understand the impact of weather conditions on crop yields. By using this method, researchers and practitioners can develop precise and reliable models that can inform decision-making and improve outcomes.

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Main results

A simple dynamic model of a short-term adjustment model is given by:

$$y_t = \alpha_0 + \gamma_0 x_t + \gamma_1 x_{t-1} + \alpha_1 y_{t-1} + \varepsilon_t \quad (341)$$

Where y_t is the dependent variable, x_t is the independent variable, y_{t-1} and x_{t-1} are lagged values of y_t and x_t respectively, $\alpha_0, \alpha_1, \gamma_0, \gamma_1$ are parameters, and ε_t is the error term assumed to follow $\varepsilon_t \sim iidN(0, \sigma^2)$

The problems associated with using the short-term model are resolved by estimating the first differences of equation (341) to obtain:

$$\Delta y_t = \alpha_0 + \gamma_0 \Delta x_t + \gamma_1 \Delta x_{t-1} + \alpha_1 \Delta y_{t-1} + \varepsilon_t \quad (342)$$

However, this approach presents problems of: Loss of information regarding long-term equilibrium; and Economic theory being undifferentiated.

The solution is to adopt the error correction mechanism (ECM) formulation of the dynamic structure. We set up the ECM as follows:

$$y_t = \alpha_0 + \gamma_0 x_t + \gamma_1 x_{t-1} + \alpha_1 y_{t-1} + \varepsilon_t \quad (343)$$

With reparametrization, equation (343), we obtain:

$$\Delta y_t = \gamma_0 x_t - (1 - \alpha_1)[y_{t-1} - \beta_0 - \beta_1 x_{t-1}] + \varepsilon_t \quad (344)$$

This is the ECM, where $\beta_0 = \frac{\alpha_0}{(1-\alpha_1)}$ and $\beta_1 = \frac{(\gamma_0+\gamma_1)}{(1-\alpha_1)}$. with $-(1 - \alpha_1)$ as the adjustment speed, and $\varepsilon_{t-1} = y_{t-1} - \beta_0 - \beta_1 x_{t-1}$ as the error correction mechanism that measures the system's distance from equilibrium.

Numerical Simulation

The data consist of random values (100 variables) of the independent variable X and the dependent variable Y , a random noise $\varepsilon \sim \mathcal{N}(0, 0.5)$; Data points are then plotted using a scatter plot (see Figure 16). The R^2 value is also calculated to evaluate the quality of the model fit. The purpose of the simulation is to demonstrate the use of CNLRM with the error correction mechanism and how it can be used to model a linear relationship between two variables while avoiding overfitting. The application is performed using the Python software.

Figure 16: CNLRM Model with Error Correction Mechanism

- The graph shows that the model reasonably captures the underlying linear relationship between the variables.

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- The $R^2 = 0.404$ indicates a moderate linear relationship between the variables, yet there is still a significant amount of unexplained variability in variable Y that is not accounted for by the model. This could be due to the presence of other factors affecting variable Y that are not included in the model, or randomness in the data.
- The simulation provides a useful demonstration of how the CNLRM with error correction mechanism can be applied to model the relationship between two variables and make predictions based on the model.

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Existence and uniqueness of asymptotically (ω, c) -periodic mild solutions of a semilinear stochastic differential equation

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Abstract: This work aims to introduce and study new concepts of square mean (ω, c) -periodic limit and square mean asymptotically (ω, c) -periodic stochastic processes. An application of the obtained theoretical result to asymptotically (ω, c) -periodic solutions of a stochastic differential equation driven by a Brownian motion is given.

Keywords: Square mean asymptotically (ω, c) -periodic stochastic process, Square mean (ω, c) -periodic limit stochastic process, Stochastic differential equation, Semigroup mild solution.

2020 Mathematics Subject Classification: 34C25, 34C27, 60H30, 34 F05.

Introduction

Let E be a complex Banach space, $\omega > 0$ and $c \in \mathbb{C} \setminus \{0\}$. The class of (ω, c) -periodic functions, denoted by $P_{\omega, c}(\mathbb{R} : E)$, has been introduced and investigated by E. Alvarez, A. Gómez and M. Pinto in [1]. Let us recall that a continuous function $f : \mathbb{R} \rightarrow E$ is said to be (ω, c) -periodic if and only if $f(t + \omega) = cf(t)$ for all $t \in \mathbb{R}$. The class of ω -periodic limit functions has recently been introduced and investigated by R. Xie and C. Zhang [4]. Let $(\mathbb{H}, \|\cdot\|)$ be a real separable Hilbert space and $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbb{P})$ be a filtered complete probability space. Denote by $L^2(\mathbb{P}, \mathbb{H})$ the Banach space of all strongly measurable square integrable $\mathbb{H}$ -valued random variables equipped with its natural norm $\|X\|_2 = \left(\int_{\Omega} \|X\|^2 d\mathbb{P}\right)^{\frac{1}{2}}, \forall X \in L^2(\mathbb{P}, \mathbb{H})$. By $\mathcal{C}(\mathbb{R}_+, L^2(\mathbb{P}, \mathbb{H}))$ (resp. $\mathcal{CB}(\mathbb{R}_+, L^2(\mathbb{P}, \mathbb{H}))$) we denote the complex Banach space of all continuous (resp. bounded and continuous) stochastic processes from $\mathbb{R}_+$ into $L^2(\mathbb{P}, \mathbb{H})$, equipped with the norm $\|X\|_{2, \infty} = \left(\sup_{t \in \mathbb{R}_+} (\mathbb{E}\|X(t)\|^2)\right)^{\frac{1}{2}}$. The class $PL_{\omega}(\mathbb{R}_+, L^2(\mathbb{P}, \mathbb{H}))$ of square mean (ω, c) -periodic limit stochastic processes was introduced and studied by S. M. Manou-Abi and W. Dimbour, see [2] and [3]. A stochastic process $X \in \mathcal{CB}_{\omega}(\mathbb{R}_+, L^2(\mathbb{P}, \mathbb{H}))$ is said to be square mean ω -periodic limit, if there exists $\omega > 0$ such that $\lim_{n \rightarrow +\infty} \mathbb{E}\|X(t + n\omega) - \tilde{X}(t)\|^2 = 0$, is well defined in $L^2(\mathbb{P}, \mathbb{H})$ for each $t \geq 0$ and for some stochastic process $\tilde{X} : \mathbb{R}_+ \rightarrow L^2(\mathbb{P}, \mathbb{H})$.

In this work, we will introduce two new classes of square mean (ω, c) -periodic limit and asymptotically (ω, c) -periodic stochastic processes. Then we will investigate their qualitative properties

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in order to study the existence of square mean asymptotically (ω, c) -periodic mild solution to stochastic differential equation driven by Brownian motion.

Main results

Square mean (ω, c) -periodic limit and square mean asymptotically (ω, c) -periodic stochastic processes.

Let $\omega > 0$ and $c \in \mathbb{C} \setminus \{0\}$. Using the principal branch of the complex Logarithm, we define $c^{\frac{t}{\omega}} := \exp\left(\left(\frac{t}{\omega}\right) \log(c)\right)$ and we use the following notations $c^\wedge(t) := c^{\frac{t}{\omega}}$ and $|c|^\wedge(t) := |c^\wedge(t)| = \exp\left(\frac{t}{\omega} \log(|c|)\right)$. Let $c \in \mathbb{C} \setminus \{0\}$ and $\omega > 0$. We start by introducing the following space:

$$\mathcal{CB}_{\omega,c}(\mathbb{R}_+, L^2(\mathbb{P}, \mathbb{H})) := \left\{ f \in C(\mathbb{R}_+, L^2(\mathbb{P}, \mathbb{H})) : \sup_{t \in \mathbb{R}_+} (\mathbb{E} \|c^\wedge(-t) f(t)\|^2) < \infty \right\}.$$

Theorem 103. $\mathcal{CB}_{\omega,c}(\mathbb{R}_+, L^2(\mathbb{P}, \mathbb{H}))$ equipped with $\|X\|_{2,\omega,c} = \|c^\wedge(-\cdot)X(\cdot)\|_{2,\infty}$ is a Banach space.

Definition 35. A stochastic process $X \in \mathcal{CB}_{\omega,c}(\mathbb{R}_+, L^2(\mathbb{P}, \mathbb{H}))$ is said to be square mean (ω, c) -periodic, if there exists $\omega > 0$ such that $\mathbb{E} \|c^\wedge(-t)(X(t+\omega) - cX(t))\|^2 = 0$. The collection of those square mean (ω, c) -periodic stochastic processes will be denoted by $P_{\omega,c}(\mathbb{R}_+, L^2(\mathbb{P}, \mathbb{H}))$.

Definition 36. Let $c \in \mathbb{C} \setminus \{0\}$. A stochastic process $X \in \mathcal{CB}_{\omega,c}(\mathbb{R}_+, L^2(\mathbb{P}, \mathbb{H}))$ is said to be square mean (ω, c) -periodic limit if there exists $\omega > 0$, such that $\lim_{n \rightarrow +\infty} \mathbb{E} \|c^\wedge(-t)(c^{-n}X(t+n\omega) - \tilde{X}(t))\|^2 = 0$ is well defined in $L^2(\mathbb{P}, \mathbb{H})$ for each $t \geq 0$ and for some stochastic process $\tilde{X} : \mathbb{R}_+ \rightarrow L^2(\mathbb{P}, \mathbb{H})$.

Proposition 9. If $X \in PL_{\omega,c}(\mathbb{R}_+, L^2(\mathbb{P}, \mathbb{H}))$, then $c^\wedge(-\cdot)\tilde{X}(\cdot) \in PL_{\omega,c}(\mathbb{R}_+, L^2(\mathbb{P}, \mathbb{H}))$.

Theorem 104. $PL_{\omega,c}(\mathbb{R}_+, L^2(\mathbb{P}, \mathbb{H}))$ equipped with the norm $\|\cdot\|_{2,\omega,c}$ is a Banach space.

The space of stochastic processes (ω, c) -vanishing at infinity is defined by

$$\mathcal{C}_{0,\omega,c}(\mathbb{R}_+, L^2(\mathbb{P}, \mathbb{H})) = \left\{ X \in \mathcal{CB}_{\omega,c}(\mathbb{R}_+, L^2(\mathbb{P}, \mathbb{H})) : \lim_{t \rightarrow +\infty} \mathbb{E} \|c^\wedge(-t)X(t)\|^2 = 0 \right\}.$$

Definition 37. Let $c \in \mathbb{C} \setminus \{0\}$. A process $X \in \mathcal{CB}_{\omega,c}(\mathbb{R}_+, L^2(\mathbb{P}, \mathbb{H}))$ is said to be square mean asymptotically (ω, c) -periodic if $X = Y + Z$ where $Y \in P_{\omega,c}(\mathbb{R}_+, L^2(\mathbb{P}, \mathbb{H}))$ and $Z \in \mathcal{C}_{0,\omega,c}(\mathbb{R}_+, L^2(\mathbb{P}, \mathbb{H}))$. By $AP_{\omega,c}(\mathbb{R}_+, L^2(\mathbb{P}, \mathbb{H}))$ we denote the space of all square mean asymptotically (ω, c) -periodic stochastic processes.

Theorem 105. The space $AP_{\omega,c}(\mathbb{R}_+, L^2(\mathbb{P}, \mathbb{H}))$ is a closed Banach subspace of $PL_{\omega,c}(\mathbb{R}_+, L^2(\mathbb{P}, \mathbb{H}))$.

An existence result of square mean asymptotically (ω, c) -periodic mild solutions to some stochastic differential equation.

Consider the following semilinear stochastic differential equation

$$\begin{cases} dX(t) = AX(t)dt + f(t, X(t))dt + g(t, X(t))dB(t), & t \in \mathbb{R}_+ \\ X(0) = X_0, & X_0 \in L^2(\mathbb{P}, \mathbb{H}) \end{cases} \quad (345)$$

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where $A : D(A) \subset L^2(\mathbb{P}, H) \longrightarrow L^2(\mathbb{P}, H)$ is a closed linear operator and $f, g : \mathbb{R}_+ \times L^2(\mathbb{P}, \mathbb{H}) \longrightarrow L^2(\mathbb{P}, \mathbb{H})$ are jointly Lipschitz continuous and bounded functions, $(B(t))_{t \in \mathbb{R}_+}$ is a two-sided standard one-dimensional Brownian motion with values in $\mathbb{H}$, $\mathcal{F}_t$ -adapted and satisfying the weak Markov property. We impose the following hypotheses:

(H1) : A is an infinitesimal generator of an exponentially stable semigroup $(T(t))_{t \in \mathbb{R}_+}$ defined on $\mathbb{H}$, i.e., there exist $M > 0, \delta > 0$: $\|T(t)\| \leq Me^{-\delta t}$, $t \geq 0$.

(H2) : There exist $L_f > 0$ and $L_g > 0$ such that

$$\mathbb{E} \|f(t, X) - f(t, Y)\|^2 \leq L_f \mathbb{E} \|X - Y\|^2, \forall t \in \mathbb{R}_+, \forall X, Y \in L^2(\mathbb{P}, \mathbb{H})$$

$$\mathbb{E} \|g(t, X) - g(t, Y)\|^2 \leq L_g \mathbb{E} \|X - Y\|^2, \forall t \in \mathbb{R}_+, \forall X, Y \in L^2(\mathbb{P}, \mathbb{H})$$

The following generalizes the Definition 2.2.

Definition 38. Let $c \in \mathbb{C} \setminus \{0\}$. A jointly stochastic process $f \in \mathcal{CB}_{\omega, c}(\mathbb{R}_+ \times L^2(\mathbb{P}, \mathbb{H}), L^2(\mathbb{P}, \mathbb{H}))$ is said to be square mean (ω, c) -periodic limit in $t \in \mathbb{R}_+$ uniformly in $X \subset K$ where K is a bounded subset of $L^2(\mathbb{P}, \mathbb{H})$, if there exists $\omega > 0$ such that

$$\lim_{n \rightarrow \infty} \mathbb{E} \|c^{\wedge}(-t)(c^{-n}f(t + n\omega, X(t))) - \tilde{f}(t, X(t))\|^2 = 0,$$

is well defined in $L^2(\mathbb{P}, \mathbb{H})$ for each $t \geq 0$ and for some stochastic process $\tilde{f} : \mathbb{R}_+ \times L^2(\mathbb{P}, \mathbb{H}) \longrightarrow L^2(\mathbb{P}, \mathbb{H})$. By $PL_{\omega, c}(\mathbb{R}_+ \times L^2(\mathbb{P}, \mathbb{H}), L^2(\mathbb{P}, \mathbb{H}))$ we denote the space of these square mean (ω, c) -periodic limit stochastic processes.

Definition 39. An $\mathcal{F}_t$ -progressively measurable process $\{X(t)\}_{t \in \mathbb{R}_+}$ is said to be a mild solution of (345) if it satisfies the following stochastic integral equation:

$$X(t) = T(t)X_0 + \int_0^t T(t-s)f(s, X(s))ds + \int_0^t T(t-s)g(s, X(s))dB(s)$$

Lemma 16. Let $X \in PL_{\omega, c}(\mathbb{R}_+, L^2(\mathbb{P}, \mathbb{H}))$ such that $\lim_{n \rightarrow +\infty} \mathbb{E} \|c^{\wedge}(-t)(c^{-n}X(t + n\omega) - \tilde{X}(t))\|^2 = 0$ is well defined for each $t \in \mathbb{R}_+$. If $\lim_{n \rightarrow +\infty} \mathbb{E} \|c^{\wedge}(-t)(c^{-n}X(t + n\omega) - \tilde{X}(t))\|^2 = 0$ uniformly on $\mathbb{R}_+$, then $X \in AP_{\omega, c}(\mathbb{R}_+, L^2(\mathbb{P}, \mathbb{H}))$.

Theorem 106. Let $f, g \in PL_{\omega, c}(\mathbb{R}_+ \times L^2(\mathbb{P}, \mathbb{H}), L^2(\mathbb{P}, \mathbb{H}))$. Under assumptions (H1), (H2), equation (345) has a unique mild solution $X \in AP_{\omega, c}(\mathbb{R}_+, L^2(\mathbb{P}, \mathbb{H}))$ whenever $2M^2(L_f \frac{1}{\delta^2} + L_g \frac{1}{\delta}) < 1$.

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Statistical inference of the Rayleigh distribution under records values using E-Bayesian method

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Abstract: In this paper, we consider the problem of estimation of the parameter and reliability function for the Rayleigh distribution using record values. The estimation methods used are the Bayesian and the E-Bayesian methods. The E-bayesian estimators obtained are compared using expected posterior mean squared error (E-PMSE). We end with a real data set to illustrate the results obtained.

Keywords: Rayleigh distribution, records values, Reliability function, E-Bayesian estimation, E-pmse.

2020 Mathematics Subject Classification: Primary xxxxxx, xxxxx, xxxxx.

Introduction

Chadler introduced the study of record values and documented many basic properties of records. Record values can be viewed as order statistics from sample whose size is determined by values and the order of occurrence of observations. In a little over thirty years, a large number of publications devoted to records have appeared. This is possibly due to the fact that we encounter this notion frequently in daily life, especially in singling out record values from a set of others and in registering and recalling record values. Record values can be viewed as order statistics from sample whose size is determined by values and order of occurrence of observations.

In this article we interest of the E-Bayesian estimation of the Rayleigh distribution using record values, Several authors have addressed the similar problems H. Piriaei and all (2020) used the expected Bayesian method to estimate parameter and some lifetime parameters such as reliability, conditional reliability, hazard rate, cumulative hazard rate, mean lifetime and mean residual life function of a system based on record data. Fatih Kizilaslan (2016) is considered the E-Bayesian estimation method obtained the estimates of the unknown parameter and reliability function of the PHR family based on record values. Alaa alhamaidah and all (2022) treated the problem of E-Bayesian estimation and prediction in the exponential distribution on the basis of record observations under the squared log error loss function. Hassan M Okasha (2016) used the E-Bayesian method for computing estimates of the parameter and reliability function of the geometric distribution under balanced loss function.

The paper is organized as follows. In section 2, we give a description of the model. The Bayesian estimators of the parameter and reliability function and their correspondent posterior risk are calculated in section 3. We obtain the E-Bayesian estimators and the E-posterior risk in section 4.

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We use a prequential analysis for comparison of the obtained E-Bayesian estimators using its E-PMSE. We end the paper by illustrative example.

Main results

Model description

The rayleigh distribution is considered to be a very useful life distribution. Rayleigh distribution is an important distribution in statistics, which formally seeks use the prior information and Bayes theorem provides the formal basis for using this information. The probability density function is given by

$$f(x, \theta) = \frac{x}{\theta^2} \exp\left(-\frac{x^2}{2\theta^2}\right), \quad x > 0, \theta > 0$$

The reliability and hazard rate function are respectively

$$R(x, \theta) = \exp\left(-\frac{x^2}{2\theta^2}\right)$$

and

$$h(x, \theta) = \frac{x}{\theta^2}$$

Based on record value, The likelihood function of the Rayleigh distribution is

$$\begin{aligned} L(x, \theta) &= f(x_m, \theta) \prod_{i=1}^{m-1} h(x_i, \theta) \\ &= \frac{1}{\theta^{2m}} \prod_{i=1}^m x_i \exp\left(-\frac{x_m^2}{2\theta^2}\right) \end{aligned}$$

We take a natural conjugate prior distribution that of $\text{Gamma}(a, b)$ on the parameter θ

$$\pi(\theta) = \frac{b^a}{\Gamma(a)} \theta^{a-1} \exp(-b\theta)$$

The posterior density is

$$\pi(\theta|x) = K^{-1} \theta^{a-2m-1} \exp\left(-\frac{x_m}{2\theta^2} - b\theta\right)$$

Or K is the normalisation constant.

Bayesian estimation

In this section we consider the Bayesian estimation of θ and $R(x, \theta)$ under squared and Linex loss function.

Under the squared loss function, the Bayesian estimator is the posterior mean. The bayesian estimators of the parameter and reliability function of the Rayleigh distribution under squared loss function are

$$\begin{aligned} \hat{\theta}_{BS} &= \int \theta \pi(\theta|x) d\theta \\ &= K^{-1} \int \theta^{a-2m} \exp\left(-\frac{x_m}{2\theta^2} - b\theta\right) d\theta \end{aligned}$$

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and

$$\begin{aligned}\hat{R}_{BS}(t) &= \int R(t)\pi(\theta|x)d\theta \\ &= K^{-1} \int \theta^{a-2m-1} \exp\left(-\frac{x_m}{2\theta^2} - b\theta - \frac{t^2}{2\theta^2}\right)\end{aligned}$$

Under Linex loss function the Bayesian estimators of the parameter θ and the reliability function are

$$\begin{aligned}\hat{\theta}_{BL} &= \frac{-1}{w} \log(E_{\pi}(\exp(-w\theta))) \\ &= \frac{-1}{w} \log\left(K^{-1} \int \theta^{a-2m-1} \exp\left(-\frac{x_m}{2\theta^2} - b\theta - w\theta\right)\right) d\theta\end{aligned}$$

and

$$\begin{aligned}\hat{R}_{BL}(t) &= \frac{-1}{w} \log(E_{\pi} \exp(-wR(t))) \\ &= \frac{-1}{w} \log\left(K^{-1} \int \theta^{a-2m-1} \exp\left(-\frac{x_m}{2\theta^2} - b\theta - \frac{t^2}{2\theta^2}\right)\right)\end{aligned}$$

E-Bayesian estimation

According to Han (2009), the prior distribution of hyperparameters a and b should be chosen so that the prior distribution $\pi(\theta)$ is a decreasing function for θ . The derivative of $\pi(\theta)$ with respect to θ is

$$\frac{d\pi(\theta)}{d\theta} = \frac{b^a}{\Gamma(a)} \theta^{a-2} \exp(-b\theta) [(a-1) - b\theta], \quad 0 < a < 1, \quad b > 0.$$

Assume that a and b are independent:

$$\pi(a, b) = \pi_1(a)\pi_2(b)$$

The E-Bayesian estimators of θ follows and $R(t)$ are respectively calculated as

$$\begin{aligned}\hat{\theta}_{EB} &= \int \int_{\Theta} \hat{\theta}_B \pi(a, b) da db \\ \hat{R}_{EB} &= \int \int_{\Theta} \hat{R}_B(t) \pi(a, b) da db\end{aligned}$$

with Θ is the area of definition of a and b , $\hat{\theta}_B$ is the Bayesian estimator of θ and $\hat{R}_B(t)$ is the Bayesian estimator of $R(t)$.

To find the E-Bayesian estimators we are used three prior distributions of the hyperparameters a and b to examine the influence of different prior distributions on the E-Bayesian estimators of θ . The prior probability density used are

$$\begin{cases} \pi_1(a, b) = \frac{2b}{c^2}, & 0 < a < 1, \quad 0 < b < c. \\ \pi_2(a, b) = \frac{3b^2}{c^3}, & 0 < a < 1, \quad 0 < b < c. \\ \pi_3(a, b) = \frac{2(c-b)}{c^2}, & 0 < a < 1, \quad 0 < b < c. \end{cases}$$

After the application of MCMC methods, we find that the E-Bayesian estimators are better than the Bayesian estimators under squared and Linex loss functions.

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APPLICATION OF INTERVAL MARKOV CHAIN IN THE PAGE RANK ALGORITHM

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Abstract: The Page Rank algorithm stands as an advanced ranking method presently employed in Google's search engine. This algorithm can be seen as a Markov chain, predicting the system's behavior as it transitions between states, relying exclusively on the current state. In this work, we suggest using Markov Interval Chain models instead of Markov chains, because after completing the crawling and indexing phase, the links between pages may change before performing a new crawling process. The user initiates his query, and the search results will be among the old pages, before changing the links between them, and here lies the issue that we seek to solve then the links between pages can be changed over time. So, the results obtained through Markov chains will be inaccurate. Overall, In the suggested model, the approach includes taking into account the discrete values of the number of links between pages, treating them as centers of symmetric intervals. Through said model, it would be possible to avoid the problem of dangling node.

Keywords: Markov chain, Interval matrix, Page rank, Dangling node

The generalized equity and exact credibility premiums

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Abstract: We consider an more generalized alternative to the usual credibility premium that arises from squared-error loss, namely, a so-called equitable credibility premium (Promislow and Young, 1999) based on exponential family. We derive an generalized formulas for the credibility weight in certain cases and give sufficient conditions for exact credibility. In this paper, we extend the work of (Promislow and Young, 2000) to the case in which an actuary is given information about a policyholder with more general exponential family. In the first section, we set our notation and assumptions and review previous work in credibility theory. In second section, we derive the class of general equitable credibility premiums.

Keywords: equity credibility, optimal premiums, exponential family, conjugate priors.

Introduction

Credibility theory began with the papers by Mowbray (1914) and Whitney (1918). In those papers, the emphasis was on deriving a premium which was a balance between the experience of an individual risk and of a class of risks. Bühlmann (1967) showed how a credibility formula can be derived in a distribution-free way, using a least squares criterion. Since then, a number of papers have shown how this approach can be extended' see particularly Bühlmann and Straub (1970), Hachemeister (1975), de Vylder (1976, 1986). The survey by (Promislow and Young, 1999) provides an excellent introduction to these papers. In this work, we deal with the problem of choosing premiums to minimize the inequity. Much of our work parallels classical credibility theory, but in place of the traditional squared error loss functions, we use the family of entropy loss functions. This is a familiar family that has frequently appeared in the economics literature for the purpose of measuring income inequality. We obtain formulas for the optimal premiums, and in certain cases, we obtain explicit formulas for the best affine approximation to the optimal premiums. A the question, then, is to ask how good the affine approximations are. A basic result of the classical squared-error approach is that they are often exact. This occurs (given certain regularity conditions) when probability distributions are chosen from the linear exponential family with conjugate priors (Jewell, 1974a,b). The purpose of this note is to investigate conditions of exactness for a particular case of an entropy loss function, knowing that the linear exponential family is generalized.

Main results

Theorem 107. Suppose that $f(X|\theta)$ is a general linear exponential family and that the natural conjugate prior satisfies the regularity conditions on its boundary given above. If v satisfies the differential

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equation

$$v' = (k/m)\mu'(\theta)\mu(\theta)$$

for some constant a , then the equitable credibility estimator Y is exact. Specifically,

$$Y = \frac{c(k\mu(\theta) + n\bar{x})}{n + k} \quad (346)$$

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Semiparametric estimation of conditional copula when margins are subject to right censoring

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Abstract: In this paper, we focus on the study case of the dependence between two random variables Y_1 and Y_2 subject to right censoring, either one or both, where the marginal distributions depend on a covariate X and the dependence structure is characterized by a parametric bivariate copula belonging to a known family of copulas that do not depend on the covariate X .

Keywords: Conditional copula, Dependence measure, Censored data, asymptotic properties.

2020 Mathematics Subject Classification: Primary 62G99, 62H20, 62N01, 62E20.

Introduction

Let Y_1 et Y_2 two random variables whose the dependence structure is defined by a parametric copula C_{theta} belonging to a known parametric copulas family. Suppose that only the margins depending in another random variable X value (called covariate). In addition, consider the cases where one or both of margins are affected by right censoring.

Let for all y_1 and y_2 in $\mathbb{R}$

$$S_x(y_1, y_2) = P(Y_1 > y_1, Y_2 > y_2 | X = x)$$

be the conditional joint survival function of (Y_1, Y_2) upon $X = x$. Let for $i = 1, 2$, S_{ix} be the marginal conditional survival function of Y_i upon $X = x$ (i.e. $S_{ix}(y_i) = P(Y_i > y_i | X = x)$). According to Sklar's theorem version given in [1], if S_{1x} and S_{2x} are continuous, there exists a unique survival copula $\mathbb{C}_\theta$, that assumed in this case belonging to a known parametric copulas' family, such that for all y_1 and y_2 in $\mathbb{R}$

$$S_x(y_1, y_2) = \mathbb{C}_\theta(S_{1x}(y_1), S_{2x}(y_2)) \quad (347)$$

Thus, from a sample of (Y_1, Y_2, X) and assuming knowledge of the parametric copula family, it suffices to estimate the parameter to estimate the copula characterizing the dependence structure.

Main results

Here we expose the case where both of margins are right-censored. The case where only one of the margins are subject to censor is treated with similar way.

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Case where both of margins are right-censored

Suppose that both of margin variables are right-censored. For $i = 1, 2$, Y_i is right-censored and define the random variable T_i as the value 'observed' of Y_i under right censoring and let δ_i be the censor index, such that, if we consider C_i the Y_i censoring random variable, we have

$$T_i = \begin{cases} Y_i & \text{if } Y_i \leq C_i, \\ C_i & \text{if } Y_i > C_i. \end{cases} \quad \text{and } \delta_i = \begin{cases} 1 & \text{if } Y_i \leq C_i, \\ 0 & \text{if } Y_i > C_i. \end{cases}$$

We observe this case the n -sample $(T_{11}, \delta_{11}, T_{21}, \delta_{21}, X_1), \dots, (T_{1n}, \delta_{1n}, T_{2n}, \delta_{2n}, X_n)$ such that for $i = 1, \dots, n$, $(T_{1i}, \delta_{1i}, T_{2i}, \delta_{2i}, X_i)$ has the same distribution as $(T_1, \delta_1, T_2, \delta_2, X)$.

Our objective is to estimate the parameter $\theta \in \Theta$ (where Θ is parameter domain given in the copulas family definition) using the maximization of loglikelihood approach as in [2]:

$$L_n(\theta) = \sum_{i=1}^n \ell(\psi^{-1}(\eta), S_{1x}(T_{1i}), S_{2x}(T_{1i})) \quad (348)$$

Where ψ is a function such that $\psi : \Theta \rightarrow \mathbb{R}$ chosen to insure that the estimate value belongs to the parameter set Θ and for $v \in \Theta$ and $(u, v) \in [0, 1]^2$

$$\begin{aligned} \ell(\theta, u_1, u_2) = & (1 - \delta_1)(1 - \delta_2) \ln \mathbb{C}_\theta(u_1, u_2) + \delta_1(1 - \delta_2) \ln \left[\frac{\partial \mathbb{C}_\theta(u_1, u_2)}{\partial u_1} \right] \\ & + (1 - \delta_1)\delta_2 \ln \left[\frac{\partial \mathbb{C}_\theta(u_1, u_2)}{\partial u_2} \right] + \delta_1\delta_2 \ln \left[\frac{\partial^2 \mathbb{C}_\theta(u_1, u_2)}{\partial u_1 \partial u_2} \right] \end{aligned} \quad (349)$$

Note that since we have no information about the margin survival distributions appearing in 348, we have to use nonparametric methods to estimate these distributions. Thus for $i=1, 2$, we will use the nonparametric estimator (so called the Beran estimator) for estimating S_{ix} , as provided in [3], given by

$$\hat{S}_{ix}(t) = \prod_{T_{ik} \leq t, \delta_{ik}=1} \left(1 - \frac{w_{nk}(x; h)}{\sum_{j=1}^n \mathbb{I}\{T_{ij} \geq T_{ik}\} w_{nj}(x; h)} \right) \quad (350)$$

where $w_{nj}(x; h)$ is the weight of j th observation often chosen as

$$w_{ni}(x; h) = \frac{K_h(X_i - x)}{\sum_{k=1}^n K_h(X_k - x)}$$

where $K_h\{\cdot\} = K\{\cdot/h\}/h$, with K is a kernel function and h the parameter bandwidth. Replacing in 348 the conditional survival functions by their estimates, we obtain the log-likelihood

$$\tilde{L}_n(\theta) = \sum_{i=1}^n \ell(\psi^{-1}(\eta), \hat{S}_{1x}(T_{1i}), \hat{S}_{2x}(T_{1i})). \quad (351)$$

and we propose $\hat{\theta}$ the estimator of θ maximizing $\tilde{L}_n(\theta)$ such that $\hat{\theta} = \psi(\hat{\eta})$ and

$$\hat{\eta} = \arg \max_{\eta} \sum_{i=1}^n \ell(\psi^{-1}(\eta), \hat{S}_{1x}(T_{1i}), \hat{S}_{2x}(T_{1i})). \quad (352)$$

We are currently determining the asymptotic properties of this estimator.

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Bandwidth selection for the conditional margin survival estimator

For the choice of the bandwidth parameter value in each conditional margin, we use the formula given in [2] based in the following minimization of the leave-one-out cross validation criterion, such that

$$h_{k,opt} = \arg \min_h \sum_{i=1}^n \sum_{j=1}^n \Delta_{kij} \left(\mathbb{I}(T_{ki} \leq T_{kj}) - \hat{F}_{kX_i}^{(-i)}(T_{kj}) \right)^2, \quad (353)$$

where $\hat{F}_{kX_i}^{(-i)}(\cdot) = 1 - \hat{S}_{kX_i}^{(-i)}(\cdot)$ is the estimator in 350 of k^{th} ($k = 1, 2$) conditional margin at bandwidth value h , obtained using the data points $(T_{kj}, \delta_{kj}, X_j)_{j \in \{1, \dots, n\} - i}$, and $\Delta_{kij} = 1$ if one of the four case is holds

- (i) $(\delta_{ki}, \delta_{kj}) = (1, 1)$;
- (ii) $(\delta_{ki}, \delta_{kj}) = (1, 0)$ and $Y_{ki} \leq Y_{kj}$;
- (iii) $(\delta_{ki}, \delta_{kj}) = (0, 1)$ and $Y_{ki} \geq Y_{kj}$; or
- (iv) $i = j$.

otherwise $\Delta_{kij} = 0$.

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MHD estimates for classic FAR(1) models, theory and simulation

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Abstract: In this work, we define the minimum Hellinger distance estimator for a Fractional Autoregressive process of order one (FAR(1)). The estimator minimizes the Hellinger distance between the probability density function of the process' innovation and a parameterized random function. Under certain conditions, we establish the asymptotic properties of this estimator.

To validate the results, simulations are carried out.

Keywords: fractional autoregressive model, Hellinger distance, estimation, simulation.

2020 Mathematics Subject Classification: 60G22.

Introduction

Among the various challenges in the field of data analysis, the estimation of unknown components is undeniably a topic of interest for researchers.

Due to its effectiveness and robustness (Beran 1977), we choose to estimate the parameters of our Fractional Autoregressive process of order 1 (FAR(1)) model using a criterion based on the method of minimum Hellinger distance.

We first introduce the model and the estimator, then we specify the assumptions and extract the main results, which are the main properties of the proposed estimator, and we finalize with a simulation study.

Model and notations

A Fractional Autoregressive process of order 1 (FAR(1)) model is a stochastic equation of the following form

$$(1 - aL)^d X_t = \varepsilon_t \quad (354)$$

whith $|a| < 1$ and $d \in \mathbb{R}$ or $|a| = \pm 1$ and $-\frac{1}{2} < d < \frac{1}{2}$, ε_t ; $t \in \mathbb{Z}$ is a white noise, sequence of independent random variables identically distributed with mean zero and finite variance σ^2 .

Any stationary solution $(X_t; t \in \mathbb{Z})$ of (354) is called a Fractional Autoregressive process of order 1.

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Considering that the process is invertible, equation (354) can be rewritten in the form of an infinite autoregressive process AR(∞) as follows

$$\varepsilon_t = \sum_{j=0}^{\infty} \Pi_j X_{t-j} \quad (355)$$

where

$$\Pi_j = \frac{\Gamma(j-d)}{\Gamma(-d)\Gamma(j+1)} a^j$$

Let $\theta = (a; d) \in \Theta \subset \mathbb{R}^2$ be the vector of parameters to be estimated, and let $X_1, \dots, X_n$ be the observations. Since the innovation ε_t is not observable, we estimate it by $\hat{\varepsilon}_t = \sum_{j=0}^{\infty} \Pi_j X_{t-j}$.

Minimum Hellinger distance estimate

This paragraph is dedicated to determining the Minimum Hellinger Distance (MHD) estimator of the parameter θ and establishing the asymptotic properties of this estimator. The true value of the parameter vector is $\theta_0 = (a_0, d_0)$.

The MHD method involves minimizing the Hellinger distance between the density function of the white noise $f_{\theta_0}(\cdot)$ and the following parameterized random function $\hat{f}_{\theta,n}(x)$

$$\hat{f}_{\theta,n}(x) = \frac{1}{nh_n} \sum_{t=1}^n K\left(\frac{x - \hat{\varepsilon}_t}{h_n}\right) \quad (356)$$

In which (h_n) is a sequence of bandwidths, K is the kernel, and $\hat{\varepsilon}_t$ is an estimator of ε_t . Let $\hat{\theta}_n$ be the MHD estimator of $\theta_0 \in \Theta$, defined as follows:

$$\hat{\theta}_n = \arg \min_{\theta \in \Theta} H_2(\hat{f}_{\theta,n}(x); f_{\theta_0}) \quad (357)$$

where

$$H_2(\hat{f}_{\theta,n}(x); f_{\theta_0}) = \left(\int_{\mathbb{R}} |\hat{f}_{\theta,n}^{\frac{1}{2}}(x) - f_{\theta_0}^{\frac{1}{2}}(x)|^2 dx \right)^{\frac{1}{2}} \quad (358)$$

We explore the main characteristics of the estimator using two theorems (consistency and asymptotic normality), which will be proved using four lemmas.

We have studied two different examples of kernel densities in our simulation research to validate the theoretical results.

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Earthquake forecasting in the Chile region based on MR and RI methods

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Abstract: This study uses the moment ratio (MR) and relative intensity (RI) methods to forecast earthquakes in Chile. MR assumes that abnormal long-term changes in background seismic activity may precede major earthquakes, while RI forecasts future events based on recent smaller earthquakes. Based on the the International Seismological Centre (ISC) database covering earthquakes for Chile from 1980 to 2023, regional forecast maps are created for earthquakes $M \geq 6.5$ from 2018 to 2023. The results demonstrate the MR method outperforms performance RI method.

Keywords: Earthquake forecasting, Moment Ratio, Relative Intensity, Molchan diagram.

2020 Mathematics Subject Classification: Primary 62-08, 86-10.

Introduction

The Moment Ratio (MR), an alarm-based earthquake forecasting uses the first and the second order moments of earthquake inter-event times. It is based on the assumption that the anomalous long-term changes in background seismicity prior to large earthquakes (Talbi et al, 2013). The Relative Intensity (RI) for earthquake forecasting is based on the hypothesis that the major earthquakes are more likely to occur in the locations where the highest seismic activity has occurred in the past (Radan et al., 2005). MR and RI forecasting maps are elaborated step by step as follows, (a) Data Preparation: an earthquake catalog database covering the study region G is prepared. (b) Spatial Gridding: the study region, G is covered by a regular grid with $N, l \times l$ cells. (c) Temporal Segmentation: the study period, is subdivided into two distinct time periods: a learning period from t_0 to t_1 ($t_0 < t_1$) and a testing period from t_1 to t_2 ($t_1 < t_2$). (d) Moment Ratio (MR) score: is calculated for each cell i within the study region G , using the ratio of the mean interevent time series $\{\xi\}_{i=1}^n$ divided by the variance during a learning period $[t_0, t_1]$. It is considered as a precursory alarm function to forecast large earthquakes during the testing period $[t_1, t_2]$, namely,

$$MR_i(t_0, t_1) = \frac{\bar{\xi}_i}{\sigma_{\xi}} \quad (1)$$

Where $\bar{\xi}_i$ and σ_{ξ} are the mean and the variance of the interevent time-series $\{\xi\}_{i=1}^n$, respectively. For a given cell $i \in G$, the standardized MR alarm function P_{MR} is defined by,

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$$P_{MR_i} = \frac{MR_i}{\max_{i \in G}(MR_i)} \quad (2)$$

(e) Relative Intensity (RI) score: is computed for each cell i by the total number of earthquakes of magnitude $M \geq m_c$ that occurred during the learning period, $n_i(t_0, t_1)$, divided by the total number of earthquakes occurred in the whole study region, G ,

$$RI_i(t_0, t_1) = \frac{n_i(t_0, t_1)}{\sum_{i=1}^n n_i(t_0, t_1)} \quad (3)$$

Similarly, the standardized RI alarm function P_{RI} is calculated for each cell $i \in G$,

$$P_{RI_i} = \frac{RI_i}{\max_{i \in G}(RI_i)} \quad (4)$$

To ensure minimum accuracy for both methods, the scores are calculated to cells and their neighbours for a minimum number of earthquakes n_{min} during the learning period.

Application and results

Figure 17: MR and RI forecasting maps for Chile during the period 2018~2023. Red circles show epicenters of magnitude $M \geq 6.5$ earthquakes.

Chile region delimited within the area -45° N to 16° N latitude and -77° E to -65° E longitude is defined as the study region. It is a high seismic activity because of near the Nazca plate boundary. The earthquake catalog data is downloaded from the ISC network database available via the link <http://www.isc.ac.uk/iscbulletin/search/catalogue/>. It spans from 1980 to 2023, with approximately 1737 earthquakes of magnitude $M \geq 5.5$ recorded. Completeness magnitude is determined as $m_c = 5.5$ using the maximum curvature method. The region is divided into grid cells of $l = 0.25^\circ$. With the parameters $t_0 = 1980, t_1 = 2018$ and $t_2 = 2023$, with $n_{min} = 20$. The MR and RI results are displayed through forecasting maps of 13 $M6.5+$ earthquakes in Figure1, showing hotspots (colored cels) with probabilities of target earthquakes. The results showed that the MR method outperforms the RI method.

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Perturbation bounds for the stationary distributions of Markov chains with application to queueing system

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Abstract: The calculation of the stationary distribution for a stochastic infinite matrix is generally difficult and do not have closed form solutions, it is desirable to have simple approximations converging rapidly to this distribution. Perturbation Theory for Markov Chains (PTMC) addresses the question of how small differences in the transition probabilities of Markov chains are reflected in differences between their stationary distributions. In this work, we use the strong stability approach, using the drift condition, to establish analytic error bounds for the generalized truncation, given by NEUTS and RAO (1990) [7], of a main multiserver retrial queue model (see [2]).

Keywords: Markov chains, perturbation theory, truncation, strong stability, drift condition, retrial queueing systems

Introduction

The class of retrial queueing systems is characterized by the following feature : a customer arriving when all servers accessible for him are busy leaves the service area but after some random time repeats his demand, and between trials a customer is said to be in orbit. This feature plays a special role in several computer and communication networks as well. Other applications include stacked aircraft waiting to land, and queues of retail shoppers who may leave a long waiting line hoping to return later when the line may be shorter, so that the area of possible applications of such systems is wide. These systems are often described by denumerable state Markov chains. The calculation of the stationary distribution for a stochastic infinite matrix is generally difficult and do not have closed form solutions, it is desirable to have simple approximations converging rapidly to this distribution. Perturbation Theory for Markov Chains (PTMC) addresses the question of how small differences in the transition probabilities of Markov chains are reflected in differences between their stationary distributions.

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The model described in this work is the $M/M/c$ retrial queueing system in which primary customers arrive according to a Poisson stream of rate λ . The service facility consists of c identical servers and customer service times are independent and exponentially distributed with parameter ν . A customer who finds all servers busy upon arrival is obliged to leave the service area and to repeat his demand after an exponential time with parameter μ , and between trials a customer is said to be in orbit. As usual, we assume that the inter-arrival periods, service times and retrial times are mutually independent. Under this assumptions, the system can be described as a regular continuous time Markov chain $\widehat{X}_1$ with state space $\mathcal{S} = \{(i, j), 0 \leq i \leq c \text{ and } j \in \mathbb{N}\}$, where i is the number of busy servers and j is the number of customers in orbit, and with the stationary distribution $\widetilde{\Pi}$. Denote by $\widetilde{X}$ the embedded Markov chain of $\widehat{X}_1$ and let $\widetilde{P}$ his transition matrix. FALIN (1984) [1] established that the ergodicity condition of Markov chain $\widetilde{X}$ is

$$\rho = \frac{\lambda}{c\nu} < 1 \quad (359)$$

Neither a closed analytical solution nor a direct algorithmic computation of $\widetilde{\Pi}$ is still available. NEUTS and RAO (1990) [7] investigated a possibility to get a numerically tractable approximation. They consider that the number of customers in orbit who are allowed to conduct retrials is restricted to an appropriate number N , so the retrial rate is $\mu_j = \min(j, N) \cdot \mu$ for $j \geq 0$. It yields an approximate process $\widehat{X}_2$ which can be reformulated as a quasi-birth-and-death (QBD) process with the same state space $\mathcal{S}$ and with the stationary distribution Π . Denote by X the embedded Markov chain of $\widehat{X}_2$ and let P his transition matrix. Note that the condition (359) does not hold for the process $\widehat{X}_2$ because the retrial rate is constant from the level N up. Following the general theory for QBD processes (see [8]), it can be proved that the ergodicity condition of the Markov process $\widehat{X}_2$, and consequently for the Markov chain X , is

$$\frac{\lambda + N\mu}{c!} \left(\frac{\lambda + N\mu}{\nu} \right)^c < N\mu \sum_{k=0}^c \frac{1}{k!} \left(\frac{\lambda + N\mu}{\nu} \right)^k \quad (360)$$

Main results

Let $\mathcal{C} = \{(0, 0)\}$ be the singleton that contains the only element $(0, 0)$, then :

Theorem 108. *The Markov chain X is strongly ν -stable if :*

$$\exists \delta \in]0, 1[, \quad \exists L > 0, \quad Pv \leq \delta \cdot \nu + L \cdot \mathbb{K}_{\mathcal{C}} \quad (361)$$

and in this case, if

$$\|\widetilde{P} - P\|_v < \frac{(1 - \delta)^2}{L + 1 - \delta} \quad (362)$$

then

$$\|\widetilde{\Pi} - \Pi\|_v \leq \frac{L(L + 1 - \delta) \cdot \|\widetilde{P} - P\|_v}{(1 - \delta)^3 - (1 - \delta)(L + 1 - \delta) \cdot \|\widetilde{P} - P\|_v} \quad (363)$$

Theorem 109. *The Markov chain X is strongly ν -stable and, under the conditions (359), (360) and other conditions on α and β , we have :*

$$\|\widetilde{\Pi} - \Pi\|_v \leq \mathbf{SSB}(\alpha, \beta) = \frac{L(L + 1 - \delta) \cdot \Delta}{(1 - \delta)^3 - (1 - \delta)(L + 1 - \delta) \cdot \Delta} \quad (364)$$

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where

$$\delta = \delta(\alpha, \beta) = \max \left\{ \frac{\alpha \lambda + \frac{1}{\alpha} \nu}{\lambda + \nu}, \frac{\alpha \lambda + \frac{\alpha}{\beta} \mu}{\lambda + \mu}, \frac{\beta \lambda + \frac{1}{\alpha} c \nu}{\lambda + c \nu} \right\}$$

$$L = L(\alpha, \beta) = \alpha - \delta(\alpha, \beta)$$

$$\Delta = \Delta(\alpha, \beta) = \max \left\{ \frac{\alpha \left(\frac{1}{\beta} + 1 \right) \lambda}{\lambda + N \mu}, \frac{\alpha \left(\frac{1}{\beta} + 1 \right) \lambda + \left(\frac{\alpha}{\beta} + \frac{1}{\alpha} \right) (c-1) \nu}{\lambda + N \mu + (c-1) \nu} \right\}$$

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Cost Optimization of $M/M/1$ Retrial Queuing System With Exponential Distribution, Exhaustive Service And Multiple Vacation

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Abstract: we consider an $M/M/1$ retrial queuing system with exponential distribution, exhaustive service and multiple vacation model . We derive the steady-state probability distributions and the expected number of customers in the system, which are then employed to develop a cost function. The parabolic method is used to minimize the expected cost of the system. To illustrate the workings of this algorithm for optimal management, we provide several examples.

Keywords: queuing system, retrial queue , multiple vacation , exhaustive service

Introduction

Retrial queueing systems are characterized by the fact that when arriving customers find the server busy, they join the retrial group/orbit and attempt their request again after random intervals. These types of queues are frequently used as mathematical models for a variety of computer systems, such as packet switching networks, shared bus local area networks operating under the Carrier-Sense Multiple Access protocol, and collision avoidance star local networks. The applications of retrial queues are broad, spanning the fields of communication networking, computer systems, and more. In retrial queues, customers who find the server occupied do not join the queue in front of the server; rather, they enter a virtual room known as the orbit and retry their request after a random period of time.

For comprehensive survey on retrial queues see Yang and Templeton [2], Falin [3], Falin and Templeton [4] and Artalejo [5].

Single server queues with vacations have been studied extensively in the past: a comprehensive survey can be found in Teghem [7], Doshi [14] and Takagi [8]. Single server queues with vacations are models that arise naturally in telephone switching systems, computer communication systems, and other similar fields. While most of the analyses for retrial queues focus on the exhaustive service schedule (as shown by Artalejo [6]) and the gated service policy (as demonstrated by Langaris [9]), research has been conducted to characterize system performance. Keilson and Servi [10] introduced a class of vacation scheduling disciplines, including the $M/G/I$ queue with

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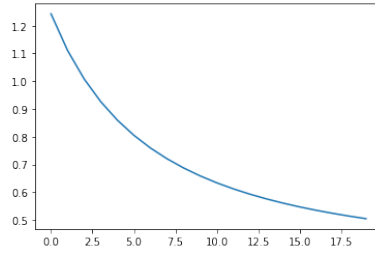


Figure 18: Effect of service rate μ on Expected orbit length L_q

Bernoulli service schedule and multiple vacations. This class of schedules allows for both performance analysis and system optimization. Bernoulli service scheduling is significant because of the existence of the control parameter p , whose extreme values correspond to the limited service discipline (for $p = 0$) and the exhaustive service discipline (for $p = 1$).

In the single-server $M/G/I$ queue model with Bernoulli service schedule and multiple vacations proposed by Keilson and Servi, customers have exponential inter-arrival times and general service times. If the queue is empty after a service completion, the server becomes inactive and begins a vacation period for a random duration with a known probability distribution. If the queue is not empty, another service begins with a specified probability p , or a vacation period commences with probability $q = 1 - p$, ($q > 0$). Service commences at the end of a vacation period if a customer is present in the queue. In this model, if the server returns from a vacation and finds no customers waiting, it takes another vacation immediately and continues in this manner until it finds at least one customer waiting (i.e., multiple vacations). The length of each vacation is assumed to be an independent and identically distributed random variable and is independent of the service times and arrival process. There is an extensive literature on various variations of this model, including work by Servi [11], Ramaswamy and Servi [12], and Tedijanto [13].

Main results

Krishna and pavi in [1] study $M^X/G/1$ Retrial Queue With Multiple Vacations and Starting Failures, and we assume $p = 1, \alpha = 1, \overline{C_1}$, retrial time is exponential distribution, service time is exponential distribution, vacation time is exponential distribution. The performance measures are given by

- The server utilization $U = \frac{\alpha}{\mu}$
- The steady-state probability that the server is idle or on vacation $D = \frac{-\alpha + \mu}{\mu}$
- The mean number of customres in the orbit

$$L_q = \frac{\alpha(\alpha^3\mu - \alpha^3\theta + 2\alpha^2\gamma\mu - 2\alpha^2\gamma\theta + \alpha\gamma^2\mu - \alpha\gamma^2\theta - \alpha\gamma\mu^2 - \gamma^2\mu^2 - \gamma\mu^2\theta)}{\mu\theta(\alpha^3 + 2\alpha^2\gamma + \alpha\gamma^2 - \alpha\gamma\mu - \gamma^2\mu)}$$

Sensitivity analysis

In the below graph, we have fixed the parameters as $\lambda = 2; \theta = 5; \gamma = 6$ From figure 18 We have observed that an increase in the value of μ , which represents the mean service time, is associated

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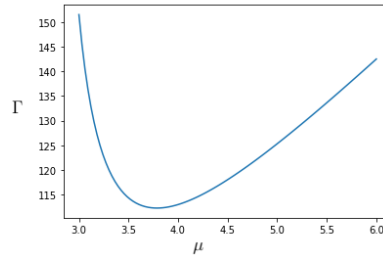


Figure 19: The effect of μ on Expected operating cost per unit time

with a decrease in the expected orbit length in a queueing system. This occurs because a larger value of μ leads to shorter service times for each customer, which reduces the queue length and waiting times. As a result, the expected orbit length for each customer decreases. In other words, the customers spend less time in the queue, and they are served more quickly, resulting in a shorter expected orbit length.

Cost analysis

The method works by generating a quadratic function through calculated points in each iteration, which approximates the function Γ . The optimal point at which Γ is minimized in the three-point pattern $\{x_1, x_2, x_3\}$ is determined by this method.

$$x_L = \frac{0.5(\Gamma(x_1)(x_2^2 - x_3^2) + \Gamma(x_2)(x_3^2 - x_1^2) + \Gamma(x_3)(x_1^2 - x_2^2))}{\Gamma(x_1)(x_2 - x_3) + \Gamma(x_2)(x_3 - x_1) + \Gamma(x_3)(x_1 - x_2)}$$

In this method, the new value obtained replaces one of the three points to improve the current three-point pattern. This process is repeated iteratively until the optimal value is obtained up to the desired degree of accuracy. 4 shows that the optimum value of $\Gamma(\mu) = 112.289$ corresponds to $\mu = 3.787$ with a permissible error of 10^{-3} , which agrees with the results shown in 30 for the values $\lambda = 2; \theta = 5; \gamma = 6$.

Table 4: The parabolic method to optimize the cost function

x_1	x_2	x_3	Γ	x_l
3	3.5	4	112.295	3.769
3.5	3.769	4	112.302	3.813
3.769	3.813	4	112.289	3.786
3.769	3.786	3.813	112.289	3.787

The convexity of the curve in Figure 19 clearly shows that there are certain values of the service rate μ that minimize the total expected cost function for the selected set of model parameters, namely Λ , θ , and γ , for different values of these parameters.

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Semi-linear PDEs Solutions and Their Application to the Existence of Weak Solutions for Markovian BSDEs

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Abstract: In this paper, we consider a system of semi-linear partial differential equations (PDE) with discontinuous coefficients. We then construct a sequence of penalized partial differential equations which converges to a solution of our initial problem. The solution v to the PDE is regular enough in a Sobolev sense. Our strategy is based on an alternative method developed in Gilbarg & Trudinger [4] and Gagliardo-Nirenberg interpolation inequality. As application, the use of Itô-Krylov formula to get the existence of weak solution for Markovian BSDEs.

Keywords: SDEs, BSDEs, PDE, Itô-Krylov's formula, weak solution.

2020 Mathematics Subject Classification: 60H20, 60H30, 35J60, 60J35.

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Algebra, number theory and Geometry

On Hom weak Hopf algebra

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Abstract: The purpose of this paper is to introduce and study weak Hom-bialgebras and weak Hom-Hopf algebras. We provide relevant definitions, properties and constructions by twisting. Moreover, we introduce Hom-coalgebras from Hom-associative algebras and construct all weak Hom-Hopf algebras in low dimension.

Keywords: Hom-associative algebra, Weak Hom-bialgebra, weak Hom-Hopf algebra

2020 Mathematics Subject Classification: Primary 16T10. Secondary 16W10.

Introduction

Quantum deformations (or q -deformations) of Lie algebras have been investigated in various domains of Mathematics and Physics. They gave rise to a great interest in quantum groups and Hopf algebras. In physical contexts appeared q -deformations of infinite-dimensional Lie algebras, primarily the Heisenberg algebras (oscillator algebras) and the Virasoro algebra. It turns out that these algebras obey some deformed (twisted) version of Jacobi identity. This kind of algebras was called by Hartwig, Larsson and Silvestrov Hom-Lie algebras. The corresponding associative algebras, called Hom-associative algebras, were introduced by the second author and Silvestrov in [3]. Hom-bialgebras and Hom-Hopf algebras were introduced and studied in [4],[5]. The weak Hom-bialgebras (resp. weak Hom-Hopf algebras) constitute a wider class which include weak bialgebras (resp. weak Hopf algebras). The categories of these algebras are monoidal categories. Recall that weak coproduct was also motivated by quantum symmetry and vector field algebras, see [2]. The weak bialgebras were introduced by Böhm et al. [1] in the context of operator algebras and quantum field theory. The aim of this paper is to generalize weak bialgebras and weak Hopf algebras to Hom-setting and study their properties.

Main results

Bialgebras and Hopf Algebras

A bialgebra is a vector space that is both an algebra and a coalgebra. Further, the algebra and coalgebra structures must be compatible. The compatibility is ensured by requiring either of the following equivalent conditions.

1. Δ and ε must be algebra morphisms.

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2. μ and η must be coalgebra morphisms

i.e; that is for $x, y \in A$

$$\Delta(\mu(x \otimes y)) = \Delta(x) \cdot \Delta(y)$$

and

$$\varepsilon(\mu(x, y)) = \varepsilon(x)\varepsilon(y).$$

Let H be a bialgebra. A linear map $S : H \longrightarrow H$ is called an antipode of the bialgebra H if S is the inverse of the identity map $id_H : H \longrightarrow H$, with respect to the convolution product $(*)$ defined by

$$id_H * S = S * id_H = \eta \circ \varepsilon.$$

$$\mu \circ (id_V \otimes S) \circ \Delta = \mu \circ (S \otimes id_V) \circ \Delta = \eta \circ \varepsilon.$$

A bialgebra H having an antipode is called a **Hopf algebra**.

Weak Hopf algebras

A weak Hopf algebra is an algebraic structure that generalizes the properties of Hopf algebras while relaxing certain conditions. A weak bialgebra $(H, \mu, \eta, \Delta, \varepsilon)$ over a field $\mathbb{K}$ is a vector space H such that :

- (H, μ, η) forms an associative algebra with multiplication $\mu : H \otimes H \rightarrow H$ and unit $\eta : k \rightarrow H$,
- (H, Δ, ε) forms a coassociative coalgebra with comultiplication $\Delta : H \rightarrow H \otimes H$ and counit $\varepsilon : H \rightarrow k$

for which the following compatibility conditions hold :

1. Multiplicativity of the Comultiplication :

$$\Delta \circ \mu = (\mu \otimes \mu) \circ (id_H \otimes \sigma_{H,H} \otimes id_H) \circ (\Delta \otimes \Delta),$$

2. Weak Multiplicativity of the Counit :

$$\varepsilon \circ \mu \circ (\mu \otimes id_H) = (\varepsilon \otimes \varepsilon) \circ (\mu \otimes \mu) \circ (id_H \otimes \Delta \otimes id_H) = (\varepsilon \otimes \varepsilon) \circ (\mu \otimes \mu) \circ (id_H \otimes \Delta^{op} \otimes id_H),$$

3. Weak Comultiplicativity of the Unit :

$$(\Delta \otimes id_H) \circ \Delta \circ \eta = (id_H \otimes \mu \otimes id_H) \circ (\Delta \otimes \Delta) \circ (\eta \otimes \eta) = (id_H \otimes \mu^{op} \otimes id_H) \circ (\Delta \otimes \Delta) \circ (\eta \otimes \eta),$$

where $\sigma_{V,W} : V \otimes W \rightarrow W \otimes V : v \otimes w \mapsto w \otimes v$ flips the two tensor factors.

Moreover $\mu^{op} = \mu \circ \sigma_{H,H}$ is the opposite multiplication and $\Delta^{op} = \sigma_{H,H} \circ \Delta$ is the opposite comultiplication.

The definition weakens the compatibility between the algebra and coalgebra structures of a bialgebra. More specifically, the unit and counit are weakened. This remains true in the axioms of a weak Hopf algebra.

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Hom Hopf and weak Algebras

Definition 40. A weak Hom-Hopf algebra is a 7-tuple $\mathcal{H} = (A, \mu, \eta, \Delta, \varepsilon, S, \alpha)$, where $(A, \mu, \eta, \Delta, \varepsilon, \alpha)$ is a weak Hom-bialgebra and S is an antipode, that is an endomorphism of A satisfying for all $x \in A$:

$$\begin{aligned} m(id \otimes S)\Delta(x) &= (\varepsilon \otimes \alpha)(\Delta(1)(x \otimes 1)), \\ \mu(S \otimes id)\Delta(x) &= (\alpha \otimes \varepsilon)((1 \otimes x)\Delta(1)), \\ \mu(\mu \otimes \alpha)(S \otimes id \otimes S)(\Delta \otimes \alpha)\Delta &= S \circ \alpha^4, \\ \alpha \circ S &= S \circ \alpha. \end{aligned}$$

Theorem 110. [?] Let $B = (A, \mu, \eta, \Delta, \varepsilon, \alpha)$ be a weak Hom-bialgebra and $\beta : A \rightarrow A$ be a weak Hom-bialgebra morphism. Then, $B_\beta = (A, \mu_\beta = \beta \circ \mu, \eta, \Delta_\beta = \Delta \circ \beta, \varepsilon, \beta \circ \alpha)$ is a weak Hom-bialgebra.

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Some remarks on additive and multiplicative decompositions of polynomials over unique factorization domains

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Abstract: In this talk, we study the analogous notions of composed addition and composed multiplication and the several results obtained in the last years, especially, in [2] (joint work with M. Benoumhani, L. Benferhat, J. Larone) . We also give some new remarks on the composed multiplication of two polynomials over unique factorization domains.

Keywords: Additive decomposition; multiplicative decomposition; Resultant; Primitive element.

2020 Mathematics Subject Classification: Primary 11T06; 13F15.

Introduction

Let $\mathbb{F}_q$ denote the finite field of q elements, let f and g be monic polynomials over $\mathbb{F}_q$, and consider the polynomials $f * g$ and $f \circ g$ defined by:

$f * g = \prod_{\alpha} \prod_{\beta} (x - (\alpha + \beta))$, $f \circ g = \prod_{\alpha} \prod_{\beta} (x - (\alpha\beta))$, where the product on the right is taken over all roots α and β of f and g respectively (including multiplicities), in the algebraic closure of $\mathbb{F}_q$. Although the roots of f and g may lie outside of $\mathbb{F}_q$, the polynomials $f * g$, $f \circ g$ have coefficients in $\mathbb{F}_q$. The operations $*$ and $\circ$ are binary operations on the set of monic polynomials over $\mathbb{F}_q$, called composed addition and composed multiplication. Additive decompositions over finite fields were widely studied by Brawley and Carlitz [4] and [5].

Main results

Theorem 111. *Let R be an integral domain. If $h \in R[x]$ has leading coefficient p where p is prime, then h is not additively decomposable over R .*

Theorem 112. *Let R be a unique factorization domain. If $h \in R[x]$ with $\deg h > 1$ has a leading coefficient that is a square-free and not a unit of R , then h is not additively decomposable over R .*

Let $\sigma : R \rightarrow S$ be a unit-preserving ring homomorphism from an integral domain R to an integral domain S . This homomorphism can naturally be extended to a homomorphism $\bar{\sigma} : R[x] \rightarrow S[x]$

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defined by $c_n x^n + \dots + c_0 \mapsto \sigma(c_n) x^n + \dots + \sigma(c_0)$. For simplicity, we denote $\bar{\sigma}$ by σ as well. Since $\text{Res}_x(f, g)$ is a polynomial in the coefficient of f and g , we have $\sigma(\text{Res}_x(f, g)) = \text{Res}_x(\sigma(f), \sigma(g))$. Let $h = f * g$ and let $a, b, c \in R$ be the leading coefficient of f, g, h respectively. Then $c = a^n b^m$ where $m, n \geq 1$. If $\deg \sigma(h) = \deg h$ for a unit-preserving homomorphism σ , then $\sigma(a)^n \sigma(b)^m = \sigma(c) \neq 0$ so that $\sigma(a) \neq 0$ and $\sigma(b) \neq 0$.

Theorem 113. *Let $\sigma : R \rightarrow S$ be a unit-preserving ring homomorphism from an integral domain R to an integral domain S , and let $h \in R[x]$. If $\deg \sigma(h) = \deg h$ and $h = f * g$ over R , then $\sigma(h) = \sigma(f) * \sigma(g)$ over S .*

The following lemma concerning linear polynomials will be used to prove that a polynomial can be additively decomposed into a finite number of indecomposable additive composition factors.

Lemma 17. *Let R be a unique factorization domain and let $h = ax + b \in R[x]$ where a is not a unit in R . Then $h = f_1 * \dots * f_r$ for some linear polynomials $f_1, \dots, f_r \in R[x]$ which are not additively decomposable over R .*

Theorem 114. *Let R be a unique factorization domain, and let $h \in R[x]$ be a non-unit with respect to composed addition. Then $h = f_1 * \dots * f_r$ for some polynomials $f_1, \dots, f_r \in R[x]$ which are not additively decomposable over R .*

Corollary 5. *Let $\sigma : R \rightarrow S$ be a unit-preserving ring homomorphism from an integral domain R to an integral domain S , and let $h \in R[x]$. If $\deg \sigma(h) = \deg h$ and $\sigma(h)$ is not additively decomposable over S , then $h = f * l$ where $f \in R[x]$ is not additively decomposable over R and $l \in R[x]$ is a linear polynomial. Moreover, if the leading coefficient of h is a unit in R , then l is a unit with respect to composed addition, so h is not additively decomposable over R .*

Theorem 115. *Let R be a unique factorization domain and $h \in R[x]$. Suppose that h is additively decomposable, that is, $h = f * g$ where*

$$f(x) = \sum_{i=0}^m f_i x^i \text{ and } g(x) = \sum_{i=0}^n g_i x^i$$

such that $\deg(f) = m$ and $\deg(g) = n$. Suppose in addition that $\gcd(\text{cont}(g), f_m) = 1$ and $\gcd(\text{cont}(f), g_n) = 1$ where f_m and g_n are the leading coefficients of f and g respectively. Then, if h is primitive in $R[x] \Rightarrow f$ and g are primitive in $R[x]$.

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On the Chebyshev polynomials and some of their new identities

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Abstract: Let $T_n(x)$ and $U_n(x)$ be the n^{th} Chebyshev polynomials of the first and second kinds, respectively. In this work, we propose a new method based on the matching polynomials of some particular graphs (paths and cycles) for the determination of new identities for the Chebyshev polynomials of the first and second kinds, namely, we give a closed form for $T_{m_1+m_2+\dots+m_s}(x)$ and $U_{m_1+m_2+\dots+m_s}(x)$ in terms of $U_{m_i+\varepsilon_i}(x)$, where s and m_i ($1 \leq i \leq s$) are positive integers and $\varepsilon_i \in \{-2, -1, 0\}$. As a result, using some properties of Chebyshev polynomials, we deduce a closed form for $\sin(m_1 + m_2 + \dots + m_s)\theta$ and $\cos(m_1 + m_2 + \dots + m_s)\theta$ in terms of $\sin(m_i + \varepsilon_i)\theta$, where θ is a real number and $\varepsilon_i \in \{-1, 0, 1\}$ for all $i \in \{1, 2, \dots, s\}$.

Keywords: Chebyshev polynomials, identities, matching polynomials, cycle, path.

2020 Mathematics Subject Classification: 05A19, 11B39, 05C38.

Introduction

Let x be a variable. The Chebyshev polynomials of the first kind $\{T_n(x)\}_{n \geq 0}$ are defined by the following recurrence relation:

$$T_0(x) = 1, T_1(x) = x, \quad \text{and} \quad T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x), \quad (n \geq 2), \quad (365)$$

Likewise, the Chebyshev polynomials of the second kind $\{U_n(x)\}_{n \geq 0}$ satisfy the same recurrence relation but with different initial values, that is:

$$U_0(x) = 1, U_1(x) = 2x, \quad \text{and} \quad U_n(x) = 2xU_{n-1}(x) - U_{n-2}(x), \quad (n \geq 2). \quad (366)$$

The relationship between trigonometric functions and Chebyshev polynomials is well-known. Combinatorially, Benjamin *et al.* [3] proved that for $n \geq 0$,

$$T_n(\cos \theta) = \cos(n\theta), \quad \text{and} \quad U_n(\cos \theta) = \frac{\sin((n+1)\theta)}{\sin \theta}. \quad (367)$$

Let $G = (V(G), E(G))$ be a simple undirected graph, where $V(G)$ is the set of its vertices, and $E(G)$ is the set of its edges. Let $n = |V(G)|$ and $m = |E(G)|$.

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Definition 41. [4] A matching of the graph G is a subset M of its edges $M \subseteq E(G)$ such that no two edges of M share the same vertex. A matching of G with exactly k edges is said to be a k -matching. We denote by $m(G, k)$ the number of k -matchings of G and assume that $m(G, 0) = 1$.

The matching polynomial of a graph G is given by:

$$\alpha(G, x) := \sum_{k=0}^m (-1)^k m(G, k) x^{n-2k}. \quad (368)$$

A path P_n from a vertex v_1 to a vertex v_n ($n \geq 2$) is a sequence of vertices $v_1, v_2, \dots, v_n$ and edges $v_i v_{i+1}$ ($1 \leq i \leq n-1$). A cycle C_n is a path with $v_1 = v_n$.

If we restrict the graph G in (368) to be C_n the cycle on n vertices and P_n the path on n vertices, we obtain:

Theorem 116. [4] Let P_n and C_n be the path and the cycle on n vertices, respectively. Then, we have $\alpha(C_n, 2x) = 2T_n(x)$ and $\alpha(P_n, 2x) = U_n(x)$.

Main results

Chebyshev polynomials have been studied by many researchers. Several identities related to Chebyshev polynomials have been established in literature; for instance, [5, 6, 8] and the closely related references therein. In our work, we are interested in some particular identities.

In 2018, Belbachir and Harik [2] gave a closed form for $F_{m_1+m_2+\dots+m_s}$ in terms of $F_{m_i+\varepsilon_i}$, where F_n is the n^{th} Fibonacci number and $\varepsilon_i \in \{-1, 0, 1\}$. Similarly but for classical Lucas numbers, Belbachir et al. [1] establish a closed form for $L_{m_1+m_2+\dots+m_s}$, in terms of $F_{m_i+\varepsilon_i}$, where L_n is the n^{th} Lucas number and $\varepsilon_i \in \{-1, 0, 1\}$.

Recently, in [7], the authors studied some nested sums to obtain closed formulas for $F_{k, m_1+m_2+\dots+m_s}$, where $F_{k, n}$ is the n^{th} k -Fibonacci number that satisfies the following recurrence relation: $F_{k, n} = kF_{k, n-1} + F_{k, n-2}$ ($n \geq 2$), with $F_{k, 0} = 0$ and $F_{k, 1} = 1$, where k is a positive integer. The main closed form presented is the following:

Theorem 117. [7] Let $n \geq 2$ be an integer. For any positive integers m_i ($1 \leq i \leq n$), we have

$$\begin{aligned} F_{k, \sum_{i=1}^n m_i} &= F_{k, n} \prod_{i=1}^n F_{k, m_i} + F_{k, n-1} \sum_{j=1}^n F_{k, m_j-1} \prod_{\substack{i=1 \\ i \neq j}}^n F_{k, m_i} \\ &+ F_{k, n-2} \sum_{j_1=1}^{n-1} \sum_{j_2=j_1+1}^n F_{k, m_{j_1}-1} F_{k, m_{j_2}-1} \prod_{\substack{i=1 \\ i \neq j_1, j_2}}^n F_{k, m_i} \\ &+ \dots \\ &+ F_{k, 1} \sum_{j_1=1}^2 \sum_{j_2=j_1+1}^3 \dots \sum_{j_{n-1}=j_{n-2}+1}^n F_{k, m_{j_1}-1} F_{k, m_{j_2}-1} \dots F_{k, m_{j_{n-1}}-1} \prod_{\substack{i=1 \\ i \neq j_1, \dots, j_{n-1}}}^n F_{k, m_i}. \end{aligned}$$

In this work, in the light of these recent works, we create closed formulas for both $T_{m_1+m_2+\dots+m_s}(x)$ and $U_{m_1+m_2+\dots+m_s}(x)$, for any positive integers s and m_i ($1 \leq i \leq s$), among others. Special cases are also discussed. Finally, we provide some new trigonometric identities by combining the closed

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formulas obtained and Equation (367).

The matching polynomials related to the path P_n on n vertices and the cycle C_n on n vertices are used in our proofs.

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Some Fundamental Concepts For Hyperlattice Ordered Commutative Semigroups

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Abstract: In this paper, we give some hyperstructures as hyperlattice ordered commutative semigroups (monoids). In this way, some results of filters and ideals in hyperlattices are extended to the framework of hyperlattice-ordered commutative semigroups.

Keywords: Semigroup, ordered semigroup, negatively (positively) ordered semigroup, hyperlattice ordered semigroup, hyperlattice ordered monoid, filter, ideal.

2020 Mathematics Subject Classification: Primary 06B20, 06F05, 20M14.

Introduction

Hyperalgebras are a kind of hyperstructures as well as hypergroups, hyperrings, hyperlattices and so on. The hyperstructures theory was studied from many points of view and applied to several areas of mathematics, computer science and logic.

The notion of filters is important to studying logical systems, algebraic and topological structures. From a logical point of view, filters correspond to sets of provable formulae. They are played a fundamental role in Stone representation theorem for Boolean lattice and in the theory of representation of distributive lattices.

Main results

Definition 42. Let A be a nonempty set. A partially ordered semigroup is a structure $(A, *, \leq)$ satisfies the following condition: (i) $(A, *)$ is a semigroup; (ii) $(A, \leq)$ is a partially ordered set; (iii) for all $a, b, x \in A$, $a \leq b$ implies $a * x \leq b * x$ and $x * a \leq x * b$.

A partially ordered semigroup is said to be negatively (positively) ordered if $x * y \leq x$ ($x \leq x * y$) and $x * y \leq y$ ($y \leq x * y$) for all $x, y \in A$.

Let L be a nonempty set and $P^*(L)$ denotes the set of all nonempty subsets of L . Maps $h : L \times L \rightarrow P^*(L)$, are called hyperoperations.

Definition 43. Let L be a nonempty set, $\wedge$ be a binary operation and $\sqcup$ be a hyperoperation on L . L is called a hyperlattice if for all $a, b, c \in L$ the following conditions hold: (i) $a \in a \sqcup a$, and $a \wedge a = a$; (ii)

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$a \sqcup b = b \sqcup a$, and $a \wedge b = b \wedge a$; (iii) $a \in [a \wedge (a \sqcup b)] \cap [a \sqcup (a \wedge b)]$; (iv) $a \sqcup (b \sqcup c) = (a \sqcup b) \sqcup c$, and $a \wedge (b \wedge c) = (a \wedge b) \wedge c$; (v) $a \in a \sqcup b \Rightarrow a \wedge b = b$. A hyperlattice L with the property $a \wedge (b \sqcup c) \subseteq (a \wedge b) \sqcup (a \wedge c)$ is called distributive, where for all nonempty subsets A and B of L we define $A \wedge B = \{a \wedge b \mid a \in A, b \in B\}$ and $A \sqcup B = \cup\{a \sqcup b \mid a \in A, b \in B\}$.

Definition 44. Let L be a non-empty set. $\wedge, *$ are binary operations and $\sqcup$ is a hyperoperation on L . L is called bounded hyperlattice ordered commutative semigroup if the following conditions hold:

- (i) $(L, \wedge, \sqcup)$ is a hyperlattice;
- (ii) $(L, *)$ is a commutative semigroup;
- (iii) for all $a, b, c \in L : a * (b \wedge c) \leq (a * b) \wedge (a * c)$ and $a * (b \sqcup c) \subseteq (a * b) \sqcup (a * c)$.

Definition 45. Let $(L, \wedge, \sqcup, *)$ be a hyperlattice ordered commutative semigroup. A non-empty subset I of L is called an ideal provided for every $x, y \in L$

- (i) $x \in I, y \leq x \Rightarrow y \in I$;
- (ii) $x, y \in I \Rightarrow x \sqcup y \subseteq I$;
- (iii) $x \in I$ or $y \in I \Rightarrow x * y \in I$.

A proper ideal I is called a prime ideal if satisfy (i) $x \wedge y \in I \Rightarrow (x \in I$ or $y \in I)$ and (ii) $x * y \in I \Rightarrow (x \in I$ or $y \in I)$ for all $x, y \in L$.

Definition 46. Let $(L, \wedge, \sqcup, *)$ be a hyperlattice ordered commutative semigroup. A non-empty subset F of L is called a filter provided for every $x, y \in L$

- (i) $x \in F, x \leq y \Rightarrow y \in F$;
- (ii) $x, y \in F \Rightarrow x \wedge y \in F$;
- (iii) $x, y \in F \Rightarrow x * y \in F$.

A proper filter F is called a prime filter if it satisfy (i) $(x \sqcup y) \cap F \neq \emptyset \Rightarrow (x \in F$ or $y \in F)$; (ii) $x * y \in F \Rightarrow (x \in F$ and $y \in F)$.

Theorem 118. Let $(L, \wedge, \sqcup, *)$ be a hyperlattice ordered commutative semigroup. If I is a prime ideal of L , then $L - I$ is a prime filter of L . Similarly, if F is a prime filter of L , then $L - F$ is a prime ideal of L .

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A square root of a triangular Toeplitz bloc matrix

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Abstract: In this paper, our goal is to find a square root of a triangular Toeplitz bloc matrix A . We give a relation to compute the entries of matrix $\sqrt{A}$ a square root of A with examples to illustrate our results.

Keywords: bloc matrix, Toeplitz matrix, triangular matrix, a square root.

Introduction

Let A be an $nN * nN$ triangular Toeplitz bloc matrix. A is given by

$$\begin{pmatrix} A_0 & A_{-1} & \dots & \dots & \dots & \dots & A_{1-n} \\ 0 & A_0 & \ddots & & \dots & \dots & A_{2-n} \\ \vdots & \ddots & \ddots & & \dots & \dots & \vdots \\ \vdots & & & \dots & & & \vdots \\ \vdots & \dots & \dots & & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & & \ddots & A_0 & A_{-1} \\ 0 & 0 & \dots & \dots & \dots & 0 & A_0 \end{pmatrix}$$

Where n, N are positive integers and $A_1, A_2, \dots, A_{n_1}$ are $n * n$ real matrices.

We denote by $\sqrt{A}$ a square root of A . The problem is to find a $\sqrt{A}$ by giving a formula to compute the entries of $\sqrt{A}$.

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Some open problems involving the difference of two powers of Padovan numbers

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Abstract: In this short note, by using some facts on linear forms in logarithms we propose to find all indices n and $n-s$ with $1 \leq s \leq N$ and all exponents x such that $P_n^x - P_{n-s}^x = 3^a$.

Keywords: Padovan numbers, linear forms in logarithms, reduction method.

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Introduction

Padovan sequence (see [2]), one of the most familiar binary recurrence sequence, is defined by $P_{n+3} = P_{n+1} + P_n$ for $n \geq 0$ where $P_0 = P_1 = P_2 = 1$. Some of the terms of the Padovan sequence are given by

$$1, 1, 1, 2, 2, 3, 4, 5, 7, 9, 12, 16, 21, 28, 37, 49, 65, 86, 114, 151, \dots$$

Its characteristic polynomial is of the form $x^3 - x - 1 = 0$ whose roots are α, β and $\gamma = \bar{\beta}$, where

$$\alpha = \frac{r_1 + r_2}{6}, \quad \beta = \frac{-r_1 - r_2 + i\sqrt{3}(r_1 - r_2)}{12},$$

and

$$r_1 = \sqrt[3]{108 + 12\sqrt{69}}, \quad r_2 = \sqrt[3]{108 - 12\sqrt{69}}.$$

Binets formula enables as to rewrite the Padovan sequence by using the roots α, β and γ

$$P_n = c_\alpha \alpha^n + c_\beta \beta^n + c_\gamma \gamma^n, \text{ for all } n \geq 0, \quad (369)$$

where

$$\begin{cases} c_\alpha = \frac{(1-\beta)(1-\gamma)}{(\alpha-\beta)(\alpha-\gamma)} = \frac{1+\alpha}{-\alpha^2+3\alpha+1}, \\ c_\beta = \frac{(1-\alpha)(1-\gamma)}{(\beta-\alpha)(\beta-\gamma)} = \frac{1+\beta}{-\beta^2+3\beta+1}, \\ c_\gamma = \frac{(1-\alpha)(1-\gamma)}{(\gamma-\alpha)(\gamma-\beta)} = \frac{1+\gamma}{-\gamma^2+3\gamma+1} = \bar{c}_\beta. \end{cases} \quad (370)$$

Further, using induction, we can prove that

$$\alpha^{n-3} \leq P_n \leq \alpha^{n-1}, \text{ for } n \geq 1. \quad (371)$$

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In this work, we consider the diophantine equation:

$$P_n^x - P_m^x = 3^a \quad (372)$$

We solve the equation (372) for $x = 1$. For this purpose, we need to some facts on algebraic numbers and linear forms in logarithms. For details, one can see [1] and [3].

Definition 47. Let η be an algebraic number of degree k with minimal polynomial

$$a_0 x^k + a_1 x^{k-1} + \dots + a_k = a_0 \prod_{i=1}^k (x - \eta_i),$$

where the a_i 's are relatively prime integers with a_0 and the η_i 's are conjugates of η . Then

$$h(\eta) = \frac{1}{k} \left(\log a_0 + \sum_{i=1}^k \log (\max \{ |\eta_i|, 1 \}) \right)$$

is called the logarithmic height of the η .

Proposition 10. Let $\eta_1, \eta_2, \dots, \eta_t$ be elements of an algebraic closure of $\mathbb{Q}$ and $m \in \mathbb{Z}$. Then

1. $h(\eta_1 \cdot \eta_2 \dots \eta_t) \leq \sum_{i=1}^t h(\eta_i)$.
2. $h(\eta_1 + \eta_2 + \dots + \eta_t) \leq (t-1) \log 2 + \sum_{i=1}^t h(\eta_i)$.
3. $h(\eta^m) = |m| \cdot h(\eta)$.

Theorem 119 (Mateev's Theorem). Let $\gamma_1, \gamma_2, \dots, \gamma_t$ be positive elements of a number field $\mathbb{L}$ of degree D , and $b_1, b_2, \dots, b_t$ be rational integers. Set

$$B = \max \{ |b_1|, |b_2|, \dots, |b_t| \}$$

and

$$\Lambda = \gamma_1^{b_1} \cdot \gamma_2^{b_2} \cdot \dots \cdot \gamma_t^{b_t} - 1.$$

If Λ is nonzero, then

$$\log |\Lambda| > -3.30^{t+4} \cdot (t+1)^{5.5} \cdot D^2 \cdot (1 + \log D) \cdot (1 + \log(B)) \cdot A_1 \cdot A_2 \dots A_t$$

where

$$A_i \geq \max \{ D \cdot h(\gamma_i), |\log \gamma_i|, 0.16 \}$$

for all $1 \leq i \leq t$. If $\mathbb{L} \subset \mathbb{R}$, then

$$\log |\Lambda| > -1.4 \cdot 30^{t+3} \cdot t^{4.5} \cdot D^2 \cdot (1 + \log D) (1 + \log B) A_1 A_2 \dots A_t.$$

Let x be a real number. In what follows, $\| \cdot \|$ denotes the distance from x to the nearest integer. In other words, $\|x\| = \min \{ |x - n|, n \in \mathbb{Z} \}$.

The next result is used in the reduction of the upper bound of n in our theorem and it is proved by Dujella and Betho [1].

Lemma 18. Let M be positive integer, and let p/q be a convergent of continued fraction of the irrational number γ such that $q > 6M$. Let A, B, μ be some real numbers with $A > 0$ and $B > 1$. If $\epsilon = \|\mu q\| - M\|\gamma q\| > 0$, then there exists no solution to the inequality

$$0 < |u\gamma - v + \mu| < AB^{-w}$$

in positive integers u, v and w with

$$u < M, \text{ and } w \geq \frac{\log(Aq/\epsilon)}{\log B}.$$

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Main results

Applying the above facts, we have:

Theorem 120. *The only triples of nonnegative integers n, m, a with $n \geq m$ satisfying the Diophantine equation $P_n - P_m = 3^a$ are the following:*

$$(n, m, a) \in \left\{ \begin{array}{l} (6, 1, 1), (6, 2, 1), (7, 3, 1), (7, 4, 1), (8, 6, 1), (10, 5, 2), (10, 9, 1), (11, 8, 2), \\ (12, 10, 2), (13, 1, 3), (13, 2, 3), (14, 13, 2), (17, 7, 4). \end{array} \right\}$$

Or, equivalently, we have

- $P_6 - P_1 = P_6 - P_2 = P_7 - P_3 = P_7 - P_4 = P_8 - P_6 = P_{10} - P_9 = 3,$
- $P_{12} - P_{10} = P_{14} - P_{13} = 3^2,$
- $P_{13} - P_1 = P_{13} - P_2 = 3^3,$
- $P_{17} - P_7 = 3^4.$

Open question

We ask whether the diophantine equation $P_n^x - P_{n-s}^x = 3^a$ has solutions for $x \geq 2$ and $a \leq 10$ or not.

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GROUPS WHOSE PROPER SUBGROUPS ARE FINITE-BY-(LOCALLY NILPOTENT

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Abstract: Let G be a group; G is called a minimal non-(finite-by-locally nilpotent) group if, and only if, all of its proper subgroups are finite-by-locally nilpotent, but G itself is not finite-by-locally nilpotent. Our main result is that a locally graded group of finite rank is a minimal non-(finite-by-locally nilpotent) group if, and only if, it is a minimal non-(finite-by-abelian) group such that $\pi(G) = p, q$ with p and q being two distinct prime numbers.

Keywords: locally graded, locally nilpotent, finite rank, Abelian.

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Introduction

Let X be a class of groups. A group G is said to be a minimal non X -group if and only if, its proper subgroups are X -groups, but G itself is not an X -group. Several results have been obtained on minimal non X -groups for different choices of X . In particular, the classification of non X -minimal groups in the class of all groups has not succeeded due to the existence of Tarski groups, which are infinite groups with all proper subgroups being finite. To avoid such groups, we are led to consider this classification in the class of locally graded groups. A group G is called locally graded if any non-trivial finitely generated subgroup of G has a proper subgroup of finite index. Miller and Moreno were the first to study minimal non X -groups. They characterized in [4] the minimal non abelian groups in the case where the group is finite. In [5] Newman and Wiegold studied the minimal non nilpotent groups and they characterized these groups in the case where they have a maximal subgroup. H. Smith [6] studied the locally nilpotent minimal non nilpotent-groups without maximal subgroup. Then Bruno and Phillips [2] in 1983 studied the locally graded minimal non (finite-by-abelian)-groups. In [7] Xu characterized the minimal non (finite-by-nilpotent) groups. Then De Giovanni and Trombetti [3] studied the minimal non hypercentral groups. Here, we consider locally graded minimal non (finite-by-locally nilpotent) groups and we obtain a result which can be stated as follows.

Main results

Proposition 11. *If G be a locally graded non-perfect minimal non-(finite-by-locally nilpotent) group of finite rank. Then G is periodic.*

Proposition 12. *If G be a locally graded non-perfect minimal non-(finite-by-locally nilpotent) group such that G' is abelian. Then G' is periodic.*

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Theorem 121. *Let G be a locally graded non-perfect group of finite rank. G is minimal non-(finite-by-locally nilpotent) group if, and only if, G is minimal non-(finite-by-abelian) group.*

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Groups whose proper subgroups are (locally finite)-by-Engel

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Abstract: Our main result is that a locally (soluble-by-finite) group G is minimal non-(finite-by-Engel) of finite rank if, and only if, it is minimal non-(finite-by-abelian) such that $|\pi(G)| = 2$. We proved also that a locally graded periodic group is minimal non-(finite-by-(n -Engel)) of finite rank if, and only if, it is a minimal non-(finite-by-abelian) group.

Keywords: Locally graded, minimal non- D , Engel, Finite rank.

2020 Mathematics Subject Classification: 20E07, 20F45.

Introduction

A group G is said to be an Engel group if for each pair of elements $x, y \in G$ we have $[x, {}_n y] = 1$ for some positive integer $n = n(x, y)$. If n can be chosen independently of x and y then G is said to be an n -Engel group. Also a group G is said to be locally graded if every non-trivial finitely generated subgroups of G has a nontrivial finite image.

Let D be a class of group, a group G is said to be a minimal non- D group if and only if every proper subgroup of G is a D -group and G is not a D -group. Among the first to study finite minimal non D -group are Miller and Moreno in 1903 [4] and Schmidt in 1924 [6], who characterized finite minimal non-abelian groups and finite minimal non-nilpotent groups respectively, while Newman and Wiegold [5] were among the first to study the infinite case. Later, in 1983, locally graded minimal non-abelian groups were characterized by B. Bruno and R. Phillips in [3]. Recently, S. Azra and N. Trabelsi proved that a locally graded group is a minimal non-(finite-by-hypercentral) group if, and only if, it is a minimal non-hypercentral group or a minimal non-(finite-by-abelian) group [2], and A. Abdollahi, N. Trabelsi and A. Zitouni showed that a locally graded group whose proper subgroups are Engel (respectively, n -Engel) is either Engel (respectively, n -Engel) or finite [1].

In our topic, we are interested in locally graded minimal non-(finite-by-Engel) groups and working on the proofs of the following results:

Main results

Theorem 122. *Let G be a locally graded group. If G is a minimal non-(finite-by-abelian) group such that $|\pi(G)| = 2$, then it is a minimal non-(finite-by-Engel) group.*

Theorem 123. *Let G be a locally (soluble-by-finite) group of finite rank minimal non-(finite-by-Engel). Then G is a minimal non-(finite-by-abelian) group and $|\pi(G)| = 2$.*

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Theorem 124. *Let G be a periodic locally graded group of finite rank. G is a minimal non-(finite-by- $(n$ -Engel)) if, and only if, it is minimal non-(finite-by-abelian).*

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Lagrange Theorem in the Bihom-group structure

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Abstract: In this paper, we introduce the notion of a Bihom group as a generalization of a non-associative group. We achieve this by altering the unitality and associativity operations through two maps. Employing the twisting principle, we provide examples of Bihom groups and examine some fundamental properties. Furthermore, we introduce the concepts of Bihom subgroups and cosets, and establish Lagrange's theorem for finite Bihom groups. This theorem asserts that the order of any Bihom subgroup H of a finite Bihom group G divides the order of G .

Keywords: Bihom associativity, Bihom group, Lagrange's theorem

2020 Mathematics Subject Classification: Primary 16T99; 16T25; 16E40.

Introduction

The motivations behind studying bihom structures are closely linked to physics and q -deformations of vector field algebras, often referred to as Hom or Bihom algebras, where certain conditions are deformed. This area has been extensively investigated in recent years [1, 5], among others. A recent study has connected these deformations to group theory by modifying the associativity condition to $(\alpha(x)(yz) = (xy)\alpha(z))$, as observed in [2, 1, 4, 4], and similar works.

In this paper, we aim to generalize the concept of a Hom group to a more generalized non-associative group structure: the Bihom group. This is achieved by introducing another map β , to modify the associativity condition as $(\alpha(x)(yz) = (xy)\beta(z))$. We explore various basic properties such as the cancellation law, the inverse of a product, and introduce the notion of Bihom subgroups and cosets. Additionally, we provide a proof of Lagrange's theorem for finite bihom groups.

Main results

Definition 48. A BiHom group $(G, 1, \alpha, \beta, \cdot)$ consists of a set G with a distinguished element $1 \in G$, bijective maps $\alpha, \beta : G \rightarrow G$, and an operation $\cdot : G \times G \rightarrow G$, satisfying:

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1. BiHom-Associativity Property:

$$\alpha(x) \cdot (y \cdot z) = (x \cdot y) \cdot \beta(z)$$

2. Multiplicativity of Maps:

$$\alpha(x \cdot y) = \alpha(x) \cdot \alpha(y), \quad \beta(x \cdot y) = \beta(x) \cdot \beta(y)$$

3. BiHom-Unitality:

$$x \cdot 1 = \alpha(x), \quad 1 \cdot x = \beta(x)$$

4. BiHom-Invertibility Condition: $\forall x \in G, \exists x_r^{-1}, x_l^{-1} \in G$ such that :

$$x \cdot x_r^{-1} = x_l^{-1} \cdot x = 1$$

Remark 6. (i) If we take $\alpha = \beta$, then we obtain an Hom-group.

(ii) If $\alpha = \beta = \text{Id}(G)$, then we have a structure of group.

Proposition 13. 1. The maps α and β commute .

2. For any $x \in G$: $\alpha^2(x_l^{-1}) = \beta^2(x_r^{-1})$.

3. Left cancellation : If $x \cdot y = x \cdot z$ then $y = z$.

4. Right cancellation : If $y \cdot x = z \cdot x$ then $y = z$.

5. the left inverse of a product : $\forall x, y \in G \quad (xy)_l^{-1} = \alpha^{-1}(\beta(y_l^{-1}))\beta^{-1}(\alpha(x_l^{-1}))$

6. the right inverse of a product : $\forall x, y \in G \quad (xy)_r^{-1} = \alpha^{-1}(\beta(y_r^{-1}))\beta^{-1}(\alpha(x_r^{-1}))$.

Theorem 125. (Lagrange's theorem) Let G be a finite Bihom group ,and H a Bihom subgroup of G ,then the ordre of H divese the ordre of G .

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Rota-Baxter Hom-groups

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Abstract: The theory of Rota–Baxter operators on algebras has been developed since 1960. In 2020, the notion of Rota–Baxter operator on a group was defined. In this work we extend the notion of Rota-Baxter operators from algebras and groups to the setting of Hom-groups. We introduce the concept of Rota-Baxter Hom-groups and discuss various fundamental properties associated with such that structure. Furthermore, we provide the notion of quotient Rota-Baxter Hom-groups and give the fundamentale theorem of homomorphism of Rota-Baxter Hom-groups. Finally, We give the factorization theorem of Rota-Baxter Hom-group.

Keywords: Hom-group, Rota-Baxter operator, Rota-Baxter Hom-group, Factorization

2020 Mathematics Subject Classification: Primary 17A99, 17B38.

Introduction

A Rota-Baxter algebra is an associative algebra equipped with a linear operator that generalizes the integral operator in analysis. Rota-Baxter operators for algebras firstly appear in the paper of G. Baxter [6] in 1960. Let A be an algebra over a field $\mathbb{k}$. A linear operator R on A is called a Rota Baxter operator of weight $\lambda \in \mathbb{k}$ if $R(x)R(y) = R\left(R(x)y + xR(y) + \lambda xy\right)$ for all $x, y \in A$. An algebra endowed with a Rota Baxter operator is called a Rota Baxter algebra. Since that time the theory of Rota-Baxter operators has been developed by different authors. In September 2020, L. Guo et al. [7], defined the notion of Rota Baxter operator on a group. A group with a Rota Baxter operator is called a Rota-Baxter group. In May 2021, V.G. Bardakov and V. Gubarev provided some general constructions of Rota-Baxter operators on a group. As well as they studied the extensions of a given maps to a Rota-Baxter operators. They established the connection between Rota-Baxter operators on a group and Rota-Baxter operators on an associated Lie ring.

The main goal of the current work is to introduce the concept of Rota-Baxter Hom-group by extending the notion of Rota-Baxter operators on algebras and groups to the Hom-groups setting where a Hom-group is the non-associative generalization of a group, whose associativity and unitality are twisted by a compatible bijective map.

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Main results

Definition 49. A Rota-Baxter Hom-group is a Hom-group (G, α) with a map $B : G \longrightarrow G$ such that $B(g)B(h) = B\left(\left(\alpha^{-1}(g)B(g)\right)\left(\alpha^{-1}(h)B(g)^{-1}\right)\right)$ for all $g, h \in G$ and $B \circ \alpha = \alpha \circ B$.

Proposition 14. If (G, α, B) is a Rota-Baxter Hom-group, then $\ker(B)$ and $\text{Im}(B)$ are both Hom-subgroups of G .

Definition 50. Let (G, α, A) , (H, β, B) be two Rota-Baxter Hom-groups. A map $f : G \longrightarrow H$ is called a Rota-Baxter Hom-group homomorphism if f is a Hom-group homomorphism such that $f \circ A = B \circ f$.

Theorem 126 (Fundamental theorem of homomorphism of Rota-Baxter Hom-groups). Let (G, α, A) and (H, β, B) be two Rota-Baxter Hom-groups. If $f : G \longrightarrow H$ is a Rota-Baxter Hom-group homomorphism, then $\frac{G}{\ker(f)} \cong \text{Im}(f)$.

Theorem 127 (Factorization theorem). If (G, α, B) is a Rota-Baxter Hom-group, then every element $g \in G$ can be uniquely expressed as $g = x\alpha^{-1}(y)$ for $(x, y) \in \text{Im}(B) \times \text{Im}(B_\star)$.

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Twistings on Superbialgebras

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Abstract: In this work, we study twist deformations and 2-cocycle deformations, extending to $\mathbb{Z}_2$ -grading case Drinfeld's theory and the work by Giaquinto and Zhang.

Keywords: superalgebra, superbialgebra, supermodule, supermodule superalgebra, twist, deformation.

2020 Mathematics Subject Classification: Primary 16T10, 16T15, 16W50.

Introduction

Twisting elements were introduced by Drinfeld [2] on quasi-Hopf algebras, in order to twist the coproduct without changing its product. They have become an important tool in the classification of finite dimensional Hopf algebras. Many works are focused on the construction of a new algebras by using twisting method. Giaquinto and Zhang [6] studied a twist applied to algebra structure, they obtain a new algebra by twisting the multiplication of a given algebra. Dually, 2-cocycle deformations is a way to modify an algebraic structure by introducing a new operation that satisfies a 2-cocycle condition. A 2-cocycle deformations of a superbialgebra is one where the coproduct is unchanged, while a new multiplication is defined via a formula which only depends on the initial multiplication and on a 2-cocycle σ of superbialgebra. The relationship between Drinfeld twists and 2-cocycles is that every Drinfeld twist corresponds to a 2-cocycle, and every 2-cocycle corresponds to a Drinfeld twist. This duality is known as the Drinfeld twist theorem, and it plays an important role in many areas of mathematics and physics, including quantum field theory, quantum groups and noncommutative geometry.

Main results

We generalize to $\mathbb{Z}_2$ -grading case the construction of a new multiplication along a twisting on a B -supermodule superalgebra, where B is a superalgebra, see [6] for the nongraded case and dually by mean of 2-cocycles.

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linearization of nonlinear operators

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Abstract: The basic question answered by tensor product construction is : is there a vector space G such that $L(G, Y)$ is isometric to $\mathcal{L}(X_1 \otimes \dots \otimes X_m; Y)$? Restating the question: Can we linearize multilinear or m -homogeneous polynomial mappings ? The solution we deliberate on in this illustration which named: Linearization of nonlinear operators.

Keywords: nonlinear operator, multilinear operator, m -homogeneous polynomial, tensor product, linearization.

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Some properties on extended eulerian numbers

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Abstract: In this paper, we will study some properties of the extended Eulerian numbers $H(n, \lambda)$, with a parameter λ , its associated Mertens type function and find the Stieltjes constants associated to their Dirichlet series.

Moreover, we define a H -champion number as a positive integer N such that if $M < N$ then $H(M, \lambda) < H(N, \lambda)$. Several properties of H -champion numbers are given, mainly about the exponents and the number of prime factors in the standard factorisation into primes. Also, we give an estimate for the number $Q(X)$ of H -champions numbers less than X .

Keywords: Extended eulerian number, Stieltjes constants, Champion numbers.

2020 Mathematics Subject Classification: Primary 11A25, 11A41, 11A67.

Introduction

Let λ be a complex number, the Extended Eulerian numbers $H(n, \lambda)$ were been introduced by L. Carlitz [2], which are defined by means of the Dirichlet series

$$\sum_{n \geq 1} \frac{H(n, \lambda)}{n^s} = \frac{1 - \lambda}{\zeta(s) - \lambda}, \quad (373)$$

where $\zeta(s) = \sum_{n \geq 1} \frac{1}{n^s}$ is the Riemann Zeta function.

- If $n = p_1 p_2 \dots p_r$, with p_i distinct primes, then $H(n, \lambda) = H_r(\lambda)$; where the generating function of $H_r(\lambda)$ is given by

$$\frac{1 - \lambda}{e^t - \lambda} = \sum_{r \geq 0} H_r(\lambda) \frac{t^r}{r!} \quad (374)$$

We put [2]

$$(\log \zeta(s))^k = \sum_{n \geq 1} \frac{\alpha_k(n)}{n^s}, \quad k = 0, 1, 2, \dots \quad (375)$$

where $\alpha_k(n) = \sum_{n=d_1 d_2 \dots d_k} \alpha(d_1) \dots \alpha(d_k), k \geq 1$, and

$$\alpha(n) = \begin{cases} \frac{1}{r} & \text{if } n = p^r, \quad r \geq 1 \\ 0 & \text{otherwise.} \end{cases} \quad (376)$$

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so

$$H(n, \lambda) = \sum_{k \geq 0}^{\Omega(n)} H_k(\lambda) \frac{\alpha_k(n)}{k!}. \quad (377)$$

- If $\lambda = 2$, then $H(n, 2) = K(n)$ is the Kalmar function which count the number of representations of n as a product of factors greater than one it was studied by M. Klazar and F. Luca [6], M. Deléglise, M.O. Hernane and J.-L. Nicolas [4].

$$\bullet \text{ If } \lambda = 0, \text{ then } H(n, 0) = \mu(n) = \begin{cases} 1 & \text{if } n = 1, \\ (-1)^k & \text{if } n = \prod_{i=1}^k p_i, p_i \text{ distinct primes,} \\ 0 & \text{if } p^2 | n \text{ for some prime } p. \end{cases}$$

- For $\lambda > 1$, then $H(n, \lambda)$ was studied by R. Evans [5] and A. Bayad, M. O. Hernane and A. Togbé [3].

Main results

Stieltjes constant for the function $\frac{1-\lambda}{\zeta(s)-\lambda}$

For $\lambda > 1$, the function $\frac{1-\lambda}{\zeta(s)-\lambda}$ has a simple pole at an $\text{Re}(s) = \rho(\lambda)$ and a simple pole at $s = \rho(\lambda)$. So we write it as a Laurent series in the neighborhood of $s = \rho(\lambda)$

$$\frac{1-\lambda}{\zeta(s)-\lambda} = \frac{S(\lambda)}{s-\rho(\lambda)} + \sum_{r \geq 0} \frac{(-1)^r}{r!} S_r(\lambda) (s-\rho(\lambda))^r,$$

where

$$S(\lambda) = \frac{1-\lambda}{\rho(\lambda)\zeta'(\rho(\lambda))}, \quad (378)$$

$\rho(\lambda)$ is the positive real solution to the equation $\zeta(s) = \lambda$, that is to say

$$\zeta^{-1}(\lambda) = \rho(\lambda) \quad (379)$$

and $S_r(\lambda)$ are the Stieltjes constants which will be given in the next theorem.

Theorem 128. Let $\lambda > 1$, $\rho(\lambda)$ and $S(\lambda)$ defined by (379) and (378). We have

$$S_r(\lambda) = \lim_{x \rightarrow \infty} \left(\sum_{n \leq x} H(n, \lambda) n^{-\rho(\lambda)} \log^r n - \frac{S(\lambda)}{r+1} \log^{r+1} x \right). \quad (380)$$

Mertens function $M(x, \lambda)$

We define an analogue to the classical Mertens function by

$$M(x, \lambda) := \sum_{n \leq x} H(n, \lambda). \quad (381)$$

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By using the Mellin transform Theorem, we obtain

$$\frac{1-\lambda}{\zeta(s)-\lambda} = s \int_1^\infty x^{-s-1} M(x, \lambda) dx.$$

The Perron's formula gives

$$M(x, \lambda) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{1-\lambda}{\zeta(s)-\lambda} x^s \frac{ds}{s}, \quad c > 1.$$

By using the Cauchy's residue theorem, we obtain for $\lambda > 1$

$$M(x, \lambda) = \frac{2(\lambda-1)}{2\lambda+1} + \frac{1-\lambda}{\rho(\lambda)\zeta'(\rho(\lambda))} x^{\rho(\lambda)} + \sum_{\rho} \operatorname{Res} \left(\frac{1-\lambda}{\zeta(s)-\lambda} \frac{x^s}{s}, \rho \right),$$

the sum runs over all the complex, different to $\rho(\lambda)$, poles of the function $s \mapsto \frac{1-\lambda}{\zeta(s)-\lambda}$.

Properties of μ_P -champions numbers

Definition 51. An H -champion number is an integer N such that

$$M < N \Rightarrow H(M, \lambda) < H(N, \lambda).$$

Several properties of H -champion numbers will be given, mainly about the exponents and the number of prime factors in the standard factorisation into primes and the number $Q(X)$ of H -champions numbers less than X .

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Symmetric and Generating Functions for k -Mersenne Numbers

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Abstract: In this paper, we consider some operators including symmetric functions. From those operators, we give new ordinary generating functions for the product of k -Mersenne numbers with some special numbers and polynomials.

Keywords: Symmetric functions, k -Mersenne numbers, k -Fibonacci numbers, k -Pell numbers, k -Lucas numbers, Fibonacci polynomials, Pell polynomials

2010 Mathematics Subject Classification: Primary 05E05; 11B39.

Introduction

Over many years, many researchers have studied well-known number sequences such as Fibonacci numbers, Lucas numbers, Pell numbers and Mersenne numbers. The Mersenne sequence $\{M_n\}_{n \in \mathbb{N}}$ is defined recursively by [3]

$$M_n = 3M_{n-1} - 2M_{n-2}, \quad n \geq 2,$$

with initial conditions $M_0 = 0$, and $M_1 = 1$. In [3] the authors defined the k -Mersenne numbers $\{M_{k,n}\}_{n \in \mathbb{N}}$ as a generalization of the Mersenne numbers by the recurrence relation

$$M_{k,n} = 3kM_{k,n-1} - 2M_{k,n-2}, \quad n \geq 2, \quad (382)$$

with initial conditions $M_{k,0} = 0$ and $M_{k,1} = 1$. The characteristic equation corresponding to the recurrence relation (382) is

$$r^2 - 3kr + 2, \quad (383)$$

whose roots are

$$r_1 = \frac{3k + \sqrt{9k^2 - 8}}{2}, \quad r_2 = \frac{3k - \sqrt{9k^2 - 8}}{2},$$

Theorem 129. [3] For every $n \in \mathbb{N}$, Binet formula for the k -Mersenne numbers is given by

$$\frac{r_1^n - r_2^n}{r_1 - r_2},$$

where r_1 and r_2 are the roots of the characteristic equation (383).

Definition 52. [3] Let n be positive integer and $A = \{a_1, a_2\}$ are set of given variables, then the n -th symmetric function $S_n(a_1 + a_2)$ is defined by

$$S_n(A) = S_n(a_1 + a_2) = \frac{a_1^{n+1} - a_2^{n+1}}{a_1 - a_2}.$$

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Definition 53. [2] The symmetrizing operator $\delta_{a_1 a_2}^k$ is defined by

$$\delta_{a_1 a_2}^k f(a_1) = \frac{a_1^k f(a_1) - a_2^k f(a_2)}{a_1 - a_2}, \quad k \in \mathbb{N}.$$

Definition 54. [3] The symmetrizing operator $\delta_{b_1 b_2}^k \delta_{a_1 a_2}^k$ is defined by

$$\delta_{b_1 b_2}^k \delta_{a_1 a_2}^k f(a_1, b_1) = \frac{a_1^k b_1^k f(a_1 b_1) - a_1^k b_2^k f(a_1 b_2) - a_2^k b_1^k f(a_2 b_1) + a_2^k b_2^k f(a_2 b_2)}{(a_1 - a_2)(b_1 - b_2)}, \quad k \in \mathbb{N}.$$

Main results

In this section, we give some new generating functions for the products of k -Mersenne numbers with some well-know numbers and polynomials.

Theorem 130. [3] Let A, B and C be three alphabets, respectively, $\{a_1, a_2\}$, $\{b_1, b_2\}$ and $\{c_1, c_2\}$ then we have

$$\sum_{n=0}^{\infty} S_n(A) S_{n+k-1}(B) S_{n+k-1}(C) z^n = b_1^k b_2^k \left[\frac{\left(\sum_{n=0}^{\infty} S_n(-A) b_2^n c_1^n z^n \right) \left(\sum_{n=0}^{\infty} S_n(-A) b_1^n c_1^n z^n \right) \sum_{n=0}^{\infty} S_n(-A) S_{n-k-1}(B) C_2^{n+k} z^n}{\left(\sum_{n=0}^{\infty} S_n(-A) b_2^n c_2^n z^n \right) \left(\sum_{n=0}^{\infty} S_n(-A) b_1^n c_2^n z^n \right) \sum_{n=0}^{\infty} S_n(-A) S_{n-k-1}(B) C_1^{n+k} z^n} \right] \quad (384)$$

$$\frac{(c_1 - c_2) \prod_{a \in A} (1 - ab_1 c_1 z) \prod_{a \in A} (1 - ab_2 c_1 z) \prod_{a \in A} (1 - ab_1 c_2 z) \prod_{a \in A} (1 - ab_2 c_2 z)}{}$$

for all $k \in \mathbb{N}_0$.

Based on the previous theorem, we obtain the following results.

Theorem 131. For $n \in \mathbb{N}$, the new generating function of the cubes of k -Mersenne numbers is given by

$$\sum_{n=0}^{\infty} M_{k,n}^3 z^n = \frac{z - (108k^2 - 24)z^3 + 432k^3 z^4 + (-864k^2 + 192)z^5 + 512z^7}{D_{M_{k,n}^3}},$$

with

$$D_{M_{k,n}^3} = 1 - 27k^3 z - (-162k^4 + (9k^2 - 4)(-36k^2 + 8))z^2 - 27k^3(108k^2 - 40)z^3 + (5832k^6 + 3888k^4 - 3456k^2 + 384)z^4 - 216k^3(108k^2 - 40)z^5 - (-10368k^4 + (64(9k^2 - 4))(-36k^2 + 8))z^6 - 13824k^3 z^7 + 4096z^8.$$

Theorem 132. For $n \in \mathbb{N}$, the new generating function for the product of square of k -Mersenne numbers with k -Fibonacci numbers is given by

$$\sum_{n=0}^{\infty} F_{k,n} M_{k,n}^2 z^n = \frac{3zk - 6k^2 z^2 + 6k(9k^2 - 4)z^3 - 108k^4 z^4 - 24k(7k^2 - 2)z^5 + 96k^2 z^6}{D_{F_{k,n} M_{k,n}^2}},$$

with

$$D_{F_{k,n} M_{k,n}^2} = -9k^3 z - (-18k^4 + (9k^2 - 4)(7k^2 - 4))z^2 - 9k^3(-32k^2 + 20)z^3 + (-324k^6 + 664k^4 - 512k^2 + 96)z^4 + 36k^3(-32k^2 + 20)z^5 - (-288k^4 + (16(9k^2 - 4))(7k^2 - 4))z^6 + 576k^3 z^7 + 256z^8.$$

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Theorem 133. For $n \in \mathbb{N}$, the new generating function for the product of square of k -Mersenne numbers with k -Pell numbers is given by

$$\sum_{n=0}^{\infty} P_{k,n} M_{k,n}^2 z^n = \frac{z - (-36k^3 + 12k + 16)z^3 - 144k^3 z^4 + (-144k^4 + 48k^2 + 64k)z^5 - 64k^3 z^7}{D_{P_{k,n} M_{k,n}^2}},$$

with

$$\begin{aligned} D_{P_{k,n} M_{k,n}^2} = & 1 - 18k^2 z - (-72k^2 + (9k^2 - 4)(9k^3 - 4k - 8))z^2 - 18k^2(-36k^3 + 20k + 16)z^3 \\ & + (648k^6 + 256 + 4k(-324k^4 - 144k^3 + 64) + 96k^2)z^4 + 72k^3(-36k^3 + 20k + 16)z^5 \\ & - (-1152k^4 + 16k^2(9k^2 - 4)(9k^3 - 4k - 8))z^6 + 1152k^5 z^7 + 256k^4 z^8. \end{aligned}$$

Theorem 134. For $n \in \mathbb{N}$, the new generating function for the product of square of k -Mersenne numbers with k -Jacobsthal numbers is given by

$$\sum_{n=0}^{\infty} J_{k,n} M_{k,n}^2 z^n = \frac{z - (-68k^2 + 24)z^3 - 144k^3 z^4 + (-544k^2 + 192)z^5 - 512z^7}{D_{J_{k,n} M_{k,n}^2}},$$

with

$$\begin{aligned} D_{J_{k,n} M_{k,n}^2} = & 1 - 9k^3 z - (-18k^4 + (9k^2 - 4)(16k^2 - 8))z^2 - 9k^3(-68k^2 + 40)z^3 + (-648k^6 + 2608k^4 \\ & - 2176k^2 + 384)z^4 + 72k^3(-68k^2 + 40)z^5 - (-1152k^4 + (64(9k^2 - 4))(16k^2 - 8))z^6 \\ & + 4608k^3 z^7 + 4096z^8. \end{aligned}$$

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Arithmetic properties of partitions enumerated by the Andrews-Göllnitz-Gordon theorem

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Abstract: In 1967, Andrews found a combinatorial generalization of the Göllnitz-Gordon identities, which can be called the Andrews-Göllnitz-Gordon theorem. He introduced the partition function $C_{k,i}(n)$ as the number of partitions of n in which no part is congruent to 2 modulo 4 and $0, \pm(2i-1)$ modulo $4k$, where $0 < i \leq k$. In this paper, we study the arithmetic behaviour of $C_{k,i}(n)$ for $k \in \{3, 9\}$ such that $k = 2i - 1$.

Keywords: Partition, Congruence, Andrews-Göllnitz-Gordon theorem

2020 Mathematics Subject Classification: 05A15, 05A17, 11P83

Introduction

A partition of a positive integer n is a finite sequence of natural numbers $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_k$, such that $\lambda_1 + \lambda_2 + \dots + \lambda_k = n$, where $\lambda_i \geq 1$. Here and throughout we will use the standard q -series notations

$$(a; q)_n = \begin{cases} \prod_{i=0}^{n-1} (1 - aq^i) & \text{if } n > 0, \\ 1 & \text{if } n = 0, \end{cases}$$

and f_k is defined by

$$f_k = (q^k, q^k)_\infty = \prod_{n \geq 1} (1 - q^{nk}).$$

In the 1960s, another set of Rogers-Ramanujan type identities, called the Göllnitz-Gordon identities, was discovered, with a story similar to that of Rogers and Ramanujan. Initially discovered by Göllnitz [2], these identities remained unknown since they were included in his 1961 unpublished honors baccalaureate thesis, until Gordon [3] independently rediscovered them in 1965.

Over time, the Göllnitz-Gordon identities have acquired considerable attention. This culminated in a major combinatorial generalization by Andrews, a general theorem [1, p. 114] that reduces to the Göllnitz-Gordon identities in the special cases in which $k = i = 2$, $k = i + 1 = 2$.

Theorem 135. (Andrews). Let i and k be integers with $0 < i \leq k$. Let $C_{k,i}(n)$ denote the number of partitions of n into parts not congruent to 2 modulo 4 and $0, \pm(2i-1)$ modulo $4k$. Let $D_{k,i}(n)$ denote the

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number of partitions $(\lambda_1^{u_1}, \lambda_2^{u_2}, \dots, \lambda_m^{u_m})$ of n in which no odd part is repeated, $\lambda_j \geq \lambda_{j+1}$, $\lambda_j - \lambda_{j+k-1} \geq 2$ if λ_j odd, $\lambda_j - \lambda_{j+k-1} > 2$ if λ_j even, and at most $i - 1$ parts are ≤ 2 . Then

$$C_{k,i}(n) = D_{k,i}(n).$$

In this paper, we present some arithmetic properties of $C_{k,i}(n)$ for $k \in \{3, 9\}$ such that $k = 2i - 1$. In the following result, for every odd $k \geq 3$ in which $k = 2i - 1$, we introduce the generating function of $C_{k,i}(n)$, the number of partitions of n into parts not congruent to 2 modulo 4 and 0, k or $3k$ modulo $4k$. Through the remainder of this paper, we will omit the parameter i from $C_{k,i}(n)$.

Theorem 136. *The generating function of $C_k(n)$, the number of partitions of n into parts not congruent to 2 modulo 4 and 0, k or $3k$ modulo $4k$ is given by*

$$\sum_{n \geq 0} C_k(n) q^n = \frac{(q^2; q^2)_\infty (q^k; q^k)_\infty (q^{4k}; q^{4k})_\infty}{(q; q)_\infty (q^4; q^4)_\infty (q^{2k}; q^{2k})_\infty}. \quad (385)$$

Main results

Theorem 137. *For any nonnegative integer n , we have*

$$C_3(6n + 4) \equiv 0 \pmod{2}. \quad (386)$$

$$C_3(18n + 8) \equiv 0 \pmod{2}. \quad (387)$$

$$C_3(18n + 14) \equiv 0 \pmod{2}. \quad (388)$$

$$C_3(54n + 38) \equiv 0 \pmod{2}. \quad (389)$$

$$C_3(162n + 74) \equiv 0 \pmod{2}. \quad (390)$$

$$C_3(162n + 128) \equiv 0 \pmod{2}. \quad (391)$$

Theorem 138. *For any prime $p \geq 5$, $\alpha \geq 0$ and $n \geq 0$, we have*

$$C_3 \left(6p^{2\alpha+1}(pn + i) + \frac{p^{2\alpha+2} - 1}{4} \right) \equiv 0 \pmod{2}, \quad (392)$$

where i is an integer and $1 \leq i \leq p - 1$.

Theorem 139. *For any nonnegative integer n , we have*

$$C_3(9n + 5) \equiv 0 \pmod{3}. \quad (393)$$

$$C_3(9n + 8) \equiv 0 \pmod{3}. \quad (394)$$

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Theorem 140. For any prime $p \equiv 3 \pmod{4}$, $\alpha \geq 0$ and $n \geq 0$, we have

$$C_3 \left(p^{2\alpha+1}(pn+i) + \frac{p^{2\alpha+2}-1}{4} \right) \equiv 0 \pmod{3}, \quad (395)$$

where i is an integer and $1 \leq i \leq p-1$.

Theorem 141. For any nonnegative integer n , we have

$$C_9(6n+3) \equiv 0 \pmod{2}. \quad (396)$$

$$C_9(6n+5) \equiv 0 \pmod{2}. \quad (397)$$

$$C_9(12n+8) \equiv 0 \pmod{2}. \quad (398)$$

$$C_9(24n+13) \equiv 0 \pmod{2}. \quad (399)$$

$$C_9(24n+19) \equiv 0 \pmod{2}. \quad (400)$$

$$C_9(24n+12) \equiv 0 \pmod{2}. \quad (401)$$

$$C_9(24n+18) \equiv 0 \pmod{2}. \quad (402)$$

Theorem 142. For all $\alpha \geq 0$ and $n \geq 0$, we have

$$C_9(6n+4) \equiv 0 \pmod{3}. \quad (403)$$

$$C_9(18n+14) \equiv 0 \pmod{3}. \quad (404)$$

$$C_9 \left(3^\alpha \cdot 54n + 4 \sum_{i=1}^{\alpha} 3^{i+2} + 44 \right) \equiv 0 \pmod{3}. \quad (405)$$

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Congruences related to the Bell numbers and Bell polynomials

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Abstract: In this paper, we use the properties of the classical umbral calculus to give some congruences related to the Bell numbers and Bell polynomials.

Keywords: Bell numbers, Bell polynomials, Congruences, Umbral calculus.

2020 Mathematics Subject Classification: 11B83, 11B73, 11A07, 05A40.

Introduction

Recall that the n -th Bell polynomials $\mathcal{B}_n(x)$ are defined by

$$\mathcal{B}_n(x) = \sum_{k=0}^n \left\{ \begin{matrix} n \\ k \end{matrix} \right\} x^k,$$

where $\left\{ \begin{matrix} n \\ k \end{matrix} \right\}$ is (n, k) -th Stirling number of the second kind which counts the number of partition of the set $[n]$ into k non-empty subsets. In a similar way the n -th r -Bell polynomials $\mathcal{B}_{n,r}(x)$ are defined by [4]

$$\mathcal{B}_{n,r}(x) = \sum_{k=0}^n \left\{ \begin{matrix} n+r \\ k+r \end{matrix} \right\}_r x^k,$$

where $\left\{ \begin{matrix} n \\ k \end{matrix} \right\}_r$ is the r -Stirling numbers of the second kind which counts the number of partition of the set $[n]$ into k non-empty subsets such that the numbers $1, \dots, r$ are in distinct subsets, see [4]. For $x = 1$, $\mathcal{B}_n(1)$ is the n -th Bell number which counts the number of all partitions of the set $[n] := \{1, \dots, n\}$ and $\mathcal{B}_{n,r}(1)$ is the n -th r -Bell number which counts the number of all partitions of set $[n+r]$, see [4]. The n -th derangement polynomials and the n -th Lah polynomials are defined respectively by:

$$\mathcal{D}_n(x) = \sum_{k=0}^n \binom{n}{k} k! (x-1)^{n-k}, \quad \mathcal{L}_n(x) = \sum_{k=0}^n L(n, k) x^k,$$

where $\mathcal{D}_n = \mathcal{D}_n(0)$ is the n -th derangement number, counting the number of permutation of the set $[n] := \{1, \dots, n\}$ without a fixed point and $L(n, k)$ is the (n, k) th Lah number counts partitions of the set $[n]$ into k ordered lists, see [1]. The number of partitions of set $[n]$ without singletons denoted by $\mathcal{V}_n$. For any non-negative integers n, r , we have [5]:

$$\mathcal{V}_n = \sum_{k=0}^n (-1)^{n-k} \binom{n}{k} \mathcal{B}_k, \quad \mathcal{B}_{n,r} = \sum_{k=0}^n r^k \binom{n}{k} \mathcal{B}_{n-k}, \quad \mathcal{B}_n = \sum_{k=0}^n \binom{n}{k} \mathcal{V}_k. \quad (406)$$

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Let $\mathbf{B}$ and $\mathbf{B}_x$ respectively be the Bell and generalized Bell umbra introduced by Rota [4] and Sun [6], given by $\mathbf{B}^n = \mathcal{B}_n$ and $\mathbf{B}_x^n = \mathcal{B}_n(x)$. Then the above numbers and polynomials can be represented umbrally as follows see [1, 5]:

$$\mathcal{B}_{n,r} = (\mathbf{B} + r)^n, \mathcal{V}_n = (\mathbf{B} - 1)^n, \mathcal{D}_n = (-1)^n (\mathbf{B} - 1)_n, \mathcal{L}_n = (\mathbf{B} + n - 1)_n,$$

and

$$\mathcal{B}_{n,r}(x) = (\mathbf{B}_x + r)^n, \mathcal{D}_n(1-x) = (-1)^n (\mathbf{B}_x - 1)_n, \mathcal{L}_n(x) = (\mathbf{B}_x + n - 1)_n.$$

For more information on the umbral calculus and its applications one can see [1, 4, 5].

Congruences via Bell umbra

In this paragraph by Bell umbra we give some congruences on the number of partitions of set $[n]$ without singleton.

Lemma 19. *Let f be a polynomial in $\mathbb{Z}[x]$. Then for any integer $s \geq 1$ and any prime number p , there holds*

$$f(\mathbf{B} - 1)((\mathbf{B} - 1)^{p^s} - (\mathbf{B} - 1)) \equiv sf(\mathbf{B} - 1) \pmod{p}.$$

Proposition 15. *For any integers $n \geq 0, m \geq 0, s \geq 1$ and any prime p , there holds*

$$\mathcal{V}_{n+ps} \equiv \mathcal{V}_{n+1} + s\mathcal{V}_n \pmod{p}, \quad (407)$$

$$\mathcal{V}_{n+mps} \equiv \sum_{k=0}^m \binom{m}{k} s^k \mathcal{V}_{n+m-k} \pmod{p}. \quad (408)$$

Corollary 6. *For any integer $n \geq 0$ and any prime p , we have*

$$\mathcal{V}_{n+p^{s+1}-p^s} \equiv \sum_{k=0}^{p-1} (-s)^k (\mathcal{V}_{n-k} + \mathcal{V}_{n-1-k}) \pmod{p}, \quad (409)$$

$$\mathcal{V}_{n+p^{s+1}-2p^s} \equiv \sum_{k=0}^{p-2} (-s)^k (1+k) (\mathcal{V}_{n-k} + \mathcal{V}_{n-1-k}) \pmod{p}, \quad (410)$$

$$\mathcal{V}_{n+p\mathcal{N}_p} \equiv \mathcal{V}_{n+\mathcal{N}_p} \pmod{p}, \quad (411)$$

$$\sum_{k=1}^{\mathcal{N}_p} \binom{\mathcal{N}_p}{k} \mathcal{V}_{n+\mathcal{N}_p-k} \equiv 0 \pmod{p}. \quad (412)$$

Where, $\mathcal{N}_p = 1 + p + \dots + p^{p-1}$.

Now by generalized Bell umbra we give some congruences related to Bell polynomials.

Lemma 20. *For any polynomials f , there holds*

$$f(\mathbf{B}_x + r) = \sum_{i=0}^r \binom{r}{i} \frac{d^i}{dx^i} f(\mathbf{B}_x).$$

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Proposition 16. For any non-negative integers n, r , there holds

$$\begin{aligned}\mathcal{B}_{n,r}(x) &= \sum_{i=0}^r \binom{r}{i} \frac{d^i}{dx^i} \mathcal{B}_n(x), \\ \mathcal{L}_r(x) &= \sum_{i=0}^r (-1)^r \binom{r}{i} \frac{d^i}{dx^i} \mathcal{D}_r(1-x), \\ \mathcal{L}_r(x) &= \sum_{i=0}^r (-1)^{r+i} i! \binom{r}{i}^2 \mathcal{D}_{r-i}(1-x), \text{ in this case for } x=1, \text{ we obtain} \\ \mathcal{L}_r &= \sum_{i=0}^r (-1)^{r+i} i! \binom{r}{i}^2 \mathcal{D}_{r-i}.\end{aligned}$$

Corollary 7. For any non-negative integer n and for any prime numbers p , there holds

$$\begin{aligned}\mathcal{L}_p(x) &\equiv (-1)^p \mathcal{D}_p(1-x) \pmod{p\mathbb{Z}[x]}, \\ \mathcal{B}_{n,p-1}(x) &\equiv \sum_{i=0}^{p-1} (-1)^i \frac{d^i}{dx^i} \mathcal{B}_n(x) \pmod{p\mathbb{Z}[x]}, \\ \mathcal{L}_{p-1}(x) &\equiv \sum_{i=0}^{p-1} (-1)^i \frac{d^i}{dx^i} \mathcal{D}_{p-1}(1-x) \pmod{p\mathbb{Z}[x]}, \\ \mathcal{L}_{p-1}(x) &\equiv \sum_{i=0}^{p-1} (-1)^i i! \mathcal{D}_{p-1-i}(1-x) \pmod{p\mathbb{Z}[x]}.\end{aligned}$$

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Y a-t-il un besoin d'introduire les espaces métriques à valeurs complexes dans la théorie du point fixe ?

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Abstract: En 2011, Azam et al. ont introduit la notion d'espace métrique à valeurs complexes et démontré l'existence de points fixes communs d'une paire de correspondances satisfaisant un type contraction impliquant des inégalités rationnelles. Dans cette communication, nous tenterons de discuter et de donner une réponse (non exhaustives) à la question suivante : Le concept d'espaces métriques à valeurs complexes offre-t-il quelque chose de nouveau dans le domaine du point fixe?.

Keywords: Espaces métriques, Espaces métriques à valeurs complexes, point fixe.

2020 Mathematics Subject Classification: Primary 47H10; 54H25; 54C30; 47H09.

Introduction

La notion d'espace métrique a été introduite en 1906 par M. Fréchet et développée peu de temps après par F. Hausdorff. En 1920, S. Banach publie un résultat significatif dans le domaine des points fixe, souvent appelé théorème du point fixe de Banach. Ce théorème fournit des conditions dans lesquelles une application contractante sur un espace métrique complet a un point fixe. Ce résultat a ouvert de nouvelles voies de généralisation et d'approfondissement la théorie des espaces métriques. Au niveau des contractions, de nombreux chercheurs ont exploré différents types d'inégalités qui généralisent le concept de contraction.

En 1989, Bakhtine a généralisé la notion d'espace métrique à celle de l'espace b-métriques et Czerwik ont étendu de nombreuses propriétés des espaces métriques en espaces b-métriques. Par la suite, plusieurs auteurs publiés des documents sur l'existence et l'unicité des points fixes pour des applications définies sur des espaces b-métriques et leurs applications.

En 2011, Azam et al. ont introduit la notion de l'espace métrique valeur complexes et ont prouvé l'existence de points fixes communs d'un paire d'application satisfaisant des conditions de type de contraction impliquant des inégalités rationnelles. Plusieurs chercheurs ont ensuite démontré des résultats liés à l'existence et à l'unicité de points fixes pour des applications définies sur des espaces métriques et b-métriques à valeurs complexes. Ces résultats sont en général connus pour les espaces métriques et b-métriques ordinaires, et certains sont même des corollaires de résultats des espaces métriques et b-métriques ordinaires, comme ce que nous verrons dans le paragraphe suivant.

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Main results

En 2019, [5] a démontré les théorèmes de Banach, Kannan et Chatterjea dans les espace b-métrique à valeurs complexes.

En utilisant les théorèmes ci-dessous, nous montrerons que les résultats démontrés dans [5] ne sont que des corollaire de théorème de Banach, Kannan et Chatterjea dans les espaces métriques ordinaires.

Theorem 143. Soient $s \geq 1$ et (X, d_c, s) un espace b-métrique à valeurs complexes. Soit $d(x, y) = |d_c(x, y)|, \forall x, y \in X$, alors $s \geq 1$ et (X, d, s) un espace b-métrique (usuel).

Theorem 144. Soient X, s, d_c et d comme dans le théorème 1. Soit $k \in [0, 1[$, et $T : X \rightarrow X$ une application.

- Si $d_c(Tx, Ty) \leq k d_c(x, y)$ alors $d(Tx, Ty) \leq k d(x, y)$.
- Si $d_c(Tx, Ty) \leq k(d_c(x, Tx) + d_c(y, Ty))$ alors $d(Tx, Ty) \leq k(d(x, Tx) + d(y, Ty))$.
- Si $d_c(Tx, Ty) \leq k(d_c(x, Tx) + d_c(y, Ty))$ alors $d(Tx, Ty) \leq k(d(x, Ty) + d(y, Tx))$.

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On the Infinite Intersection Property of Groups

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Abstract: A group G is said to satisfy the infinite trivial intersection property (ITIP for short), if for every pair of finite subgroups U, V such that $U \cap V = 1$, there exist infinite subgroups X and Y of G such $U \leq X$ and $V \leq Y$ and $X \cap Y = 1$. We say that a group G satisfies the infinite non-trivial intersection property (INIP) if every pair of infinite subgroups of G intersect non-trivially. In this talk, we shall give classes of groups that satisfy ITIP. We show, among other things, that every periodic locally nilpotent non-Chernikov group satisfies ITIP. The Prüfer-by-finite p -groups are examples of locally nilpotent Chernikov groups that do not satisfy ITIP. We then describe locally nilpotent groups that satisfy INIP and structure theorems are given in the periodic and the non-periodic case.

Keywords: Locally nilpotent groups, Chernikov groups, infinite intersection property.

2020 Mathematics Subject Classification: 20E07, 20F50.

Introduction

The structure of groups with an intersection property on given systems of subgroups has been investigated for several different choices of the property. See for example [1], [2], [3], [4], [5]. In particular, in [5] G. Walls classified finite p -groups all of whose proper subgroups have trivial intersection. The description of groups with non-trivial intersection property for every pair of non-trivial subgroups was a further step forward in this direction. Abelian groups with non-trivial intersection of every pair of non-trivial subgroups are isomorphic to a subgroup of either the additive group $\mathbb{Q}$, or the Prüfer group C_{p^∞} for some prime p . A.I. Sozutov in [4] described the non-abelian residually finite groups with non-trivial intersection of every pair of non-trivial subgroups.

A group G is said to have the infinite trivial intersection property (ITIP for short), if for every pair of finite subgroups U, V of G such that $U \cap V = 1$, there exist infinite subgroups X and Y of G such that $U \leq X$ and $V \leq Y$ and $X \cap Y = 1$. It is not difficult to see that ITIP is held for groups that are the direct product of infinitely many non-trivial subgroups. On the other hand, there are groups for which this property fails. Tarski groups, that is infinite simple groups whose proper non-trivial subgroups have prime order, are examples of groups that do not satisfy ITIP. Other examples of such groups include the additive group $\mathbb{Q}$ of rational numbers, Prüfer groups and the Prüfer-by-finite groups. In this talk, we first present criteria for a periodic locally nilpotent group to satisfy ITIP. The results obtained will be used to characterize locally nilpotent groups that satisfy the infinite nontrivial intersection property (INIP). In particular, we will show that such groups are (locally cyclic)-by-finite. A group G is said to have the infinite non-trivial intersection property (abbreviated INIP) if the intersection of every pair of infinite subgroups of G is nontrivial.

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Main results

Our first main result is as follows.

Theorem 145. *Let G be a periodic non-Chernikov locally nilpotent group. Then G satisfies ITIP.*

As a consequence, we have the following:

Corollary 8. *Every periodic locally nilpotent group with trivial center satisfies ITIP.*

We consider now the case of a mixed locally nilpotent group G that does not satisfy ITIP such that $G = T \times M$, where T is the torsion subgroup of G and M is a torsion-free subgroup. In particular, our next theorem shows that such groups are soluble of finite rank.

Theorem 146. *Let G be a locally nilpotent group such that $G = T \times M$, where T is the torsion subgroup of G and M is a torsion-free subgroup of G . If G does not satisfy ITIP, then T is a Chernikov group and $M \simeq B \leq \mathbb{Q}$.*

We finish by describing the structure of a locally nilpotent group satisfying the infinite non-trivial intersection property (INIP). The proof requires the fact that periodic non-Chernikov locally nilpotent groups satisfy ITIP.

Theorem 147. *An infinite locally finite group G satisfies INIP if and only if it is a Prüfer-by-finite group.*

Theorem 148. *A mixed locally nilpotent group G satisfies INIP if and only if it is a (torsion-free locally cyclic)-by-finite group.*

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A New Digital Signature Using Matrices Over Grouping

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Abstract: Digital signature methods represent one of the most significant outcomes of public key cryptography. These techniques enable digital communication while ensuring authenticity, integrity, and non-repudiation. In recent years, researchers have extended this concept by combining signatures with specific protocols to achieve functionalities beyond basic signature schemes. This has led to the development of various new signature schemes within the traditional public key framework. In a standard digital signature system, a signer signs the message, and the receiver verifies its authenticity. Representing messages as matrices instead of integers can reduce key sizes and storage requirements without compromising security. This work presents a new and efficient digital signature scheme in a group over group ring using circulant matrices and factoring, the security of the proposed scheme based on RSA assumptions (Rivest, Adi Shamir, and Leonard Adleman algorithm), also the group ring is chosen as a platform to enhance the security. We prove that our signature scheme is secure against the known key attacks.

Keywords: Digital signatures, Circulant matrices, Grouping, RSA cryptosystem.

2020 Mathematics Subject Classification: 94A60, 15B33, 20G35, 20G45.

Introduction

In 1976, public-key cryptography and digital signatures were introduced, revolutionizing secure communication. The RSA algorithm [4], proposed in 1978 and based on the difficulty of factoring large numbers, remains challenging despite many variations. Recent matrix-based schemes aim to reduce key sizes without compromising security, but still rely on the factorization problem. This study proposes a new RSA digital signature scheme using circulant matrices over a group ring. The scheme is validated, proven secure under RSA assumptions, and offers performance benefits. In this work, we propose a New Digital Signature Scheme based on RSA using circulant matrix over grouping. The scheme is based on the RSA signature scheme. The proposed signature scheme is validated, and its security is proven under the RSA assumptions.

"Preliminaries" briefly explains fundamental definitions related to group rings, circulant matrices and RSA signature scheme, on which our scheme is based. Subsequently, we present our "Proposed Scheme" a New Digital Signature Scheme based on RSA utilizing circulant matrices over grouping. "Analysis of the Proposed Scheme" provides the proof of correctness, security analysis, and performance analysis of the proposed scheme. "Conclusion" summarizes and concludes the work.

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Main results

Preliminaries

Definition 55 (Group ring). Let $\mathbb{F}$ be a field, and G be a group. Then the group ring $\mathbb{F}[G]$, is defined to be the set of all linear combinations

$$\gamma = \sum_{g \in G} b_g g,$$

where $b_g \in \mathbb{F}$. The sum and the product in group ring are defined as follows :

$$\begin{aligned} \gamma + \delta &= \left(\sum_{g \in G} b_g g \right) + \left(\sum_{g \in G} c_g g \right) \\ &= \sum_{g \in G} (b_g + c_g) g \end{aligned} \quad \text{and} \quad \begin{aligned} \gamma \sigma &= \left(\sum_{g \in G} b_g g \right) \left(\sum_{h \in G} d_h h \right) \\ &= \sum_{g, h \in G} b_g d_h (gh) \\ &= \sum_{u \in G} a_u u \end{aligned}$$

where $\delta = \sum_{g \in G} c_g g$, $\sigma = \sum_{h \in G} d_h h$, $gh = u$ and $a_u = \sum_{gh=u} b_g d_h$.

In this work, we propose to choose as a platform group the group of invertible matrices over group ring: $GL_2(\mathbb{F}_q[S_r])$, we have $|GL_2(\mathbb{F}_q[S_r])| > q^{16}$. Thus even for small sizes of q and r , essential security can be obtained easily. The main advantage of using $GL_n(\mathbb{F}_q[S_r])$ is that the multiplication of matrices is very efficient [2, 3].

Definition 56 (Circulant matrix). A $n \times n$ circulant matrix, is a special type of square matrix over a field $\mathbb{R}$. All rows except the first are right circular shifts of the row above. In other words, a circulant matrix C is defined by its first row.

For example: $n = 3$, a circulant matrix $C = \begin{pmatrix} c_1 & c_2 & c_3 \\ c_3 & c_1 & c_2 \\ c_2 & c_3 & c_1 \end{pmatrix}$.

It is known that the group of circulant matrices offer security equivalent to finite fields of similar size, but with half the computational cost.

RSA digital signature scheme

The RSA scheme [4] uses a private key for signing messages and a public key for verification.

Proposed Scheme

In this section, a new signature scheme is proposed based on the group of invertible matrices over a group ring, RSA and [1]. A simple example will also be given.

Suppose that Alice "A" wants to generate a signature on a message M and send it to Bob "B" for verification. Let $G = GL_n(\mathbb{F}_q[S_r])$ be the group of $n \times n$ invertible matrices over the group ring $\mathbb{F}_q[S_r]$ and L be the subgroup of circulant matrices of G

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. Key generation:

- Chose 2 large random prime numbers p and q and Compute the modulus $n = p \times q$ such that the determinant of M and modulus n are relatively prime.
- Calculate $N = (p^h - 1)(q^h - 1)$, which gives the number of matrices that are relatively prime to n , where h represents the order of the matrix.
- Select a random integer e , such that $1 < e < N$ and $\gcd(e, N) = 1$.
- determine d as $d = e^{-1} \bmod N$ and $1 < d < N$.
- Public and private keys are, therefore e and d respectively.

. Signature generation:

- Let the plain text message $M \in L$, with all the entries under modulo n such that the determinant of M is relatively prime to n .
- Then select a random matrix $R \in G$ and compute the Signature $S = M^d \bmod n \times R$, Signature is now (S, R) .

. Signature verification:

- Verifier computes $M = (S \times R^{-1})^e \bmod n$.

Simple Example:

Analysis of the Proposed Scheme

In this subsection, we study the security of the proposed signature scheme.

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d -orthogonal polynomials of Laguerre type

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Abstract: In this paper, we deal with introducing a d -orthogonal polynomials of Laguerre type ($d \geq 2$) with two parameters α and β by utilizing a suitable generating function from Sheffer type. Furthermore, we derive several identities satisfied by these polynomials. Moreover, for the specific case $d = 2$, we present some important formulas represented in explicit, connection and inversion formulas. In addition, we deal with the d -orthogonality property, in this regard, we give the $(d + 1)$ -order recurrence relation ($\alpha \neq n, n \geq 0$, if $d = 2$). Also for a particular case, we establish the $(d + 1)$ -order linear differential equation.

Keywords: d -orthogonal polynomials of Laguerre type, generating function, $(d + 1)$ -order recurrence relation, $(d + 1)$ -order linear differential equation.

2020 Mathematics Subject Classification: 33C45, 39A70, 41A58, 42C05.

Introduction

The vector space of polynomials with coefficients in $\mathbb{C}$ is denoted by $\mathcal{P}$, and its algebraic dual is denoted by $\mathcal{P}'$.

Among the well-known polynomials that have attracted the curiosity of many researchers are Laguerre orthogonal polynomials given by

$$\sum_{n=0}^{+\infty} L_n^{(\alpha)}(x) t^n = (1-t)^{-\alpha-1} \exp\left(\frac{-xt}{1-t}\right).$$

In literature, there are several d -orthogonal polynomials of Laguerre type. Among them, we cite the following:

$$\begin{aligned} \sum_{n=0}^{+\infty} P_n(x; \alpha; d) \frac{t^n}{n!} &= \exp(\pi_{d-1}(t)) (1-t)^{-\alpha-1} \exp\left(\frac{-xt}{1-t}\right), \\ \sum_{n=0}^{+\infty} P_n(x) \frac{t^n}{n!} &= \exp(\pi_{d-2}(t)) (1-t)^{-\alpha-1} \exp\left(-\frac{x}{2} \frac{t(2-t)}{(1-t)^2}\right). \end{aligned}$$

In this work, we present a d -orthogonal polynomials of Laguerre type, and we will discuss its different properties.

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Main results

Definition 57. We define a polynomial set of Laguerre type $\left\{\mathcal{L}_n^{(\alpha,\beta)}\right\}_{n \geq 0}$ by

$$\begin{aligned} G_{(\alpha,\beta)}(x,t) &= \sum_{n=0}^{+\infty} \mathcal{L}_n^{(\alpha,\beta)}(x) \frac{t^n}{n!} \\ &= (1-t)^\alpha \exp(\pi_{d-2}(t)) \exp\left(\frac{\beta t}{1-t}\right) \exp\left(\frac{x}{2} \frac{t(2-t)}{(1-t)^2}\right), \\ \alpha, \beta &\in \mathbb{C}, d \geq 2, \end{aligned} \quad (413)$$

where $\pi_{d-2}(t) = \sum_{k=0}^{d-2} \alpha_k \frac{t^k}{k!}$, $\alpha_{d-2} \neq 0$ and $\pi_{d-2}(0) = 1$.

Theorem 149. The following relations hold for $n \geq 0$:

$$\begin{aligned} \mathcal{L}_n^{(\alpha+1,\beta)}(x) &= \mathcal{L}_n^{(\alpha,\beta)}(x) - n \mathcal{L}_{n-1}^{(\alpha,\beta)}(x), \\ \mathcal{L}_n^{(\alpha,\beta)}(x) &= \sum_{m=0}^n \binom{n}{m} \mathcal{L}_{n-m}^{(\alpha-m,\beta)}(x), \\ \frac{\mathcal{L}'_n^{(\alpha,\beta)}(x)}{(n+1)} &= \mathcal{L}_n^{(\alpha-2,\beta)}(x) - \frac{1}{2} n \mathcal{L}_{n-1}^{(\alpha-2,\beta)}(x), \\ 2\mathcal{L}'_n^{(\alpha,\beta)}(x) &= \mathcal{L}_n^{(\alpha-2,\beta)}(x) - \mathcal{L}_n^{(\alpha,\beta)}(x), \\ \mathcal{L}_n^{(\alpha,\beta)}(x) + \mathcal{L}_n^{(\alpha-1,\beta)}(x) &= 2 \left(\frac{\mathcal{L}'_{n+1}^{(\alpha,p)}(x)}{(n+1)} - \mathcal{L}'_n^{(\alpha,\beta)}(x) \right). \end{aligned}$$

Theorem 150. For $d = 2$, we have the following

- Explicit formula:

$$\mathcal{L}_n^{(\alpha,\beta)}(x) = \sum_{m=0}^n \frac{1}{m!} \sum_{k=0}^n \sum_{i=0}^m \frac{(-m)_i}{i!} \frac{(-n)_k (-\beta)^k}{k!} (2i+k-\alpha)_{(n-k)} \left(\frac{-x}{2}\right)^m.$$

- Connection formula:

$$\begin{aligned} \mathcal{L}_n^{(\alpha,\beta)}(x) &= \sum_{m=0}^n c_m(n) \mathcal{L}_m^{(\lambda,\mu)}(x), \\ c_m(n) &= \frac{n!}{m!} \sum_{i=0}^{n-m} \frac{(\beta-\mu)^i}{i!} \frac{(\lambda-\alpha+i)_{(n-m-i)}}{(n-m-i)!}. \end{aligned}$$

- Inversion formula:

$$\begin{aligned} x^n &= n! \sum_{m=0}^n \frac{b_m(n)}{m!} \mathcal{L}_m^{(\alpha,\beta)}(x), \\ b_m(n) &= \sum_{k=0}^{+\infty} \sum_{i=0}^k \sum_{j=0}^m \frac{(-2)^n}{n!} \frac{(\beta)^k}{k!} \frac{(-m)_j}{j!} \frac{(-k)_i}{i!} \left(\frac{j-\alpha-i}{2}\right)_n. \end{aligned}$$

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Theorem 151. $\left\{\mathcal{L}_n^{(\alpha,\beta)}\right\}_{n \geq 0}$ is a d -orthogonal polynomial set and satisfies the following $(d+1)$ -order recurrence relation ($\alpha \neq n, n \geq 0$, if $d = 2$) :

$$\begin{aligned}\mathcal{L}_{n+1}^{(\alpha,\beta)}(x) = & (x + 3n + \beta - \alpha)\mathcal{L}_n^{(\alpha,\beta)}(x) + n(2\alpha + 3 - 3n)\mathcal{L}_{n-1}^{(\alpha,\beta)}(x) \\ & + n(n-1)(n-\alpha-2)\mathcal{L}_{n-2}^{(\alpha,\beta)}(x) \\ & + \sum_{k=0}^{d-3} \alpha_{k+1} \left[\binom{n}{k} \mathcal{L}_{n-k}^{(\alpha,\beta)}(x) - 3n \binom{n-1}{k} \mathcal{L}_{n-k-1}^{(\alpha,\beta)}(x) \right. \\ & \left. + 3n(n-1) \binom{n-2}{k} \mathcal{L}_{n-k-2}^{(\alpha,\beta)}(x) - n(n-1)(n-2) \right. \\ & \left. \times \binom{n-3}{k} \mathcal{L}_{n-k-3}^{(\alpha,\beta)}(x) \right], \quad n \geq 0,\end{aligned}$$

with $\mathcal{L}_{-n}^{(\alpha,\beta)}(x) = 0, n \geq 1$.

For the particular case $A(t) = (1-t)^\alpha \exp\left[(1-t)^{d-2}\right] \exp\left(\frac{\beta t}{1-t}\right)$, we have

Theorem 152. $\left\{\mathcal{L}_n^{(\alpha,\beta)}\right\}_{n \geq 0}$ satisfies the following $(d+1)$ -order linear differential equation:

$$\begin{aligned}n\mathcal{L}_n^{(\alpha,\beta)}(x) = & (x + \beta - \alpha)D\mathcal{L}_n^{(\alpha,\beta)}(x) + \frac{1}{2}(3x + \alpha + \beta)D^2\mathcal{L}_n^{(\alpha,\beta)}(x) - xD^3\mathcal{L}_n^{(\alpha,\beta)}(x) \\ & + (2-d) \sum_{k=1}^{d+1} \frac{(-2)^k}{k!} \left[\left(\frac{3-d}{2}\right)_k - \left(\frac{2-d}{2}\right)_k \right] D^k \mathcal{L}_n^{(\alpha,\beta)}(x), \quad n \geq 0.\end{aligned}$$

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Several results on homogeneous weights over the finite ring $\mathcal{R}_{3,3}$, alongside their practical implications in linear codes over said ring

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Abstract: The aim of this work is to characterize the different properties of the finite ring $\mathcal{R}_{3,3} = \{\bar{\omega}_i = \xi_0^i + u_1\xi_1^i + u_2\xi_2^i + u_3\xi_3^i \mid \xi_0^i, \xi_1^i, \xi_2^i, \xi_3^i \in \mathbb{F}_3, 1 \leq i \leq 81\}$ and provide valuable insights into the homogeneous weights over this particular ring. Finite rings, such as $\mathcal{R}_{3,3}$, have unique algebraic structures that play a significant role in various mathematical and computational applications. By studying the properties of this specific ring, we can better understand its algebraic and structural features. Additionally, investigating homogeneous weights over $\mathcal{R}_{3,3}$ is a crucial aspect of this research, as it offers practical implications for coding theory, error correction, and cryptography.

Keywords: homogeneous weight, gray map, linear codes, images Gray

2020 Mathematics Subject Classification: Primary 94B05, 94B15, 11T71, 14G50, 15Axx.

Introduction

Contemporary information and communication systems heavily depend on the reliable and efficient transmission of data. However, practical channels are frequently affected by disturbances, such as noise in mobile radio channels or scratches on optical discs. Consequently, there is a pressing need to employ suitable channel coding schemes to identify or rectify errors in transmitted data. Channel coding theory provides these schemes, focusing not just on their effectiveness in error detection and correction, but also on the complexity of the architectures required for encoding and decoding algorithms, which is vital for real-world applications. Researchers acknowledged the importance of linear codes over finite rings in the realm of algebraic coding theory, which led to the development of numerous mathematical methodologies. For example, we focused on the concept of homogeneous weight, which was first introduced in the context of coding over finite rings and integer residue rings by Constantinescu and Heise[1].

This paper aims to establish the fundamental principles of linear codes over a specific class of finite rings defined by the following expression $\mathcal{R}_{3,3} = \mathbb{F}_3 + u_1\mathbb{F}_3 + u_2\mathbb{F}_3 + u_3\mathbb{F}_3$, with $u_i u_j = u_j u_i = 0$ and $u_i^2 = u_i$, for $1 \leq j \neq i \leq 3$. The notion of homogeneous weight extends the concept of Hamming

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weight to finite rings. Hence, this paper delves deeply into investigating this weight on $\mathfrak{R}_{3,3}$. To ascertain the homogeneous weight on $\mathfrak{R}_{3,3}$, we utilize the definition of homogeneous weight on $\mathbb{F}_3$ [1, 2].

The following sections of the paper are structured as follows: Section 2 focuses on introducing definitions and properties of linear codes over $\mathfrak{R}_{3,3} = \mathbb{F}_3 + u_1\mathbb{F}_3 + u_2\mathbb{F}_3 + u_3\mathbb{F}_3$. In Section 3, we present several findings regarding the homogeneous weight on $\mathfrak{R}_{3,3} = \mathbb{F}_3 + u_1\mathbb{F}_3 + u_2\mathbb{F}_3 + u_3\mathbb{F}_3$, and conclude this section with some illustrative examples.

Linear codes over $\mathfrak{R}_{3,3}$

A finite ring $\mathfrak{R}_{3,3}$ is defined as $\mathfrak{R}_{3,3} = \mathbb{F}_3 + u_1\mathbb{F}_3 + u_2\mathbb{F}_3 + u_3\mathbb{F}_3$. Each element in this ring is expressed as

$$x = (1 + 2u_1 + 2u_2 + 2u_3)a_0 + u_1(a_1 + a_0) + u_2(a_2 + a_0) + u_3(a_3 + a_0). \quad (414)$$

Consider C as a linear code of length n over $\mathfrak{R}_{3,3}$. We can decompose C into C_0, C_1, C_2 , and C_3 , which are linear codes of length n over $\mathbb{F}_3$. In essence, the code C can be uniquely represented as:

$$C = (1 + 2u_1 + 2u_2 + 2u_3)C_0 \oplus (u_1)C_1 \oplus (u_2)C_2 \oplus (u_3)C_3 \quad (415)$$

where

$$C_i = \{a_0 + a_i \in \mathbb{F}_3^n \mid \exists a_j \in \mathbb{F}_3^n, i \neq j, a_0 + u_1a_1 + u_2a_2 + u_3a_3 \in C\}, \quad (416)$$

with $1 \leq i, j \leq 3$.

Theorem 153. Let $C = (1 + 2u_1 + 2u_2 + 2u_3)C_0 \oplus (u_1)C_1 \oplus (u_2)C_2 \oplus (u_3)C_3$ be a linear code of length n over $\mathfrak{R}_{3,3}$. Then,

$$C^\perp = (1 + 2u_1 + 2u_2 + 2u_3)C_0^\perp \oplus (u_1)^\perp C_1 \oplus (u_2)C_2^\perp \oplus (u_3)C_3^\perp \quad (417)$$

Homogeneous weights over the finite ring $\mathfrak{R}_{3,3}$

Drawing from the simplified model presented in [4], we will calculate the homogeneous weight on the ring $\mathfrak{R}_{3,3}$. Subsequently, these findings will be utilized to define and explore various applications. The homogeneous weight ω on $\mathfrak{R}_{3,3}$, with generating character χ , is expressed as:

$$\begin{aligned} \omega : \mathfrak{R}_{3,3} &\longrightarrow \mathbb{R} \\ \tau &\longrightarrow \omega(\tau) = \eta \cdot \left(1 - \frac{1}{|\mathfrak{R}_{3,3}^\times|} \cdot \sum_{u \in \mathfrak{R}_{3,3}^\times} \chi(\tau u)\right), \end{aligned} \quad (418)$$

where $\mathfrak{R}_{3,3}^\times$ is the group of units of $\mathfrak{R}_{3,3}$.

Theorem 154. If the rings $R_1, R_2, \dots, R_n$ with identity and $\mathcal{I}$ an ideal in $\prod_{j=1}^n R_j$, then $\mathcal{I} = \prod_{j=1}^n A_j$ with A_j is an ideal in R_j , where

$$\prod_{j=1}^n R_j = \{(\omega_1, \omega_2, \dots, \omega_n) \mid \omega_j \in R_j\}. \quad (419)$$

By utilizing the definition (3.1) and theorems 154 previously discussed, we formulate the following theorem for computing the homogeneous weight on $\mathfrak{R}_{3,3}$.

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Theorem 155. *The homogeneous weight on $\mathcal{R}_{3,3}$ is obtained as*

$$\omega_{hom}(\varpi) = \begin{cases} 0 & \text{if } \varpi = 0 \\ \frac{9}{8}\eta_1 & \text{if } \varpi \text{ is divisor of zero} \\ \frac{15}{16}\eta_2 & \text{if } \varpi \text{ is unit.} \end{cases}$$

In this paper, we explore linear codes over a specific $\mathcal{R}_{3,3} = \mathbb{F}_3 + u_1\mathbb{F}_3 + u_2\mathbb{F}_3 + u_3\mathbb{F}_3$, with $u_i u_j = u_j u_i = 0$ and $u_i^2 = u_i$, for $1 \leq j \neq i \leq 3$. We recognize the importance of studying the homogeneous weight, which has played a pivotal role in defining the covering radius.

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Some identities and generating functions of binary products of certain Gaussians numbers with some (p, q) numbers and bivariate polynomials

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Abstract: In this paper, by using properties of the symmetric functions, we introduce a new generating functions for the product of Gaussian numbers such as Gaussian Tribonacci, Gaussian Tribonacci Lucas, and certain numbers and bivariate polynomials.

Keywords: Generating functions, Gaussian Tribonacci, Gaussian Tribonacci Lucas, bivariate polynomials.

Introduction

There are several uses for the generating functions in various areas of mathematical physics and mathematics. Our goal in this work is to use some numbers and polynomials to calculate the generating functions, there are many of numbers can be define by third-order recurrence relation, such as : Gaussian Tribonacci, Gaussian Tribonacci Lucas, Gaussian Padovan.

Main results

In this section we are going to establish a new theorem. One of the essential tools of the proof of our main finding is the following theorem.

Theorem 156. Given two alphabets $E = \{e_1, e_2, e_3\}$ and $A = \{a_1, a_2\}$, we have

$$\sum_{n=0}^{\infty} S_n(E) S_{n+k-3}(A) z^n = \frac{S_{k-3}(A) + S_1(-E) a_1 a_2 S_{k-4}(A) z + S_2(-E) a_1^2 a_2^2 S_{k-5}(A) z^2 - a_1^{k-2} a_2^{k-2} z^{k-1} \sum_{n=0}^{\infty} S_{n+k-1}(-E) S_n(A) z^n}{\left(\sum_{n=0}^{\infty} S_n(-E) a_1^n z^n \right) \left(\sum_{n=0}^{\infty} S_n(-E) a_2^n z^n \right)}, \quad (2.1)$$

for all $n \in \mathbb{N}_0$ and $k \in \{2, 3, 4, 5\}$.

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Theorem 157. For $n \in \mathbb{N}$, the new generating function of the product of Gaussian Tribonacci numbers with (p, q) Lucas numbers is given by:

$$\sum_{n=0}^{\infty} GT_n L_{p,q,n} t^n = \frac{pt + (q + (p^2 + 2q)i)t^2 + (pq(1 - 3i))t^3 + (2q^2 + p^2q - 2q^2i)t^4 - pq^2it^5}{1 - pt - (p^2 + 3q)t^2 - (p^3 + 4pq)t^3 - (p^2q + q^2)t^4 + pq^2t^5 - q^3t^6}. \quad (3.1)$$

Theorem 158. For $n \in \mathbb{N}$, the new generating function of the product of Gaussian Tribonacci Lucas numbers with (p, q) Lucas numbers is given by:

$$\sum_{n=0}^{\infty} GK_n L_{p,q,n} t^n = \frac{6 - 2i - 5p(1 - i)t + [(-4p^2 - 12q) + 8qi]t^2 + [(-3p^3 - 12pq) + i(p^3 + 6pq)]t^3 + [(-2q^2 - 2p^2q) + i(4p^2q + 10q^2)]t^4 + pq^2(1 + i)t^5}{1 - pt - (p^2 + 3q)t^2 - (p^3 + 4pq)t^3 - (p^2q + q^2)t^4 + pq^2t^5 - q^3t^6}. \quad (3.2)$$

Theorem 159. For $n \in \mathbb{N}$, the new generating function of the product of Gaussian Padovan numbers with (p, q) Lucas numbers is given by:

$$\sum_{n=0}^{\infty} GP_n L_{p,q,n} t^n = \frac{2 + p(1 + i)t + [(p^2 - 2q) + (-p^2 + 2q)i]t^2 + [(-p^3 - 2pq) + (pq)i]t^3 + [(2q^2 + p^2q) + (p^2q - q^2)i]t^4 - pq^2it^5}{1 - (p^2 + 2q)t^2 - (p^3 + 3pq)t^3 + q^2t^4 + pq^2t^5 - q^3t^6}. \quad (3.3)$$

Theorem 160. For $n \in \mathbb{N}$, the new generating function of the product of Gaussian Tribonacci numbers with (p, q) Lucas numbers is given by:

$$\sum_{n=0}^{\infty} GT_n F_{p,q,n} t^n = \frac{t + ipt^2 + -(q - iq)t^3 - pqt^4 + q^2it^5}{1 - pt - (p^2 + 3q)t^2 - (p^3 + 4pq)t^3 - (p^2q + q^2)t^4 + pq^2t^5 - q^3t^6}. \quad (3.4)$$

Theorem 161. For $n \in \mathbb{N}$, the new generating function of the product of Gaussian Tribonacci Lucas numbers with (p, q) Fibonacci numbers is given by:

$$\sum_{n=0}^{\infty} GK_n F_{p,q,n} t^n = \frac{(1 + 3i)t + 2p(1 - i)t^2 + [(3p^2 + 4q) + (-p^2 - 6q)i]t^3 + pq(2 - 4i)t^4 - q^2(1 + i)t^5}{1 - pt - (p^2 + 3q)t^2 - (p^3 + 4pq)t^3 - (p^2q + q^2)t^4 + pq^2t^5 - q^3t^6}. \quad (3.5)$$

Theorem 162. For $n \in \mathbb{N}$, the new generating function of the product of Gaussian Padovan numbers with (p, q) Fibonacci numbers is given by:

$$\sum_{n=0}^{\infty} GP_n F_{p,q,n} t^n = \frac{(1 + i)t + p(1 + i)t^2 + (p^2 - qi)t^3 - pq(1 + i)t^4 + iq^2t^5}{1 - (p^2 + 2q)t^2 - (p^3 + 3pq)t^3 + q^2t^4 + pq^2t^5 - q^3t^6}. \quad (3.6)$$

Theorem 163. For $n \in \mathbb{N}$, the new generating function of the product of Gaussian Tribonacci numbers with (p, q) Pell Lucas numbers is given by:

$$\sum_{n=0}^{\infty} GT_n Q_{p,q,n} t^n = \frac{4pt + [2q + (12p^2 + 2q)i]t^2 + -(2pq + 6pqi)t^3 + [(2q^2 + 4p^2q) - 2q^2i]t^4 + 2pq^2it^5}{1 - 2pt - (4p^2 + 3q)t^2 - (8p^3 + 8pq)t^3 - (4p^2q + q^2)t^4 + 2pq^2t^5 - q^3t^6}. \quad (3.7)$$

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On constacyclic and Quasi-twisted codes over R

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Abstract: In this paper, we investigate the algebraic structure and properties of constacyclic and quasi-twisted (QT) codes over the ring $R = \mathbb{Z}_q + u\mathbb{Z}_q$ with $u^2 = 1$. We show that the image of a constacyclic code over R under a natural Gray map is a QT code of index 2 over $\mathbb{Z}_q$. We find the decomposition of QT code over R .

Keywords: Constacyclic codes, Gray map, quasi-twisted codes.

Introduction

Constacyclic codes are one of the most important classes of codes in algebraic coding theory due to their rich algebraic structures. In addition, codes over $\mathbb{Z}_4$ play a special role in recent developments on codes over rings. It was shown that certain good nonlinear binary codes can be constructed from linear codes over $\mathbb{Z}_4$ and this sparked significant research activity in this area. The construction of constacyclic codes over $\mathbb{Z}_4 + u\mathbb{Z}_4$, $v^2 = v$.

Quasi-twisted (QT) codes are a remarkable generalization of constacyclic codes that contain the class of quasi-cyclic (QC) codes as a special case. They have been shown to be promising in addressing one of the most important problems in coding theory, namely to construct codes with best possible parameters.

The ring R we consider in this paper is a generalization of $\mathbb{Z}_4[u]/\langle u^2 - 1 \rangle$. The ring $\mathbb{Z}_q[u]/\langle u^2 - 1 \rangle$ is semi-local ring, it has two maximal ideals $(\langle 1 + u \rangle, \langle 1 - u \rangle)$. However, when $q = 2$, the ring is a principal ideal ring with maximal ideal $\langle 1 + u \rangle$, it can be shown that $R/\langle 1 + u \rangle$ and $R/\langle 1 - u \rangle$ are isomorphic to $\mathbb{Z}_q$.

Main results

Definition 58. Let R be a commutative ring with identity. A linear code of length n over R is an R -submodule of R^n . Let λ be a unit in R , ℓ be a positive integer and let $\tau_{\ell, \lambda}$ be the permutation of R^n given by $\tau_{\ell, \lambda}(c) = (\lambda(c_{n-\ell}), \dots, \lambda(c_{n-1}), c_0, \dots, c_{n-\ell-1})$ for any $c = (c_0, \dots, c_{n-1}) \in R$. For a linear code C of length n over R , if $\tau_{\ell, \lambda}(C) = C$, then C is called an ℓ -quasi twisted (QT) code (or a QT code of index ℓ), over R . For $\ell = 1$, we obtain the important special case $\tau_{1, \lambda} = \rho_\lambda$ of a λ -constacyclic code of length n over R . The unit λ is called the shift constant.

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Constacyclic codes over $\mathbb{R}$

Recall that $R = \mathbb{Z}_q + u\mathbb{Z}_q$ where $u^2 = 1$. For $r \in R$ and $\epsilon_2 = \frac{1+u}{2}$. It is easy to check that $\epsilon_i^2 = \epsilon_i$, $\epsilon_i\epsilon_j = 0$ and $\epsilon_1 + \epsilon_2 = 1$ for $i = 1, 2$ and $i \neq j$. Therefore, $R = \epsilon_1\mathbb{Z}_q \oplus \epsilon_2\mathbb{Z}_q$ and any $r \in R$ can be expressed as $r = \epsilon_1 r_1 + \epsilon_2 r_2$ where $r_1, r_2 \in \mathbb{Z}_q$. Now we define a Gray map on R_q^n as follows

$$\Psi : R^n \longrightarrow \mathbb{Z}_q^{2n}$$

$$(r_0, r_1, \dots, r_{n-1}) \longrightarrow (r_{0,1}, r_{1,1}, \dots, r_{n-1,1}, r_{0,1} + r_{0,2}, r_{1,1} + r_{1,2}, \dots, r_{n-1,1} + r_{n-1,2}),$$

where $(r_0, r_1, \dots, r_{n-1}) \in R^n$ and $r_i = \epsilon_1 r_{i,1} + \epsilon_2 r_{i,2} \in R$ for $i = 0, 1, \dots, n-1$.

Let C be a linear code of length n over R . Then we define

$$C_1 = \{r_1 \in \mathbb{Z}_q^n : \epsilon_1 r_1 + \epsilon_2 r_2 \in C, \text{ for some } r_2 \in \mathbb{Z}_q^n\},$$

$$C_2 = \{r_2 \in \mathbb{Z}_q^n : \epsilon_1 r_1 + \epsilon_2 r_2 \in C, \text{ for some } r_1 \in \mathbb{Z}_q^n\}.$$

Let $\sigma : \mathbb{Z}_q^{2n} \rightarrow \mathbb{Z}_q^{2n}$ be the QT shift operator defined by

$$\sigma(a^{(0)} \mid a^{(1)}) = (\rho_\lambda(a^{(0)}) \mid \rho_\lambda(a^{(1)})),$$

where $a^{(0)}, a^{(1)} \in \mathbb{Z}_q^n$, $\mid$ is vector concatenation, and ρ_λ is the constacyclic shift operator defined in the previous section.

Proposition 17. [1] With the notation above, we have $\Psi \circ \rho_\lambda = \sigma \circ \Psi$.

Proposition 18. [1] Let C be a linear code of length n over R . Then C is a constacyclic code of length n over R if and only if $\Psi(C)$ is a QT code of length $2n$ over $\mathbb{Z}_q$ of index 2.

Proposition 19. [1] An element $\lambda = \lambda_1 + u\lambda_2$ is a unit of R if and only if α_1, α_2 are units in $\mathbb{Z}_q^*$, where $\alpha_1 = \lambda_1 - \lambda_2$ and $\alpha_2 = \lambda_1 + \lambda_2$.

Theorem 164. [1] Let C be a linear code over R of length n and let $C = \epsilon_1 C_1 \oplus \epsilon_2 C_2$ be its decomposition where C_1 and C_2 are codes over $\mathbb{Z}_q$ of length n . Then C is a λ -constacyclic code over R if and only if C_1 and C_2 are α_j -constacyclic codes over $\mathbb{Z}_q$ for $j = 1, 2$.

A decomposition of quasi-twisted codes over R

We assume in the rest of the paper that $\gcd(n, p) = 1$ where $q = p^m$ for a prime p and a positive integer m . Let $R_s = R[x]/\langle x^s - \lambda \rangle$ and suppose C is a QT code of length $n = s\ell$ and index ℓ over R . We now consider the factorization of $x^n - \lambda$ over R . We denote by $\bar{\cdot}$ the natural ring homomorphism

$$\bar{\cdot} : R \longrightarrow \mathbb{Z}_q$$

$$a + bu \longrightarrow a$$

Now extend $\bar{\cdot}$ to a ring homomorphism from $R[x]$ into $\mathbb{Z}_q[x]$ by

$$\bar{f} = \sum \bar{u}_k x^k, \text{ for all } f = \sum u_k x^k \in R[x] \text{ where } u_k \in R.$$

A monic polynomial $f \in R[x]$ is said to be monic basic irreducible over R if $\bar{f}$ is an irreducible polynomial over $\mathbb{Z}_q$.

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Lemma 21. Let $R = \mathbb{R}$ and $g(x)$ be an irreducible polynomial over $\mathbb{Z}_q$, where $g(x) \mid (x^n - \lambda)$. Then there exists a unique basic irreducible polynomial $f(x) \in R[x]$ such that $\overline{f(x)} = g(x)$ and $f(x) \mid (x^n - \lambda)$ over R .

Since $\gcd(n, p) = 1$, it follows that $x^s - \lambda$ can be factored uniquely into pairwise coprime basic irreducible polynomials over R , i.e., for $i = 1, \dots, t$

$$x^s - \lambda = f_1(x)f_2(x)\dots f_t(x), \quad (420)$$

where $f_i(x)$ is a basic irreducible polynomial over R . Therefore, since R_s is a principal ideal ring, it can be decomposed into a direct sum of local rings. Hence, the Chinese Remainder Theorem gives the following decomposition

$$R_s = R[x]/\langle x^s - \lambda \rangle = R[x]/\langle f_1(x) \rangle \oplus R[x]/\langle f_2(x) \rangle \oplus \dots \oplus R[x]/\langle f_t(x) \rangle \quad (421)$$

The direct sum on the right hand side is endowed with coordinate-wise addition and multiplication. For convenience, we denote the ring $R[x]/\langle f_i(x) \rangle$ by R_i for $1 \leq i \leq t$. It follows that

$$R_s^\ell \simeq \bigoplus_{i=1}^t R_i^\ell.$$

Then the following theorem is an immediate consequence

Theorem 165. [1] Let C be a QT code of length $n = s\ell$ over R . Then C is a linear code over R_s of length ℓ which can be decomposed as the direct sum

$$C \simeq \bigoplus_{i=1}^m C_i, \quad (422)$$

where C_i is a linear code over R_i of length ℓ for $1 \leq i \leq t$.

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Combinatorial interpretation and some positivities of Hyper-Leonardo numbers

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Abstract: In this paper, we establish the combinatorial interpretations of the hyper-Leonardo numbers $Le_n^{(r)}$ and Leonardo numbers Le_n . We investigate the log-concavity of the Leonardo numbers for $n \geq 3$ and the hyper-Leonardo numbers for $n, r \geq 1$. In addition, we prove the log-balancedness of the hyper-Leonardo numbers for $r = 1, 2$. Furthermore, we prove the q -log-concavity of the polynomial $\sum_{k=0}^n Le_k^{(r)} q^k$ for $n, r \geq 1$.

Keywords: Hyper-Leonardo numbers, tiling, log-concavity, log-convexity, q -log-concavity.

2020 Mathematics Subject Classification: Primary: 05A20; Secondary: 05A19, 11B37, 11B39.

Introduction

The sequence $\{Le_n\}_{n \geq 0}$ of *Leonardo numbers* was introduced by Catarino and Borges [1] and is defined recursively by

$$Le_n = Le_{n-1} + Le_{n-2} + 1, \quad n \geq 2,$$

with initial values $Le_0 = Le_1 = 1$.

This sequence has the following generating function:

$$\sum_{n \geq 0} Le_n x^n = \frac{1 - x + x^2}{(1 - x)(1 - x - x^2)}.$$

The previous authors [1] proved that this sequence has some relations with the Fibonacci $\{F_n\}$, Lucas $\{L_n\}$ sequences such as

$$Le_n = 2F_n - 1$$

and

$$Le_n = L_{n+2} - F_{n+2} - 1.$$

Alp, and Kocer obtained Cassini's identity for Leonardo numbers as

$$(Le_n)^2 - Le_{n-1} Le_{n+1} = Le_{n-1} - Le_{n-2} + 4(-1)^n.$$

As an extension of Leonardo numbers, Kuhapatanakul and Chobson [2] defined the generalized Leonardo numbers $\mathcal{L}_{k,n}$ recursively by

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$$\mathcal{L}_{k,n} = \mathcal{L}_{k,n-1} + \mathcal{L}_{k,n-2} + k, \quad n \geq 2,$$

with initial values $\mathcal{L}_{k,0} = \mathcal{L}_{k,1} = 1$, and the parameter k can be a fixed positive integer or an indeterminate. When $k = 1$, we obtain the classical Leonardo numbers $\mathcal{L}_{1,n} = Le_n$.

Bahsi and Mersin [3] introduced the hyper-Leonardo numbers as follows

$$Le_n^{(r)} = \sum_{k=0}^n Le_k^{(r-1)} \quad \text{with} \quad Le_n^{(0)} = Le_n, \quad Le_0^{(r)} = 1, \quad Le_1^{(r)} = r + 1.$$

This relation yields the recurrence relation

$$Le_n^{(r)} = Le_{n-1}^{(r)} + Le_n^{(r-1)}.$$

Let n and s be two positive integers, the binomial coefficients $\binom{n}{k}_s$ are defined as the k^{th} coefficients in the expansion,

$$(1 + x + \cdots + x^s)^n = \sum_{k=0}^{sn} \binom{n}{k}_s x^k,$$

with $\binom{n}{k}_s = 0$ unless $sn \geq k \geq 0$.

Main results

Definition 59. A k -tile is a rectangular piece coming in one of k colors which

- must occur as the first piece in a tiling, if it occurs at all,
- has arbitrary length greater than or equal two.

A k -tile of length l will be denoted by k_l for all $l \geq 2$.

Proposition 20. [4, Proposition 2.1] The generalized Leonardo number $\mathcal{L}_{k,n}$ counts the number of n -board tilings using squares, dominos and k -tiles.

This proposition allows us to propose the following combinatorial interpretation for the hyper-Leonardo numbers $Le_n^{(r)}$.

Theorem 166. The hyper-Leonardo number $Le_n^{(r)}$ counts the number of $n + 2r$ -board tilings using squares, one 1-tile and at least r dominos.

By setting $r = 0$ in Theorem 166, we obtain immediately the tiling interpretation of the Leonardo numbers.

Corollary 9. The Leonardo number Le_n counts the number of n -board tilings using squares, one 1-tile and dominos.

Proposition 21. If $n \geq 1$ and $r \geq 1$, we have

$$Le_n^{(r)} = \sum_{k=0}^n \binom{n+r-k-1}{r-1} Le_k. \quad (423)$$

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Let $\{a_n\}_{n \geq 0}$ be a sequence of nonnegative numbers. The sequence is called log-concave (resp. log-convex) if $a_n a_{n+2} \leq a_{n+1}^2$ (resp. $a_n a_{n+2} \geq a_{n+1}^2$) for all $n \geq 0$.

We say that $\{a_n\}_{n \geq 0}$ is log-balanced [5] if $\{a_n\}_{n \geq 0}$ is log-convex and $\{a_n/n!\}_{n \geq 0}$ is log-concave.

For two polynomials with real coefficients $f(q)$ and $g(q)$, denote $f(q) \geq_q g(q)$ if the difference $f(q) - g(q)$ has only nonnegative coefficients. For a polynomial sequence $(f_n(q))_{n \geq 0}$, it is called q -log-concave (resp. q -log-convex, introduced by Liu and Wang [6]), first suggested by Stanley [7], if

$$f_n(q)^2 \geq_q f_{n-1}(q)f_{n+1}(q) \quad (\text{resp. } f_n(q)^2 \leq_q f_{n-1}(q)f_{n+1}(q)).$$

Theorem 167. The Leonardo numbers $\{Le_n\}_n$ form a log-concave sequence for $n \geq 3$.

Corollary 10. The sequences $\{\sum_{k=0}^n \binom{n}{k} Le_k\}_{n \geq 3}$ and $\{\sum_{k=0}^{sn} \binom{n}{k}_s Le_k\}_{n \geq 3}$ are log-concave.

Theorem 168. The hyper-Leonardo numbers $\{Le_n^{(r)}\}_{n \geq 1}$ ($r \geq 1$) form a log-concave sequence.

Corollary 11. The sequences $\{\sum_{k=0}^n \binom{n}{k} Le_k^{(r)}\}_{n \geq 1}$ and $\{\sum_{k=0}^{sn} \binom{n}{k}_s Le_k^{(r)}\}_{n \geq 1}$ ($r \geq 1$) are log-concave.

Theorem 169. The sequence $\{n!Le_n^{(1)}\}_{n \geq 1}$ is log-balanced.

Lemma 22. For $n \geq 0$, we have

$$\begin{aligned} \left(Le_{n+1}^{(2)}\right)^2 - Le_n^{(2)}Le_{n+2}^{(2)} &= \frac{1}{2}(n^2 + 11n + 16)Le_{n+2} - (2n + 7)Le_{n+3} \\ &\quad + \frac{1}{2}(3n^2 + 21n + 36) + 4(-1)^{n+5}. \end{aligned} \quad (424)$$

Theorem 170. The sequence $\{n!Le_n^{(2)}\}_{n \geq 1}$ is log-balanced.

Theorem 171. For $r \geq 1$, let

$$Le_{n,r}(q) = \sum_{k=0}^n Le_k^{(r)} q^k.$$

The polynomial $Le_{n,r}(q)$ is q -log-concave for $n \geq 0$.

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Theoretical challenges in picture fuzzy sets: new definitions based on the $\mathbb{D}^*$ structure

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Abstract: Recently, Cuong introduced the concept of picture fuzzy sets as a generalization of fuzzy sets and intuitionistic fuzzy sets to overcome further uncertainties. In this work, Cuong's basic definitions are re-examined and scrutinized. This analysis reveals that there are some specific contradictions that are resolved in this work as well.

Keywords: picture fuzzy sets, picture fuzzy sets operations, the set $\mathbb{D}^*$.

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Introduction

A direct generalization of Zadeh's fuzzy sets [4] and Atanassov's intuitionistic fuzzy sets [1] is the following notion of picture fuzzy sets.

Definition 60. [2] Let X be a non-empty set. A picture fuzzy set A on X is an object of the form $A = \{\langle x, \mu_A(x), \eta_A(x), \nu_A(x) \rangle \mid x \in X\}$, where $\mu_A(x) \in [0, 1]$ is called "the degree of positive membership of x in A ", $\eta_A(x) \in [0, 1]$ is called "the degree of neutral membership of x in A " and $\nu_A(x) \in [0, 1]$ is called "the degree of negative membership of x in A ". μ_A, η_A and ν_A satisfy $\mu_A(x) + \eta_A(x) + \nu_A(x) \leq 1$, for any $x \in X$.

The set of all the picture fuzzy sets in the universe X will be denoted by $PFS(X)$.

Main results

According to Cuong's definitions of picture fuzzy sets and their operations (see, [2]) and respecting the condition "for all $A \in PFS(X)$, $\emptyset \subseteq A$ and $A \subseteq X$ ". The definitions of empty and universal picture fuzzy sets should be respectively as follows: $\emptyset = \{\langle x, 0, 0, 1 \rangle \mid x \in X\}$ and $X = \{\langle x, 1, 1, 0 \rangle \mid x \in X\}$.

There is a contradiction in Definition 60 since $1 + 1 + 0 \not\leq 1$, i.e., $X \notin PFS(X)$. On the other hand, $A \cup \emptyset \neq A$, $A \cup X \neq X$. and $A \not\subseteq A \cup B$.

So, picture fuzzy sets operations should be redefined to overcome these inconsistencies.

Consider the set $\mathbb{D}^*$ defined by:

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$$\mathbb{D}^* = \{a = (a_1, a_2, a_3) \in [0, 1]^3, a_1 + a_2 + a_3 \leq 1\}.$$

This set plays the role of a prototype of a picture fuzzy set, and the study of this set allows us to perform picture fuzzy sets operations using these of $\mathbb{D}^*$.

Definition 61. [3] For all $a, b \in \mathbb{D}^*$, we define the order relation $\leq$ on $\mathbb{D}^*$ by

$a \leq b$ if and only if $(a_1 < b_1 \text{ and } a_3 \geq b_3)$ or $(a_1 = b_1 \text{ and } a_3 > b_3)$ or $(a_1 = b_1 \text{ and } a_3 = b_3 \text{ and } a_2 \leq b_2)$, for all $a, b \in \mathbb{D}^*$.

Note that $(\mathbb{D}^*, \leq, \vee, \wedge, 1_{\mathbb{D}^*}, 0_{\mathbb{D}^*})$ is a bounded lattice with top element $1_{\mathbb{D}^*} = (1, 0, 0)$ and bottom element $0_{\mathbb{D}^*} = (0, 0, 1)$. And for each $a, b \in \mathbb{D}^*$, $a \wedge b$ and $a \vee b$ are given by

$$a \wedge b = \begin{cases} a, & \text{if } a \leq b, \\ b, & \text{if } b \leq a, \\ (a_1 \wedge b_1, 1 - a_1 \wedge b_1 - a_3 \vee b_3, a_3 \vee b_3), & \text{otherwise.} \end{cases}$$

$$a \vee b = \begin{cases} b, & \text{if } a \leq b, \\ a, & \text{if } b \leq a, \\ (a_1 \vee b_1, 0, a_3 \wedge b_3), & \text{otherwise.} \end{cases}$$

In what follows, we redefine the picture fuzzy sets operations and investigate some of their properties.

Definition 62. Let $X \neq \emptyset$ and let $A, B \in PFS(X)$. Using the laws of $\mathbb{D}^*$, we define

- $A \subseteq B$ iff $(\mu_A(x) < \mu_B(x) \text{ and } v_A(x) \geq v_B(x))$ or $(\mu_A(x) = \mu_B(x) \text{ and } v_A(x) > v_B(x))$ or $(\mu_A(x) = \mu_B(x) \text{ and } v_A(x) = v_B(x) \text{ and } \eta_A(x) \leq \eta_B(x))$, for all $x \in X$.
- $A \cap B = \{\langle x, \min\{\mu_A(x), \mu_B(x)\}, \eta_{A \cap B}(x), \max\{v_A(x), v_B(x)\} \rangle \mid x \in X\},$

$$\text{where } \eta_{A \cap B}(x) = \begin{cases} \eta_A(x) & \text{if } A \subseteq B, \\ \eta_B(x) & \text{if } B \subseteq A, \\ 1 - \min\{\mu_A(x), \mu_B(x)\} - \max\{v_A(x), v_B(x)\}, & \text{otherwise.} \end{cases}$$

- $A \cup B = \{\langle x, \max\{\mu_A(x), \mu_B(x)\}, \eta_{A \cup B}(x), \min\{v_A(x), v_B(x)\} \rangle \mid x \in X\},$

$$\text{where } \eta_{A \cup B}(x) = \begin{cases} \eta_B(x) & \text{if } A \subseteq B, \\ \eta_A(x) & \text{if } B \subseteq A, \\ 0 & \text{otherwise.} \end{cases}$$

- $A^C = \{\langle x, v_A(x), 1 - \mu_A(x) - \eta_A(x) - v_A(x), \mu_A(x) \rangle \mid x \in X\}.$
- $\emptyset = \{\langle x, 0, 0, 1 \rangle \mid x \in X\}$ and $X = \{\langle x, 1, 0, 0 \rangle \mid x \in X\}.$

Proposition 22. Let X be a non-empty set and let $A, B, C \in PFS(X)$. Then

$$(i) \ A \cap (B \cap C) = (A \cap B) \cap C, A \cup (B \cup C) = (A \cup B) \cup C.$$

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- (ii) $A \cap B = B \cap A, A \cup B = B \cup A.$
- (iii) $A \cup B \supseteq A \supseteq A \cap B, A \cup B \supseteq B \supseteq A \cap B.$
- (iv) $A \cup \emptyset = A, A \cup X = X, A \cap \emptyset = \emptyset$ and $A \cap X = A$, for all $A \in PFS(X).$
- (v) $(A \cap B)^C = A^C \cup B^C, (A \cup B)^C = A^C \cap B^C.$
- (vi) $(\emptyset)^C = X, (X)^C = \emptyset.$
- (vii) $(A^C)^C = A.$
- (viii) $A \subseteq B$ implies $B^C \subseteq A^C.$
- (ix) $A^C \cap A \neq \emptyset, A^C \cup A \neq X.$

Proposition 23. $(PFS(X), \subseteq, \cap, \cup, \emptyset, X)$ is a bounded lattice.

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Projective Spaces Over a Finite Field

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Abstract: Let $V = V(n+1, q)$ be an $(n+1)$ -dimensional vector space over the field $K = \mathbb{F}_q$. In $\text{PG}(n, q)$, a subspace of dimension m , or m -space, is a set of points, all of whose representing vectors form, together with the origin, a subspace of dimension $m+1$ of $V(n+1, q)$; it is denoted by Π_m . Subspaces of dimension zero, one, two and three are called a point, a line, a plane, and a solid. A subspace of dimension $n-1$ is called a prime or a hyperplane. The purpose of this work is to give some definitions and some properties on m -space, of $\text{PG}(n, q)$ and give some information about the number of points of quadrics, as well as some related properties.

Keywords: Finite field, Quadric, Projective space, Coding theory.

2020 Mathematics Subject Classification: 05B25, 51E20, 94B60.

Introduction

Let $V = V(n+1, q)$ be an $(n+1)$ -dimensional vector space over the field K with zero element 0. Consider the equivalence relation on the elements of $V \setminus \{0\}$ whose equivalence classes are the one-dimensional subspaces of V with the zero deleted. Thus, if $X, Y \in V \setminus \{0\}$, then $X = (x_0, \dots, x_n)$ is equivalent to $Y = (y_0, \dots, y_n)$ if $Y = tX$ for some t in $K_0 = K \setminus \{0\}$; that is, $y_i = tx_i$ for all i . Then the set of equivalence classes is the n -dimensional projective space over K and is denoted by $\text{PG}(n, K)$; when $K = \mathbb{F}_q$, it is $\text{PG}(n, q)$ or $\mathbb{P}^n$.

A point with coordinate vector $X = (x_0, \dots, x_n)$ is denoted by $P(X) = P(x_0, x_1, \dots, x_n)$, where $x_0, x_1, \dots, x_n \in \mathbb{F}_q$ and not all zero.

The set of projectivities fixing $\text{PG}(n, q)$ is the group $\text{PGL}(n+1, q)$.

The theory of quadrics and their number of points in $\text{PG}(n, q)$ is motivated by many important applications, especially in the theory of error-correcting codes; see [2, 4, 1].

The theory of non-singular quadrics and their number of points was studied by Segre [5]. A comprehensive summary may be found in [3, Chapter 1].

Main results

Theorem 172. *As for non-singular quadrics, singular quadrics are of three types.*

- Let $2m < n$, $m \geq 0$.

A singular parabolic quadric has the standard form,

$$\mathcal{P} = V(X_0^2 + X_1X_2 + \dots + X_{2m-1}X_{2m}).$$

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Its number of points is

$$q^{n-1} + q^{n-2} + \cdots + \cdots + q + 1 = |\mathbf{P}^{n-1}|.$$

- Let $2m - 1 < n$, $m \geq 1$.

1. A singular hyperbolic quadric has the standard form,

$$\mathcal{H} = V(X_0X_1 + X_2X_3 + \cdots + X_{2m-2}X_{2m-1}).$$

Its number of points is

$$q^{n-1} + q^{n-2} + \cdots + 2q^{n-m} + q^{n-m-1} + \cdots + 1 = |\mathbf{P}^{n-1}| + q^{n-m}.$$

2. A singular elliptic quadric has the standard form,

$$V(f(X_0, X_1) + X_2X_3 + \cdots + X_{2m-2}X_{2m-1}),$$

with f irreducible over $\mathbf{F}_q$. Its number of points is

$$q^{n-1} + q^{n-2} + \cdots + q^{n-m+1} + q^{n-m-1} + \cdots + 1 = |\mathbf{P}^{n-1}| - q^{n-m}.$$

Proposition 24. In $\text{PG}(n, q)$, let Π_r be a subspace of dimension r . If $\mathcal{Q} = V(F^s)$ is a quadric given by F in $\text{PG}(s, q)$, then

$$|V(F^n)| = |V(F^r)| \cdot q^{n-r} + |\text{PG}(n-r-1, q)|.$$

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Construction of a new key using circulant matrices and a chaotic logistic maps

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Abstract: To ensure confidentiality and avoid human attacks on our data, we exchange keys for encryption and decryption. In this work, we will take advantage of the commutativity of the circular matrix composition to create a common encryption key by applying the Diffie-Hellman exchange protocol via a conventional channel. To increase the security level of our system, we will introduce chaotic logistics maps.

Keywords: Toeplitz matrices, Circulant matrices, Cryptography, Chaotic logistic maps, Non-linear systems, BB84 protocol, Diffie-Hellman protocol.

2020 Mathematics Subject Classification: Primary 37D45, 35K55, 15B05, 15B33, 14G50, 11T71, 94A60.

Introduction

How can we protect our information? and especially how can we exchange this information (text or images) with those we want without it being hacked?

The answer to this question is to find a way to share one or more keys that will be used for encryption and decryption of transmitted messages.

Based on the same techniques of our previous work [1], we choose a logistic maps :

$$(x_0, \mu_x) : x_{n+1} = \mu_x x_n (1 - x_n); \quad 3 < \mu_x < 4; \quad x_0 \text{ initially chooses} \quad (425)$$

and we create a circulant matrix of order n .

But before that, we give some definitions

Definition 63. [1, 2] A Toeplitz matrix is a matrix $T \in M_n(\mathbb{R})$ or $M_n(\mathbb{C})$ of the form

$$T = \begin{bmatrix} t_0 & t_1 & \cdots & t_{-(n-1)} \\ t_1 & t_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & t_{-1} \\ t_{n-1} & \cdots & t_1 & t_0 \end{bmatrix}; \quad \text{where } t_i \in \mathbb{R} \text{ (or } \mathbb{C} \text{) for all } i \in \{-(n-1), \dots, n-1\}.$$

We can distinct the two particular cases in the fallowing remark

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1. If $t_{n-j} = t_{-j}$ for $j \in \{0, \dots, n\}$, the matrix $\mathcal{T}$ is said to be circular matrix.
2. If $t_{n-j} = -t_{-j}$ for $j \in \{0, \dots, n\}$, so we are talking about anti-circular matrix.

In this work, we are interested by circulant matrices, so we give the definition:

Definition 64. A circular matrix is a Toeplitz matrix given by:

$$C = C_n(c_1, \dots, c_n) = \begin{bmatrix} c_1 & c_2 & \cdots & c_n \\ c_n & c_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & c_2 \\ c_2 & \cdots & c_n & c_1 \end{bmatrix} \in \mathcal{M}_n(\mathbb{R}) \quad (426)$$

Theorem 173. Let G_C be the set of all circulant matrices, then for any A and B elements of G_C , we have $AB = BA$.

Proof See [1, 2] ■

The construction of the key

The idea is to create and share a high complexity key, we assume two interlocutors Brahim and Salah. Both individuals complete the following steps:

}}

}}

Brahim will fix the a logistic map (x_0, μ_x) . the n first successive terms of the sequence form the components of the vector $U = (x_1, x_2, \dots, x_n)$.

}}

Salah will fix the logistic map (y_0, μ_y) . the n first successive terms of the sequence form the components of the vector $V = (x_1, x_2, \dots, x_n)$.

}}

We obtain a circulant matrix B generated by the vector U :

}}

$$B = \langle U \rangle$$

$$(427)$$

}}

}}

$$S = \langle V \rangle$$

$$(428)$$

Let P be a public circulant matrix, the two interlocutors Brahim and Salah proceed to the following operations :

1. Brahim create the matrix

}}

}}

$$K_B = BPB$$

$$(429)$$

}}

}}

and send it to Salah.

}}

}}

2. Brahim receives the matrix K_S from Salah and repeats the same operation, so he gets

}}

}}

$$K'_B = BK_S B$$

$$(430)$$

}}

}}

1. Salah create the matrix

$$K_S = SPS$$

$$(431)$$

and send it to Brahim.

2. Salah also receives the matrix K_B from Brahim and do the operation :

$$K'_S = SK_B S$$

$$(432)$$

Theorem 174. The two keys K'_B and K'_S in (430) and (432) are equal.

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Proof We know that the multiplication in $(G_C, \times)$ the monoïde of circulant matrices is commutative, so

$$\begin{aligned} K'_B &= BK_S B \quad \text{from (430)} \\ &= B S P S B \quad \text{from (431)} \\ &= S B P B S \quad \text{from the theorem 173} \\ &= S K_B S \quad \text{from (429)} \\ &= K'_S \quad \text{from (432)} \end{aligned}$$

■

Finally, if we note by $K := K'_S = K'_B$. K is the key that we will use for encryption and decryption of messages exchanged between the two interlocutors.

Remark 7. *the key K must be invertible, for this the choice of the initial parameters (x_0, μ_x) in (425) must be made in a specific way so that the invertibility is ensured.*

Application

Let IM be a digital image of $n \times n$ pixel. (if the image size is $n \times n$ pixel, then the vector sizes in (427) and (428) are fixed by n too). and Brahim wont to send the image I to Salah.

- Brahim converts IM to a square matrix $I \in \mathcal{C}_n(\mathbb{R})$ using Matlab, Scilab or Python. Then he encrypts I to get $I' = KIK^{-1}$ and send it to Salah.
- Salah decrypts I' to get $K^{-1}KIK^{-1}K = I$.

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A nonlinear chaotic map to generate keys in cryptography

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Abstract: The work focuses on the generation of encryption keys based on chaos. These schemes use, on the transmitter side, nonlinear dynamic systems exhibiting chaotic behavior. The complex sequence thus produced is used to hide information. Several key exchange protocols are studied in the case of discrete chaotic systems. For these schemes, the reconstruction of a common key requires the synchronization of the transmitter and receiver. We propose a symmetric key exchange method, taking into account the specificity related to chaos. This new key will be used to encrypt and decrypt our secret data (text or digital image). Finally, encryption analysis by inclusion is performed.

Keywords: Encryption keys, Chaotic systems, Digital image, Cryptography.

2020 Mathematics Subject Classification: 70K55, 37D45.

Introduction

Cryptography, a real science governing the coding of information [1], has experienced a real explosion with the development of computer systems, moving from an artisanal era to very high-tech systems requiring significant computing power. It has experienced even greater growth with the arrival of modern communications systems (Internet,...etc.) [2], where there is an absolute need to protect data exchanged individuals. At the same time, another cryptographic enemy branch called cryptanalysis has been developed, which is the art of revealing plaintext texts that have been encrypted without knowing the decryption key.

Nowadays, cryptologists are very interested in the study of chaotic phenomena, so much work has been done on chaos in general [2]. This work has led to the generalization of its use in various fields of science. The use of chaos for the secure transmission of information has been considered in recent years as a very promising solution to increase the performance of current transmission systems. Thanks to its quasi-stochastic characteristics, chaos offers a possible solution for transmitting large amounts of secure information.

Chaotic systems in discrete time

The Hénon system is a model proposed in 1976 by the mathematician Michel Hénon. It is a discrete-time system that introduces iterations into the plane [3]. These iterations are defined by the following relationships.

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$$\begin{cases} x_{k+1} = a - x_k^2 + by_k \\ y_{k+1} = x_k \end{cases}$$

The parameter values proposed by Michel Hénon to observe the chaotic phenomenon are:
 $a \in [1, 2]$ and $b \in [0, 1]$ (to obtain chaotic phenomenon).

Construction of a matrix $\mathcal{H}$ using Hénon chaotic map

Consider the square matrix $\mathcal{H} = (\mathcal{H}_{ij})$ generated by the terms of the Hénon chaotic map. And we obtain the following matrix $\mathcal{H}$:

$$\mathcal{H} = \begin{pmatrix} \frac{x_1+y_1}{2} & x_2 & x_3 & x_4 & \cdots & \cdots & \cdots & x_n \\ y_2 & \frac{x_2+y_2}{2} & x_{n+1} & x_{n+2} & \cdots & \cdots & \cdots & x_{2n-3} \\ y_3 & y_{n+1} & \frac{x_3+y_3}{2} & x_{2n-2} & \cdots & \cdots & \cdots & x_{3n-6} \\ y_4 & y_{n+2} & y_{2n-2} & \frac{x_4+y_4}{2} & \cdots & \cdots & \cdots & x_{4n-9} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & y_{\frac{n^2-n}{2}} \\ y_n & y_{2n-3} & y_{3n-6} & y_{4n-9} & \cdots & \cdots & y_{\frac{n^2-n}{2}} & \frac{x_n+y_n}{2} \end{pmatrix}$$

Application in encryption data using the matrix $\mathcal{H}$

We recall that the BB84 protocol (proposed by Charles Bennett and Gilles Brassard in 1984) is a quantum key distribution protocol (QKD) that guarantees the security of the key generation process against attacks. The protocol uses the principles of quantum mechanics to establish a secure key between two interlocutors, while detecting the presence of an attempted espionage.

Using the BB84 protocol two person shared secretly the four parameters a, b, x_0 and y_0 then they construct the same matrix $\mathcal{H}$.

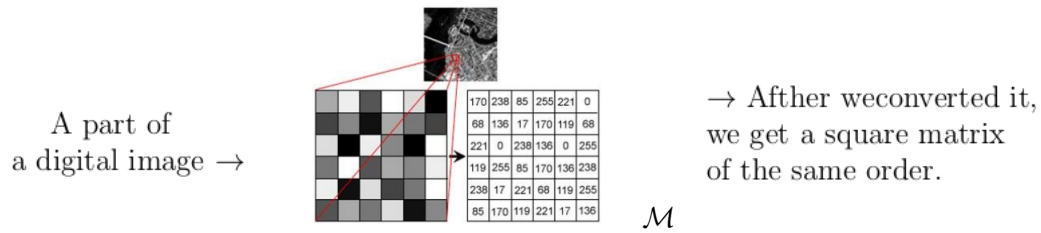
Digital image and how we convert it to a matrix

Image processing is a discipline of computer science and applied mathematics that studies digital images and their transformations.[3]

To convert an image to a matrix we should firstly identifies the size of the image (how many pixels in image).then, recognize the type of this image.

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Encryption and decryption of a digital image

Two interlocutors want to share some information(digital image):

1st step: The emitor converts this digital image to a matrix $\mathcal{M}$.

2nd step: The two interlocutors must shared secretly the same key $\mathcal{H}$.

3rd step: Encryption and decryption

Encryption: The emitor uses the formula $\mathcal{M} \oplus \mathcal{H} = \mathcal{M}_c =$ Cipher matrix ($\oplus$ is the xor operation).

Decryption: The receiver receives $\mathcal{M}_c$ and and decrypt it using the formula $\mathcal{M}_c \oplus \mathcal{H} = \mathcal{M}$.

Proof: $\mathcal{M}_c \oplus \mathcal{H} = (\mathcal{M} \oplus \mathcal{H}) \oplus \mathcal{H} = \mathcal{M} \oplus \mathcal{H} \oplus \mathcal{H} = \mathcal{M} \oplus \mathcal{O} = \mathcal{M}$.

Main results

In our work we study the chaotic systems in particular the 2-dimensions system of *Hnon*. Then we made the encryption and the decryption of a digital image to study the cryptography by the chaos so that we are interested to study and realize a system transmitter and receiver of chaotic message first we studied the system of 2-dimensions adapt this system with the encryption of an image $\mathcal{M}$, after having built and shared a common key for the encryption by the sender and send the coded image, then the second step was made the decryption to determine the initial image. At the end we can say that we succeeded has laid a symmetric cryptographic system reinforced by chaos and through cryptanalysis we have ensured that our system resists attacks especially the brute force attack.

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On Arithmetic Function Related to the Least Common Multiple

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Abstract: Let the prime factorization of the positive integer $n > 1$ be $\prod_{i=1}^r p_i^{e_i}$. For a positive integer α , we defined f_α to be the arithmetic function such that $f_\alpha(n) = \prod_{i=1}^r p_i^{[e_i, \alpha]}$, where $[e_i, \alpha]$ is the least common multiple of e_i and α . some properties of f_α are given in this talk. This paper based on [2].

Keywords: arithmetic function, lcm, gcd.

2020 Mathematics Subject Classification: 11A25.

Introduction

Throughout this paper, we let (a, b) and $[a, b]$ denote, respectively, the greatest common divisor and the least common multiple of any two integers a and b . Let $\prod_{i=1}^r p_i^{e_i}$ be the prime factorization of the positive integer $n > 1$. The author [1, 1] introduced and studied some properties of the following arithmetic function, for a positive integer α :

$$f_\alpha(n) = \prod_{i=1}^r p_i^{(e_i, \alpha)}, \quad f_\alpha(1) = 1.$$

In different paper the author defined new arithmetic function similar to f_α as:

$$g_\alpha(n) = \prod_{i=1}^r p_i^{[e_i, \alpha]}, \quad g_\alpha(1) = 1.$$

In our presentation, we will discuss some properties of the functions g_α .

Main results

Clearly, g_α is a multiplicative function, i.e., if $(m, n) = 1$, then:

$$g_\alpha(mn) = g_\alpha(m)g_\alpha(n). \quad (433)$$

The next theorem gives a condition for m and n (which are not necessarily coprime) to be satisfied the equation (433), for all odd positive integers α .

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Theorem 175. Let α be an odd positive integer. Then

$$g_{\alpha}(mn) = g_{\alpha}(m)g_{\alpha}(n)$$

for all square-free positive integers m and n .

Theorem 176. For each natural number $n \geq 2$, there are finitely many positive integers α such that $g_{\alpha}(n) = n$.

Theorem 177. Let α and β be positive integers. Then for every n :

$$g_{\alpha}(g_{\beta}(n)) = g_{[\alpha, \beta]}(n).$$

Theorem 178. Let $\alpha > 1$ be a positive integer. Then the two following systems of inequalities:

$$\left\{ \begin{array}{l} a < b \\ g_{\alpha}(a) < g_{\alpha}(b) \end{array} \right. \quad \text{and} \quad \left\{ \begin{array}{l} a < b \\ g_{\alpha}(a) > g_{\alpha}(b) \end{array} \right.$$

hold for infinitely many positive integers a and b .

The next theorems give, respectively, the values of g_2 for perfect numbers and the values of g_4 for even perfect numbers.

Theorem 179. Let $N > 6$ be a perfect number. Then

$$\frac{g_2(N)}{N} = \begin{cases} m & \text{if } N = 2^{p-1}m \text{ is even,} \\ p^e & \text{if } N = p^e m^2 \text{ is odd.} \end{cases}$$

Theorem 180. Let $N = 2^{p-1}m$ be an even perfect number such that $N > 28$. Then

$$\frac{g_4(N)}{N} = \begin{cases} m^3 & \text{if } p \equiv 1 \pmod{4}, \\ 2^{p-1}m^3 & \text{if } p \equiv 3 \pmod{4}. \end{cases}$$

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On single valued neutrosophic mappings

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Abstract: Recently, Wang et al. [8] introduced the concept of single-valued neutrosophic sets, as a subclass of neutrosophic sets, which has grown rapidly due to its broad theoretical richness and uses in many applied fields. In this work, we introduce the notion of single-valued neutrosophic mapping, which is defined by single-valued neutrosophic relation and considered as a generalization of fuzzy mapping. Additionally, many properties related to this concept were examined.

Keywords: Binary relation, mapping, Neutrosophic set, Single-valued neutrosophic set.
2020 Mathematics Subject Classification: Primary 03E20, 30C25, 03B52, 54E35.

Introduction

In the mathematics literature, mappings in crisp set theory are among the oldest knowledge of modern mathematics and play an important role in many branches of pure and applied mathematics, such as in group theory, computer science, and others. For this concept, extended approaches are proposed based on the Zadeh fuzzy set previously introduced in [10] using two membership functions for the elements of the discourse universe. In [6], S. Heilpern proposed the notion of fuzzy mapping and proved a fixed point theorem for fuzzy contraction mappings. Since then, the concept of fuzzy mapping has attracted widespread attention. It appears in many researches, like Azam [1] and more others. Among them, Ismail and Massa'deh [3] defined L -fuzzy mappings and studied their operations, also they developed many properties of classical mappings into L -fuzzy cases.

In recent years, Smarandache [5] generalized the concepts of intuitionistic fuzzy sets and other types of sets to the notion of neutrosophic sets. He introduces this concept to deal with imprecise and indeterminate data. Neutrosophic sets are characterized by three independent components, the truth, indeterminacy, and falsity membership functions. Especially, to properly train neutrosophic sets in real-world applications, Wang et al. [8] introduced the concept of single-valued Neutrosophic sets serve as a subcategory of neutrosophic sets. Studies, both theoretical and applied on single-valued neutrosophic sets, are progressing rapidly. For example [2, 4, 7, 9], and many others. In this work, we generalized the concept of fuzzy mapping defined by fuzzy relation introduced by Ismail and Massa'deh to the setting of single valued neutrosophic sets. Furthermore, we studied some properties relevant to this concept.

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Main results

Definition 65. Let A be a single valued neutrosophic set on X and B be a single valued neutrosophic set on Y , let $f : \text{Supp } A \rightarrow \text{Supp } B$ be an ordinary mapping and R be a single valued neutrosophic relation on $X \times Y$. Then f_R is called a single valued neutrosophic mapping if for all $(x, y) \in \text{Supp } A \times \text{Supp } B$ the following condition is satisfied:

$$\mu_R(x, y) = \begin{cases} \min(\mu_A(x), \mu_B(f(x))), & \text{if } y = f(x) \\ 0, & \text{Otherwise,} \end{cases}$$

and

$$\sigma_R(x, y) = \begin{cases} \min(\sigma_A(x), \sigma_B(f(x))), & \text{if } y = f(x) \\ 0, & \text{Otherwise,} \end{cases}$$

and

$$\nu_R(x, y) = \begin{cases} \max(\nu_A(x), \nu_B(f(x))), & \text{if } y = f(x) \\ 1, & \text{Otherwise,} \end{cases}$$

Remark 8. Through the following way, we can construct the single-valued neutrosophic mapping.

- (i) We determine the $\text{Supp } A$ and $\text{Supp } B$.
- (ii) We determine the ordinary mapping from $\text{Supp } A$ to $\text{Supp } B$.
- (iii) We determine the single valued neutrosophic relation by its membership function, indeterminacy function and non-membership function.
- (iv) Finally, we conclude the construction of the single valued neutrosophic mapping.

Definition 66. Let f_R, g_S be two single valued neutrosophic mappings, then f_R and g_S are equal if and only if $f = g$ and $R = S$ i.e., $(\mu_R(x, f(x)) = \mu_S(x, g(x)), \sigma_R(x, f(x)) = \sigma_S(x, g(x)), \text{ and } \nu_R(x, f(x)) = \nu_S(x, g(x)))$.

Definition 67. Let f_R, g_S be two single valued neutrosophic mappings, then f_R and g_S are equal if and only if $f = g$ and $R = S$ i.e., $(\mu_R(x, f(x)) = \mu_S(x, g(x)), \sigma_R(x, f(x)) = \sigma_S(x, g(x)), \text{ and } \nu_R(x, f(x)) = \nu_S(x, g(x)))$.

Definition 68. Let A be a single valued neutrosophic set on X , let $f : \text{Supp } A \rightarrow \text{Supp } A$ be an ordinary mapping such that $f(x) = x$ and R be a single valued neutrosophic relation on $X \times X$. Then f_R is called a single valued neutrosophic identity mapping if for all $x, y \in \text{Supp } A$ the following conditions are satisfied:

$$\mu_R(x, y) = \begin{cases} \mu_A(x), & \text{if } x = y \\ 0, & \text{Otherwise,} \end{cases}$$

and

$$\sigma_R(x, y) = \begin{cases} \sigma_A(x), & \text{if } x = y \\ 0, & \text{Otherwise,} \end{cases}$$

and

$$\nu_R(x, y) = \begin{cases} \nu_A(x), & \text{if } x = y \\ 1, & \text{Otherwise,} \end{cases}$$

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Hom-compatible order and ordering Hom-group

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Abstract: In recent papers, many authors have studied the Hom-groups as a non-associative generalization of the groups. In this work, we extend the concept of ordered groups to the Hom-groups setting by introducing the notion of Hom-compatible order. Some properties, characterizations, and related results are investigated. In particular, the positive cone relative to a Hom-compatible order on Hom-group (resp. quotient Hom-group) is characterized.

Keywords: Hom-group, Hom-subgroup, Normal-Hom subgroup, Hom-compatible order, ordered Hom-group, positive cone of Hom-ordered group.

2020 Mathematics Subject Classification: Primary 18B35, 06F15, 06F15, 20F60 .

Introduction

A Hom-group G is a generalization of a group where associativity, invertibility, and unitarity are deformed by a homomorphism $\alpha : G \rightarrow G$ satisfies the Hom-associative condition $\alpha(g)(ht) = (gb)\alpha(t)$, for all $g, h, t \in G$, so G may not have a group structure since the associativity is failed, but it is a Hom-group by deformation.

Recently, many authors have worked on the Hom-groups such as Hassan zadah in [3, 4] used Cayley table of quasigroups to provide some examples of Hom-groups, he introduced the notions of Hom-subgroups and cosets and proved Lagrange's theorem for finite Hom-groups as well as he introduced two types of modules over a Hom-group G . In 2020 Jun Jiang et al. introduced the notion of a (regular) Hom-Lie group. Chen et al. in [5] produced some new examples of Hom-groups, and shown the first and the second isomorphism fundamental theorems of homomorphisms on Hom-groups. They also introduced the notion of Hom-group action, and as an application, they given the first Sylow theorem for the Hom-groups.

Partially ordered algebraic systems play an important role in algebra and adjacent disciplines such as logic and computer science. This notion has been appeared in the early days of lattice ordered groups by Birkhoff in [7, 8] and followed by many other researchers in different areas of mathematics. Some important concepts in partially ordered systems are compatible order and ordered groups. These concepts play a major role in the study of lattice-ordered groups, lattice-ordered rings, vector lattices, ordered topology, rings of continuous functions, functional analysis, and the fuzzy set theory of lattice-ordered algebraic structures, besides being a major branch in algebra. Therefore, it appears that the study of the ordered Hom-groups and lattices ordered

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Hom-groups algebraic structures are necessary, to get a better insight into Hom-algebra. In this work we extend the notion of compatible order to the setting which we call "Hom-compatible order" and we give some important properties, characterizations, and related results. Particularly, the positive cone of an ordered Hom-group is characterized.

Main results

In this work, we have introduced the concept of Hom-compatible order and extended the concept of ordered groups to the setting of Hom groups. In this regard, we have presented some characterizations and related results that help us identify a subset of the ordered Hom-group that can be a positive cone of an Hom-compatible order. Some other results are also given.

Definition 69. Let $(E, \cdot, \alpha)$ be a Hom-magma.

- (i) By an **Hom-compatible order** on E , we mean an order $\leq$ on E with respect to which all translations $y \mapsto x\alpha^{-1}(y)$ and $y \mapsto \alpha^{-1}(y)x$ are isotone.
- (ii) By an ordered Hom-group we mean a Hom-group on which a Hom-compatible order is defined, that is, a structure $(G, \cdot, 1, \alpha, \leq)$ such that $(G, \cdot, 1, \alpha)$ is a Hom-group and $\leq$ is a Hom-compatible order on G .

Note that in Definition 69, if $\alpha = id_E$, then we get the notion of the compatible order (resp. ordered group) on E . In this way, the notion a Hom-compatible order (resp. ordered group) is a generalized notion of a compatible order (resp. ordered group) (see, e.g., [2]).

Proposition 25. Let $(G, \cdot, 1, \alpha)$ be a Hom-group. The following statements hold:

- i) For all $g, h, k \in G$, $g\alpha^{-1}(hk) = \alpha^{-1}(gh)k$;
- (ii) For all $g, h \in G$, $g\alpha^{-1}(g^{-1}h) = \alpha^{-1}(hg^{-1})g = \alpha(h)$;
- (iii) For all $g, h, k \in G$, $(gh)(h^{-1}k) = \alpha(gk)$;
- iv) For all $x, g, h \in G$, $gx = h$ if and only if $x = \alpha^{-1}(g^{-1})\alpha^{-2}(h)$;
- v) For all $x, g, h \in G$, $xg = h$ if and only if $x = \alpha^{-2}(h)\alpha^{-1}(g^{-1})$;
- vi) For all $x, g, h, k \in G$, $(hx)g = k$ if and only if $x = \alpha^{-1}(h^{-1})\left(\alpha^{-4}(k)\alpha^{-3}(g^{-1})\right)$.

Proposition 26. Let $(G, \cdot, 1, \alpha)$ be a Hom-group and $\leq$ be an order relation on G . The following statements are equivalent:

- (i) $(G, \cdot, 1, \alpha, \leq)$ is an ordered Hom-group;
- (ii) If, moreover, $(G, \leq)$ is a $\vee$ -semilattice, then for all $x, y \in G$,

$$\alpha^{-1}(x \vee y)z = \alpha^{-1}(x)z \vee \alpha^{-1}(y)z \text{ and } z\alpha^{-1}(x \vee y) = z\alpha^{-1}(x) \vee z\alpha^{-1}(y);$$

- (iii) For all $x, y, z \in G$, $x < y$ implies that $\alpha^{-1}(x)z < \alpha^{-1}(y)z$ and $z\alpha^{-1}(x) < z\alpha^{-1}(y)$.

Theorem 181. Let $(G, \mu, 1, \alpha, \leq)$ be an ordered Hom-group and $\beta : G \rightarrow G$ be a bijective map. It holds that

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- (1) If β is multiplicative and commute with α , then $(G, \beta \circ \mu, 1, \beta \circ \alpha, \leq)$ is also an ordered Hom-group
- (2) If β is multiplicative and commute with α , then for any $n \in \mathbb{N}$, $(G, \beta^n \circ \alpha^n \circ \mu, 1, \beta^n \circ \alpha^{n+1}, \leq)$ is an ordered Hom-group.

Proposition 27. Let $(G, \cdot, 1, \alpha, \leq)$ be an ordered Hom-group and $x, y \in G$. The following statements hold:

- (i) If $x, y \in N_G$, then $\alpha^{-1}(xy) \in N_G$, $\alpha^{-1}(xy) \leq x$ and $\alpha^{-1}(xy) \leq y$;
- (ii) If $x, y \in P_G$, then $\alpha^{-1}(xy) \in P_G$, $\alpha^{-1}(xy) \geq x$ and $\alpha^{-1}(xy) \geq y$.

The following theorem characterize the positive cone relative to a Hom-compatible order on Hom-group.

Theorem 182. Let $(G, \cdot, 1, \alpha)$ be a Hom-group and P be a subset of G . Then P is the positive cone relative to a Hom-compatible order on G if and only if

- (1) $P \cap P^{-1} = \{1\}$;
- (2) $\alpha^{-1}(P^2) = P$;
- (3) $\forall x \in G, \left(\alpha^{-1}(x)P \right) x^{-1} = P$.

Moreover, $P \cup P^{-1} = G$, if and only if this order is total.

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Differential equations and Dynamical systems

Initial value problem of a system of generalized Sturm-Liouville and Langevin Hilfer-Katugampola fractional differential equations

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Abstract:

In this work, we study initial value problem of system of generalized Sturm-Liouville and Langevin Hilfer-Katugampola fractional differential equations.

We establish the existence and uniqueness of solution of proposed system in the weighted Banach space by using Schauder fixed point theorem and Banach contraction principle. Finally, we provide an example to illustrate our results.

Keywords: Sturm-Liouville problem, fractional Langevin equation, Hilfer-Katugampola derivative, Schauder's fixed point theorem, Banach contraction principle.

2020 Mathematics Subject Classification: 34A08, 34B18.

Introduction

The Langevin equation (first formulated by Langevin in [2]) is found to be an effective tool to describe the evolution of physical phenomena in fluctuating environments. On the other hand, the Sturm-Liouville problem has many applications in different areas of science and engineering. Combining both fractional Sturm-Liouville and fractional Langevin equations would provide adequate description for dynamical processes defined in a fractal medium in which the fractal and memory properties with a dissipative memory kernel are incorporated. In this work we investigate the existence as well as the uniqueness of the solution for the following problem

$$\begin{cases} {}^{\rho}D_{a^{+}}^{\alpha_i, \beta_i} \left[p_i(t) {}^{\rho}D_{a^{+}}^{\alpha'_i, \beta'_i} + r_i(t) \right] u_i(t) = f_i(t, u_1(t), u_2(t)) & t \in [a, T], a > 0, i = 1, 2, \\ u_i(a) = 0, \end{cases} \quad (434)$$

where, $(0 < \alpha_i, \alpha'_i < 1)$ and $(0 \leq \beta_i, \beta'_i \leq 1)$.

${}^{\rho}D^{\alpha, \beta}$ is a Hilfer-Katugampola fractional derivative of order α , $(0 < \alpha < 1)$ and type β , $(0 \leq \beta \leq 1)$. $f_i : [a, T] \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ are continuous functions, $p_i \in C([a, T], \mathbb{R} \setminus \{0\})$ and $r_i \in C([a, T], \mathbb{R})$ for $i = 1, 2$.

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Main results

Notations : $p_i^* = \inf_t |p_i(t)|$, $r_i^* = \sup_t |r_i(t)|$ and we assume

$$K_i = 2A_i N_i + B_i < 1, \quad i = 1, 2. \quad (435)$$

with

$$A_i = \left(\frac{T^\rho - a^\rho}{\rho} \right)^{1-\gamma_i+\alpha_i+\alpha_i'} \frac{1}{p_i^* \rho^2 \Gamma(\alpha_i + 1) \Gamma(\alpha_i' + 1)},$$

$$B_i = \left(\frac{T^\rho - a^\rho}{\rho} \right)^{\alpha_i'} \frac{r_i^*}{p_i^* \rho \Gamma(\alpha_i' + 1)}.$$

Theorem 183. We suppose that $f_i, i = 1, 2$ satisfies the following hypothesis.

(H1) There exists a constant $N_i > 0$ such that :

$$|f_i(t, u_1, u_2) - f_i(t, v_1, v_2)| \leq N_i (\|u_1 - v_1\| + \|u_2 - v_2\|), \forall t \in [a, T], \forall u_i, v_i \in \mathbb{R} \quad i = 1, 2.$$

(H2) For all $u = (u_1, u_2)$, there exists $\delta_i > 0$ such that

$$|f_i(t, u)| \leq \delta_i, \quad \forall t \in I_0, (i = 1, 2).$$

and moreover condition (435) is verified then (434) has a unique solution in $\mathcal{U}$,

where

$$\mathcal{U} = \{u \in C_{1-\gamma_1, \rho} \times C_{1-\gamma_2, \rho} : \|u\|_{C_{1-\gamma, \rho}} \leq \omega\}.$$

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Parabolic variational inequalities with Volterra type operators

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Abstract: The objective of this paper is to demonstrate the existence and uniqueness of solutions for parabolic variational inequalities involving Volterra-type operators. To this end, we will establish the equivalence between a parabolic variational inequality and an integro-differential sweeping process of Volterra type.

Keywords: Variational inequalities, sweeping process, maximal monotone operator, differential inclusions.

2020 Mathematics Subject Classification: 49J40, 47J20, 47J22, 34G25.

Introduction

Let H be a Hilbert space and let Ω be a bounded subset of $\mathbb{R}^n$. We consider the following parabolic variational inequalities with a moving obstacle :

find a function $x(t) \in C(t)$ such that there exists a positive constant $\delta_t > 0$ satisfying

$$\begin{aligned} \int_{\Omega} \dot{x}(t)(v(t) - x(t))dy + \int_{\Omega} \nabla x(t)(\nabla v(t) - \nabla x(t))dy \\ + \int_{\Omega} \left(\int_0^t B(t-s)x(s)ds \right) (v(t) - x(t))dy \geq -\delta_t \|v(t) - x(t)\|^2, \quad \forall v(t) \in C(t), \end{aligned} \quad (436)$$

where the initial value is prescribed as

$$x(0) = x_0 \in C(0),$$

and the set $C(t)$ is the union of two disjoint closed, convex sets.

To demonstrate the existence and uniqueness of solutions for (436) we present the connection between (436) and the following integro-differential sweeping process

$$\begin{cases} -\dot{x}(t) \in N_{C(t)}(x(t)) + Ax(t) + \int_0^t f(t,s,x(s))ds \quad \text{a.e. } t \in [0, T], \\ x(0) = x_0 \in C(0). \end{cases} \quad (437)$$

where $A : H \rightrightarrows H$ is a set-valued maximal monotone operator and $N_{C(t)}(x(t))$ denotes the proximal normal cone operator of the closed prox-regular set $C \subset H$.

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Main results

We now proceed to prove the main results about the existence and uniqueness of the solution. But first we need the following auxiliary theorem, which is proved in [1].

Theorem 184. *Let $C(t)$ be nonempty, closed and r -prox-regular set in H which moving in an absolutely continuous way for each $t \in [0, T]$. Assume that the measurable function $f : Q_\Delta \times H \rightarrow H$ and the maximal monotone operator A confirmed the following hypothesis.:*

- (1) $\bigcup_{t \in [0, T]} C(t) \subset \text{int}(\text{Dom}A)$;
- (2) *there exists a non-negative functions $\alpha(\cdot), \delta(\cdot) \in L^2([0, T], \mathbb{R}_+)$ with $\alpha(\cdot)$ is a continuous function, such that*

$$\|A^0 x\| \leq \alpha(t)\|x\| + \delta(t), \text{ for a.e. } t \in [0, T] \text{ and } x \in C(t)$$

- (3) *there exists a non-negative function $\beta(\cdot, \cdot) \in L_1(Q_\Delta, \mathbb{R}_+)$ such that*

$$\|f(t, s, x)\| \leq \beta(t, s)(1 + \|x\|), \text{ for all } (t, s) \in Q_\Delta \text{ and for any } x \in \bigcup_{t \in [0, T]} C(t).$$

Moreover, there exists $\gamma_1(\cdot), \gamma_2(\cdot) \in L^1([0, T], \mathbb{R}_+)$ such that $\beta(t, s) \leq \gamma_1(t) \cdot \gamma_2(s)$ for a.e. $t \in [0, T]$ where

$$Q_\Delta := \{(t, s) \in [0, T] \times [0, T] : s \leq t\}.$$

- (4) *for each real $\eta > 0$ there exists a non-negative function $L^\eta(\cdot) \in L^1([0, T], \mathbb{R}_+)$ such that for all $(t, s) \in Q_\Delta$ and for any $(x, y) \in B[0, \eta] \times B[0, \eta]$,*

$$\|f(t, s, x) - f(t, s, y)\| \leq L^\eta(t)\|x - y\|.$$

Then for all $x_0 \in C(0)$, there exists a unique absolutely continuous solution for (437).

Theorem 185. *Let Ω be a bounded subset of $\mathbb{R}^n$ and $H = L^2(\Omega)$, $U = H^2(\Omega) \cap H_0^1(\Omega)$. Let be given $\phi \in L^2(0, T; U)$, $M(\cdot)$ is a constant function and $B : [0, T] \rightarrow L^\infty(\Omega)$ such that $\phi(\cdot)$ is k -Lipschitz continuous with respect to the supremum norm.*

For each $t \in [0, T]$, we define

$$C(t) := C_1(t) \cup C_2(t)$$

where

$$C_1(t) := \{v \in U : v \geq \phi(t) \text{ a.e. on } \Omega \text{ and } \|\Delta v\| \leq M(t)\} \text{ is closed and convex set.}$$

And

$$C_2(t) := \{v \in U : v \leq \phi(t) - 1 \text{ a.e. on } \Omega \text{ and } \|\Delta v\| \leq M(t)\} \text{ is closed and convex set.}$$

Then for all $x_0 \in C(0)$, there exists a unique absolutely continuous solution for (436).

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Explicit-implicit finite scheme approximation equation of the one dimensional variable-order time-fractional diffusion equation

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Abstract: In this paper, we investigate explicit-implicit finite scheme approximation equation of the one dimensional variable-order time-fractional diffusion equation. The numerical method of solving differential equations by approximating them with difference equations is called the implicit finite difference method. The first-order numerical scheme, stability, consistency and convergence of the method are proven. Moreover, the theoretical analysis of the above scheme and illustrate the effectiveness of the suggested schemes.

Keywords: fractional derivatives; discretization; implicit numerical scheme; stability; convergence.

2020 Mathematics Subject Classification: 70K75, 93A30, 34K37, 65N06.

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Multiplicity of positive solutions for a second order quasilinear equations

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Abstract: In this paper we discuss existence and multiplicity of positive solutions for a class of second order quasilinear equations with an autonomous nonlinearity. To obtain our results we will use Ekeland Variational Principle and Mountain Pass Theorem.

Keywords: critical point, Ekeland variational principle, Mountain Pass Theorem, Palais Smale condition, positive solution.

2020 Mathematics Subject Classification: 35A15, 35B38, 30E25, 58E30, 49K35.

Introduction

Our aim in this paper is to obtain at least two positive solutions for the problem

$$\begin{cases} -u'' + u = \lambda h(x)|u|^{\beta-2}u + q(x)f(u), & x \in (0, +\infty), \\ u(0) = u(+\infty) = 0, \end{cases} \quad (438)$$

where $f \in C(\mathbb{R}, \mathbb{R})$, β and λ are positive real parameters with $1 < \beta < 2$ and $\lambda > 0$.

Main results

Preliminary

Theorem 186. [5] (Weak Ekeland variational principle) Let (E, d) be a complete metric space and let $J : E \rightarrow \mathbb{R}$ be a functional that is lower semi-continuous, bounded from below. Then, for each $\varepsilon > 0$, there exists $u_\varepsilon \in E$ with

$$J(u_\varepsilon) \leq \inf_E J + \varepsilon,$$

and whenever $w \in E$ with $w \neq u_\varepsilon$, then

$$J(u_\varepsilon) < J(w) + \varepsilon d(u_\varepsilon, w).$$

Definition 70. Let E be a Banach space and $J : E \rightarrow \mathbb{R}$ a C^1 -functional, and $c \in \mathbb{R}$. The functional J is said to satisfy the (local) Palais-Smale condition at the level c , denoted by $(P.S)_c$, if any sequence (u_n) in E such that

$$J(u_n) \rightarrow c, \text{ and } J'(u_n) \rightarrow 0, \quad (439)$$

admits a convergent subsequence.

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Main existence results

Now we define the Euler-Lagrange functional associated to problem (438). Let $J_\lambda : H_0^1(0, +\infty) \rightarrow \mathbb{R}$ be defined by

$$J_\lambda(u) = \frac{1}{2}\|u\|^2 - \frac{\lambda}{\beta} \int_0^{+\infty} h(x)|u|^\beta(x)dx - \int_0^{+\infty} q(x)F(u)dx. \quad (440)$$

Theorem 187. Suppose that $(H_0) - (H_4)$ hold. Then, there exists $\xi > 0$ such that, for $0 < \lambda < \xi$, the problem (438) has at least two positive solutions.

Example 9. Consider the problem

$$\begin{cases} -u'' + u = \lambda h(x)|u|^{\beta-2}u + q(x)f(u), & x \in [0, +\infty), \\ u(0) = u(+\infty) = 0, \end{cases} \quad (441)$$

Here we have

$$f(u) = \begin{cases} \frac{3}{2}|u| + |u|^\beta, & \text{if } |u| \leq 1; \\ 4|u|^\beta - \frac{3}{2}, & \text{if } |u| \geq 1; \end{cases}$$

$q(x) = e^{-\frac{3}{2}x}$ and $h(x) = e^{-\frac{4}{3}x}$. Choosing $p(x) = e^{-\frac{1}{4}x}$, we see that

$$\frac{q}{p}(x) = e^{-\frac{5}{4}x}, \quad \frac{q}{p^2}(x) = e^{-x}, \quad \frac{h}{p^{\beta-1}}(x) = e^{\frac{1}{4}(\beta-\frac{19}{3})x}, \quad \frac{q}{p^\beta}(x) = e^{\frac{1}{4}(\beta-6)x},$$

$$\frac{h}{p^\beta}(x) = e^{\frac{1}{4}(\beta-\frac{4}{3})x}, \quad \text{and} \quad \frac{q}{p^{\beta+1}}(x) = e^{\frac{1}{4}(\beta-5)x}$$

are in $L^1[0, +\infty)$ for all $\beta \in (1, 2)$. Therefore by some calculation, we obtain

$$M_{2,q} = \frac{2\sqrt{2}}{\sqrt{15}}, \quad A := \lim_{u \rightarrow 0^+} \frac{f(u)}{|u|} = \frac{3}{2} \quad \text{and} \quad B := \lim_{u \rightarrow +\infty} \frac{f(u)}{|u|^\beta} = 4.$$

It is easy to see that conditions $(H_0) - (H_4)$ hold. So by Theorem 187, the problem (441) has at least two positive solutions for each $\lambda \in (0, \xi)$.

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Crossing limit cycles created by piecewise linear system without equilibria and hamiltonian saddle

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Abstract: In this work, we study the maximum number of crossing limit cycles for discontinuous piecewise differential systems separated by one straight line $x = -1$ and formed by a linear system without equilibrium points and hamiltonian saddle. We are going to prove that this class has at most one non-algebraic crossing limit cycle.

Keywords: limit cycles, piecewise linear system, first integrals.

2020 Mathematics Subject Classification: 34C05, 34C07, 34C23.

Introduction

An important problem of the qualitative theory of differential equations is to determine the limit cycles of differential equations in the plane. We usually only ask for the number of such limit cycles. In the last few years, there has been an increasing interest in the study of piecewise differential systems, see [1, 2] for instance. This interest has been mainly motivated by their wide range of application in modelling real phenomena (e.g., control theory, biology, chemistry, physics).

The goal of this study is to give a discontinuous piecewise linear differential systems separated by one straight line with two linear systems, such that one of them is a hamiltonian saddle and the second one has no equilibria. Moreover, the main purpose of this paper is to investigate the existence, number and the explicit expression of crossing period orbits for these systems.

We consider planar piecewise linear systems with two linearity regions separated by a straight line $\Sigma = \{(x, y) \in \mathbb{R}^2 : x = -1\}$, where we assume that the two linearity regions in the phase plane are the left and right half-planes

$$\Sigma_L = \{(x, y) \in \mathbb{R}^2 : x < -1\}, \quad \Sigma_R = \{(x, y) \in \mathbb{R}^2 : x > -1\}.$$

More precisely, in the region R_R we consider an arbitrary linear system

$$\dot{x} = c_0x + c_1y, \quad \dot{y} = \eta x + d_1y, \quad (442)$$

with $c_i, d_1, \eta \in \mathbb{R}$ for $i = 0, 1$, and in the region R_L we consider

$$\dot{x} = ax + by + c, \quad \dot{y} = \mu(ax + by) + d. \quad (443)$$

with $a, b, c, \mu, d \in \mathbb{R}$.

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Main results

In the following, we need a lemma on the explicit expression of the first integral of an arbitrary linear differential system without equilibrium points.

Lemma 23. *For system (443) the following statements hold.*

i) *If $a + b\mu = 0$, then system (443) is Hamiltonian and all its solutions are algebraic and given by parabolas. Moreover, this system has the first integral*

$$H_L(x, y) = b\mu^2 x^2 - 2b\mu xy - 2dx + by^2 + 2cy \quad (444)$$

ii) *If $(a + b\mu) \neq 0$ and $d - \mu c \neq 0$, the only algebraic invariant curve of is an invariant line. Moreover, this system has the first integral*

$$H_L(x, y) = ((a + b\mu)(ax + by) + ac + bd) e^{\frac{(a+b\mu)(\mu x - y)}{d - c\mu}} \quad (445)$$

It is known that the system (442) having a saddle is given in the newt lemma

Lemma 24. *the piecewise linear system in R_R is Hamiltonian saddle if $c_0 = -d_1$ and can be written as*

$$\dot{x} = -d_1 x + c_1 y, \quad \dot{y} = \eta x + d_1 y, \quad (446)$$

with $d_1^2 + c_1 \eta > 0$. Moreover, this system has the first integral

$$H_R(x, y) = -\frac{1}{2}c_1 y^2 + d_1 xy + \frac{1}{2}\eta x^2 \quad (447)$$

Now, the piecewise linear vector field with two zones to study is one formed by the following two linear systems

$$\begin{aligned} \dot{x} &= -d_1 x + c_1 y, & \dot{y} &= \eta x + d_1 y, & \text{if } (x, y) \in R_R, \\ \dot{x} &= ax + by + c, & \dot{y} &= \mu(ax + by) + d. & \text{if } (x, y) \in R_L, \end{aligned} \quad (448)$$

Our main results are the following.

Theorem 188. *A discontinuous planar piecewise linear differential systems (448) has at most one crossing limit cycle intersecting Σ . Moreover, inside this class of discontinuous piecewise linear differential systems, there are systems with exactly 1 such limit cycle.*

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Numerical Solution of Fredholm Integro-Differential Equations Using Legendre Collocation Method

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Abstract: The aim of the current paper is to present an effective simple method using the shifted Legendre polynomials to solve first-order nonlinear Fredholm integro-differential equations of the second kind on the interval $[0,1]$. The properties of shifted Legendre polynomials together with the shifted Legendre-Gauss nodes and weights are utilized to transform the main problem to a nonlinear system of algebraic equations that can be solved by usual iterative Newton's method. The method is then tested on some examples. Numerical results are included and comparisons with other methods are made to demonstrate the validity and applicability of the proposed method.

Keywords: Nonlinear Fredholm integro-differential equations, shifted Legendre polynomials, collocation method, Gaussian quadrature, Legendre-Gauss nodes and weights.

2020 Mathematics Subject Classification: 44A15, 45B05, 45J05, 65D30, 65R20.

Introduction

Integro-differential equations provide an important tool in the modeling of real-life phenomena in engineering and science fields, such as: heat and mass transfer, fluid dynamics, viscoelasticity, economics, biological models and in many other phenomena [2]. The study of integro-differential equations has recently attracted much attention of research. This class of equations is solved by several numerical methods such as: hybrid function approximations [1], Chebyshev collocation method [4], Radial basis functions [3], Taylor polynomial [5], Bernstein polynomials [7]. The Legendre polynomials have been used also successfully in [6] to solve integral and integro-differential equations. These methods have some advantages, they are easy to implement in a computer and the approximating functions are the sum of Legendre polynomials with coefficients easy to calculate.

In the present paper, we consider the first-order nonlinear Fredholm integro-differential equations of the type :

$$\begin{cases} y'(x) = \int_0^1 K(x,t)\varphi(t,y(t))dt + g(x), & 0 \leq x \leq 1, \\ y(0) = y_0. \end{cases} \quad (449)$$

where $K \in L^2[0,1]^2$ and $g \in L^2[0,1]$ are given and y is an unknown function to be determined.

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Legendre polynomials

The Legendre polynomials $P_n(x)$ are defined as

$$\begin{aligned} P_0(x) &= 1, \quad P_1(x) = x, \\ P_{i+1}(x) &= \left(\frac{2i+1}{i+1} \right) x P_i(x) - \left(\frac{i}{i+1} \right) P_{i-1}(x), \quad i = 1, 2, \dots, \quad x \in [-1, 1]. \end{aligned}$$

So the shifted Legendre polynomials $P_n^*(x)$ are defined as

$$\begin{aligned} P_0^*(x) &= 1, \quad P_1^*(x) = \frac{2x - (b+a)}{b-a}, \\ P_{i+1}^*(x) &= \left(\frac{2i+1}{i+1} \right) \left(\frac{2x - (b+a)}{b-a} \right) P_i^*(x) - \left(\frac{i}{i+1} \right) P_{i-1}^*(x), \quad i = 1, 2, \dots, \quad x \in [a, b]. \end{aligned}$$

Function approximation

A function $y(x) \in L^2[a, b]$, may be expressed in terms of shifted Legendre polynomials as $y(x) = \sum_{i=0}^{\infty} c_i P_i^*(x)$, where the coefficients $c_i = \langle f(x), P_i^*(x) \rangle$, in which $\langle \cdot, \cdot \rangle$ denotes the inner product in $L^2[a, b]$. Considering only the first $(n+1)$ -terms of shifted Legendre polynomials we have $y(x) \simeq \sum_{i=0}^n c_i P_i^*(x) = C^T P^*(x)$ where $C = [c_0, c_1, \dots, c_n]^T$ and $P^*(x) = [P_0^*(x), P_1^*(x), \dots, P_n^*(x)]^T$.

Main results

Consider the one-dimensional nonlinear Fredholm integro-differential equations with the given condition in Eq(449). At first, we have

$$\varphi(t, y(t)) = \varphi \left(t, \int_0^t \int_0^1 K(x, s) \varphi(s, y(s)) ds dx + \int_0^t g(x) dx + y_0 \right). \quad (450)$$

Then, we approximate $\varphi(t, y(t))$ by using shifted Legendre polynomials as follows

$$\varphi(t, y(t)) \simeq \sum_{i=0}^n c_i P_i^*(t) = C^T P^*(t). \quad (451)$$

Substituting (451) into Eq. (450), and taking collocation points $t_j = a + \frac{(b-a)j}{n}$, $j = 0, 1, \dots, n$ into the obtained equation, we have

$$C^T P^*(t_j) = \varphi \left(t_j, \int_0^{t_j} \int_0^1 K(x, s) C^T P^*(s) ds dx + \int_0^{t_j} g(x) dx + y_0 \right). \quad (452)$$

In order to use the Gaussian integration formula for Eq.(452), we convert the intervals $[0, 1]$ and $[0, t_j]$ into interval $[-1, 1]$ by means of the transformations

$$\eta_1 = 2s - 1, s \in [0, 1], \quad \eta_2 = \frac{2}{t_j}x - 1, x \in [0, t_j].$$

Let $H(x, s) = K(x, s) C^T P^*(s)$. Then, eq.(452) can be rewritten as

$$C^T P^*(t_j) = \varphi \left[t_j, \frac{t_j}{4} \int_{-1}^1 \int_{-1}^1 H \left(\frac{t_j}{2}(\eta_2 + 1), \frac{1}{2}(\eta_1 + 1) \right) d\eta_1 d\eta_2 + \frac{t_j}{2} \int_{-1}^1 g \left(\frac{t_j}{2}(\eta_2 + 1) \right) d\eta_2 + y_0 \right].$$

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By using the Legendre-Gauss quadrature formula, we have

$$C^T P^*(t_j) = \varphi \left[t_j, \frac{t_j}{4} \sum_{l=0}^n \sum_{k=0}^n \omega_k \omega_l H \left(\frac{t_j}{2} (\eta_l + 1), \frac{1}{2} (\eta_k + 1) \right) + \frac{t_j}{2} \sum_{l=0}^n g \left(\frac{t_j}{2} (\eta_l + 1) \right) + y_0 \right]. \quad (453)$$

for $j = 0, 1, \dots, n$. Eq.(453) generates a nonlinear system of $(n + 1)$ equations that can be solved by usual iterative Newton's method to obtain unknown vector. Therefore, $y'(x)$ can be approximated by

$$y'(x) = \frac{1}{2} \sum_{i=0}^n \omega_i K \left(x, (\eta_i + 1)/2 \right) C^T P^* \left((\eta_i + 1)/2 \right) + g(x).$$

Also, we approximate $y(t)$ as follows

$$y(t) = \frac{1}{2} \sum_{i=0}^n \omega_i \int_0^t K \left(x, (\eta_i + 1)/2 \right) C^T P^* \left((\eta_i + 1)/2 \right) dx + \int_0^t g(x) dx + y_0.$$

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Some Numerical Approximation methods to Fractional Integrals by means of derivatives

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Abstract: In this work, we will introduce numerical approximation of fractional integrals using several techniques, such as the fractional linear multistep method, and some special polynomials.

The fractional integral can often be viewed as an algebraic generalization of the traditional integration of integer order. Considering the terminology, the $f(x)$ (or $f'(x)$) function of variable x in $\mathbb{R}$ is taken as the function in the α -inner product with kernel $(x-t)^{\alpha-1}$. Here, it is often taken that $\alpha > 0$ and $\alpha \notin \mathbb{N}$.

We consider only the integral on the left side. Since the methods for deriving the right-sided integrals are nearly identical to those for the left-sided ones, the numerical procedures for them are not included here.

Keywords: fractional integral, fractional derivative, numerical approximation, Error estimation.

2020 Mathematics Subject Classification: Primary 26A33, 65D25, 65D30.

Introduction

Fractional integrals were initially defined by Riemann-Liouville about a century ago, which gave them a theoretical foundation. But still, they have not gained recognition, despite the fact that a large part of practically useful theorems and formulas in the field of integral transforms, special functions, integral equations, potential theory, and in other areas of pure and applied sciences. It is just the beginning, and today it is a challenging potential for future researchers to design such numerical methods that will attract them and others to work on the problems associated with fractional integrals and derivatives. During the last quarter of a century, the theory of fractional differential equations (DE's) and inclusions has undergone rapid development as regards both basic theory and applications and has come in a variety of fields to use these fundamental operators. It is necessary to be able to find an appropriate approximation of a solution, which will often be a function of a very special type; the development of a theory of numerical methods that will approximate such solutions seems to be an interesting and challenging topic for future research, the difficulty is concerning the availability of a simple and efficiently computable method of obtaining the fractional integrals of functions or of determining "good" approximations of them has become the motivation of the present work.

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Main results

The numerical approximations to fractional integrals and derivatives are the main topic of this work . Nearly all of the findings are included in this regard. For the benefit of applied scientists and the engineering community, existing results and a few observations on fractional calculus are summarized. We supported these results with some examples that confirm the importance of result obtained in this work.

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Dynamics of Catuto Fractional Order SIR Epidemic Model with Stability Analysis

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Abstract: This study presents a mathematical model known as the fractional SIR model, which represents the dynamics of transmission and spread of infectious diseases. Additionally, the study addresses the existence unique solutions and the stability of the proposed mathematical model, along with analyzing various stability outcomes related to specific conditions to prevent the spread of infectious diseases.

Keywords: SIR model, fractional derivative, system, existence, stability analysis.

2020 Mathematics Subject Classification: 34A08; 62P10; 92-10; 92B05; 93-10.

Introduction

The infectious diseases have been a persistent challenge for thousands of years, with scientists and researches continuously striving to understand and combat them. These diseases vary in their causes, transmission, and impacts, encompassing a wide range of conditions, from common illnesses to global pandemics. Studying these diseases and analyzing their effects is of utmost importance for understanding ways to address and prevent their spread.

This model requires the division of the total population N into three epidemiological classes namely Susceptible S , Infected I , and Recovered class R .

Parameters' definition of the SIR model could be presented as follows:

- Λ : indicates the rate of increase in the susceptible individuals.
- $\mu \leq \Lambda$: is the rate of natural death population.
- ν represents the death rate due to the disease.
- γ is the vaccine rate of the suspected population.
- β is the rate of contacts between subceptible S and infectious population I .
- δ is the recovery rate from infectious populations I .
- κ is the rate of transfer of recovered population R even those who were vaccinated to the susceptible population S .

In recent decades, studying fractional calculus has been regarded as essential as it provides an effective alternative to integer-order models, and can describe and process various structures' properties.

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For $0 \leq t \leq T < \infty$, and $0 < \alpha \leq 1$, we have:

$$\begin{cases} {}^C D_{0+}^{\alpha,p} S(t) = \Lambda N(t) + \kappa R(t) - \left(\frac{\beta}{N(t)} I(t) + \gamma + \mu \right) S(t) \\ {}^C D_{0+}^{\alpha,p} I(t) = \frac{\beta}{N(t)} S(t) I(t) - (\delta + v + \mu) I(t) \\ {}^C D_{0+}^{\alpha,p} R(t) = \gamma S(t) + \delta I(t) - (\kappa + \mu) R(t) \end{cases} \quad (454)$$

The alterations in infectious disease transmission within the SIR model (454) can be interpreted through reference to the following chart:

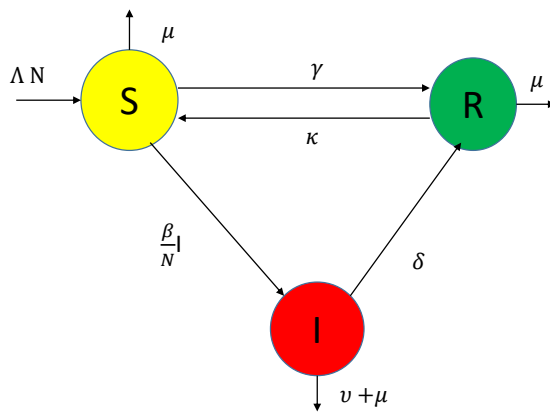


Figure 20: Flowchart of SIR.

Our primary objective is to efficiently find the solution to the system of nonlinear fractional differential equations (454). We aim to determine the solution's main properties, including stability, existence, boundedness, and computation of equilibrium points. Using the fixed point theory, we establish the necessary conditions for obtaining at least one solution or its uniqueness.

Main results

The SIR model(454) will be investigated within a biologically feasible region in $\mathbb{R}^3$, as defined in the subsequent lemma.

Lemma 25. *Let N_0 be the initial total population at $t = 0$, where $0 \leq t \leq T < \infty$, then the solution of the model under consideration is restricted to the feasible region given by*

$$(S, I, R) \in \mathbb{R}_+^3, \quad N(t) \leq N_0 \exp\left(\frac{(\Lambda - \mu) T^\alpha}{\Gamma(\alpha + 1)}\right).$$

This shows the boundedness of the considered population dynamic model.

In the following, we assume the existence of a constant $\mathcal{N} > 0$

$$\mathcal{N} \leq N_0 \exp\left(\frac{(\Lambda - \mu) T^\alpha}{\Gamma(\alpha + 1)}\right),$$

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To normalize the model (454), we put:

$$\mathcal{S}(t) = \frac{S(t)}{N}, \quad \mathcal{I}(t) = \frac{I(t)}{N}, \quad \mathcal{R}(t) = \frac{R(t)}{N}, \quad (455)$$

then we obtain

$$\begin{cases} {}^C\mathcal{D}_{0+}^{\alpha}\mathcal{S}(t) &= \Lambda + \kappa\mathcal{R}(t) - (\beta\mathcal{I}(t) + \gamma + \mu)\mathcal{S}(t), \\ {}^C\mathcal{D}_{0+}^{\alpha}\mathcal{I}(t) &= \beta\mathcal{I}(t)\mathcal{S}(t) - (\delta + v + \mu)\mathcal{I}(t), \\ {}^C\mathcal{D}_{0+}^{\alpha}\mathcal{R}(t) &= \gamma\mathcal{S}(t) + \delta\mathcal{I}(t) - (\kappa + \mu)\mathcal{R}(t), \end{cases} \quad (456)$$

along with the positive initial conditions

$$\mathcal{S}(0) = u_1, \mathcal{I}(0) = u_2, \mathcal{R}(0) = u_3. \quad (457)$$

Theorem 189. Let $\beta, \delta, \kappa, \gamma, \tau, v, \mu, \alpha, T \in \mathbb{R}_+$, be such that $\alpha \in (0, 1]$ and

$$T < \left(\frac{\Gamma(\alpha + 1)}{\mu + \max\{\beta + \gamma, \beta + \delta + v, \kappa\}} \right)^{\frac{1}{\alpha}}, \quad (458)$$

then the problem (456)–(457) has at least one solution on $[0, T]$.

Theorem 190. We give $\alpha \in (0, 1]$, $T > 0$, and

$$\eta = \max\{\beta + \gamma, \beta + \delta + v, \kappa\},$$

for some $\beta, \delta, \kappa, \gamma, v, \mu \in \mathbb{R}_+$. If

$$\frac{(\mu + \eta)T^{\alpha}}{\Gamma(\alpha + 1)} < 1, \quad (459)$$

thus, there is a unique solution to the problem (456)–(457) on $[0, T]$.

Theorem 191. Assuming that (459) holds, system (456) is Ulam-Hyers stable. Furthermore, it can also be asserted that (456) is a generalized Ulam-Hyers stable system.

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Zero-Hopf Bifurcation in a Modified Hyperchaotic Chen System

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Abstract:

We study the zero-Hopf bifurcation of a modified hyperchaotic Chen system

$$\begin{aligned}\dot{x} &= \alpha(a_1y - a_2x + w), \\ \dot{y} &= dx + cy - xz, \\ \dot{z} &= xy - bz, \\ \dot{w} &= yz + rw,\end{aligned}$$

where $\alpha, a_1, a_2, b, c, d$ and r are real arbitrary parameters. The dot denotes the derivative with respect to an independent variable t . Using the averaging theory, we prove analytically the existence of two periodic orbits for this system that born from a zero-Hopf bifurcation.

Keywords: Hyperchaotic Chen system, periodic orbit, averaging theory, zero-Hopf bifurcation.

2020 Mathematics Subject Classification: Primary xxxxxx, xxxxx, xxxxx.

Introduction

Chaos has been broadly investigated in mathematics and physics since the discovery of the first chaotic attractor in a third-order autonomous system by Lorenz in 1963. Through this investigation, hyperchaos also became a center of interest when in 1979 Otto Rossler proposed one of the first hyperchaotic attractors. A hyperchaotic system is typically defined as a chaotic system with at least two positive Lyapunov exponents, implying that its chaotic dynamics are extended in many different directions at the same time. This system is more complex than the traditional chaotic systems, and with such complexity, hyperchaos became a feature in enhancing security in chaotic communication systems. Also, it can be useful in encryption, fluid mixing, nonlinear circuits, and many other fields. Furthermore, The minimal dimension for a hyperchaotic system is four. Hyperchaotic systems such as the hyperchaotic Lorenz-Hakan system, the hyperchaotic attractor of Rossler, the hyperchaotic L²u system, and Chua's circuit are known systems that are extended from third-order chaotic systems, which is a typical way of constructing hyperchaotic systems.

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In [2], the authors introduced the hyperchaotic Chen system

$$\begin{aligned}\dot{x} &= a(y - x) + w, \\ \dot{y} &= dx + cy - xz, \\ \dot{z} &= xy - bz, \\ \dot{w} &= yz + rw,\end{aligned}$$

and its study has provided significant development in many areas such as secure communications and power system protection. In [1] S. Maza considered this system in a zero-Hopf bifurcation study, in which she did an analytic analysis on the existence of periodic orbits by applying the averaging theory of first order.

Here, we consider a modified hyperchaotic Chen system

$$\begin{aligned}\dot{x} &= \alpha(a_1 y - a_2 x + w), \\ \dot{y} &= dx + cy - xz, \\ \dot{z} &= xy - bz, \\ \dot{w} &= yz + rw,\end{aligned}$$

in which we will prove analytically the existence of periodic orbits bifurcating from the zero-Hopf equilibrium located at the origin of the system, by using the averaging method of first order.

Main results

The main result of this work is summed up in the following theorem.

Theorem. Let $(r, b) \rightarrow (\epsilon r, \epsilon b)$ where ϵ is a small parameter and $c = \alpha a_2$. Then the hyperchaotic Chen system (1) has a zero-Hopf bifurcation at the equilibrium point localized at the origin. Assume that $\alpha(a_2^2 \alpha + a_1 d) < 0$ and $\alpha a_2 r b > 0$. Then, when $\epsilon = 0$ the system exhibits two periodic orbits bifurcating from the zero-Hopf equilibria as parameters vary.

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The dynamics of three classes of planar Gradient systems

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Abstract: The aim of this work is to provide the global phase portraits in the Poincaré disc for three classes of homogeneous Gradient polynomial differential systems of degrees 1, 2 and 4, having a finite number of equilibrium points. More precisely, we notice that these polynomial differential systems have exactly ten phase portraits topologically equivalents.

Keywords: Homogeneous Gradient system, Phase portrait, Poincaré disc

2020 Mathematics Subject Classification: Primary 34C05.

Introduction

The classical Center-Focus Problem posed by H. Poincaré in 1880's is concerned on the existence of a center or focus at a singular point of a polynomial differential system of the form

$$\dot{x} = -y + P_n(x, y), \quad \dot{y} = x + Q_n(x, y), \quad (460)$$

with P_n and Q_n homogeneous polynomials of degree n . The center is a type of singular point where the trajectories of the system are periodic, while the focus is a type of singular point where the trajectories spiral around it. There are several approaches to solving the center-focus problem, including algebraic methods and geometric methods. The centers of the polynomial differential systems (460) have been studied for $n = 2, 3, 4$ and 5 . For $n = 2$ in 1993, D.Schlomiuk clarify the global geometrical phenomena corresponding to the notion of center for plane quadratic vector fields, see [1] and for $n = 3$ see [4]. In 1996 J. Chavarriga and J. Giné studied the integrability of a two-dimensional system with linear part of center type and non-linear part given by homogeneous polynomials of forth and fifth degree, see [2] and [3] respectively. While the phase portraits of systems with centers of degrees 2 and 3 have been classified in the Poincaré disc [1] and [5], this is not the case for the centers of degrees 4 and 5. In this paper we study the classification of the phase portrait of the homogeneous Gradient systems of degree 1, 2, 4 in the Poincaré disc. These systems are represented by the equations

$$\dot{x} = \frac{\partial G_n(x, y)}{\partial x}, \quad \dot{y} = \frac{\partial G_n(x, y)}{\partial y}, \quad (461)$$

where $G_n(x, y)$ is a homogeneous polynomial of degree n with $n \in \{1, 2, 4\}$.

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Main results

Theorem 192. *The phase portraits in the Poincaré disc of two compactified polynomial differential systems $p(\chi)$ and $p(\mathcal{Y})$ with finitely many separatrices are topologically equivalent if and only if their separatrix configurations $S(p(\chi))$ and $S(p(\mathcal{Y}))$ are topologically equivalent.*

Theorem 193. *The homogeneous Gradient differential system of degree 1, 2 and 4 with finitely many equilibria has ten phase portraits topologically equivalent.*

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Linear Quadratic Regulator (LQR) and Luenberger Observer for Dynamic Solar Tracking System

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Abstract: In this paper, we present the Linear Quadratic Regulator (LQR) approach to the design of a state feedback controller, exploiting information about the current state of the system to make control decisions.

The aim of this study is to estimate the angle of incident sunlight on the surface of solar panel and optimize it with respect to the sun, thus increasing the efficiency of solar energy collection using LQR in conjunction with the Luenberger observer. LQR, an optimal control method for linear systems with quadratic criteria, is used to solve control problems. The Luenberger observer, on the other hand, is used to estimate parameters that are difficult to measure directly. In our case, the Luenberger observer is used to estimate the position and angular velocity of the solar tracker.

Keywords: Linear Quadratic Regulator, Luenberger Observer, Optimal control, Solar tracker.

2020 Mathematics Subject Classification: 37N35 Dynamical systems in control.

Introduction

In the field of dynamical systems control, the combined use of a state observer and a regulator plays a crucial role in ensuring optimal performance.

The Luenberger observer is an indispensable tool for estimating the unmeasured internal states of a system from available output measurements. Concurrently, LQR, uses these state estimates along with the reference setpoint to generate appropriate control commands, the requirements to design LQR are the system must be controllability and observability [3]. This closed-loop approach stabilizes the system, tracks desired references, and mitigates the effects of disturbances. In this paper, we will discuss the significance of the Luenberger observer and LQR in the control of a solar tracking system, that many studies have discussed about it [1, 2].

Solar panel is a device that can convert sun energy into electricity. It can be modified by being given a DC motor, in order to detect the direction of the sun's light. It is solar tracker.

This research [1] is simulation of two axes solar tracker (horizontal and vertical rotary axis) included all component (LDR Sensors, DC Motor, gear transmission and solar panel itself).

The aim of this work is to estimate the angle of incident sunlight on the surface of the photovoltaic (PV) cell of a solar tracker and optimize it with respect to the sun, thus increasing the efficiency of solar energy collection using LQR and Luenberger observer.

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The system input is the LDR sensor, which is used to detect the position of the sun (i.e. its angle: azimuth and elevation), and this information will be used to orientate a solar panel which powers a DC motor. The mathematical model of the solar tracker is determined using the fundamental laws of physics [3], applying Kirchhoff's law to the DC motor, the transfer function is obtained as follows:

$$G(s) = \frac{K_t K_s K N}{L_a J_g s^3 + (b L_a + R_a J_g) s + (R_a b + K_t K_b) s}$$

Converting this function into a linear dynamic system, in state-space form [3].

$$\begin{cases} \dot{x} &= Ax + Bu \\ y &= Cx \end{cases}$$

Once the system matrices are defined, we check observability and controllability [3] to apply LQR method (to provide a compromise solution between energy consumption efficiency and response speed to reach a stable value) and Luenberger observer (to estimate position and angular velocity on the solar tracker, we get a gain matrix L based on the pole placement method).

The block diagram of the solar tracker system is shown in the figure below:

Figure 21: Diagram Block of solar tracking system [1]

Main results

In this section, we highlight important aspects of the results.

1. We apply the following LQR algorithm to our system:
Our objective is to find the optimal cost function, given by: $J = \int_0^\infty [\mathbf{x}^T \mathbf{Q} \mathbf{x} + \mathbf{u}^T \mathbf{R} \mathbf{u}] dt$,
 - (a) Verify system controllability and observability,
 - (b) Selected matrices $\mathbf{Q}$ and $\mathbf{R}$ ($\mathbf{Q} = \mathbf{Q}^T \geq 0, \mathbf{R} = \mathbf{R}^T > 0$) with trial and error method,
 - (c) find the $\mathbf{P}$ matrix by solving the Riccati equation: $\mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A} - \mathbf{P} \mathbf{B} \mathbf{R}^{-1} \mathbf{B}^T \mathbf{P} + \mathbf{Q} = 0$,
 - (d) Calculate the control matrix $\mathbf{K}$: $\mathbf{K} = \mathbf{R}^{-1} \mathbf{B}^T \mathbf{P}$,
 - (e) Optimal control is: $\mathbf{u} = -\mathbf{K} \mathbf{x}$
2. State feedback control include prefilter which make LQR turn into LQR Servo, which means it can control the output with reference input [4].
The new system will be:

$$\begin{cases} \dot{x}(t) &= (\mathbf{A} - \mathbf{B} \mathbf{K}) x(t) + \mathbf{B} \mathbf{M} r(t) \\ y(t) &= \mathbf{C} x(t) \end{cases}$$

$\Rightarrow$ The new control is: $\mathbf{u}(t) = -\mathbf{K} \mathbf{x}(t) + \mathbf{M} r(t)$

where $\mathbf{M} = (\mathbf{C}(-\mathbf{A} + \mathbf{B} \mathbf{K})^{-1} \mathbf{B})^{-1}$ is a matrix gain used to set the static gain of the closed-loop system ; $r(t)$: reference signal.

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3. To obtain the Luenberger observer matrix L , we use the pole placement method:

$$|\lambda I - A + LC| = (\lambda - \lambda_1)(\lambda - \lambda_2) \dots (\lambda - \lambda_n)$$

Where λ_i desired pole observer.

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Boundary Value Problems of Hadamard Fractional Differential Equations of variable ordre

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Abstract: In this work, the existence, uniqueness and stability of solutions to a boundary value problem of nonlinear FDEs of variable order are established. To do this, we first investigate some aspects of variable order operators of Hadamard type. Then, with the help of the generalized intervals and piecewise constant functions, we convert the variable order Hadamard FBVP to an equivalent standard Hadamard BVP of the fractional constant order. Further, two fixed point theorems due to Schauder and Banach are used and, finally, the stability of the Ulam-Hyers-Rassias stability of the given variable order Hadamard FBVP is examined. These results are supported with the aid of a comprehensive example.

Keywords: boundary value problem; Hadamard derivatives of variable order; piecewise constant functions; fixed point theorems.

Introduction

The primitive idea of fractional calculus is to constitute the rational numbers in the order of derivation operators with natural numbers. Although this idea is preliminary, but it involves remarkable effects and outcomes which describe some physical and natural phenomena accurately. Benchohra *et al.* [1] investigate the existence for nonlinear implicit fractional differential equations for fractional of constant order

$$\begin{cases} {}^H D_{1+}^w x(t) = \psi(t, x(t), {}^H D_{1+}^w), & t \in I := [1, T], \quad T > 1, \quad 0 < w \leq 1, \\ x(1) = x_1, \end{cases}$$

where ${}^H D_{1+}^{w(t)}$ is the Hadamard fractional derivative of constant order w and $\psi : I \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ is a given function with some properties defined for it. Motivated by the above mentioned articles and by the given Benchohra's FBVP [1], in this work, we deal with the some qualitative aspects of solutions to the following FBVP of Hadamard variable ordre type with terminal conditions as

$$\begin{cases} {}^H D_{1+}^{w(t)} x(t) = \psi(t, x(t)), & t \in I := [1, T], \\ x(1) = x(T) = 0, \end{cases} \quad (462)$$

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where $1 < T < +\infty$, $1 < w(t) \leq 2$, $\psi : I \times \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function and ${}^H D_{1^+}^{w(t)}$ is the Hadamard fractional derivative of variable-order $w(t)$.

The purpose of this study is to propose new criteria on the uniqueness and existence for solutions of the Hadamard variable order FBVP(462). Additionally, we investigate the stability criterion of the obtained solution of the Hadamard variable order FBVP (462) in the sense of Ulam-Hyers-Rassias. Ultimately, the remaining part of our research is arranged as follows. In Section 2, some preliminaries and properties of the variable order operators are introduced. In Section 3, new existence conditions are obtained based on the standard functional analysis techniques. The Ulam-Hyers-Rassias stability behavior is investigated in the sequel of this section. An example is given in Section 4 to illustrate the application of our main results. In Section 5, we indicate conclusions.

Main results

Definition 71. [1] Let $k \in \mathbb{N}$ and $w : [a, b] \rightarrow (k-1, k)$. The Hadamard derivative of variable order $w(t)$ for ψ is defined by

$$({}^H D_{a^+}^{w(t)} \psi)(t) = \frac{1}{\Gamma(k-w(t))} \left(t \frac{d}{dt} \right)^k \int_a^t \left(\log \frac{t}{s} \right)^{k-w(t)-1} \frac{\psi(s)}{s} ds, \quad t > a. \quad (463)$$

if the right-hand side integral exists.

Let us introduce the following assumptions:

(H1) For $n \in \mathbb{N}$ define,

$$P = \{I_1 := [0, T_1], I_2 := (T_1, T_2], I_3 := (T_2, T_3], \dots, I_n := (T_{n-1}, T]\}$$

as a partition of the interval I , and assume that $w(t) : I \rightarrow (1, 2]$ be a piecewise constant with respect to P ; in other words

$$w(t) = \sum_{i=1}^n w_i l_i(t) = \begin{cases} w_1, & \text{if } t \in I_1, \\ w_2, & \text{if } t \in I_2, \\ \cdot & \\ \cdot & \\ \cdot & \\ w_n, & \text{if } t \in I_n, \end{cases}$$

in which $1 < w_i \leq 2$ belong to $\mathbb{R}$, and l_i illustrates the indicator of $I_i := (T_{i-1}, T_i]$, $i \in \mathbb{N}_1^n$, (by assuming $T_0 = 1, T_n = T$) so that

$$l_i(t) = \begin{cases} 1, & \text{if } t \in I_i, \\ 0, & \text{if } t \notin I_i. \end{cases}$$

(H2) For $0 \leq g \leq 1$, let $(\log t)^g \psi \in C(I \times \mathbb{R}, \mathbb{R})$ and $\exists l > 0$ so that $|(\log t)^g (\psi(t, x_1) - \psi(t, x_2))| \leq l |x_1 - x_2|$, for any $x_1, x_2 \in \mathbb{R}$ and $t \in I$. In addition, by $E_i = C(I_i, \mathbb{R})$, we denote the class of functions which form a Banach space via

$$\|x\|_{E_i} = \sup_{t \in I_i} |x(t)|, \text{ where } i \in \{1, 2, \dots, n\}$$

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By using the definition (71) and according to (H1), (H2) Going through several basic and necessary steps From now on, we follow our study on the equivalent standard Hadamard Fractional FBVP which takes the form

$$\begin{cases} ({}^H D_{T_{i-1}^+}^{w_i} x)(t) = \psi(t, x(t)), & t \in I_i, \\ x(T_{i-1}) = 0, & x(T_i) = 0, \end{cases} \quad (464)$$

Proposition 28. *The function $x \in E_i$ is a solution of the equivalent standard Hadamard Fractional FBVP (464) if x satisfies*

$$x(t) = -(\log \frac{T_i}{T_{i-1}})^{1-w_i} (\log \frac{t}{T_{i-1}})^{w_i-1} {}^H I_{T_{i-1}^+}^{w_i} \psi(T_i, x(T_i)) + {}^H I_{T_{i-1}^+}^{w_i} \psi(t, x(t)). \quad (465)$$

Theorem 194. *Consider (H1) and (H2).if*

$$\frac{2l \left[(\log T_i)^{1-g} - (\log T_{i-1})^{1-g} \right]}{(1-g)\Gamma(w_i)} \left(\log \frac{T_i}{T_{i-1}} \right)^{w_i-1} < 1, \quad (466)$$

then the variable ordre Hadamard Fractional FBVP (464) involves a solution in E_i uniquely.

Theorem 195. *Consider the hypotheses (H1), (H2) and the inequality (466). Assume:*

(H3) $\exists \phi \in C(I_i, \mathbb{R}^+)$ as an increasing mapping and $\exists \lambda_\phi > 0$ so that $\forall t \in I_i$,

$${}^H I_{T_{i-1}^+}^{w_i} \phi(t) \leq \lambda_{\phi(t)} \phi(t).$$

Then, the variable ordre Hadamard Fractional FBVP (462) is Ulam-Hyers-Rassias stable with respect to ϕ .

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Analysis for unilateral contact problem with friction in electro-visco-plasticity

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Abstract: A dynamic contact problem is considered in this work. The material behavior is described by electro-viscoplastic law with piezoelectric effects. The body is in contact with damage and an obstacle. The contact is frictional and bilateral with a moving rigid foundation which results in the wear of the contacting surface. The damage of the material caused by elastic deformations. The evolution of the damage is described by an inclusion of parabolic type[1]. The problem is formulated as a coupled system of an elliptic variational inequality for the displacement, variational equation for the electric potential and a parabolic variational inequality for the damage. We establish a variational formulation for the model and we prove the existence of a unique weak solution to the problem[2],[3].

Keywords: electro-viscoplastic; dynamic; weak solution

2020 Mathematics Subject Classification: Primary74M10; 74M15; 74F15; 49J40.

Introduction

The piezoelectric effect is the apparition of electric charges on surfaces of particular crystals after deformation. Its reverse effect consists of the generation of stress and strain in crystals under the action of the electric field on the boundary. A deformable material which presents such a behavior is called a piezoelectric material. Piezoelectric materials are used extensively as switches and actually in many engineering systems. Different models have been developed to describe the interaction between the electric and mechanical fields. General models for elastic materials with piezoelectric effects can be found in [1-2] and more recently in [3], viscoelastic piezoelectric materials in [3]. In this work, we consider a general model for the dynamic process of frictional contact between a deformable body and a rigid obstacle. The material obeys an electro-viscoelastic constitutive law with piezoelectric effects, which is set as a system coupling a variational second order evolution inequality. We establish the existence of a unique weak solution of the model. The idea is to reduce the second order evolution inequality of the system to first order evolution inequality. After this, we use classical results on first order evolution inequalities and equations and the fixed point arguments.

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Main results

We consider a mathematical model of a contact problem in electro-viscoplastic(see [1])

$$\sigma(t) = \mathcal{A}(\varepsilon(\dot{u}(t))) + \mathcal{B}(\varepsilon(u(t)), \beta(t)) + \int_0^t \mathcal{G}(\sigma(s) - \mathcal{A}(\varepsilon(\dot{u}(s))), \varepsilon(u(s))) ds - \xi^* E(\varphi),$$

$$D = BE(\varphi) + \xi \varepsilon(u), \text{ in } \Omega \times [0, T],$$

Where $u : \Omega \times [0, T] \rightarrow \mathbb{R}^d$ is a displacement field, $\sigma : \Omega \times [0, T] \rightarrow \mathbb{S}^d$ is a stress field $\varphi : \Omega \times [0, T] \rightarrow \mathbb{R}$ is the an electric potentiel field, $D : \Omega \times [0, T] \rightarrow \mathbb{R}^d$ is the an electric displacement field
The body is in contact with an obstacle. The contact is frictional and bilateral with a moving rigid foundation.

$$\begin{cases} \sigma_\nu = -\alpha |\dot{u}_\nu|, & |\sigma_\tau| = -\mu \sigma_\nu, \\ \sigma_\tau = -\lambda (\dot{u}_\tau - v^*), & \lambda \geq 0, \text{ on } \Gamma_3 \times [0, T], \end{cases}$$

We establish a variational formulation for the model and we prove the existence of a unique weak solution to the problem.

$$\begin{aligned} & (\ddot{u}(t), w - \dot{u}(t))_{V' \times V} + (\sigma(t), \varepsilon(w - \dot{u}(t)))_{\mathcal{H}} + \\ & j(\dot{u}(t), w) - j(\dot{u}, \dot{u}(t)) + \phi(\dot{u}, w) - \\ & \phi(\dot{u}, \dot{u}(t)) \geq (f(t), w - \dot{u}(t)), \forall u, w \in V, \\ & (D(t), \nabla \varphi)_{\mathbb{L}^2(\Omega)^d} + (q(t), \varphi)_W = 0, \text{ for all } \varphi \in W \end{aligned}$$

The proof is based on a classical existence and uniqueness result on parabolic inequalities, differential equations and fixed point arguments.

our main results on existence and uniqueness which state the unique weak solvability.

Theorem 196. *The Problem PV has a unique solution (u, σ, φ, D) which satisfies*

$$\begin{aligned} u & \in C^1(0, T, H) \cap W^{1,2}(0, T, V) \cap W^{2,2}(0, T, V') \\ \sigma & \in \mathbb{L}^2(0, T, \mathcal{H}_1), \text{Div } \sigma \in \mathbb{L}^2(0, T, V') \\ \varphi & \in W^{1,2}(0, T, W) \\ D & \in W^{1,2}(0, T, \mathcal{W}_1) \end{aligned}$$

Conclusion

We establish a variational formulation for the model and we prove the existence of a unique weak solution to the problem.

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Solution of a coupled system of fractional differential equations with nonlocal conditions

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Abstract: In this work, we study and investigate an interesting we application of fixed point theorem appropriate, to obtain the existence and uniqueness solution of a coupled system involving right Caputo and left Riemann-Liouville of fractional differential supplemented with nonlocal three-point coupled boundary conditions. First, we obtain an equivalent to a system of integral equations. Subsequently, then we prove existence. And We have in addition presented in detail uniqueness of solution, using some of reliable theorems and lemmas. Finally, Further, also an one example is provided to show the utilization of primary outcomes and the effectiveness of the main theory.

Keywords: Caputo Fractional derivative; Rimann-Liouville Fractional derivative; Systems; Existence of solution; Fixed point theorems

Introduction

Let us introduce the space $A = \{u(s) | u(s) \in C([0, T], \mathbb{R})\}$ endowed with the norm

$\|u\| = \sup |u(s)|, s \in [0, T]$ and note that $(A, \|\cdot\|)$ is a Banach space.

Then the product space $(A \times A, \|(u, v)\|)$ is also a Banach space equipped with the norm $\|(u, v)\| = \|u\| + \|v\|$.

In this paper, they investigated a new coupled system of nonlinear fractional differential equations involving both right Caputo and left Riemann-Liouville fractional derivatives, equipped with nonlocal coupled boundary conditions given by

$$\left\{ \begin{array}{l} {}^c D_{1-}^r D_{0+}^p u(s) = h_1(s, u(s), v(s)), s \in I := [0, T], \infty > T \geq 1, \\ {}^c D_{1-}^n D_{0+}^m v(s) = h_2(s, u(s), v(s)), s \in I := [0, T], \infty > T \geq 1, \\ u(0) = u'(0) = 0, u(1) = \mu v(\tau), 0 < \tau < 1, \\ v(0) = v'(0) = 0, v(1) = \sigma u(\lambda), 0 < \lambda < 1, \end{array} \right. \quad (467)$$

where ${}^c D_{1-}^r, {}^c D_{1-}^n$ denote the right Caputo fractional derivatives of order $r, n \in (1, 2]$ and D_{0+}^p, D_{0+}^m denote the left Riemann-Liouville fractional derivative of order $p, m \in (0, 1]$ respectively,

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$h_1, h_2 : I \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ are given functions and $\mu, \sigma \in \mathbb{R}$ are appropriate constants. The existence and uniqueness of solutions for the given problem will be derived by applying the well-known methods of functional analysis such as the Banach fixed point theorem. For computational convenience, They set

$$\Omega_1 = \frac{1}{\Gamma(r+1)\Gamma(p+1)} \left[1 + \frac{1}{|\wedge| \Gamma(p+2)\Gamma(m+2)} (1 + |\mu| |\sigma| \tau^{m+1} \lambda^p) \right] \quad (468)$$

$$\Omega_2 = \frac{|\mu| \tau^m}{|\wedge| \Gamma(n+1)\Gamma(m+1)\Gamma(p+2)\Gamma(m+2)} (1 + \tau), \quad (469)$$

$$\Omega_3 = \frac{|\sigma| \lambda^p}{|\wedge| \Gamma(r+1)(p+1)\Gamma(p+2)\Gamma(m+2)} (1 + \lambda), \quad (470)$$

$$\Omega_4 = \frac{1}{\Gamma(n+1)\Gamma(m+1)} \left[1 + \frac{1}{|\wedge| \Gamma(p+2)\Gamma(m+2)} (1 + |\mu| |\sigma| \tau^m \lambda^{p+1}) \right]. \quad (471)$$

Main results

Theorem 197. $(H_1) h_1, h_2 : [0, T] \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ are continuous functions and there exist positive constants e_1 and e_2 such that, for all $s \in [0, T]$ and $u_i, v_i \in \mathbb{R}, i = 1, 2$,

$$|h_1(s, u_1, u_2) - h_2(s, v_1, v_2)| \leq e_1 (|u_1 - v_1| + |u_2 - v_2|),$$

$$|h_2(s, u_1, u_2) - h_2(s, v_1, v_2)| \leq e_2 (|u_1 - v_1| + |u_2 - v_2|).$$

If

$$(\Omega_1 + \Omega_3)e_1 + (\Omega_2 + \Omega_4)e_2 < 1,$$

where $\Omega_i, i = 1, 2, 3, 4$ are given by (468)-(471), then system (467) has a unique solution $[0, T]$.

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A generalized viral infection model with non-local diffusion

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Abstract: In this work, We study global Dynamics of a viral infection model with nonlocal diffusion . We first prove that the solution of the model exists globally and the model system possesses a global attractor. We then identify the basic reproduction number R_0 for the model and prove its threshold role: if $R_0 < 1$, the disease free equilibrium is globally asymptotically stable (by the Lyapunov direct method); if $R_0 > 1$, the solution of the model is uniformly persistent and there exists a positive steady state.

Introduction

differential equations as epidemic models have attracted much attention of many researchers. The heterogeneity (position, age, sex, etc.) of each individual is known to be an important factor in spread of infectious diseases, and hence, motivates more realistic model for disease dynamics. In particular, the spatial heterogeneity would give insights into disease spread and control, and there have been quite a few publications along this line (see, for instance, [1,2] and the references therein). Recently, Wang et al . [3] proposed the following hybrid system of two ODEs and one PDE.

The previously mentioned works next to model [3,system (1.2)] used the Laplacian operator to model the virus dispersal, which represents that the virus diffuses in a small range. However, it is well-known that in different infectious diseases, the virus moves freely and in relatively high concentrations within tissues. Also, the virus uses the blood to move through the host. Therefore, the Laplacian operator does not consider this possibility of long-range mobility. It makes more sense to consider the nonlocal dispersal to describe this long-range mobility. Usually, the nonlocal dispersal is modeled as

$$\int_{\Omega} J(x-\epsilon)(\varphi(\epsilon)-\varphi(x))dy$$

This basic model has been modified to study different viral infections, in this work we propose the following model :

$$\left\{ \begin{array}{l} \frac{\partial U_1(t,x)}{\partial t} = \lambda(x) - \beta_2(x)U_1(t,x)U_3(t,x) - \beta_1(x)U_1(t,x)U_2(t,x) - b_1(x)U_1(t,x) \\ \frac{\partial U_2(t,x)}{\partial t} = \beta_2(x)U_1(t,x)U_3(t,x) + \beta_1(x)U_1(t,x)U_2(t,x) - b_2(x)U_2(t,x) \\ \frac{\partial U_3(t,x)}{\partial t} = \delta \int_{\Omega} J(x-\epsilon)(U_3(\epsilon) - U_3(x))dy + c(x)U_2(t,x) - b_3(x)U_3(t,x) \\ \frac{\partial U_3(t,x)}{\partial t} = 0, x \in \partial\Omega, t > 0 \end{array} \right. \quad x \in \partial\Omega, t > 0 \quad (1.2)$$

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Here, $x \in \overline{\Omega}$ represents the position, and t denotes the time. $U_1(t, x)$, $U_2(t, x)$, and $U_3(t, x)$ are, respectively, the densities of the uninfected cells, infected cells, and free virus.

$\lambda(x)$ is the newly reproduced susceptible cells. $b_1(x)$, $b_2(x)$, and $b_3(x)$ are the mortality rates for uninfected cells, infected cells, and free viruses, respectively. $\beta_1(x)$, and $\beta_2(x)$ are the coefficients of cell-to-cell transmission and virus-to-cell transmission, respectively. δ is the diffusion coefficient. $c(x)$ is the rate of virus production due to the lysis of the infected cells. $\Omega \subset \mathbb{R}^n, n \geq 1$ is a bounded set with smooth boundary $\partial\Omega$

with initial data

$$U_1(0, x) = U_{10}(x), U_2(0, x) = U_{20}(x), U_3(0, x) = U_{30}(x), x \in \overline{\Omega} \quad (1.3)$$

supposed to be space-dependent and positive and Holder continuous on $\overline{\Omega}$. Assume that

$\diamond J(x)$ is Lipschitz on $\mathbb{R}^N$ with support in $\overline{\Omega}$.

$\diamond \int_{\mathbb{R}^N} J(\epsilon) dy = 1, J(\epsilon) > 0$, on $\overline{\Omega}$, $J(-\epsilon) = J(\epsilon)$ on $\mathbb{R}^N$.

Keywords: basic reproduction number, nonlocal diffusion, uniform persistence, cell-to-cell transmissions.

2020 Mathematics Subject Classification: 37, 37, 37, 37.

Main results

Theorem 198. Suppose that $U_0 = (U_{10}, U_{20}, U_{30}) \in \mathbb{Y}$. Hence, there exists $T_0 > 0$ such that (1.2) has unique positive solution $U(\cdot; U_0) \in C([0, T_0], \mathbb{X}) \cap C^1((0, T_0), \mathbb{X})$, such that either $T_0 = \infty$ or $\lim_{t \rightarrow T_0^-} \sup \|U(t, U_0)\| = \infty$. Moreover, $U(t, U_0) \in \mathbb{X}_+$ for $t \in (0, T_0)$ if $U_0 \in \mathbb{X}_+$.

Theorem 199. The solution of (1.2-1.3) is globally defined.

Theorem 200. The semiflow $\Phi(t)$ has a global compact attractor C that attracts all bounded sets of $\mathbb{X}^+$.

$\diamond$ To determine the expression of the basic reproduction number R_0 . Hence, we have the following.

Lemma 26. $R_0 = r(-FB^{-1}) = r(A_1 + A_2)$ with $A_1 = \frac{\lambda\beta_1}{b_1b_2}$ and $A_2 = \frac{\lambda c}{b_1b_2} L_2^{-1}$

$\diamond$ for the **cell-to-cell** transmission only is : $R_0^1 = r(A_1) = \max_{x \in \Omega} \frac{\lambda(x)\beta_1(x)}{b_1(x)b_2(x)}$.

$\diamond$ Besides, the basic reproduction number becomes in the case of the **virus-to-cell** only (used variational formula)

$$R_0^2 = r(A_2) = \sup_{\phi \in L^2(\Omega), \phi \neq 0} \frac{\int_{\Omega} \frac{\beta_2(x)\lambda(x)c(x)}{b_1(x)b_2(x)} \phi^2(x) dx}{\delta \int_{\Omega} \int_{\Omega} \frac{1}{2} J(x-\epsilon) (\phi(\epsilon) - \phi(x))^2 d\epsilon dx + \int_{\Omega} b_3(x) \phi^2(x) dx}$$

Theorem 201. $\diamond$ If $R_0 < 1$, then $E_0(\frac{\lambda(x)}{b_1(x)}, 0, 0)$ is globally asymptotically stable.

$\diamond$ if $R_0 > 1$, then (1.2-1.3) is strongly uniformly persistent. Moreover, system (1.2-1.3) has at least one positive steady state which is also uniformly persistent in Σ_0 .

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We propose a generalization of a model with age of infection in a heterogeneous environment. Firstly, we give the well-posedness of the model and prove that the solutions are bounded and positive. The difficult mathematical issue in this research is that the model is partially degenerate, and the solution map is not compact. In addition, we constru

Introduction

From **Lemma 2** we deduce that the semiflow $\Phi(t)$ is point dissipative. **Lemma 4** implies that $\Phi(t)$ is κ -contraction. Therefore, **Lemma 2.3.5** in [4] shows that κ -contraction semiflow is asymptotically smooth. **Theorem 2.6** in Magal and Zhao [5] insures that $\Phi(t)$ has global compact attractor denoted by C which attracts all bounded sets of X^+ .

On a nonlinear differential problem under certain physical conditions

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Abstract: We are interested here in a nonlinear differential inclusion. In a previous paper, we proved an existence result for the considered inclusion when the second member is a set valued mapping having nonempty closed values, measurable with respect to the first variable and satisfying some conditions, in the framework of a particular space. Then, we gave an approximation for the solutions set. The study was inspired by another interesting article ([3]). As a continuity of our previous paper, here, we prove the existence of solutions for the same inclusion but in the context of an unbounded time interval and under other suitable conditions. Other results also arise from this.

Keywords: locally integrable functions, nonlinear differential equations, set-valued functions, integration of set-valued functions, measurable selections, Hausdorff distance

2020 Mathematics Subject Classification: Primary 34A34, 26E25, 28B20.

Introduction

Consider the nonlinear differential inclusion

$$(\mathcal{P}) \begin{cases} \dot{X}(t) \in \Phi(t, X(t)), & t \in I \\ X(0) = A_0, \end{cases}$$

where I is a real interval, $\Phi : I \times \mathcal{E} \rightrightarrows \mathcal{E}$ is a set-valued mapping, $A_0 \in \mathcal{E}$, $\dot{X}(t)$ is the derivative of X at t , and $\mathcal{E}$ is the nonlinear complete separable metric space consisted of the family of nonempty compact convex subsets of the space $\mathbb{R}^d$, endowed with the Hausdorff distance $\mathcal{H}$, and suitable differentiability, integrability and convexity.

In [2] we proved that when the time interval is compact, i.e. $I = [0, T]$, under some conditions, the problem $(\mathcal{P})$ admits at least a solution, and that the solutions set of $(\mathcal{P})$, $S_{\mathcal{P}}$, is dense in the solutions set of a related (relaxed) problem $(co\mathcal{P})$, for the distance of uniform convergence on $[0, T]$.

We were mainly inspired by the paper [3], where Chuong demonstrated similar results in the framework of a separable Banach space.

In this paper, we demonstrate that some results of paper [2] are true when the time interval I is unbounded, which can be beneficial for certain situations in Physics. This, always drawing inspiration from the ideas of Chuong [3], adapting them to our context and with changes to the conditions.

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$\mathcal{H}$ is the Hausdorff distance associated with the norm $\|\cdot\|_{\mathbb{R}^d}$ defined for every two nonempty closed subsets A, B of $\mathbb{R}^d$, by

$$\mathcal{H}(A, B) = \max(e(A, B), e(B, A)),$$

where

$$e(A, B) = \sup_{x \in A} d(x, B) = \sup_{x \in A} (\inf_{y \in B} \|x - y\|_{\mathbb{R}^d}).$$

however $\mathfrak{H}$ is the Hausdorff distance associated with $\mathcal{H}(\cdot, \cdot)$, defined for every two nonempty closed sub-families $\mathcal{A}, \mathcal{B} \subset \mathcal{E}$ by

$$\mathfrak{H}(\mathcal{A}, \mathcal{B}) = \max(e(\mathcal{A}, \mathcal{B}), e(\mathcal{B}, \mathcal{A}))$$

where

$$e(\mathcal{A}, \mathcal{B}) = \sup_{A \in \mathcal{A}} d(A, \mathcal{B}) = \sup_{A \in \mathcal{A}} (\inf_{B \in \mathcal{B}} \mathcal{H}(A, B)).$$

Let $C(I, \mathcal{E})$ be the complete metric space of all continuous mappings $F : I \rightarrow \mathcal{E}$ equipped with the distance of uniform convergence on I , defined by

$$d_u(F_1, F_2) = \max_{t \in I} \mathcal{H}(F_1(t), F_2(t)).$$

A mapping $F : I \rightarrow \mathcal{E}$ is said to be measurable if for each $C \in \mathcal{E}$ the set $\{t \in I : F(t) \cap C \neq \emptyset\}$ is Lebesgue measurable. A measurable mapping $F : I \rightarrow \mathcal{E}$ is said to be H-integrable if the single-valued function $\mathcal{H}(F(\cdot), \{0_{\mathbb{R}^d}\})$ is Lebesgue integrable. A measurable mapping $F : [0, +\infty[\rightarrow \mathcal{E}$ is said to be **locally** H-integrable if it is H-integrable on each **compact sub-interval** of $[0, +\infty[$. By the magnitude of $B \in \mathcal{E}$ we mean $\|B\| = \mathcal{H}(B, \{0_{\mathbb{R}^d}\})$. A mapping $F : I \rightarrow \mathcal{E}$ is said to be **bounded** if the single-valued function $\|F(\cdot)\|$ is bounded. A mapping $F : [0, +\infty[\rightarrow \mathcal{E}$ is said to be **locally bounded** if it is bounded on each **compact sub-interval** of $[0, +\infty[$. A set-valued mapping $\Gamma : I \rightrightarrows \mathcal{E}$ is said to be measurable if for each open set $\mathcal{V}$ in $(\mathcal{E}, \mathcal{H})$ the set $\{t \in I : \Gamma(t) \cap \mathcal{V} \neq \emptyset\}$ is Lebesgue measurable.

Main results

Consider the problem $(\mathcal{P})$ on the **unbounded** interval $[0, +\infty[$, i.e. assume that $I = [0, +\infty[$, $\Phi : [0, +\infty[\times \mathcal{E} \rightrightarrows \mathcal{E}$. In this case we have the following existence result

Theorem 202. Assume that the following conditions hold:

(a0) Φ has nonempty closed values.

(a1) For every $B \in \mathcal{E}$, $\Phi(\cdot, B)$ is measurable.

(a2)' There exists a measurable, **locally** bounded mapping $U : [0, +\infty[\rightarrow \mathcal{E}$, such that the set-valued mapping $t \mapsto \Phi(t, U(t))$ admits a selection $F_U : [0, +\infty[\rightarrow \mathbb{R}$, **locally** H-integrable on $[0, +\infty[$.

(b)'' There exists a **locally** Lebesgue-integrable function $L : [0, +\infty[\rightarrow \mathbb{R}$ such that

for a.e. $t \in [0, +\infty[$,

$$\mathfrak{H}(\Phi(t, B_1), \Phi(t, B_2)) \leq L(t) \mathcal{H}(B_1, B_2), \quad \forall B_1, B_2 \in \mathcal{E}.$$

Then, problem $(\mathcal{P})$ admits at least a solution on $[0, +\infty[$.

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Study of an elliptic differential equation in three habitats

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Abstract: We will study an elliptic differential equation set in three habitats with skewness boundary conditions at the interfaces. It represents the linear stationary case of dispersal problems of population dynamics which incorporate responses at interfaces between the habitats. Existence, uniqueness and regularity of the solution of these problems are obtained in holder spaces under necessary and sufficient conditions on the data. Our techniques are based on the semigroup theory, the fractionnal powers of linear operators, the H^∞ functional calculus for sectorial operators in Banach spaces and some proprieties of reel interpolation spaces.

Keywords: Population dynamics, habitats type, skew brownian motion, diffusion equation, semigroup

2020 Mathematics Subject Classification: 34K30, 35J25, 47D06, 92D25, 92D50

Introduction

The objective in this works is to study a problem of Diffusion with interfaces coming from (concrete) situations in population dynamics. In stationary mode this problem is reduced to an operational from the type:

$$(P) \begin{cases} (Equats) & v''(x) + Av(x) = h(x) \text{ in }]-l, 0[\cup]0, 2l[\cup]2L, 2L + l[\\ (Bounda.C.) & v_-(-l) = f_-, \quad v_+(2L + l) = f_+ \\ (Transmission.C.) & v_-(0) = v_0(0), \quad v_0(2L) = v_+(2L) \\ (SkewnessC.) & (1 - p)dv'_-(0) = pd_0v'_0(0), \quad pd_0v'_0(2L) = (1 - p)dv'_+(2L). \end{cases}$$

The function g is such that

$$\begin{cases} h_- = h|_{[-l, 0]} \in C^\theta([-l, 0]; E) \\ h_0 = h|_{[0, 2L]} \in C^\theta([0, 2L]; E) \\ h_+ = h|_{[2L, 2L + l]} \in C^\theta([2L, 2L + l]; E), \end{cases}$$

The operator A verifies the following ellipticity hypothesis:

$$\rho(A) \supset]0, \infty[\text{ et } \exists C > 0 : \forall \lambda \geq 0, \|(A - \lambda I)^{-1}\|_{L(E)} \leq \frac{C}{1 + |\lambda|}. \quad (472)$$

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Where $\rho(A)$ denotes the resolvent set of A .

The operator $B := -(-A)^{\frac{1}{2}}$ generates an analytic semigroup which makes it possible to find a representation of the solution. The analysis of this representation makes it possible to find necessary and sufficient conditions on the data for existence and regularity.

Main results

Our main results concerning problem (P) are summarized in the following Theorems

Theorem 203. Let $h_- \in C^\theta([-1, 0]; E)$, $h_0 \in C^\theta([0, 1]; E)$ and $h_+ \in C^\theta([1, 2]; E)$ with $0 < \theta < 1$, and $f_- \in D(B^2)$, $f_+ \in D(B^2)$. Assume (472). then problem (P) has a unique solution

$$\begin{cases} v_- : [-1, 0] \longrightarrow E \\ v_0 : [0, 1] \longrightarrow E \\ v_+ : [1, 2] \longrightarrow E, \end{cases}$$

such that $v_- \in C([-1, 0]; D(B^2)) \cap C^2([-1, 0]; E)$,

$v_0 \in C([0, 1]; D(B_0^2)) \cap C^2([0, 1]; E)$,

$v_+ \in C([1, 2]; D(B^2)) \cap C^2([1, 2]; E)$,

if and only if

$$\begin{cases} B^2 f_- + \frac{h_-(-1)}{d} \in \overline{D(B)} \\ B^2 f_+ + \frac{h_+(2)}{d} \in \overline{D(B)} \\ \left(\frac{h_-(0)}{d} + \frac{h_0(0)}{d_0} \right) \in \overline{D(B)} \\ \left(\frac{h_0(1)}{d_0} - \frac{h_+(1)}{d} \right) \in \overline{D(B)}. \end{cases}$$

Theorem 204. Assume (472), let $h_- \in C^\theta([-1, 0]; E)$, $h_0 \in C^\theta([0, 1]; E)$ and $h_+ \in C^\theta([1, 2]; E)$ with $0 < \theta < 1$, and $f_- \in D(B^2)$, $f_+ \in D(B^2)$. then $B^2 v_-(\cdot)$, $v_-'' \in C^\theta([-1, 0]; E)$, $B_0^2 u_0(\cdot)$, $v_0'' \in C^\theta([0, 1]; E)$, $B^2 v_+(\cdot)$, $v_+'' \in C^\theta([1, 2]; E)$

if and only if

$$\begin{cases} B^2 f_- + \frac{h_-(-1)}{d} \in D|_B(0, +\infty) \\ B^2 f_+ + \frac{h_+(2)}{d} \in D_B(0, +\infty) \\ \left(\frac{h_-(0)}{d} + \frac{h_0(0)}{d_0} \right) \in D_B(0, +\infty) \\ \left(\frac{h_0(1)}{d_0} - \frac{h_+(1)}{d} \right) \in D_B(0, +\infty). \end{cases}$$

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Analysis of Solutions for Nonlinear Fractional Differential Systems Using Caputo's q -Derivative

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Abstract: Fractional differential equations are a versatile tool for modeling complex systems, and their study is a rapidly growing field of research, as they are used in a variety of applied areas, including control engineering, physics, biomathematics, chemistry, and materials science [1], [2]. The study of quantum calculus, or infinite calculus, dates back to Jackson's article [3]. Q -calculus has since been used to bridge the gap between mathematics and physics, and the recent interest in EDFs has led to a renewed interest in q -fractional equations. Q -calculus has bridged the gap between mathematics and physics, and q -fractional equations have become increasingly important in recent years due to their applications in a variety of fields, including quantum theory, mechanics, capacitor theory, electrical circuits, particle physics, electroanalytical chemistry, and neurology [1, 2].

Keywords: q -Derivative, q -fractional, EDFs, fixed point theorem

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New Results on the Solvability of Abstract Sequential Caputo Fractional Differential Equations and Applications

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Abstract: This work aims to establish the existence and uniqueness of the mild solution for abstract sequential fractional differential equations. The approach employed involves the utilization of resolvent operators and the fixed point theorem. Additionally, we investigate a specific example concerning a partial differential equation incorporating the Caputo fractional derivative..

Keywords: Caputo fractional derivative, Abstract sequential fractional differential Equation, Resolvent operators, Nonlocal condition.

2020 Mathematics Subject Classification: Primary 26A33, 34A08, 34B15.

Introduction

In this work, we extend the sequential fractional equations studying in [1]

$$(\mathcal{D}^\alpha + \lambda \mathcal{D}^{\alpha-1})\varphi(t) = f(t, \varphi(t)), \quad t \in [0, T], 1 < \alpha \leq 2, \lambda \in \mathbb{R}^+, \quad (473)$$

by considering the case where λ represents a closed linear operator A . We investigate the following problem with abstract sequential fractional differential equation of the form:

$$\begin{cases} \mathcal{D}^\beta (\mathcal{D}^\alpha - A)\varphi(t) = \mathcal{H}_\omega(t, \mathcal{I}^\sigma(\varphi(t)), \varphi(t)), & t \in [0, T], \\ \varphi(0) = g(\varphi), \\ \varphi(T) = \mathcal{I}_T^\alpha(A\varphi). \end{cases} \quad (474)$$

where $\mathcal{D}^\alpha, \mathcal{D}^\beta$ are two Caputo fractional derivatives and $\mathcal{I}^\sigma$ is the Riemann-Liouville fractional integral with $0 < \alpha < 1, 0 < \beta, \sigma < 1$. A is a closed linear unbounded operator with domain $D(A)$ contained in a Banach space X and $\mathcal{H}_\omega$ depending on a parameter $\omega \geq 0$ with $\mathcal{H}_\omega : [0, T] \times X^2 \rightarrow X, g : C(J, X) \rightarrow X, J \subset \mathbb{R}$ are continuous functions.

The main objective of this paper is to examine the existence and uniqueness of mild solutions.

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Our investigation utilizes the Caputo fractional derivative and Riemann-Liouville fractional integral operators, with a particular focus on the role of resolvent operators. To establish uniqueness, we apply the Banach contraction principle, while for existence, we utilize the Krasnoselskii fixed point theorem.

Main results

The main results of this work:

Uniqueness of the Mild Solution

theorem

Under the assumptions of this work, there exists a unique mild solution of problem (474) in the interval $[0, T]$.

Existence of the Mild Solution

Theorem

Under the assumptions of this work, there exists a mild solution to problem (474) in the interval $[0, T]$.

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Y a-t-il un besoin d'introduire les espaces métriques à valeurs complexes dans la théorie du point fixe ?

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Abstract: En 2011, Azam et al. ont introduit la notion d'espace métrique à valeurs complexes et démontré l'existence de points fixes communs d'une paire de correspondances satisfaisant un type contraction impliquant des inégalités rationnelles. Dans cette communication, nous tenterons de discuter et de donner une réponse (non exhaustives) à la question suivante : Le concept d'espaces métriques à valeurs complexes offre-t-il quelque chose de nouveau dans le domaine du point fixe?.

Keywords: Espaces métriques, Espaces métriques à valeurs complexes, point fixe.

2020 Mathematics Subject Classification: Primary 47H10; 54H25; 54C30; 47H09.

Introduction

La notion d'espace métrique a été introduite en 1906 par M. Fréchet et développée peu de temps après par F. Hausdorff. En 1920, S. Banach publie un résultat significatif dans le domaine des points fixe, souvent appelé théorème du point fixe de Banach. Ce théorème fournit des conditions dans lesquelles une application contractante sur un espace métrique complet a un point fixe. Ce résultat a ouvert de nouvelles voies de généralisation et d'approfondissement la théorie des espaces métriques. Au niveau des contractions, de nombreux chercheurs ont exploré différents types d'inégalités qui généralisent le concept de contraction.

En 1989, Bakhtine a généralisé la notion d'espace métrique à celle de l'espace b-métriques et Czerwik ont étendu de nombreuses propriétés des espaces métriques en espaces b-métriques. Par la suite, plusieurs auteurs publiés des documents sur l'existence et l'unicité des points fixes pour des applications définies sur des espaces b-métriques et leurs applications.

En 2011, Azam et al. ont introduit la notion de l'espace métrique valeur complexes et ont prouvé l'existence de points fixes communs d'un paire d'application satisfaisant des conditions de type de contraction impliquant des inégalités rationnelles. Plusieurs chercheurs ont ensuite démontré des résultats liés à l'existence et à l'unicité de points fixes pour des applications définies sur des espaces métriques et b-métriques à valeurs complexes. Ces résultats sont en général connus pour les espaces métriques et b-métriques ordinaires, et certains sont même des corollaires de résultats des espaces métriques et b-métriques ordinaires, comme ce que nous verrons dans le paragraphe suivant.

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Main results

En 2019, [5] a démontré les théorèmes de Banach, Kannan et Chatterjea dans les espace b-métrique à valeurs complexes.

En utilisant les théorèmes ci-dessous, nous montrerons que les résultats démontrés dans [5] ne sont que des corollaire de théorème de Banach, Kannan et Chatterjea dans les espaces métriques ordinaires.

Theorem 205. Soient $s \geq 1$ et (X, d_c, s) un espace b-métrique à valeurs complexes. Soit $d(x, y) = |d_c(x, y)|, \forall x, y \in X$, alors $s \geq 1$ et (X, d, s) un espace b-métrique (usuel).

Theorem 206. Soient X, s, d_c et d comme dans le théorème 1. Soit $k \in [0, 1[$, et $T : X \rightarrow X$ une application.

- Si $d_c(Tx, Ty) \leq k d_c(x, y)$ alors $d(Tx, Ty) \leq k d(x, y)$.
- Si $d_c(Tx, Ty) \leq k(d_c(x, Tx) + d_c(y, Ty))$ alors $d(Tx, Ty) \leq k(d(x, Tx) + d(y, Ty))$.
- Si $d_c(Tx, Ty) \leq k(d_c(x, Tx) + d_c(y, Ty))$ alors $d(Tx, Ty) \leq k(d(x, Ty) + d(y, Tx))$.

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Homotopy Perturbation Method for Solving Nonlinear Fractional Model

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Abstract: We consider the fractional model of Newell-Whitehead-Segel equation [1]

$$D_t^\alpha U(x, t) = aU_{xx}(x, t) + bU(x, t) - cU^q(x, t), 0 < \alpha \leq 1,$$

where α is a parameter which describes the order of the time-fractional derivative in the sense of Caputo and a, b and c are real constants with $a, b > 0$, q is a positive integer.

If $\alpha = 1$, this equation becomes the classical Newell-Whitehead-Segel equation.

The Newell-Whitehead-Segel equation is an important model arising in biology, finance, fluid-mechanics and some more processes. Many researchers have worked to find an approximate solution to this model using different methods. In this work, the fractional model of Newell-Whitehead-Segel equation is solved using the homotopy perturbation method (HPM). Illustrative examples are presented to demonstrate the accuracy and effectiveness of the proposed method.

Keywords: Fractional partial differential equations, Caputo fractional derivative, Homotopy Perturbation Method, Approximate analytical solution

2020 Mathematics Subject Classification: 34A08, 26A33, 35F40, 35C10.

Preliminaries and Notations

In this section, we give some important definitions and properties of the theory of fractional calculus which are used further in this work.

Definition 72. [5] The Riemann-Liouville fractional integral operator I_t^α of order α for a function $U(x, t)$ is defined as

$$I_t^\alpha U(x, t) = \begin{cases} \frac{1}{\Gamma(\alpha)} \int_0^t (t - \xi)^{\alpha-1} U(x, \xi) d\xi, & \alpha > 0, \\ U(x, t), & \alpha = 0, \end{cases}$$

where $\Gamma(\cdot)$ is the well-known gamma function.

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Definition 73. [5] The time-fractional derivative of $U(x, t)$ in the Caputo sense is defined as

$$\begin{aligned} D_t^\alpha U(x, t) &= I^{n-\alpha} D^n U(x, t) \\ &= \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\xi)^{n-\alpha-1} U^{(n)}(x, \xi) d\xi, \end{aligned}$$

where $n-1 < \alpha \leq n$, $n \in \mathbb{N}$.

For the Riemann-Liouville fractional integral and Caputo time-fractional derivative, we have the following relation

$$I_t^\alpha D_t^\alpha U(x, t) = U(x, t) - \sum_{k=0}^{n-1} U^{(k)}(x, 0^+) \frac{t^k}{k!}.$$

Analysis the HPM

In this section, an analytical solution of the fractional model of Newell-Whitehead-Segel equation is proposed by the homotopy perturbation method (HPM).

Theorem 207. Consider the fractional model of Newell-Whitehead-Segel equation

$$D_t^\alpha U(x, t) = aU_{xx}(x, t) + bU(x, t) - cU^q(x, t), 0 < \alpha \leq 1, \quad (475)$$

with the initial condition

$$U(x, 0) = U_0(x). \quad (476)$$

Using HPM, the solution of Eqs. (475)-(476) can be expressed as an infinite series as follows

$$U(x, t) = \lim_{p \rightarrow 1} \sum_{n=0}^{\infty} p^n U_n(x, t) = \sum_{n=0}^{\infty} U_n(x, t)$$

where

$$\sum_{n=0}^{\infty} p^n U_n(x, t) = U_0(x) + I_t^\alpha \left(a \sum_{n=0}^{\infty} p^n U_{nxx}(x, t) + b \sum_{n=0}^{\infty} p^n U_n(x, t) - c \sum_{n=0}^{\infty} p^n H_n(U) \right),$$

and $H_n(U)$ is He's polynomials [3] of the nonlinear term $U^q(x, t)$ and it can be calculated by formula given below

$$H_n(U_0, U_1, \dots, U_n) = \frac{1}{n!} \frac{\partial^n}{\partial p^n} \left[N \left(\sum_{i=0}^{\infty} p^i U_i \right) \right]_{p=0}, n = 0, 1, 2, \dots$$

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Existence of solutions for a quasilinear boundary value problem driven by the one-dimensional curvature operator

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Abstract: This paper is concerned with the following one-dimensional quasilinear Neumann problem involving the curvature operator:

$$-\left(u'/\sqrt{1+(u')^2}\right)' = \lambda a(x)f(u) \text{ for } x \in (0,1) \text{ and } u'(0) = \alpha, u'(1) = \beta,$$

where $\lambda \in \mathbb{R}$ is a parameter, $a \in C([0,1])$ and $f \in C(\mathbb{R})$. Under some suitable assumptions on a and f , we prove that the above equation has at least one solution by means of the Schauder's fixed point theorem.

Keywords: Quasilinear equation; Neumann boundary condition; Existence; Schauder's fixed Point theorem.

2020 Mathematics Subject Classification: Primary 35J93, 34B15, 37C25.

Introduction

In this paper, we study the existence of solution for the following quasilinear equation with Neumann boundary condition

$$\begin{cases} -\left(\frac{u'}{\sqrt{1+(u')^2}}\right)' = \lambda a(x)f(u), & x \in (0,1), \\ u'(0) = \alpha, u'(1) = \beta, \end{cases} \quad (477)$$

where $\lambda \in \mathbb{R}$ is a parameter, $\alpha, \beta \in \mathbb{R}$ with $(\alpha, \beta) \neq (0, 0)$, $a \in C([0,1])$ and $f \in C(\mathbb{R})$.

Problem (477) is a one dimensional prototype of the following high-dimensional elliptic problem

$$\begin{cases} -\operatorname{div}\left(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}}\right) = g(x, u), & \text{in } \Omega, \\ -\frac{\nabla u \cdot \nu}{\sqrt{1+|\nabla u|^2}} = \sigma, & \text{on } \partial\Omega, \end{cases} \quad (478)$$

where Ω is a bounded regular domain in $\mathbb{R}^N$ with outward pointing normal ν , and $g : \Omega \times \mathbb{R} \rightarrow \mathbb{R}$ and $\sigma : \partial\Omega \rightarrow \mathbb{R}$ are given functions. Indeed, let $\gamma(x) = (x, u(x))$ be the graph of a smooth function

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u , defined on an open interval $I \subset \mathbb{R}$. Then the curvature function of the plane curve $\gamma(I)$ is given by

$$\kappa(x) = \frac{u''(x)}{(1 + u'(x)^2)^{3/2}} = \left(\frac{u'(x)}{\sqrt{1 + (u'(x))^2}} \right)'.$$

Therefore, problem (477) problem is driven by the one-dimensional curvature operator.

Problem (478) arisen in some geometrical and physical contexts, such us the analysis of capillary surfaces and when describing capillarity phenomena for incompressible fluids. For more information on the geometrical and physical relevance of these topics, the reader may see [2] and the references therein.

Main results

Our main purpose is to obtain couples (λ, u) solving the problem (477). Using the Schauder's fixed Point theorem, we prove that the problem (477) has at least a nontrivial solution (λ_0, u_0) .

In order to state the main results of this paper, we make the following assumptions:

(H_a) $a \in C([0, 1])$ and $a(x) > 0$ for all $x \in (0, 1)$.

(H_f) $f \in C(\mathbb{R})$ such that $f(0) \geq 0$ and $f(u) > 0$ for all $u \in \mathbb{R} \setminus \{0\}$.

Our main results are the following.

Theorem 208. *Suppose that (H_a) and (H_f) hold, and $\alpha < \beta$ or $\alpha > \beta$. Then, there exists $(\lambda_0, u_0) \in \mathbb{R} \times E$ solution of problem (477).*

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Existence solutions of ODEs involving two generalized Caputo fractional derivatives with boundary value problems by Laplace transform

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Abstract: This research inscription gets to grips with novel class of boundary value problems. We study existence of solutions for nonlinear ordinary differential equations en capsuling a collective fractional derivative known as the Caputo-Katugampola fractional operator (Generalized Caputo fractional derivative) Such operators are generated by iterating a local integral of a function. The existence of the solutions of the aforehand equations is tackled by using monotonicity, continuity and explicit estimation of Mittag-Leffler functions via fixed point theorems. Two pertinent examples are presented to corroborate the reported results.

Keywords: Generalized Caputo fractional derivative, Existence and uniqueness, Mittag-Leffler functions , ρ -Laplace transform, Fixed points theorem.

2020 Mathematics Subject Classification: 34A34, 65L05.

Introduction

In the present study, we analyze the following Caputo fractional differential equations to obtain uniqueness and existence criteria of solutions

$$\begin{cases} {}^c\mathcal{D}_0^{\alpha_1, \rho} u(t) - \lambda {}^c\mathcal{D}_0^{\alpha_2, \rho} u(t) = v(t, u(t)) & \text{for a.e } t \in [0, T] \\ \alpha u(0) + \beta u(T) = d \end{cases} \quad (479)$$

where ${}^c\mathcal{D}^{\alpha_i}$ is the generalized Caputo fractional derivative, for $i = 1, 2$, where $\alpha_i \in (0, 1]$, $\lambda > 0$ and $f : J \times \mathbb{R} \rightarrow \mathbb{R}$ is given function and α, β are real with $\alpha + \beta \neq 0$ and $d \in \mathbb{R}$.

Main results

The first result is based on the Banach contraction mapping principle.

Lemma 27. Let $\alpha_i \in (0, 1]$ with $i = 1, 2$ and $\rho > 0$ then the linear problem (479) has a unique solution

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in terms of the integral given by

$$\begin{aligned} u(t) &= \frac{d}{\alpha + \beta} - \frac{1}{\alpha + \beta} \int_0^T \left(\frac{T^\rho - s^\rho}{\rho} \right)^{\alpha_1 - 1} E_{\alpha_1 - \alpha_2, \alpha_1} \left(\lambda \left(\frac{T^\rho - s^\rho}{\rho} \right)^{\alpha_1 - \alpha_2} \right) v(s) \frac{ds}{s^{1-\rho}} \\ &+ \int_0^t \left(\frac{t^\rho - s^\rho}{\rho} \right)^{\alpha_1 - 1} E_{\alpha_1 - \alpha_2, \alpha_1} \left(\lambda \left(\frac{t^\rho - s^\rho}{\rho} \right)^{\alpha_1 - \alpha_2} \right) v(s) \frac{ds}{s^{1-\rho}}. \end{aligned}$$

We establish the uniqueness criteria for solution of (479) with the help of Banach contraction principal.

Theorem 209. Assume (H_1) and (H_2) hold. If

$$\begin{aligned} L &= k \left(m(\alpha_1 - \alpha_2, \alpha_1, \lambda) \left(\frac{1}{2\alpha_2 - \alpha_1} \left(\frac{T^\rho}{\rho} \right)^{2\alpha_2 - \alpha_1} + \frac{1}{\alpha_2} \left(\frac{T^\rho}{\rho} \right)^{\alpha_2} \right) \right. \\ &\quad \left. + \frac{1}{\alpha_1 - \alpha_2} \lambda^{1-\alpha_1} \exp \left(\lambda^{\frac{1}{\alpha_1 - \alpha_2}} \left(\frac{T^\rho}{\rho} \right) \right) \right) \left(1 + \frac{1}{|\alpha + \beta|} \right) < 1. \end{aligned}$$

Then the problem (479) has a unique solution.

Our second result is based on Krasnoselskii fixed point theorem.

Theorem 210. Let $\alpha_1, \alpha_2 \in (0, 1]$ and $\lambda > 0$. Assume that (H_1) , (H_2) and (H_2) hold. If

$$\begin{aligned} \ell &= \frac{k}{|\alpha + \beta|} \left\{ m(\alpha_1 - \alpha_2, \alpha_1, \lambda) \left(\frac{1}{2\alpha_2 - \alpha_1} \left(\frac{T^\rho}{\rho} \right)^{2\alpha_2 - \alpha_1} + \frac{1}{\alpha_2} \left(\frac{T^\rho}{\rho} \right)^{\alpha_2} \right) \right. \\ &\quad \left. + \frac{1}{|\alpha_1 - \alpha_2|} \lambda^{1-\alpha_1} \exp \left(\lambda^{\frac{1}{\alpha_1 - \alpha_2}} \left(\frac{T^\rho}{\rho} \right) \right) \right\} < 1. \end{aligned}$$

Then, problem (479) has at least one solution.

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Caputo Tempered Fractional Differential Equations with Mixed Boundary Condition Under the Weak Topology

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Abstract: This work is a subject about some results of the existence of weak solutions for the following boundary value problem involving Caputo tempered fractional differential equations with mixed boundary conditions. First we introduce some preliminaries, definitions, lemmas and auxiliary results that are used throughout this work. Next, we give some existence results for the our problem that are based on the concept of measure of weak noncompactness combined with Mönch's type fixed point theorem on Banach space. Finally we present an example to show the validity of our results.

Keywords: Caputo tempered fractional derivative, boundary value problem, weak solution, Pettis integral, measure of weak noncompactness, fixed point.

2020 Mathematics Subject Classification: 26A33, 34A08, 34B15, 34K37.

Introduction

Differential equations are crucial for understanding natural phenomena and engineering systems. Recent focus extends classical theory to complex scenarios. Buschman's research[3]h introduced fractional integration definitions, incorporating weak singular and exponential kernels.

Tempered fractional derivatives, which incorporate exponentially decaying tails, provide a more accurate representation of underlying dynamics in fields like viscoelastic materials, nonlocal physics models, and fractional-order control systems.

De Blasi introduced the weak noncompactness measure, which has been refined and widely applied in various studies. Recent research has further utilized this method, as demonstrated by Álvarez[1], Benchohra[2], Guo[4], and others.

Boundary value problems with integral boundary conditions are a significant category, encom-

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passing two-point, three-point, multi-point, and nonlocal problems. They are commonly used in population dynamics and cellular systems analysis, with numerous authors exploring this area.

we investigate the existence of weak solutions for the following boundary value problem involving Caputo tempered fractional differential equations with mixed boundary conditions

$$({}_0^C \mathcal{D}_t^{\zeta, \omega} x)(t) = f(t, x(t)); \quad t \in I := [0, T], \quad (480)$$

$$x(0) + \eta \int_0^T e^{\omega s} x(s) ds = x(T), \quad (481)$$

where $0 < \zeta < 1$, $\omega \geq 0$, $f : I \times E \rightarrow E$ is given function and $\eta \in \mathbb{R}^*$, ${}_0^C \mathcal{D}_t^{\zeta, \omega}$ is the Caputo tempered fractional derivative of order ζ and E is a Banach space with the norm $\|\cdot\|_E$ and dual E^* .

Main results

Definition 74 (The Caputo tempered fractional derivative [5]). Let $n - 1 < \zeta < n$; $n \in \mathbb{N}^+$, $\omega \geq 0$. The Caputo tempered fractional derivative is defined as

$${}_0^C \mathcal{D}_t^{\zeta, \omega} f(t) = e^{-\omega t} {}_0^C \mathcal{D}_t^{\zeta} (e^{\omega t} f(t)) = \frac{e^{-\omega t}}{\Gamma(n - \zeta)} \int_0^t (t - s)^{n - \zeta - 1} \frac{d^n}{ds^n} (e^{\omega s} f(s)) ds,$$

where ${}_0^C \mathcal{D}_t^{\zeta, \omega}$ denotes the Caputo fractional derivative, given by

$${}_0^C \mathcal{D}_t^{\zeta} f(t) = \frac{1}{\Gamma(n - \zeta)} \int_0^t (t - s)^{n - \zeta - 1} \frac{d^n}{ds^n} f(s) ds.$$

Lemma 28. Let $K : I \rightarrow E$ be continuous function, $0 < \zeta < 1$ and $\omega \geq 0$. Then the fractional boundary value problem

$$({}_0^C \mathcal{D}_t^{\zeta, \omega} x)(t) = K(t); \quad t \in I,$$

$$x(0) + \eta \int_0^T e^{\omega t} x(s) ds = x(T),$$

has a unique solution defined by

$$x(t) = \int_0^T G(t, s) K(s) ds,$$

where $G(t, s)$ is the Green's function defined by

$$G(t, s) = \begin{cases} \frac{e^{-\omega(t-s)}(t-s)^{\zeta-1}}{\Gamma(\zeta)} + \frac{e^{-\omega(T+t-s)}(T-s)^{\zeta-1}}{(1+\eta T - e^{-\omega T})\Gamma(\zeta)} \\ - \frac{\eta e^{-\omega(t-s)}(T-s)^{\zeta}}{(1+\eta T - e^{-\omega T})\Gamma(\zeta+1)}, & 0 \leq s \leq t, \\ \frac{e^{-\omega(T+t-s)}(T-s)^{\zeta-1}}{(1+\eta T - e^{-\omega T})\Gamma(\zeta)} - \frac{\eta e^{-\omega(t-s)}(T-s)^{\zeta}}{(1+\eta T - e^{-\omega T})\Gamma(\zeta+1)}, & t \leq s < T. \end{cases}$$

To establish our main result concerning the existence of solutions of (480)-(481), we introduce the following hypotheses:

(A₁) For each $t \in I$, the function $z(t, \cdot)$ is weakly sequentially continuous.

(A₂) For each $x \in \mathcal{C}$, the function $f(\cdot, x(\cdot))$ is Pettis integrable on I .

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(A₃) There exist $f^* \in L^\infty(I, \mathbb{R}^+)$ and a continuous nondecreasing function $\psi : [0, \infty) \rightarrow [0, \infty)$, such that

$$\|f(t, x)\|_E \leq f^*(t)\psi(\|x\|_E).$$

(A₄) There exists a constant $\mathcal{R} > 0$ such that

$$\frac{\mathcal{R}}{\|f^*\|_{L^\infty} \psi(\mathcal{R}) G^*} \geq 1.$$

(A₅) For each bounded set $\Xi \subset E$ and each $t \in I$,

$$\beta(f(t, \Xi)) \leq f^*(t)\beta(\Xi).$$

Theorem 211. Assume that the hypotheses (A₁) – (A₅), and the condition

$$\|f^*\|_{L^\infty} G^* \leq 1,$$

hold. Then the boundary value problem (480)-(481) has at least one solution defined on I .

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Analysis of a dynamic frictional contact problem with adhesion

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Abstract: The aim of this work is to establish the existence and uniqueness of the weak solution for a dynamic frictional contact problem between a viscoelastic body and an adhesive foundation. The contact is modeled with normal damped response condition and the associated new version of Coulomb's law of dry friction in which the adhesion is taken into account. The model given differs from the standard model because of the term $(1 - \beta)$ in the friction law. This means that friction is not active when adhesion is still complete and that the friction threshold progressively increases as the adhesion decreases. The variational formulation is given as a system coupling an evolution inequality of the second order for the displacement and a differential equation of the first order for the bonding field.

Keywords: Dynamic frictional contact, viscoelasticity, adhesion, existence and uniqueness

2020 Mathematics Subject Classification: 74M15, 74M10, 74H20, 74H25.

Introduction

Processes of adhesion are important in many industrial settings where parts, usually nonmetallic, are glued together. For this reason, adhesive contact between bodies, when a glue is added to prevent the surfaces from relative motion, has recently received increased attention in the literature. In this work we introduce the surface internal variable, the bonding field, which describes the fractional density of active bonds on the contact surface and which is denoted by β . The bonding field satisfies the restriction $0 \leq \beta \leq 1$. If $\beta = 0$, all the bonds are inactive, severed, and there is no adhesion, if $\beta = 1$, the adhesion is complete and all the bonds are active, if $0 < \beta < 1$, the adhesion is partial and only a fraction of the bonds is active. General models with adhesion can be found in [2, 3, 4, 5] and the references therein. The contact is modeled with normal damped response condition and the associated new version of Coulomb's law of dry friction in which the adhesion is taken into account. The model given differs from the standard model because of the term $(1 - \beta)$ in the friction law, see [1].

Main results

We consider a mathematical model of a contact problem in viscoplasticity

$$\sigma = \mathcal{A}\varepsilon(\dot{u}) + \mathcal{F}\varepsilon(u) \quad \text{in} \quad \Omega \times (0, T), \quad (482)$$

where $u : \Omega \times [0, T] \rightarrow \mathbb{R}^d$ is the displacement field, $\sigma : \Omega \times [0, T] \rightarrow S_d$ is the stress field and $\beta : \Gamma_3 \times [0, T] \rightarrow [0, 1]$ is a the bonding field. The body is in contact with an obstacle. The process is

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dynamic

$$\rho \ddot{\mathbf{u}} = \text{Div} \boldsymbol{\sigma} + \mathbf{f}_0 \quad \text{in } \Omega \times (0, T). \quad (483)$$

Since the body is clamped on $\Gamma_1 \times (0, T)$ and surface tractions of density $\mathbf{f}_2$ act on $\Gamma_2 \times (0, T)$

$$\mathbf{u} = \mathbf{0} \quad \text{on } \Gamma_1 \times (0, T), \quad (484)$$

$$\boldsymbol{\sigma} \boldsymbol{\nu} = \mathbf{f}_2 \quad \text{on } \Gamma_2 \times (0, T). \quad (485)$$

The contact is modeled with a normal damped response condition with adhesion associated to a new version of Coulomb's law of dry friction in which the adhesion of contact surfaces is taken into account

$$-\sigma_\nu = p_\nu(\dot{u}_\nu) - \gamma_\nu \beta^2 R_\nu(u_\nu) \quad \text{on } \Gamma_3 \times (0, T), \quad (486)$$

$$\left. \begin{aligned} |\boldsymbol{\sigma}_\tau + \gamma_\tau \beta^2 \mathbf{R}_\tau(\mathbf{u}_\tau)| &\leq (1 - \beta) \mu p_\nu(\dot{u}_\nu), \\ -\boldsymbol{\sigma}_\tau - \gamma_\tau \beta^2 \mathbf{R}_\tau(\mathbf{u}_\tau) &= (1 - \beta) \mu p_\nu(\dot{u}_\nu) \frac{\dot{\mathbf{u}}_\tau}{|\dot{\mathbf{u}}_\tau|} \quad \text{if } \dot{\mathbf{u}}_\tau \neq \mathbf{0} \end{aligned} \right\} \quad \text{on } \Gamma_3 \times (0, T), \quad (487)$$

$$\dot{\beta} = H_{ad}(\beta, \psi_\beta, \mathbf{R}(|\mathbf{u}_\tau|)) \quad \text{on } \Gamma_3 \times (0, T), \quad (488)$$

We have the initials conditions

$$\mathbf{u}(0) = \mathbf{u}_0, \quad \dot{\mathbf{u}}(0) = \mathbf{v}_0 \quad \text{in } \Omega, \quad (489)$$

$$\beta(0) = \beta_0 \quad \text{on } \Gamma_3. \quad (490)$$

We establish a variational formulation PV for the mechanical problem (482)-(490).

Problem PV . Find a displacement field $\mathbf{u} : [0, T] \rightarrow V$ and a bonding field $\beta : [0, T] \rightarrow L^\infty(\Gamma_3)$ such that for a.e. $t \in (0, T)$

$$\begin{aligned} & \left(\ddot{\mathbf{u}}(t), \mathbf{v} - \dot{\mathbf{u}}(t) \right)_{V' \times V} + \left(\mathcal{A} \varepsilon(\dot{\mathbf{u}}(t)), \varepsilon(\mathbf{v}) - \varepsilon(\dot{\mathbf{u}}(t)) \right)_{\mathcal{H}} + \left(\mathcal{F}(\varepsilon(\mathbf{u}(t))), \varepsilon(\mathbf{v}) - \varepsilon(\dot{\mathbf{u}}(t)) \right)_{\mathcal{H}} \\ & + j_{ad}(\beta(t), \dot{\mathbf{u}}(t), \mathbf{v} - \dot{\mathbf{u}}(t)) + j_{nc}(\dot{\mathbf{u}}(t), \mathbf{v} - \dot{\mathbf{u}}(t)) + j(\beta(t), \dot{\mathbf{u}}(t), \mathbf{v}) - j(\beta(t), \dot{\mathbf{u}}(t), \dot{\mathbf{u}}(t)) \\ & \geq (\mathbf{f}(t), \mathbf{v} - \dot{\mathbf{u}}(t))_{V' \times V} \quad \forall \mathbf{v} \in V, \\ & \dot{\beta}(t) = H_{ad}(\beta(t), \psi_\beta, \mathbf{R}(|\mathbf{u}_\tau(t)|)), \quad 0 \leq \beta(t) \leq 1, \\ & \mathbf{u}(0) = \mathbf{u}_0, \quad \dot{\mathbf{u}}(0) = \mathbf{v}_0, \quad \beta(0) = \beta_0, \end{aligned}$$

and we prove the existence of a unique weak solution to the problem.

Theorem 212. *The problem PV has a unique solution $(\mathbf{u}, \boldsymbol{\sigma}, \beta)$ satisfying $\mathbf{u} \in W^{1,2}(0, T; V) \cap C^1(0, T; H) \cap W^{2,2}(0, T; V')$, $\beta \in W^{1,\infty}(0, T; L^\infty(\Gamma_3)) \cap Z$.*

The proof will be carried out in several steps and is based on arguments of time-dependent variational inequalities, parabolic variational inequalities of the second kind, differential equations and fixed point theorem.

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Discrete Memristive based Map with Fractional Order and Hidden Dynamics

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Abstract: This paper introduces and explores the dynamics of a novel three-dimensional (3D) fractional map with hidden dynamics. The map is constructed through the integration of a mathematical operator, namely a fractional variable order Caputo-like difference operator to establish the fractional form of the map with short memory. The numerical simulation results highlight its excellent dynamical behavior, revealing that the addition of the piecewise fractional order makes the memristive-based Duffing map even more chaotic. It is characterized by some special features, including the absence of an equilibrium point and the presence of multiple hidden chaotic attractors.

Keywords: Discrete fractional calculus, Variable order, Discrete memristive, Chaos.

Introduction

Fractional calculus is the study of derivatives and difference operators with non-integer orders, applied to both continuous and discrete systems. It provides increased accuracy in characterizing memory effects in various materials and processes. Recent research has focused on exploring chaotic behaviors in discrete fractional systems, such as nonlinear maps described by non-integer order difference equations. Variable-order fractional maps, where the fractional order changes over discrete time, have also been investigated, showing chaotic behavior in systems like the Hopfield neural network. Additionally, there's been exploration into chaos in discrete systems involving fractional discrete memristors, nonlinear circuit elements linking electric charge and magnetic flux. This paper introduces a novel 3D fractional discrete memristive map with a variable order and short memory effect, integrating a discrete memristive with the Duffing map. The resulting system exhibits distinctive features, including the coexistence of multiple hidden chaotic attractors. The manuscript outlines the development of this map and analyzes its chaotic dynamics.

Main results

This study introduces a variation of the Duffing map by adding fractional variable order and a discrete sinusoidal memristor. Theoretical analyses suggest that this modified map can produce

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hidden attractors. The new fractional map demonstrated the presence of many attractors and chaos, according to numerical results.

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Existence of Periodic Oscillations and Stability in a Delayed Virotherapy Model

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Abstract: In this paper, we propose a delayed mathematical model which models the virotherapy treatment of a cancer dynamics with logistic growth. The total cancer population is divided into two types of cells: uninfected and infected cells. Considering time delay as a parameter, we establish the positivity and boundedness of solutions and the stability of the possible equilibria. We prove that, time delay can lead to periodic oscillations with small amplitude called "Jeff's phenomenon" which observed in laboratory and causes oscillations in cancer size via Hopf bifurcation theory. We give an algorithm determining the direction of Hopf bifurcation via center manifold and normal form theories. We end with some numerical simulations illustrating our obtained results.

Keywords: Virotherapy; delay differential equations; stability; Hopf bifurcation; center manifold; normal form; Jeff's phenomenon.

2020 Mathematics Subject Classification: 34K05, 34K18, 34K20, 34K60, 92B05.

Introduction

Oncolytic virotherapy also called gene therapy, based essentially on modifying genetically viruses that have the ability to suppress and kill cancer cells and boosting the immune system to realise oncolytic cytokines in order to activate cells killers. These oncolytic viruses are capable to replicate within the cancer cells to cause viral burden to the cell which in turn leads to infected cancer cell death [14, 15]. Moreover, they are released from the virus-infected and lysed cancer cells that can infect and kill other cancer cells. Many oncolytic viruses are presently under clinical experimental trials and their evaluations, and some clinical trials has proved that oncolytic viruses can significantly improve the prognosis of cancer patients such as cancers of ovary, sarcoma, pancreas, prostate, and bladder [15].

Adenoviruses, retroviruses, herpes viruses, paramyxoviruses, measles, vesicular stomatitis virus, etc., are ones of the oncolytic viruses used to facilitate cancer therapy. An examples of this variety of viruses which have shown promising results in clinical trials [4] are the adenovirus Onyx-015 [22], herpes simplex virus HSV-1 [20], Newcastle disease virus NDV [3] and M1 virus [4]. The FDA has already given its approval for the use of oncolytic virus therapy (T-VEC, Imlygic) in 2015 [15].

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In conclusion, virotherapy based anti-cancer approaches make use of the potential of oncolytic viruses to [10, 17, 23]:

- replicate repeatedly in the cancer cells and cause infected cell death,
- produce cytotoxic protein and cause cell death,
- infect the cancer cells and activate anti-cancer immunity .

Such oncolytic viruses were tested for head, neck cancers [22] and metastatic colon carcinoma [27, 28] and for other tumour types [25]. In literature, a lot of researchers use ordinary differential equations to model therapy of different kinds of cancers. [8, 24, 3, 4] However, there are very few mathematical models which are dealing with virotherapy and delay differential equations [1, 6, 31] and references therein.

Main results

In the therapy with viruses, there exist two modes of spreading direct and indirect, the first is between cells and the second is from environmental (or other) to cells. The mode of transmission is a very important factor that determines the efficacy of treatment [23]. As a first step, we consider that direct mode of spreading and the growth of the two types of cancer follow the logistic law at a rate r and s for the uninfected cells and infected cells respectively. The maximal sizes of the two cancer cells T_u and T_i are given with environmental carrying capacity k and β is the transmission rate of the virus into the cancer tissue. Infected cells are eliminated by the virus at a cytotoxicity rate a . The time delay τ is the time needed for a susceptible cell to be infectious. The delayed oncolytic model considered in this paper is given by the following 2-dimensional delay differential system

$$\begin{cases} \frac{dT_u(t)}{dt} = rT_u(t)\left(1 - \frac{T(t)}{k}\right) - dT_u(t) - \beta T_u(t)T_i(t), \\ \frac{dT_i(t)}{dt} = sT_i(t)\left(1 - \frac{T(t)}{k}\right) + \beta T_u(t - \tau)T_i(t - \tau) - aT_i(t), \\ T = T_u + T_i, \\ T_u(s) = \phi(\vartheta) \geq 0, T_i(s) = \chi(\vartheta) \geq 0, \vartheta \in [-\tau, 0). \end{cases} \quad (491)$$

$T = T_u + T_i$:= Total size of cancer, where T_u and T_i are, respectively, the total number of uninfected cancer and infected cancer cells.

To the best of our knowledge, the first model given in this optic was introduced by Wodarz [31] by a simple ODE system [31]. The author studied the effect of cytotoxicity a on the dynamics of uninfected and infected cells from a biological and numerical point of view. The corresponding cross-diffusion model was studied in [21]. In their work, the authors proved the occurrence of the Turing bifurcation and determined the amplitude equation and pattern formation. Other than these models, some works considered the indirect virus spreading (see [31, 6] and reference therein). In [31, 4], the authors did not provide a rigorous mathematical proof of the stability of equilibria of the above model without delay. The study of the effect of delays have attracted a number of researchers [30, 8, 7, 12, 13, 26]. Other authors use delay differential equations to establish oscillations in biology for example see [29, 32, 33]. In the present paper, we give a detailed mathematical and numerical analysis of the proposed model with and without delay.

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Study of a Generalized Proportional Fractional Creep Problem with Multiple Delays

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Abstract: We study a class of nonlinear Generalized Proportional fractional differential equations with multiple delays, which is represented by the Voigt creep fractional model of viscoelasticity. We discuss two Voigt models, the first being linear and the second being nonlinear. The linear Voigt model give us the physical interpretation and is associated with important results since the creep function characterizes the viscoelastic behavior of stress and strain. For the nonlinear model of Voigt, our theoretical study and analysis provides existence and stability, where time delays are expressed in terms of Boltzmann's superposition principle. By means of the Banach contraction principle, we prove existence of a unique solution and investigate its continuous dependence upon the initial data as well as Ulam stability.

Keywords: generalized proportional fractional derivatives; differential equations; creep function; Ulam stability; fixed-point theorems.

2020 Mathematics Subject Classification: 26A33; 34A12; 47H10.

Introduction

Recently, fractional Differential Equations have emerged as a powerful tool for dealing with viscoelastic phenomena: For example, stress-strain behavior of polymeric materials.

To describe the behavior of materials and exhibit their viscoelastic properties, rheological models are used: These models can be of the Voigt or Maxwell type, or a combination of these basic models. On the other hand, there are many materials that are difficult to describe in rheological models that contain finite elements of elastic and viscous components. Therefore, it is common to resort to the use of fractional models, which allow us to use fewer elements while still providing an accurate description.

In 2017, Jarad et al. [1] introduced a new generalized proportional derivative which is well-behaved and has several advantages over the classical derivatives, such as being an accurate generalization of the existing fractional derivatives and integrals of Caputo and Riemann-Liouville.

We will use this derivative to generalize the work done by Ammar et al.[2] where they studied the fractional Voigt creep problem using the Caputo fractional derivative. we study the Generalized

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Proportional Fractional Differential Equation

$$({}^{PC}D_{0+}^{\alpha,\rho}x)(t) + \lambda x(t) = \sum_{j=1}^n b_j(t)g_j(x(t-\tau_j)), t \in [0, T] \quad (492)$$

where ${}^{PC}D_{0+}^{\alpha,\rho}$ denotes the generalized proportional fractional derivative of Caputo type of order $0 < \alpha, \rho < 1$; λ is real positive constant; b_j, g_j are given function; and τ_j represents the time delays. we take

$$x(t) = \psi(t), t \in [-v, 0], \quad (493)$$

where $0 < v = \max_{j=1, \dots, n} \{\tau_j\} \leq T$ and $x(0) = \psi(0) = 0$.

The rest of the paper is organized as follows: in the results firstly, we obtain an integral representation of our problem and then extract the fractional creep function associated with the generalized linear problem, and secondly, we use the Banach contraction principle to show the existence and uniqueness of the solution, finally we establish continuous dependence of the solution with the initial data and we study the Ulam stability.

Main results

Linear Generalized Proportional Fractional Differential Equation

Now we take our linear fractional problem with the fractional proportional derivative as the fractional Voigt model.

$$({}^{PC}D_{0+}^{\alpha,\rho}x)(t) + \lambda x(t) = \varphi(t), x(0) = 0; (t > 0, \lambda > 0) \quad (494)$$

Theorem 213. Let $0 < \alpha < 1, \lambda > 0$ and $\varphi \in C([0, T]; \mathbb{R})$. Then $x(t)$ is the strain and the continuous solution of (494)

$$x(t) = \int_0^t e^{\frac{\rho-1}{\rho}(t-s)} (t-s)^{\alpha-1} E_{\alpha,\alpha}(-\lambda(t-s)^\alpha) \varphi(s) ds \quad (495)$$

Moreover, (494) and (495) are equivalent in $C[0, T]$.

Theorem 214. The creep function associated with the fractional problem (494) is given by

$$k_{\alpha,\rho}(t) = \frac{t^\alpha}{\rho} e^{\frac{\rho-1}{\rho}t} E_{\alpha,\alpha+1}(-\lambda t^\alpha) \quad (496)$$

Existence and Uniqueness Results

We now turn our attention to the nonlinear problem (492) and (493) as a generalization of the linear fractional problem.

Let us introduce the following hypotheses.

(H1). $b_j : [0, T] \rightarrow \mathbb{R}$ are continuous functions with $B_j = \sup_{t \in [0, T]} |b_j(t)|$.

(H2). $g_j : C[-v, T] \rightarrow C[-v, T]$ are Lipschitz functions, i.e., there exists $l_j > 0$ such that

$$\|g_j(x) - g_j(y)\|_{C[-v, T]} \leq l_j \|x - y\|_{C[-v, T]}; x, y \in C[-v, T] \quad (497)$$

with $g_j(0) \neq 0$.

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Theorem 215. :Assume that (H1)and (H2) are satisfied. If

$$T^\alpha \sum_{j=1}^n B_j l_j \leq \Gamma(\alpha + 1) \quad (498)$$

then problem (492) and (493) has a unique solution $x \in C([-v, T], R)$.

Stability results

In this section, we study the continuous dependence upon the data of the initial function $\psi(t)$ and Ulam stability of the problem (492) and (493).

Theorem 216. Under the conditions of Theorem (215), the unique solution of (492) and (493) depends continuously on function $\psi(t)$.

Theorem 217. If problem (492) and (493) has a unique solution, then it is Ulam–Hyers stable

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Application and Efficacy of the Generalized Kudryashov Method for Exact Traveling Wave Solutions of Some Non-linear Evolution Equations

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Abstract: This study delves into the practicality and efficiency of the generalized Kudryashov method, a widely-employed technique for deriving exact traveling wave solutions in both physical and mathematical sciences. Specifically, we apply this method to two prominent systems: the Couple Boiti-Leon-Pempinelli equations system and the Drinfeld–Sokolov–Satsuma–Hirota equation. Our analysis underscores the method’s efficacy in efficiently handling nonlinear evolution equations (NLEEs). Additionally, we demonstrate that specific parameter values can lead to solitary wave solutions derived from these exact traveling wave solutions. These newly obtained solutions possess significant implications across various domains, particularly in physics and applied sciences. Visualizing the obtained analytic solutions using Maple software reveals a diverse range of 2D, 3D, contour, and density graphical patterns, thereby facilitating a deeper understanding of physical phenomena. Our findings affirm that the proposed method is not only effective but also straightforward and adaptable to various types of nonlinear partial differential equation systems.

Keywords: The generalized Kudryashov method, the Couple Boiti-Leon-Pempinelli equations system, the Drinfeld–Sokolov–Satsuma–Hirota equation, traveling wave solution.

2020 Mathematics Subject Classification: 35C07, 35C08,

Introduction

Nonlinear evolution equations (NLEEs) are prevalent in various scientific disciplines, including physics and mathematics. Finding exact solutions to these equations is often challenging but crucial for understanding underlying physical phenomena and predicting real-world behavior. The generalized Kudryashov method has emerged as a promising approach for deriving exact traveling wave solutions in NLEEs. In this study, we investigate the practicality and efficiency of this method, focusing on its application to the Couple Boiti-Leon-Pempinelli equations system and the Drinfeld–Sokolov–Satsuma–Hirota equation.

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Methodology

We begin by introducing the generalized Kudryashov method and its underlying principles. Subsequently, we apply this method to the aforementioned systems of NLEEs, aiming to derive exact traveling wave solutions. We explore the behavior of these solutions under different parameter values and assess their significance in understanding physical phenomena. Additionally, we utilize Maple software to visualize the obtained analytic solutions, employing various graphical techniques such as 2D plots, 3D plots, contour plots, and density plots.

Results and Discussion

Our analysis reveals that the generalized Kudryashov method proves to be highly effective in handling NLEEs, yielding exact traveling wave solutions for the studied systems. Notably, we find that specific parameter values lead to the emergence of solitary wave solutions from these exact solutions. These solitary wave solutions carry significant implications across multiple domains, including physics and applied sciences. Moreover, the visualization of analytic solutions using Maple software provides valuable insights into the underlying physical phenomena, enhancing our understanding of the studied systems.

Conclusion

In conclusion, our study demonstrates the practicality and efficiency of the generalized Kudryashov method for deriving exact traveling wave solutions of nonlinear evolution equations. Through its application to prominent systems such as the Couple Boiti-Leon-Pempinelli equations system and the Drinfeld–Sokolov–Satsuma–Hirota equation, we highlight the method's effectiveness in handling NLEEs and its ability to unveil solitary wave solutions under specific parameter conditions. The visualization of obtained analytic solutions using Maple software further enhances the comprehensibility of the results. Overall, our findings affirm the utility of the proposed method and its potential for application in various scientific disciplines.

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Optimal energy decay for a viscoelastic equation with distributed delay acting on nonlinear frictional damping

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Abstract: In this work, we have analyzed the influence of viscoelastic and frictional damping on the decay rate of solutions for a viscoelastic wave equation with a distributed delay acting on nonlinear internal damping with the Dirichlet boundary condition. Taking the relaxation function of a fairly large class and using the method of energy in which we introduce an adapted Lyapunov functional and by exploiting certain properties of convex functions, under certain assumptions on the constants of system, we obtain the optimal decay rate of energy in the sense that it is compatible with the decay rate of the relaxation function.

Keywords: viscoelastic wave equation, frictional damping, nonlinear distributed delay, optimal decay rate

2020 Mathematics Subject Classification: 35A01 35B35, 35G16.

Introduction

Time-delay systems appear frequently in engineering, they are a source of instability and degraded performance, and they greatly growing the difficulty of stability analysis and control of dynamical systems. Consequently, time-delay systems have attracted the attention of many authors and have become an active area of research in recent years. In this presentation, we would like to study the following viscoelastic wave equation with a distributed delay term acting on a non-linear internal damping

$$\begin{cases} u_{tt} - \Delta u + h * \Delta u + a_1 g_1(u_t) + a_2 \int_{\kappa_1}^{\kappa_2} \mu(s) g_2(u_t(x, t - \tau(s, t))) ds = 0 & \text{in } \Omega \times \mathbb{R}_+, \\ u = 0 & \text{on } \partial\Omega \times \mathbb{R}_+, \\ u(x, 0) = u_0(x), \quad u_t(x, 0) = u_1(x) & \text{in } \Omega, \\ u_t(x, -t) = f_0(x, -t) & \text{in } \Omega \times (0, \tau_2). \end{cases} \quad (499)$$

where Ω is a bounded domain in $\mathbb{R}^n$, $n \in \mathbb{N}^*$, with smooth boundary $\partial\Omega$, and $*$ means the usual convolution product $(h * \varphi)(t) = \int_0^t h(t-s)\varphi(s)ds$, the kernel h is a positive non-increasing function, and it is considered under certain general conditions in order to cover a rather large class of relaxation functions. a_1 and a_2 are positive constants satisfying some hypotheses. The functions g_1 and g_2 are satisfying some assumptions, $\mu : (\kappa_1, \kappa_2) \rightarrow \mathbb{R}$ is an L^∞ function, κ_1 and κ_2 are two

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real numbers satisfying $0 \leq \kappa_1 < \kappa_2$, τ is a positive function defined on $(\kappa_1, \kappa_2) \times \mathbb{R}_+$ and check some assumptions. Moreover, the initial data (u_0, u_1, f_0) belong to a suitable function space.

Our objective is to see the influence of frictional and viscoelastic damping on the decay rate of the solutions of a viscoelastic wave equation with a distributed delay in the general forme $\int_{\kappa_1}^{\kappa_2} \mu(s) g_2(u_t(x, t - \tau(s, t))) ds$ acting on the non-linear internal damping. To stabilise the system (499), we consider appropriate assumptions on the initial data, the coefficient a_1 and a_2 and on the function tau. The relaxation functions h of a more general type. We prove the main result by exploiting some techniques of convex functions and using the energy method, where we introduce an appropriate Lyapunov function.

Main results

In the following proposition, under necessary hypotheses on the data of the problem (see [1]), we state a well-posedness result of our problem, which can be established by using the standard Faedo Galerkin method (see for example: [2, 3, 4]).

Proposition 29. *Let $u_0 \in H^2(\Omega) \cap H_0^1(\Omega)$, $u_1 \in H_0^1(\Omega)$, $f_0 \in H_0^1(\Omega, H^1(0, 1))$ satisfy the compatibility condition $f_0(\cdot, 0) = u_1$. Under some hypotheses hold. Then for any $T > 0$, the problem (499) has a weak solution*

$$u \in L^\infty(0, T; H^2(\Omega) \cap H_0^1(\Omega)), u_t \in L^\infty(0, T; H_0^1(\Omega)), u_{tt} \in L^\infty(0, T; L^2(\Omega)).$$

The modified energy to the problem (499) is defined by

$$\begin{aligned} E(t) = & \frac{1}{2} \|u_t\|^2 + \frac{1}{2} \left(1 - \int_0^t h(\sigma) d\sigma \right) \|\nabla u\|^2 + \frac{1}{2} (h \square \nabla u)(t) \\ & + a_2 \int_\Omega \int_0^1 \int_{\kappa_1}^{\kappa_2} \tau(s, t) (|\mu(s)| + \xi) G(z(x, \rho, s, t)) ds d\rho dx, \end{aligned} \quad (500)$$

where

$$(h \square \nabla u)(t) = \int_0^t h(t - \sigma) \|\nabla u(t) - \nabla u(\sigma)\|_2^2 ds.$$

We will fix ξ such that

$$0 < \xi < \frac{a_1 - 2\alpha_5 a_2 \int_{\kappa_1}^{\kappa_2} |\mu(s)| ds}{\alpha_5 a_2 (\kappa_2 - \kappa_1)}. \quad (501)$$

General decay

For prove the main result, we check that the energy of system (499) is decreasing and we consider the following Lyapunov functional

$$\begin{aligned} L(t) = & NE(t) + \varepsilon_1 \int_\Omega u_t u dx - \varepsilon_2 \int_\Omega u_t \int_0^t h(t - \sigma) (u(t) - u(\sigma)) d\sigma dx \\ & + \varepsilon_3 \int_\Omega \int_0^1 \int_{\kappa_1}^{\kappa_2} \tau(s, t) e^{-\tau(s, t)\rho} (|\mu(s)| + \xi) G(z(x, \rho, s, t)) ds d\rho dx, \end{aligned}$$

where $N, \varepsilon_1, \varepsilon_2$ and ε_3 are positive constants to be chosen later. Now, we are going to see that the Lyapunov functional equivalent to the energy functional.

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Proposition 30. *The functionals $E(t)$ and $L(t)$ satisfy, for some positive constants ς_1 and ς_2 ,*

$$\varsigma_1 E(t) \leq L(t) \leq \varsigma_2 E(t), \quad \forall t \geq 0, \quad (502)$$

for N large enough.

By derivation of the Lyapunov functional, we obtain the following inequality.

Lemma 29. *The functional $L(t)$ satisfies*

$$L'(t) \leq -\omega E(t) - 4(1 - \eta) \|\nabla u\|_2^2 + c \|g_1(u_t)\|_2^2 + \frac{1}{4} (h \square \nabla u)(t), \quad \forall t \geq t_1. \quad (503)$$

Now, by using formula (503), and by exploiting some properties of convex functions, we are able to prove the main result of this work (see [1]).

Theorem 218. *Under some hypotheses hold. Then, there exist strictly positive constants k_1 and k_2 such that the energy of (499) satisfies*

$$E(t) \leq k_2 H_1^{-1} \left(k_1 \int_0^t \zeta(s) ds \right) \quad \text{for all } t \geq 0, \quad (504)$$

where

$$H_1(t) = \int_t^r \frac{1}{sH'(s)} ds.$$

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Geometry Solutions of a New Class of Nonlinear Planar Differential systems with as Solution a Cubic Algebraic Curve

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Abstract: In this paper, we classify the global phase portraits in the Poincaré disc of all quadratic polynomial differential systems having a classical cubic algebraic Anguinea as invariant algebraic curve.

Keywords: Quadratic differential system, Poincaré disc, Invariant algebraic curve.

2020 Mathematics Subject Classification: Primary 34C29, 34C25, 47H11.

Introduction

We consider two-dimensional vector fields is given by

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y), \quad (505)$$

where P and Q are polynomials of $\mathbb{R}[x, y]$ ($\mathbb{R}[x, y]$ denotes the ring of polynomials in the variables x and y with real coefficients). We recall that the degree of the differential system (505) is given by $m = \max(\deg\{P\}, \deg\{Q\})$. An algebraic curve $H(x; y) = 0$ is called an invariant curve for (505) if and only if there exists a cofactor $k(x; y)$ which is a polynomial satisfying

$P(x; y)\frac{\partial f}{\partial x} + Q(x; y)\frac{\partial f}{\partial y} = k(x; y)f(x; y)$ where k is a polynomial in x and y . For more details on the invariant curves of a polynomial differential system see Chapter 8 of [5].

The quadratic differential systems or (simply **QS**) which are the differential systems of the form

$$\begin{aligned} \dot{x} &= a_{00} + a_{10}x + a_{01}y + a_{20}x^2 + a_{11}xy + a_{02}y^2, \\ \dot{y} &= b_{00} + b_{10}x + b_{01}y + b_{20}x^2 + b_{11}xy + b_{02}y^2. \end{aligned} \quad (506)$$

Here the dot denotes the derivative with respect to an independent variable t , usually called time. Appear frequently in many areas of applied mathematics, electrical circuits, astrophysics, in population dynamics, chemistry, neural networks, laser physics, hydrodynamics, etc. Although these differential systems are the simplest nonlinear polynomial systems, they are also important as a basic testing ground for the general theory of the nonlinear differential systems. There are more

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Figure 22: Global phase portraits of differential system (507).

than one thousand papers published on QS. The difficulty of studying these differential systems is due to the fact that they depend on twelve parameters.

The present article is to classify all qualitative dynamics in the Poincaré disc of all **QS** which have the cubic *Anguinea* invariant algebraic curve of degree 3, by examining its phase portraits in the Poincaré disc, it appears this yields 5 topologically different phase portraits.

Main results

The next theorems characterize the main results

Theorem 219. *Qualitative dynamics of the QS exhibiting Anguinea invariant curve of degree 3 $H(x, y) = y(x^2 + d^2) - adx = 0$, $ad \neq 0$, with its associated cofactor $k(x, y) = x + (2d + a)y$, is topologically equivalent to the phase portraits of the following systems:*

$$\begin{aligned}\dot{x} &= d^2(c - 1) + cx^2, \\ \dot{y} &= ad(c - 1) + (1 - 2c)xy + (2dy^2)/a.\end{aligned}\tag{507}$$

Remark 9. *System (507) is invariant under the changes $(x, y, t, a, d, c) \rightarrow (-x, y, -t, a, -d, c)$ and $(x, y, t, a, d, c) \rightarrow (x, -y, t, -a, d, c)$, then we only need to study this systems for $d > 0$ and $a > 0$.*

Theorem 220. *Qualitative dynamics of QS (507) in the Poincaré disc are:*

- *system (507) when $c \in (0, 1/3) \cup (1/3, 1)$ and $ad \neq 0$ has the phase portrait 1 in figure 22.*
- *system (507) when $c = 1/3$ and $ad \neq 0$ has the phase portrait 2 in figure 22.*
- *system (507) when $c = 0$ and $ad \neq 0$ has the phase portrait 3 in figure 22.*
- *system (507) when $c = 1$ and $ad \neq 0$ has the phase portrait 4 in figure 22.*
- *system (507) when $c \in (-\infty, 0) \cup (1, \infty)$ and $ad \neq 0$ has the phase portrait 5 in figure 22.*

Finite and infinite singularities

This section is dedicated to the analysis of the singular points (or simply **SP**) of all quadratic systems 507.

Proposition 31. *For the finite SP of systems (507) the following statements hold.*

- (1) *If $c \in (0, 1)$ systems (507) has four hyperbolic singularities, a stable node at $q_1 = (-d\sqrt{(1-c)/c}, -a\sqrt{c(1-c)})$, an unstable node at $q_2 = (d\sqrt{(1-c)/c}, a\sqrt{c(1-c)})$, and two saddles at $q_3 = (-d\sqrt{(1-c)/c}, (a\sqrt{(1-c)/c})/2)$ and $q_4 = (d\sqrt{(1-c)/c}, -(a\sqrt{(1-c)/c})/2)$.*

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- (2) If $c = 1$ it has one finite linearly zero **SP** at $(0, 0)$ and by doing a blow-up, we know that its local phase portrait is formed by two parabolic and two hyperbolic sectors.
- (3) If $c \neq 1$ the systems has no finite **SP**.

For the infinite singularities of systems (507) we have the following Proposition

Proposition 32.

- In U_1 systems (507) have two **SP**; $p_1 = (0, 0)$ which is an unstable node when $c < 0$, a hyperbolic saddle if $0 < c < \frac{1}{3}$, an hyperbolic stable node if $c > \frac{1}{3}$, a semi-hyperbolic saddle-node if $\frac{1}{3}$ and a semi-hyperbolic unstable node when $c = 0$; the seconde singularity at $p_2 = (a(3c - 1)/(2d), 0)$ which is a saddle if $c \in (-\infty, 0) \cup (\frac{1}{3}, \infty)$, a stable node if $c \in (0, \frac{1}{3})$, a semi-hyperbolic saddle-node if $c = \frac{1}{3}$ and a semi-hyperbolic saddle if $c = 0$.
- In U_2 systems (507) have an hyperbolic stable node for all $c \in \mathbb{R}$ at $(0, 0)$.

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Investigation of nonlinear fractional differential equation with integro-multipoint boundary conditions

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Abstract: We investigated the existence and uniqueness of solutions to a nonlinear fractional differential equation containing the Caputo operator supplemented with separated multipoint and multi-term integral boundary conditions. The existence results are obtained by utilizing the Banach contraction principle and the well-known fixed point theorem of Krasnoselskii for a sum of contraction and compact mappings in Banach spaces. Some examples are also presented to illustrate the main results.

Keywords: Caputo fractional derivative, Krasnoselskii's fixed point theorem, solution, Banach contraction principle, compact.

2020 Mathematics Subject Classification: Primary 34A08 34A34 34B10

Introduction

In recent years, the interest in the study of fractional differential equations has been growing rapidly due their importance since they provided valuable tools for their applications in various sciences, such as gas dynamics, nuclear physics, electrodynamics of complex medium, polymer rheology. With this advantage, fractional order models are regarded as more realistic and practical. In this paper, we discuss the existence and uniqueness of positive solutions for the following nonlinear fractional differential equation

$${}^c D_{0+}^{\alpha} u(t) + f(t, u(t), u'(t)) = 0, 0 \leq t \leq 1, \quad (508)$$

supplemented with separated multipoint and multi-term integral boundary conditions

$$\begin{cases} u^{(i)}(0) = 0, i = 2, \dots, n-1, \\ u'(0) = \sum_{i=1}^{m-2} b_i u'(\xi_i), u(1) = \sum_{i=1}^{m-2} a_i \int_{\eta_{i-1}}^{\eta_i} u(s) \end{cases} \quad (509)$$

where ${}^c D_{0+}^{\alpha}$ is the Caputo fractional derivatives, $n-1 < \alpha \leq n, n \geq 3$, is an integer, $0 = \eta_0 < \eta_1 < \eta_2 < \dots < \eta_{m-2} < 1$, $0 = \xi_0 < \xi_1 < \xi_2 < \dots < \xi_{m-2} < 1$, $a_i \geq 0$, $b_i \geq 0$, ($i=1, \dots, m-2$), $0 \leq \sum_{i=1}^{m-2} b_i < 1$ and $0 \leq \sum_{i=1}^{m-2} a_i(\eta_i - \eta_{i-1}) < 1$, where $m > 2$ is an integer, and $f : [0, 1] \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ is continuous.

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Main results

Lemma 30. For $y \in [0, T]$, the following boundary value problem

$$\begin{cases} D_{0+}^{\alpha} u(t) + y(t) = 0, t \in (0, 1), \\ u^{(i)}(0) = 0, i = 2, \dots, n-1, \\ u'(0) = \sum_{i=1}^{m-2} b_i u'_i(\xi_i), u(1) = \sum_{i=1}^{m-2} a_i \int_{\eta_{i-1}}^{\eta_i} u(s) ds, \end{cases} \quad (510)$$

has the unique solution

$$u(t) = c_0 + c_1(t) - \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} y(s) ds,$$

wer

$$\begin{aligned} c_0 &= \frac{\int_0^1 (1-s)^{\alpha-1} y(s) ds}{(1 - \sum_{i=1}^{m-2} a_i (\eta_i - \eta_{i-1})) \Gamma(\alpha)} \\ &\quad - \frac{\sum_{i=1}^{m-2} a_i [\int_0^{\eta_i} (\eta_i - s)^{\alpha} - \int_0^{\eta_{i-1}} (\eta_{i-1} - s)^{\alpha}] y(s) ds}{(1 - \sum_{i=1}^{m-2} a_i (\eta_i - \eta_{i-1})) \Gamma(\alpha + 1)} \\ &\quad + \frac{(1 - \sum_{i=1}^{m-2} a_i \frac{\eta_i^2 - \eta_{i-1}^2}{2}) \sum_{i=1}^{m-2} b_i \int_0^{\xi_i} (\xi_i - s)^{\alpha-2} y(s) ds}{(1 - \sum_{i=1}^{m-2} a_i (\eta_i - \eta_{i-1})) (1 - \sum_{i=1}^{m-2} b_i) \Gamma(\alpha - 1)} \\ c_1 &= - \frac{\sum_{i=1}^{m-2} b_i \int_0^{\xi_i} (\xi_i - s)^{\alpha-2} y(s) ds}{(1 - \sum_{i=1}^{m-2} b_i) \Gamma(\alpha - 1)}. \end{aligned}$$

Throughout the paper, we let

$$\begin{aligned} M_1 &= \frac{1}{(1 - \sum_{i=1}^{m-2} a_i (\eta_i - \eta_{i-1})) \Gamma(\alpha + 1)} + \frac{\sum_{i=1}^{m-2} a_i (\eta_i^{\alpha+1} - \eta_{i-1}^{\alpha+1})}{(1 - \sum_{i=1}^{m-2} a_i (\eta_i - \eta_{i-1})) \Gamma(\alpha + 2)} + \frac{(1 - \sum_{i=1}^{m-2} a_i \frac{\eta_i^2 - \eta_{i-1}^2}{2}) \sum_{i=1}^{m-2} b_i \xi_i^{\alpha-1}}{(1 - \sum_{i=1}^{m-2} a_i (\eta_i - \eta_{i-1})) (1 - \sum_{i=1}^{m-2} b_i) \Gamma(\alpha)} \\ &\quad + \frac{\sum_{i=1}^{m-2} b_i \xi_i^{\alpha-1}}{(1 - \sum_{i=1}^{m-2} b_i) \Gamma(\alpha)} + \frac{1}{\Gamma(\alpha + 1)}, \\ M_2 &= \frac{\sum_{i=1}^{m-2} b_i \xi_i^{\alpha-1}}{(1 - \sum_{i=1}^{m-2} b_i) \Gamma(\alpha)} + \frac{1}{\Gamma(\alpha)} \end{aligned}$$

Theorem 221. Assume (C_1) there exists constant $K > 0$ such that

$$|f(t, u, v) - f(t, \bar{u}, \bar{v})| \leq K(|u - v| + |\bar{u} - \bar{v}|), \text{ for any } u, v, \bar{u}, \bar{v} \in \mathbb{R}, t \in [0, 1]$$

If

$$K(M_1 + M_2) < 1,$$

then the problem (508) – (509) has a unique solution on $[0, 1]$.

Theorem 222. (Krasnoselskii's fixed point theorem) Let M be a closed convex and nonempty subset of a Banach space E . Let A, B be the operators such that:

- (i) $Au + Bv \in M$ whenever $u, v \in M$,
- (ii) A is compact and continuous,
- (iii) B is a contraction mapping. Then there exists $w \in M$ such that $w \in Aw + Bw$.

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Theorem 223. Assume that (C_1) and the following hypothesis holds (C_2) There exists $\mu \in L^1([0, 1] \times \mathbb{R}^+)$ such that

$\|f(t, u(t), v(t))\| \leq \mu(t)$, for all $(t, u, v) \in [0, 1] \times X \times X$.

If

$$K \left(M_1 + M_2 - \frac{\alpha + 1}{\Gamma(\alpha + 1)} \right) < 1.$$

Then, the boundary value problem (508) – (509) has at least one solution on $[0, 1]$.

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Existence results for a fourth order differential boundary value problem at two points

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Abstract:

In this work, we discuss the existence of nonnegative solutions for a fully fourth order differential system with boundary conditions at two points, where the considered nonlinearity $f : [0, 1] \times \mathbb{R}_+^n \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ is assumed continuous and includes derivatives up to order three. We established new conditions that ensures the existence of at least one solution and the existence of at least two nonnegative solutions by using recent fixed point theorem for the operator sum $T + S$ on Banach spaces, firstly by considering T linear and $(I - S)$ compact, and secondly by taking this sum such that T is an expansive operator and S is completely continuous one.

Keywords: Fully fourth-order boundary value problem, fixed point, sum of operators, positive solution, banach space, cone.

2020 Mathematics Subject Classification: 47H10, 34B10.

Introduction

In this work, we investigate the following boundary value problem (BVP)

$$u^{(4)}(t) = f(t, u(t), u'(t), u''(t), u'''(t)), \quad t \in [0, 1], \quad (511)$$

$$u(0) = u(1) = u''(0) = u''(1) = 0, \quad (512)$$

where $f = (f_1, f_2, \dots, f_n) \in \mathcal{C}([0, 1] \times \mathbb{R}_+^n \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n, \mathbb{R}^n)$

In the case where $n = 1$, the boundary value problem (511)-(512) describes the deformation of an elastic beam in an equilibrium state, whose two ends are simply supported, the study of this problem is very interesting due to its importance in physics where only positive solution is significant. We will use a recent fixed point theorem for the sum of two operators $T + S$ on cones of Banach spaces. Firstly, we obtained the existence of at least one solution to the problem (511)-(512) using a recent fixed point theorem for the sum of two operators; one of them is required to be compact on an open of a closed, convex set of a Banach space. Secondly, we discussed the

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existence of multiple nonnegative solutions to the problem (511)-(512) using fixed point theorem for the sum of two operators on cones of a Banach space by taking the sum $T + S$ such that T is an expansive operator and S is completely continuous one. The solutions are localized in a conical hull. Note that our approach for the sum of two operators has been applied recently to various kinds of boundary value problems. see [1, 2, 3, 5, 7], and references therein.

Main results

Assume that we have the following two assumptions:

(H1) $f = (f_1, f_2, \dots, f_n) \in C([0, 1] \times \mathbb{R}_+^n \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n, \mathbb{R}^n)$ is nonnegative and

$$\|f(t, u, v, w, z)\|_{\mathbb{R}^n} \leq a(t) + b_1(t)\|u\|_{\mathbb{R}^n}^{p_1} + b_2(t)\|v\|_{\mathbb{R}^n}^{p_2} + b_3(t)\|w\|_{\mathbb{R}^n}^{p_3} + b_4(t)\|z\|_{\mathbb{R}^n}^{p_4},$$

with $(t, u, v, w, z) \in [0, 1] \times \mathbb{R}_+^n \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n$, $a, b_j \in C([0, 1])$, $0 \leq a, b_j \leq A$ on $[0, 1]$, $j \in \{1, 2, 3, 4\}$, $A > 0$ is a given constant.

$\mathbb{R}_+^n = \{u = (u_1, u_2, \dots, u_n), u_i \geq 0, u_i \in \mathbb{R} \text{ for } i = 1, 2, \dots, n\}$.

(H2) Let $m > 0$ be large enough and A, r, L, R_1 be positive constants that satisfy the following inequalities

$$\begin{cases} r < L < R_1, & R_1 > \left(\frac{1+\|Y\|_{\mathbb{R}^n}}{5m} + 1\right)L, \\ C\left(A\left(1 + R_1^{p_1} + R_1^{p_2} + R_1^{p_3} + R_1^{p_4}\right) + R_1\right) < \frac{L}{5}. \end{cases}$$

Our main existence results are the following

Theorem 224. Suppose that (H1) holds. Then the BVP (511)-(512) has at least one solution in $C^4([0, 1], \mathbb{R}^n)$.

Theorem 225. Suppose that (H1) and (H2) hold. Then the BVP (511)-(512) has at least two nonnegative solutions in $C^4([0, 1], \mathbb{R}^n)$.

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Regularity of the solution of a population dynamics model governed by a differential equation with operator coefficients

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Abstract: In this work we study an elliptic differential equation set in two habitats with skewness boundary conditions at the interface. This problem represents the linear stationary case of dispersal problems of population dynamics which incorporate responses at interfaces between the habitats.

We prove that this operator generates an analytic semigroup in an adapted space of Hölder-continuous functions.

Keywords: population dynamics, semigroup.

2020 Mathematics Subject Classification: Partial differential equations, Dynamical systems and ergodic theory, Operator theory.

Introduction

Our goal in this work is to analyze the analogous situation as [2] in two dimension space. More precisely, we will be concerned with the study of the analyticity of the C_0 -semigroup generated by the dispersal process in two habitats under some skewness condition and continuous dispersal condition at the interface which represent the behavior of the individuals at boundaries. Our problem is reduced to an operational form of the type:

$$\begin{cases} u''(x) - B^2 u(x) = G(x) & \text{on }]-l, 0[\cup]0, L[\\ u_-(-l) = 0 \text{ and } u'_+(L) = 0 \\ \begin{cases} d_- [u''(0^-) + Au_-(0^-)] - r_- u_-(0^-) \\ = d_+ [u''(0^+) + Au_+(0^+)] + r_+ u_+(0^+) \end{cases} \\ (1-p)d_- u'_-(0^-) = p d_+ u'_+(0^+). \end{cases}$$

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The operator A verifies the following ellipticity hypothesis:

$$\rho(A) \supset [0, +\infty[\text{ et } \exists C > 0 : \forall \lambda \geq 0, \|(A - \lambda I)^{-1}\|_{L(E)} \leq \frac{C}{1 + |\lambda|}.$$

Where $\rho(A)$ denotes the resolvent set of A .

The operator $B := -(-A)^{\frac{1}{2}}$ generates an analytic semigroup which makes it possible to find a representation of the solution.

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Fractional Tent map: Bifurcation and chaos

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Abstract: In this work, based on a Tent map and a Caputo-like delta difference operator, the Tent map with fractional order is presented. The dynamic behaviours of this map are studied through the use of bifurcation diagrams and 0-1 test. Finally, we found that dynamic of the proposed fractional order map is affected by both the map parameter and the fractional order.

Keywords: Chaos, fractional, Tent map , Bifurcation

2020 Mathematics Subject Classification: Primary 26A33, 34H10, 39A33.

Introduction

In the introduction to this work, we will introduce some definitions and concepts necessary of discrete fractional calculus. Let $\mathbb{N}_b = \{b, b+1, b+2, \dots\}$ denotes the isolated time scale ($b \in \mathbb{R}$ fixed) [2]. For $\chi(n)$, the delta difference operator Δ is defined as

$$\Delta\chi(n) = \chi(n+1) - \chi(n).$$

Definition 75. [2] Let $\chi : \mathbb{N}_b \longrightarrow \mathbb{R}$ and $v > 0$. Then, the fractional sum of v order is defined by

$$\Delta_b^{-v}\chi(t) = \frac{1}{\Gamma(v)} \sum_{s=b}^{t-v} (t - \sigma(s))^{(v-1)} \chi(s), \quad t \in \mathbb{N}_{b+v},$$

where b is the starting point, $\sigma(s) = s+1$ and $t^{(v)}$ is the falling function defined in terms of the Gamma function as

$$t^{(v)} = \frac{\Gamma(t+1)}{\Gamma(t+1-v)}.$$

Definition 76. [1] For $v > 0$, $v \notin \mathbb{N}$ and $\chi(t)$ defined on $\mathbb{N}_b$, the Caputo-like delta difference is defined by

$${}^c\Delta_b^v\chi(t) = \Delta_b^{-(m-v)}\Delta^m\chi(t) \tag{513}$$

$$= \frac{1}{\Gamma(m-v)} \sum_{s=b}^{t-(m-v)} (t - \sigma(s))^{(m-v-1)} \Delta_s^m\chi(s), \tag{514}$$

where $t \in \mathbb{N}_{b+m-v}$, $m = [v] + 1$.

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Theorem 226. [3] For the delta fractional difference equation

$$\begin{cases} {}^c\Delta_b^v \chi(t) = f(t+v-1, \chi(t+v-1)) \\ \Delta^k \chi(b) = X_k, m = [v] + 1, k = 0, \dots, m-1, \end{cases}$$

the equivalent discrete integral equation can be obtained as

$$\chi(t) = \chi_0(t) + \frac{1}{\Gamma(v)} \sum_{s=b+m-v}^{t-v} (t-\sigma(s))^{(v-1)} \times f(s+v-1, \chi(s+v-1)), t \in \mathbb{N}_{b+m}, \quad (515)$$

where

$$\chi_0(t) = \sum_{k=0}^{m-1} \frac{(t-b)^{(b)}_{k!}}{\Delta^k \chi(k)}.$$

Fractional chaotic Tent map

The Tent map is defined as

$$T_a(x) = a \min\{x, 1-x\}, \quad x \in [0, 1], \quad (516)$$

The first order difference of (516) can be formulated as:

$$x(n+1) = \min\{ax(n), a(1-x(n))\}, \quad (517)$$

$$x(n+1) - x(n) = \min\{ax(n), a(1-x(n))\} - x(n), \quad (518)$$

$$\Delta x(n) = \min\{ax(n) - x(n), a(1-x(n)) - x(n)\}, \quad (519)$$

$$\Delta x(n) = \min\{(1-a)x(n), -(1+a)x(n) + a\}, \quad (520)$$

Using the Caputo-like delta difference with b as the starting point, the fractional-order difference of (520) is given by

$${}^c\Delta_b^v x(t) = \min\{(1-a)x(t+v-1), -(1+a)x(t+v-1) + a\}, \quad (521)$$

where a is the bifurcation parameter and v is the fractional order.

For $0 < v \leq 1$ and $t \in \mathbb{N}_{b+1-v}$. Following Theorem 226, using the discrete kernel function (522):

$$(t-\sigma(s))^{v-1} = \frac{\Gamma(t-s)}{\Gamma(t-s-v+1)}, \quad (522)$$

and assuming that $b = 0$, the numerical formula of fractional map (521) is expressed as:

$$x(n) = x(0) + \frac{1}{\Gamma(v)} \sum_{j=1}^n \frac{\Gamma(n-j+v)}{\Gamma(n-j+1)} \min\{(1-a)x(j-1), -(1+a)x(j-1) + a\}. \quad (523)$$

For $v = 1$, the discrete fractional map (523) can be reduced to the classical one (516).

For the map (523), set the initial condition $x_0 = 0.3$ and $v = 0.5$; the bifurcation diagram is depicted in figure 23.

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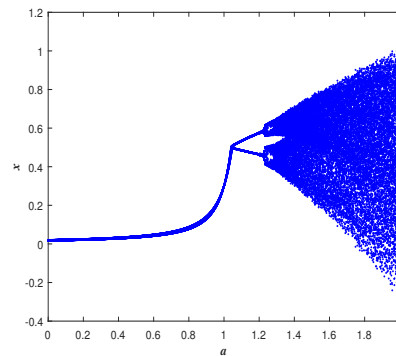


Figure 23: Bifurcation diagram of the fractional Tent map (523) for $v = 0.5$.

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Attractivity of Solutions for First Order Impulsive Integro-Differential Equations

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Abstract: We explore the existence of mild solutions for non-instantaneous integro-differential equations using Grimmer's concept of resolvent operators. Additionally, we demonstrate the attractiveness of solutions to our problem. Lastly, we provide an example to validate the theoretical framework.

Keywords: Attractivity, fixed point theorem, integrodifferential equation, measures of noncompactness, mild solution, resolvent operator.

2020 Mathematics Subject Classification: Primary 34D23, 34D23, 47H10, 45J05, 47H08.

Introduction

In domains such as population dynamics and optimal control, impulsive integral equations, impulsive integro-differential equations, and impulsive differential equations naturally occur (see the monographs [1, 2, 12, 10, 11]). In 1982, Grimmer started by utilizing resolvent operators to demonstrate the existence of integro-differential systems in [8, 9]. The resolvent operator with fixed point technique is the most convenient and appropriate approach for solving integrodifferential equations. Motivated by the above mentioned works, we investigate the existence and attractivity of mild solutions to the following impulsive integrodifferential equations using resolvent operators:

$$\begin{cases} x'(t) = Ax(t) + f(t, x(t), (\mathcal{K}x)(t)) + \int_0^t B(t-s)x(s)ds; \text{ if } t \in \mathbb{I}_k; k = 0, 1, \dots, \\ x(t) = g_k(t, x(t_k^-)); \text{ if } t \in \mathbb{J}_k; k = 1, 2, \dots, \\ x(0) = x_0, \end{cases} \quad (524)$$

where $\mathbb{I}_0 = [0, t_1]$, $\mathbb{I}_k := (s_k, t_{k+1}]$ and $\mathbb{J}_k = (t_k, s_k]$ with $0 = s_0 < t_1 \leq s_1 \leq t_2 < \dots < s_{m-1} \leq t_m \leq s_m \leq t_{m+1} \leq \dots \leq +\infty$, $A : D(A) \subset E \rightarrow E$ is the infinitesimal generator of a strongly continuous semigroup $\{T(t)\}_{t \geq 0}$, $B(t)$ is a closed linear operator with domain $D(A) \subset D(B(t))$, the operator $\mathcal{K}$ is defined by $(\mathcal{K}x)(t) = \int_0^a h(t, s, x(s))ds$, for $a > 0$, $D_h = \{(t, s) \in \mathbb{R}^2; 0 \leq s \leq t \leq a\}$, and $h : D_h \times E \rightarrow E$.

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The nonlinear term $f : \mathbb{I}_k \times E \times E \rightarrow E$; $k = 0, 1, \dots$, $g_k : \mathbb{J}_k \times E \rightarrow E$; $k = 1, 2, \dots$, are a given functions, and $(E, \|\cdot\|)$ is a Banach space. They base their arguments on the fixed point theory and the concept of measure of noncompactness with the help of the resolvent operator.

Now, consider the space

$$PC(\mathbb{R}^+, E) = \left\{ x : \mathbb{R}^+ \rightarrow E : x|_{\mathbb{I}_k} = g_k; k = 1, 2, \dots, x|_{\mathbb{J}_k}; k = 0, 1, \dots, \text{ are continuous, } \right. \\ \left. x(s_k^-), x(s_k^+), x(t_k^-) \text{ and } x(t_k^+) \text{ exist with } x(t_k^-) = x(t_k^+) \right\},$$

and

$$\mathcal{X} = \{x \in PC(\mathbb{R}^+, E) : x \text{ is bounded on } \mathbb{R}^+\},$$

Main results

Definition 77. A function $x \in \mathcal{X}$ is called a mild solution of problem (524) if it satisfies

$$x(t) = \begin{cases} \mathcal{R}(t)x_0 + \int_0^t \mathcal{R}(t-s)f(s, x(s), (\mathcal{K}x)(s))ds; & \text{if } t \in \mathbb{I}_0, \\ \mathcal{R}(t-s_k)[g_k(s_k, x(t_k^-))] + \int_{s_k}^t \mathcal{R}(t-s)f(s, x(s), (\mathcal{K}x)(s))ds; & t \in \mathbb{I}_k, \\ g_k(t, x(t_k^-)), & \text{if } t \in \mathbb{J}_k, \end{cases}$$

The hypotheses:

(H₁) $f : \mathbb{J} \times E \times E \rightarrow E$ is a Carathéodory function and there exist a function $\zeta \in L^1(\mathbb{J}, \mathbb{R}^+)$ and a continuous nondecreasing function $\varphi : \mathbb{J} \rightarrow (0, +\infty)$, such that :

$$\|f(t, u, \bar{u})\| \leq \zeta(t)\varphi(\|u\| + \|\bar{u}\|), \quad \text{for } u, \bar{u} \in E.$$

(H₂) The function $h : D_h \times E \rightarrow E$ is continuous and there exists $c_h > 0$, such that

$$\|h(t, s, u) - h(t, s, \bar{u})\| \leq c_h\|u - \bar{u}\|, \quad \text{for each } (t, s) \in D_h \text{ and } u, \bar{u} \in E.$$

(H₃) $g_k : \mathbb{J}_k \times E \rightarrow E$ are continuous and there exists $\beta_{g_k} \in (0, 1)$; $k \in \mathbb{N}$, such that

$$\|g_k(t, u) - g_k(t, v)\| \leq \beta_{g_k}\|u - v\|, \quad \text{for all } u, v \in E, k = 1, 2, \dots$$

$$\text{and } \max_{k \in \mathbb{N}} \{\beta_{g_k}\} = \beta_{g_k}^*.$$

(H₄) Assume that there exist $\mathcal{M}_{\mathcal{R}} \geq 1$ and $b \geq 0$, such that

$$\|\mathcal{R}(t)\|_{B(E)} \leq \mathcal{M}_{\mathcal{R}}e^{-bt}, \quad \text{and } \lim_{t \rightarrow +\infty} \sup_{t \in \mathbb{J}} \int_0^t e^{-b(t-s)} \zeta(s)ds = 0.$$

Theorem 227. Assume that the conditions (H₁) – (H₄) are satisfied. If $\mathcal{M}_{\mathcal{R}}\beta_{g_k}^* < 1$, then the system (524) has at least one mild solution.

Theorem 228. Suppose that hypotheses (H₁) – (H₄) hold, with

$$\max \left\{ 2\mathcal{M}_{\mathcal{R}}(\|x_0\| + \varphi(K_{\bar{r}}^*)\|\zeta\|_{L^1}), 2\mathcal{M}_{\mathcal{R}}(\beta_{g_k}^* \bar{r} + g_0 + \varphi(K_{\bar{r}}^*)\|\zeta\|_{L^1}) \right\} \leq \bar{r},$$

and $\mathcal{M}_{\mathcal{R}} \max\{\beta_{g_k}^*\} < \frac{1}{2}$, then, the solutions of problem (524) are uniformly locally attractive.

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Exploring Chaos in the Discrete Fractional Gauss Map

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Abstract:

A novel discrete fractional Gauss map is introduced based on the left Caputo discrete delta's definition, incorporating discrete memory. Bifurcation diagrams are presented, and numerical simulations demonstrate its chaotic dynamics.

Keywords: Bifurcation diagram, Chaotic attractor, Fractional, Caputo-like.

2020 Mathematics Subject Classification: 34C23, 65P20, 34F10, 74H65..

Introduction

Fractional calculus, although rooted in a history spanning over 300 years, has experienced increased application across various scientific and engineering domains since the 1990s. Its ability to interpolate behavior between integer-order systems and effectively address memory effects in dynamical systems through fractional-order derivatives has been instrumental. In recent years, discrete fractional calculus has emerged as a focal point of research interest. The discrete fractional difference operator, which captures memory effects, has become a valuable tool for modeling numerous physical phenomena.

This paper aims to explore the chaotic behavior exhibited by fractional difference equations. Specifically, we undertake a numerical analysis of the chaotic dynamics inherent in the discrete Gauss map. This map holds significance as a standard one-dimensional model renowned for its characteristic bifurcation patterns and chaos in integer-order systems. Its versatility is evident, with applications spanning diverse fields from biology to number theory.

Main results

Let $a \in \mathbb{R}$ fixed and let $\mathbb{N}_a = \{a, a+1, a+2, \dots\}$ denotes the isolated time scale. For the function $u(n)$, the delta difference operator Δ is defined as

$$\Delta u(n) = u(n+1) - u(n). \quad (525)$$

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Definition: [1] Let $u : \mathbb{N}_a \rightarrow \mathbb{R}$ and $v > 0$. Then the fractional sum of v order is defined by

$$\Delta_a^{-v} u(t) = \frac{1}{\Gamma(v)} \sum_{s=a}^{t-v} (t - \sigma(s))^{(v-1)} u(s), \quad t \in \mathbb{N}_{a+v}, \quad (526)$$

where a is the starting point, $\sigma(s) = s + 1$ and t^v is the falling function defined in terms of the Gamma function as

$$t^{(v)} = \frac{\Gamma(t+1)}{\Gamma(t+1-v)}. \quad (527)$$

Definition: [2] For $v > 0$, $v \notin \mathbb{N}$ and $u(t)$ defined on $\mathbb{N}_a$, the Caputo-like delta difference is defined by

$${}^c \Delta_a^v u(t) = \Delta_a^{-(m-v)} \Delta^m u(t) \quad (528)$$

$$= \frac{1}{\Gamma(m-v)} \sum_{s=a}^{t-(m-v)} (t - \sigma(s))^{(m-v-1)} \Delta_s^m u(s), \quad (529)$$

where $t \in \mathbb{N}_{a+m-v}$, $m = [v] + 1$.

theorem: [3] For the delta fractional difference equation

$$\begin{cases} {}^c \Delta_a^v u(t) = f(t+v-1, u(t+v-1)) \\ \Delta^k u(a) = u_k, m = [v] + 1, k = 0, \dots, m-1, \end{cases}$$

the equivalent discrete integral equation can be obtained as

$$u(t) = u_0(t) + \frac{1}{\Gamma(v)} \sum_{s=a+m-v}^{t-v} (t - \sigma(s))^{(v-1)} \times f(s+v-1, u(s+v-1)), \quad t \in \mathbb{N}_{a+m}, \quad (530)$$

where

$$u_0(t) = \sum_{k=0}^{m-1} \frac{(t-a)^{(a)}_k}{k!} \Delta^k u(a). \quad (531)$$

The fractional-order version of the proposed map:

Gauss map is defined as

$$f(x) = \exp(-\alpha * x^2) + \beta \quad (532)$$

The first-order difference of (532) can be easily formulated as

Firstly we fix $\alpha = 7.5$

$$\Delta u(n) = \exp(-7.5 * u(n)^2) + \beta - u(n). \quad (533)$$

Using the Caputo-like delta difference with d as the starting point, the fractional-order difference of (533) is given by

$${}^c \Delta_d^v u_n = \exp(-7.5 * u(t-1+v)^2) + \beta - u(t-1+v) \quad (534)$$

assuming that $d=0$, the numerical formulas for fractional map (534) may be obtained as

$$u(n) = u_0 + \frac{1}{\Gamma(v)} \sum_{j=1}^n \frac{\Gamma(n-j+v)}{\Gamma(n-j+1)} (\exp(-7.5 * u(j-1)^2) + \beta - u(j-1)), \quad (535)$$

In order to understand more about the behaviour of fractional map (535), we employ bifurcation diagrams as follows:

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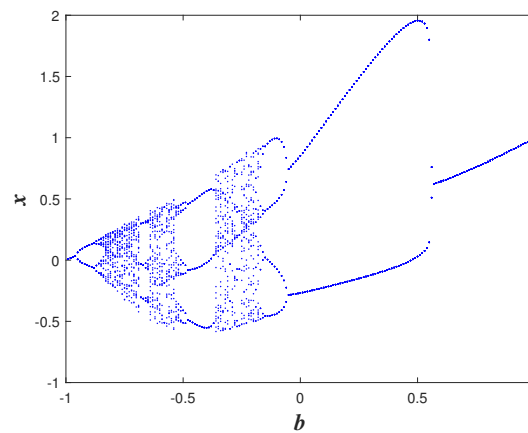


Figure 24: Bifurcation diagram of fractional map (535) .

Figure 24 represents the bifurcation diagram of the Gauss map obtained for $\nu = 0.6$ with initial condition as $u_0 = 0$.

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On Chaos in discrete fractional Financial system: Bifurcation, entropy, 0–1 test

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Abstract: This study describes an innovative discrete-time financial system with incommensurate fractional order. It involves an exploration of the zero equilibrium point stability to demonstrate that the system exhibits chaotic behavior. Furthermore, the non-linear dynamics of this incommensurate fractional discrete financial system are investigated using various numerical techniques, including the Lyapunov exponent, phase portraits and bifurcation diagrams. Finally, an entropy algorithm is used to measure the complexity and 0-1 test confirm the presence of chaos in the proposed financial model.

keywords: discrete fractional calculus, chaos, financial system, complexity.

Keywords: Dynamical systems, Fractional, Chaos

2020 Mathematics Subject Classification: Primary 74Hxx, 90C32, 74H65.

Introduction

In recent years, discrete fractional calculus has received great interest from many researchers due to their memory properties, interest has also been concentrated on fractional difference operators and discrete fractional calculus within the past decade, In comparison to the integer-order derivative, the fractional-order derivative is more exact. Thanks to these unique characteristics, fractional order iterated maps have been deeply studied in academic fields including engineering, physics, and economics, among others. financial models are a useful tool designed to conduct theoretical investigations of financial markets. Chaotic discrete-time system has become an attractive research subject in nearly all kinds of fields Recently, a few studies have reported the dynamics of the fractional forms of some conventional chaotic maps by applying fractional discrete calculus. The main advantage of this type of calculus is the memory effect incurred in the fractional differentiation, which makes for a more flexible modeling of natural phenomena the structure of the work: In Section 1, we define some basic preliminaries to discrete fractional calculus and provide the fractional discrete version of the system. The mathematical model for financial is presented in Section 2. In Section 3, the model's dynamics and the stability of its equilibrium points are explored in the context of the incommensurate instances. In Section 4, the 0-1 test approach and the ApEn algorithm are employed to validate the presence of chaos.we describes the following

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discrete-time system as follows

$$\begin{cases} x_1(s+1) = x_1(s) + \beta(x_3(s) + (x_2(s) - a)x_1(s)), \\ x_2(s+1) = x_2(s) + \beta(1 - bx_2(s) - x_1^2(s)), \\ x_3(s+1) = x_3(s) + \beta(-x_1(s) - cx_3(s)). \end{cases} \quad (536)$$

$\beta \in]0, 1[$ is step size, and a, b, c are positive constant parameters.

Main results

The objective of this paper is to investigate and analyze the chaotic dynamics of a newly proposed incommensurate discrete financial system as follows: The characterization of the incommensurate fractional financial map can be defined as:

$$\begin{cases} {}^c\Delta_t^{\gamma_1} x_1(s) = \beta(x_3(s-1+\gamma_1) + (x_2(s-1+\gamma_1) - a)x_1(s-1+\gamma_1)), \\ {}^c\Delta_t^{\gamma_2} x_2(s) = \beta(1 - bx_2(s-1+\gamma_2) - x_1^2(s-1+\gamma_2)), \\ {}^c\Delta_t^{\gamma_3} x_3(s) = \beta(-x_1(s-1+\gamma_3) - cx_3(s-1+\gamma_3)) \end{cases} \quad (537)$$

where $\gamma_1, \gamma_2, \gamma_3$ is fractional orders. The study revealed that the system exhibits intricate and diverse dynamical characteristics. The incommensurate fractional-order cases are examined using various methods such as plotting phase portraits, creating bifurcation charts, calculating the Lyapunov exponent and using the 0–1 test. Complexity was measured using the ApEn algorithm, which confirmed chaos in the model. In conclusion, the results indicate that the proposed model produces chaotic behaviour with a higher complexity degree and a broader range of chaotic regions when fractional orders are varied.

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Third-Order Nonlinear IBVPs on the Half-Line

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Abstract: In this paper, we study the existence of solution, by using the Schauder fixed point theorem, to the third-order impulsive boundary value problem.

$$\begin{cases} u'''(t) + g(t, u(t), u'(t), u''(t)) = 0, & t \in [0, +\infty) \setminus \{t_1, t_2, \dots\}, \\ u(0) = A, u'(0) = B, & u''(+\infty) = C, \\ \Delta u(t_k) = I_{1k}(t_k, u(t_k), u'(t_k), u''(t_k)), \\ \Delta u'(t_k) = I_{2k}(t_k, u(t_k), u'(t_k), u''(t_k)), \\ \Delta u''(t_k) = I_{3k}(t_k, u(t_k), u'(t_k), u''(t_k)). \end{cases}$$

Keywords: Boundary conditions, differential equations, fixed point, impulsive conditions

2020 Mathematics Subject Classification: Primary 34B37, 47H10.

Introduction

The investigation into third-order Boundary Value Problems (BVPs) over both finite and infinite intervals has emerged as a focal point of research interest in recent years, yielding a rich tapestry of diverse and intriguing findings. This exploration is exemplified in a plethora of studies, a selection of which include references [2, 3, 4, 5, 8]. These problems stand as pivotal constructs in elucidating a myriad of phenomena spanning across disciplines such as mechanics, physics, and beyond. With their intricate nature and profound implications, third-order BVPs serve as indispensable tools for modeling and understanding complex systems and behaviors. This introduction sets the stage for a deeper exploration into the significance and ramifications of these boundary value problems within the realm of scientific inquiry and practical application. In this work, we study the existence of solutions to IBVP on the half-line with the third-order fully differential equation

$$u'''(t) + g(t, u(t), u'(t), u''(t)) = 0, \quad t \in [0, +\infty) \setminus \{t_1, t_2, \dots\}, \quad (538)$$

where $g : [0, +\infty) \times \mathbb{R}^3 \rightarrow \mathbb{R}$ is an L^1 -Carathéodory function, and the boundary conditions are

$$u(0) = A, u'(0) = B, \quad u''(+\infty) = C, \quad (539)$$

$A, B, C \in \mathbb{R}$ and $u''(+\infty) := \lim_{t \rightarrow +\infty} u''(t)$,

and the impulsive condition

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$$\begin{cases} \Delta u(t_k) = I_{1k}(t_k, u(t_k), u'(t_k), u''(t_k)), \\ \Delta u'(t_k) = I_{2k}(t_k, u(t_k), u'(t_k), u''(t_k)), \\ \Delta u''(t_k) = I_{3k}(t_k, u(t_k), u'(t_k), u''(t_k)), \end{cases} \quad (540)$$

$\Delta u^{(i-1)}(t_k) = u^{(i-1)}(t_k^+) - u^{(i-1)}(t_k^-)$, $I_{ik} \in C([0, +\infty) \times \mathbb{R}^3, \mathbb{R})$, $i = 1, 2, 3$ where $k = 1, 2, \dots$ and $0 < t_1 < t_2 < \dots < t_k < \dots$ such that $t_n \rightarrow +\infty$ as $n \rightarrow +\infty$ and $J = [0, +\infty)$, $J' = [0, +\infty) \setminus \{t_1, t_2, \dots\}$. We take $J_0 = [0, t_1]$ and $J_k = (t_k, t_{k+1}]$ for $k \in \mathbb{N}^*$.

Definitions and preliminary results

Let

$$PC^m[0, +\infty) = \{u \in C^m([0, +\infty), \mathbb{R}), \text{ for } t \neq t_k, u^{(i)}(t_k) = u^{(i)}(t_k^-), u^{(i)}(t_k^+) \text{ exist} \\ \text{for } k = 1, 2, 3, \dots, \text{ for } i = 0, 1, \dots, m\}.$$

Consider the space

$$X = \left\{ u \in PC^2[0, +\infty) : \lim_{t \rightarrow +\infty} u''(t) \text{ exists in } \mathbb{R} \right\},$$

and the norm $\|x\|_X := \max\{\|x\|_0, \|x'\|_1, \|x''\|_2\}$, where

$$\|\omega^{(i)}\|_i = \sup_{0 \leq t < +\infty} \left| \frac{\omega^{(i)}(t)}{1 + t^{2-i}} \right|, \quad i = 0, 1, 2.$$

From [3] and [9], we have that $(X, \|\cdot\|_X)$ is a Banach space such that $\lim_{t \rightarrow +\infty} u''(t) = \lim_{t \rightarrow +\infty} \frac{u'(t)}{1+t}$ and $\frac{1}{2} \lim_{t \rightarrow +\infty} u''(t) = \lim_{t \rightarrow +\infty} \frac{u(t)}{1+t^2}$ for all $u \in X$.

We set the next definitions for the nonlinearity and the impulsive conditions.

Definition 78. A function $g : [0, +\infty) \times \mathbb{R}^3 \rightarrow \mathbb{R}$ is called an L^1 -Carathéodory function if it satisfies:

- (i) for each $(x, y, z) \in \mathbb{R}^3$, $t \mapsto g(t, x, y, z)$ is measurable on $[0, +\infty)$;
- (ii) for almost every $t \in [0, +\infty)$, $(x, y, z) \mapsto g(t, x, y, z)$ is continuous in $\mathbb{R}^3$;
- (iii) for each $\rho > 0$, there exists a positive function $\varphi_\rho \in L^1[0, +\infty)$ such that for all $(x(t), y(t), z(t)) \in \mathbb{R}^3$ with $\sup_{0 \leq t < +\infty} \left\{ \left| \frac{x(t)}{1+t^2} \right|, \left| \frac{y(t)}{1+t} \right|, \left| \frac{z(t)}{2} \right| \right\} < \rho$, then

$$|g(t, x(t), y(t), z(t))| \leq \varphi_\rho(t), \quad \text{a.e. } t \in [0, +\infty).$$

Definition 79. A sequence $(w_n)_{n \in \mathbb{N}} : [0, +\infty) \times \mathbb{R}^3 \rightarrow \mathbb{R}$ is a Carathéodory sequence if

- (i) for each $u, v, w \in \mathbb{R}$, $(u, v, w) \rightarrow w_n(t, u, v, w)$ is continuous for all $n \in \mathbb{N}$;
- (ii) for each $\rho > 0$, there are nonnegative constants $\psi_{n,\rho} \geq 0$ with $\sum_{n=1}^{+\infty} \psi_{n,\rho} < +\infty$ such that, for $|u| < \rho(1+t^2)$, $|v| < \rho(1+t)$, $t \in [0, +\infty)$, and $|\frac{w}{2}| < \rho$, we have

$$|w_n(t, u, v, w)| \leq \psi_{n,\rho} \quad \text{for all } n \in \mathbb{N}, t \in [0, +\infty).$$

In the next lemma, we see the solution of the linear impulsive boundary value problem associated with problem (538)-(539)-(540).

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Lemma 31. Let $\eta \in L^1[0, +\infty)$ and $\sum_{k=1}^{+\infty} I_{3k}$ is a Carathéodory sequence. Then the linear impulsive boundary value problem

$$u'''(t) + \eta(t) = 0, \quad t \in J', \quad (541)$$

with boundary conditions (539)-(540), has a unique solution in X . Moreover, this solution can be expressed as

$$\begin{aligned} u(t) = & A + Bt + \frac{t^2}{2}C + \sum_{t_k < t} I_{1k}(u(t_k), u'(t_k), u''(t_k)) + \sum_{t_k < t} I_{2k}(u(t_k), u'(t_k), u''(t_k))(t - t_k) \\ & + \int_0^t (t-s) \left(\int_s^{+\infty} \eta(\tau) d\tau - \sum_{t_k > s} I_{3k}(u(t_k), u'(t_k), u''(t_k)) \right) ds. \end{aligned} \quad (542)$$

Proof Integrating (541) from 0 to t , we have that

$$u''(t) = u''(0) - \int_0^t \eta(s) ds + \sum_{t_k < t} I_{3k}(t_k, u(t_k), u'(t_k), u''(t_k)).$$

Passing to the limit, as $t \rightarrow +\infty$, we obtain

$$u''(t) = C + \int_t^{+\infty} \eta(s) ds - \sum_{t_k > t} I_{3k}(t_k, u(t_k), u'(t_k), u''(t_k)). \quad (543)$$

By repeating the same process as above, we arrive at

$$\begin{aligned} u(t) = & A + Bt + \frac{t^2}{2}C + \sum_{t_k < t} I_{1k}(t_k, u(t_k), u'(t_k), u''(t_k)) + \sum_{t_k < t} I_{2k}(t_k, u(t_k), u'(t_k), u''(t_k))(t - t_k) \\ & + \int_0^t (t-s) \left(\int_s^{+\infty} \eta(\tau) d\tau - \sum_{t_k > s} I_{3k}(t_k, u(t_k), u'(t_k), u''(t_k)) \right) ds. \end{aligned}$$

■

To show the existence of solutions, we see the next two results.

Lemma 32. [1] A set $M \subset X$ is relatively compact if the following three conditions hold:

all functions from M are uniformly bounded, equicontinuous and all functions from M are equiconvergent at infinity.

Theorem 229. [7, Schauder's Fixed Point Theorem] Let E be a Banach space, and $\Omega \subset E$ a nonempty, bounded, closed, and convex subset of E . Let $T : \Omega \rightarrow E$ be a completely continuous operator with $T(\Omega) \subset \Omega$. Then T has a fixed point.

Main result

In this section, we see the existence of solutions to problem (538), (539)-(540).

Theorem 230. Let $g : [0, +\infty) \times \mathbb{R}^3 \rightarrow \mathbb{R}$ be an L^1 -Carathéodory function and $I_{1k}, I_{2k}, I_{3k} : [0, +\infty) \times \mathbb{R}^3 \rightarrow \mathbb{R}$ are Carathéodory sequences with constants $\psi_{1,k,\rho}, \psi_{2,k,\rho}, \psi_{3,k,\rho} \geq 0$ with $\sum_{k=1}^{+\infty} \psi_{i,k,\rho} < +\infty$ for $i = 1, 2, 3$ such that

$$|I_{ik}(t_k, x, y, z)| \leq \psi_{i,k,\rho}, \quad i = 1, 2, 3, \quad (544)$$

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for $|x| < \rho(1+t^2)$, $|y| < \rho(1+t)$, for all $t \in [0, +\infty)$, and $|\frac{z}{2}| < \rho$. If there exist $\rho_1 > 0$ and $M_{\rho_1} > 0$ such that

$$\rho_1 \geq |A| + |B| + |C| + \sum_{k=1}^{+\infty} \psi_{1,k,\rho_1} + \sum_{k=1}^{+\infty} \psi_{2,k,\rho_1} + M_{\rho_1} + \sum_{k=1}^{+\infty} \psi_{3,k,\rho_1}, \quad (545)$$

and for all $u \in X$ satisfying $\|u\|_X < \rho_1$ it is valid

$$\|g(., u, u', u'')\|_{L^1[0, +\infty)} \leq M,$$

then problem (538)-(539)-(540) has at least a solution $u \in X$.

Proof The impulsive boundary value problem (538), (539)-(540) has at least one solution:

Consider the operator $T : X \rightarrow X$ by

$$\begin{aligned} Tu(t) = & A + Bt + \frac{C}{2}t^2 + \sum_{t_k < t} I_{1k}(t_k, u(t_k), u'(t_k), u''(t_k)) + \sum_{t_k < t} I_{2k}(t_k, u(t_k), u'(t_k), u''(t_k))(t - t_k) \\ & + \int_0^t (t-s) \left(\int_s^{+\infty} G_u(\mu) d\mu - \sum_{t_k > s} I_{3k}(t_k, u(t_k), u'(t_k), u''(t_k)) \right) ds \end{aligned}$$

where

$$G_u(s) := g(s, u(s), u'(s), u''(s)),$$

and with $u \in X$ such that $\|u\| < \rho$ there is $0 \leq \varphi_\rho \in L^1[0, +\infty)$, $\int_0^{+\infty} |G_u(s)| ds \leq \int_0^{+\infty} \varphi_\rho(s) ds = M_\rho < +\infty$

By Lemma 31, the fixed points of T are solutions of IBVP (538), (539)-(540).

(i) $T : X \rightarrow X$ is well defined. Let $u \in X$. As g is an L^1 -Carathéodory function, so, for

And, G is well defined on X ,

$$\begin{aligned} \lim_{t \rightarrow +\infty} (Tu)''(t) &= \lim_{t \rightarrow +\infty} \left(C - \sum_{t_k > t} I_{3k}(t_k, u(t_k), u'(t_k), u''(t_k)) \right) = C \\ &= \lim_{t \rightarrow +\infty} \frac{(Tu)'(t)}{1+t} = 2 \lim_{t \rightarrow +\infty} \frac{Tu(t)}{1+t^2}. \end{aligned}$$

So $Tu \in X$.

(ii) T is **continuous**. Let $u_n \rightarrow u$ in X , there exists $\rho > 0$ such that $\sup_n \|u_n\|_X < \rho$,

$$\begin{aligned} |(Tu_n)''(t) - (Tu)''(t)| &\leq \int_0^{+\infty} |G_{u_n}(\mu) - G_u(\mu)| d\mu + \sum_{k=1,2,\dots} |I_{3k}(t_k, u_n(t_k), u'_n(t_k), u''_n(t_k)) \\ &\quad - I_{3k}(t_k, u(t_k), u'(t_k), u''(t_k))|. \end{aligned}$$

By using Lebesgue Dominated Convergence Theorem, we get

$$\int_0^{+\infty} |G_{u_n}(\mu) - G_u(\mu)| d\mu \rightarrow 0,$$

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as $n \rightarrow +\infty$. Moreover,

$$\begin{aligned} & \sum_{k=1,2,\dots} \left| I_{3k}(u_n(t_k), u'_n(t_k), u''_n(t_k)) - I_{3k}(t_k, u(t_k), u'(t_k), u''(t_k)) \right| \\ & \leq 2 \sum_{k=1}^{\infty} \psi_{3,k,\rho} < +\infty \end{aligned}$$

By the dominated Convergence Theorem for series, we have

$$\sum_{k=1,2,\dots} \left| I_{3k}(u_n(t_k), u'_n(t_k), u''_n(t_k)) - I_{3k}(t_k, u(t_k), u'(t_k), u''(t_k)) \right| \rightarrow 0,$$

as $n \rightarrow +\infty$. So, $\|(Tu_n)'' - (Tu)''\|_2 \rightarrow 0$, as $n \rightarrow +\infty$. Also,

$$\begin{aligned} & \sup_{0 \leq t < +\infty} \left| \frac{(Tu_n)'(t)}{1+t} - \frac{(Tu)'(t)}{1+t} \right| \\ & \leq \sup_{0 \leq t < +\infty} \frac{1}{1+t} \sum_{k=1,2,\dots} \left| I_{2k}(t_k, u_n(t_k), u'_n(t_k), u''_n(t_k)) - I_{2k}(t_k, u(t_k), u'(t_k), u''(t_k)) \right| \\ & + \sup_{0 \leq t < +\infty} \frac{2}{1+t} \int_0^t \|(Tu_n)'' - (Tu)''\|_2 ds \rightarrow 0, \end{aligned}$$

as $n \rightarrow +\infty$.

$$\begin{aligned} \sup_{0 \leq t < +\infty} \left| \frac{(Tu_n)(t)}{1+t^2} - \frac{(Tu)(t)}{1+t^2} \right| & \leq \sum_{k=1,2,\dots} \left| I_{1k}(t_k, u_n(t_k), u'_n(t_k), u''_n(t_k)) - I_{1k}(t_k, u(t_k), u'(t_k), u''(t_k)) \right| \\ & + \sum_{k=1,2,\dots} \left| I_{2k}(t_k, u_n(t_k), u'_n(t_k), u''_n(t_k)) - I_{2k}(t_k, u(t_k), u'(t_k), u''(t_k)) \right| \\ & + 2\|(Tu_n)'' - (Tu)''\|_2 \rightarrow 0, \end{aligned}$$

as $n \rightarrow +\infty$.

(iii) T is **compact**. Let $U \subset X$ be any bounded subset, there is $\rho > 0$ such that $\|u\|_X < \rho$ for all $u \in U$ and

$$L_\rho = M\rho + \sum_{k=1}^{\infty} \psi_{3,k,\rho}.$$

For each $u \in U$, one has

$$\begin{aligned} \|Tu\|_0 & \leq |A| + |B| + |C| + \left| \sum_{t_k < t} I_{1k}(t_k, u(t_k), u'(t_k), u''(t_k)) \right| + \left| \sum_{t_k < t} I_{2k}(t_k, u(t_k), u'(t_k), u''(t_k)) \right| \\ & + \sup_{0 \leq t < +\infty} \int_0^t \frac{t-s}{1+t^2} L_{\rho_1} ds \leq |A| + |B| + \sum_{k=1}^{\infty} \psi_{1,k,\rho} + \sum_{k=1}^{\infty} \psi_{2,k,\rho} + \frac{L_\rho}{2} < +\infty, \end{aligned}$$

$\|(Tu)\|_0 < +\infty$. Also,

$$\begin{aligned} \|(Tu)'\|_1 & \leq |B| + |C| + \left| \sum_{t_k < t} I_{2k}(t_k, u(t_k), u'(t_k), u''(t_k)) \right| + \sup_{0 \leq t < +\infty} \int_0^t \frac{1}{1+t} L_{\rho_1} ds \\ & \leq |B| + |C| + \sum_{k=1}^{\infty} \psi_{2,k,\rho} + L_\rho < +\infty, \end{aligned}$$

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$\|(Tu)'\|_1 < +\infty$. Also,

$$\|(Tu)''\|_2 = \frac{1}{2} \sup_{0 \leq t < +\infty} \left| \left(\int_t^{+\infty} G_u(\mu) d\mu - \sum_{t_k > t} I_{3k}(t_k, u(t_k), u'(t_k), u''(t_k)) \right) \right| \leq L_\rho < +\infty,$$

$\|(Tu)''\|_2 < +\infty$. And, therefore,

$$\|Tu\|_X = \max \{ \|Tu\|_0, \|(Tu)'\|_1, \|(Tu)''\|_2 \} < +\infty,$$

that is, TU is uniformly bounded. By (545), if $U \subset X$ is the closed bounded ball of radius $r = \rho_1$ and center 0, then we have $TU \subset U$.

TU is equicontinuous, for $L > 0$ and $t_1, t_2 \in [0, L] \cap J_k$ for $k \in \mathbb{N}$, $t_1 < t_2$, we get

$$\left| (Tu)''(t_2) - (Tu)''(t_1) \right| \leq \left| - \int_{t_1}^{t_2} G_u(\mu) d\mu + \sum_{t_k > t_2 > t_1} I_{3k}(t_k, u(t_k), u'(t_k), u''(t_k)) \right| \rightarrow 0,$$

as $t_1 \rightarrow t_2$, then $\left| (Tu)''(t_2) - (Tu)''(t_1) \right| \rightarrow 0$, as $t_1 \rightarrow t_2$. Moreover,

$$\begin{aligned} \left| \frac{(Tu)'(t_2)}{1+t_2} - \frac{(Tu)'(t_1)}{1+t_1} \right| &\leq \left| \frac{1}{1+t_2} - \frac{1}{1+t_1} \right| \left\| B + \sum_{t_k < t_1} I_{2k}(t_k, u(t_k), u'(t_k), u''(t_k)) \right\| \\ &\quad + \frac{1}{1+t_2} \sum_{t_1 < t_k < t_2} \left| I_{2k}(t_k, u(t_k), u'(t_k), u''(t_k)) \right| + \left| \frac{t_2}{1+t_2} - \frac{t_1}{1+t_1} \right| \left\| C \right\| \\ &\quad + \int_0^{t_1} \left| \frac{1}{1+t_2} - \frac{1}{1+t_1} \right| L_\rho ds + \int_{t_1}^{t_2} \left| \frac{1}{1+t_2} \right| L_\rho ds \rightarrow 0, \end{aligned}$$

as $t_1 \rightarrow t_2$.

By same technique we get

$$\left| \frac{Tu(t_2)}{1+t_2^2} - \frac{Tu(t_1)}{1+t_1^2} \right| \rightarrow 0,$$

as $t_1 \rightarrow t_2$.

Moreover, $TU \subset X$ is equiconvergent at infinity, we use (544) and L^1 -Carathéodory function G . for all $u \in U$, $\lim_{t \rightarrow +\infty} (Tu)''(t) = C$, then, $\lim_{t \rightarrow +\infty} \frac{(Tu)'(t)}{1+t} = C$ and $\lim_{t \rightarrow +\infty} \frac{(Tu)(t)}{1+t^2} = \frac{C}{2}$

$$\begin{aligned} &\left| (Tu)''(t) - C \right| \\ &= \left| \int_t^{+\infty} G_u(\mu) d\mu - \sum_{t_k > t} I_{3k}(t_k, u(t_k), u'(t_k), u''(t_k)) \right| \rightarrow 0, \end{aligned}$$

as $t \rightarrow +\infty$. For the first derivatives,

$$\begin{aligned} \lim_{t \rightarrow +\infty} \left| \frac{(Tu)'(t)}{1+t} - C \right| &= \lim_{t \rightarrow +\infty} \left| \frac{1}{1+t} \left(B + \sum_{t_k < t} I_{2k}(t_k, u(t_k), u'(t_k), u''(t_k)) + Ct \right) \right. \\ &\quad \left. + \left(\int_t^{+\infty} G_u(\mu) d\mu - \sum_{t_k > t} I_{3k}(t_k, u(t_k), u'(t_k), u''(t_k)) \right) - C \right| = 0. \end{aligned}$$

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By the same way,

$$\left| \frac{Tu(t)}{1+t^2} - \frac{C}{2} \right| \rightarrow 0,$$

as $t \rightarrow +\infty$.

From Lemma 32, the set TU is relatively compact. As T is completely continuous. By Schauder's Fixed Point Theorem, T has at least one fixed point $u \in X$. ■

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Lyapunov's Method for Stability Analysis of Nonlinear Fractional Differential Equations with Incommensurate Orders

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Abstract: This paper introduces new criteria to evaluate stability in nonlinear fractional-order systems employing the Caputo differential operator. Through the Lyapunov function method, it provides rigorous proofs for these criteria. Numerical verification is conducted to confirm the stability of the systems, validating the theoretical results.

Keywords: Incommensurate order, Caputo differential operator.

2020 Mathematics Subject Classification: 37-00, 37C25, 37C75.

Introduction

In recent years, fractional-order operators have emerged as pivotal tools for enhancing control systems and modeling various real-world phenomena. These systems can broadly be classified into commensurate and incommensurate fractional-order systems. Commensurate systems have identical fractional-order values, while incommensurate systems feature distinct fractional-order values, often necessitating the introduction of auxiliary variables for stability analysis. The stability assessment of incommensurate systems presents greater complexity compared to commensurate ones, as traditional methods like Lyapunov analysis and eigenvalue verification may not directly apply. Despite these challenges, incommensurate systems have garnered significant research attention. This paper proposes novel stability conditions for nonlinear incommensurate fractional-order continuous systems, employing the Lyapunov function method. It delineates the basics of fractional calculus, presents theoretical frameworks for stability analysis using Lyapunov methods, validates the findings through numerical examples, and concludes by summarizing the contributions of the study.

Preliminaries

First, we present some basic concepts regarding the Caputo fractional-order differential operator.

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Definition 80. [1] The α -order integral for ϕ can be expressed as:

$$I_t^\alpha \phi(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^t (t-\tau)^{\alpha-1} \phi(\tau) d\tau, \quad t \geq t_0, \quad (546)$$

$\Gamma(\cdot)$ is the gamma function, and ϕ is a function that makes the integral given in (546) convergent.

Definition 81. [1] The α ($0 < \alpha < 1$) Caputo fractional-order differential for ϕ can be expressed as:

$${}^C D_t^\alpha \phi(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^t (t-\tau)^{-\alpha} \frac{d}{d\tau} \phi(\tau) d\tau, \quad (547)$$

where ϕ is a differentiable function that makes the integral given in (547) convergent.

Definition 82. [2] A function $\phi(t)$ is said to belong to the class $\mathcal{K}$ if and only if $\phi \in C([0, \rho], \mathbb{R}_+)$, $\phi(0) = 0$ and $\phi(t)$ is strictly monotonically increasing in t .

Throughout this paper, we use the notation ${}^C D_t^\alpha$ instead of the notation ${}^C_{t_0} D_t^\alpha$. Next, we state some properties of the operator at hand for completeness.

Proposition 33. For all $\alpha, \beta \geq 0$, we have $I_t^\alpha I_t^\beta = I_t^{\alpha+\beta}$.

Proposition 34. For all $t \geq t_0$, we have $I_t^\alpha {}^C D_t^\alpha \phi(t) = \phi(t) - \phi(t_0)$.

Remark 10. Let $\phi : \mathbb{R}_+ \rightarrow \mathbb{R}$ (a continuous function) and $\alpha \in (0, 1)$. If $\phi(t) \geq 0$ or $\phi(t) \leq 0$ for all $t \in \mathbb{R}_+$, then

$$I_t^\alpha \phi(t) = 0 \Leftrightarrow \phi(t) = 0.$$

Main results

In this section, we derive some stability results for the Caputo incommensurate fractional-order systems in their continuous case using a Lyapunov function. To achieve this, let us consider the fractional-order system described by:

$${}^C D_t^{\bar{\alpha}} x(t) = f(t, x(t)), \quad (548)$$

where

$${}^C D_t^{\bar{\alpha}} x(t) = ({}^C D_t^{\alpha_1} x_1(t), {}^C D_t^{\alpha_2} x_2(t), \dots, {}^C D_t^{\alpha_n} x_n(t))^T$$

such that ${}^C D_t^{\bar{\alpha}} x(t) \in \mathbb{R}^n$, $f : \mathbb{R}^{n+1} \rightarrow \mathbb{R}^n$ is a continuous function, and $\alpha_i \in (0, 1)$ such that $1 \leq i \leq n$.

Theorem 231. [3] Suppose that $V(t, x)$ is a function, and let ψ_1, ψ_2 , and ψ_3 be class $\mathcal{K}$ functions such that:

$$\psi_1(\|x\|) \leq V(t, x(t)) \leq \psi_2(\|x\|), \quad (549)$$

$${}^C D_t^\beta V(t, x(t)) \leq -\psi_3(\|x\|), \quad (550)$$

for at least one value of $\beta \in (0, 1)$. Then, $x = 0$ is asymptotically stable.

Lemma 33. [4] Suppose that $x(t)$ is differentiable. Then:

$$\frac{1}{2} {}^C D_t^\alpha x^2(t) \leq x(t) {}^C D_t^\alpha x(t), \quad (551)$$

for all $\alpha \in (0, 1)$, and for all $t \geq t_0$.

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Theorem 232. Let $\beta = \min_{1 \leq i \leq n} \alpha_i$. If $\forall x \in \mathbb{R}^n$:

$$\sum_{i=1}^n I_t^{\alpha_i - \beta} x_i(t) f_i(t, x(t)) \leq 0. \quad (552)$$

Then $x = 0$ of (548) is stable. Additionally, if $\forall x \neq 0$:

$$\sum_{i=1}^n I_t^{\alpha_i - \beta} x_i(t) f_i(t, x(t)) < 0, \quad (553)$$

then $x = 0$ is asymptotically stable.

Proof To prove this result, let's consider $V(t, x(t))$ as follows:

$$V(x(t)) = \frac{1}{2} \sum_{i=1}^n x_i^2(t). \quad (554)$$

Obviously, this function is positive definite. However, according to Lemma 33 and Proposition 33, one can obtain:

$$\begin{aligned} {}^C D_t^\beta V(x(t)) &= \frac{1}{2} \sum_{i=1}^n {}^C D_t^\beta x_i^2(t) \\ &= \frac{1}{2} \sum_{i=1}^n I_t^{1-\beta} \frac{dx_i^2(t)}{dt} \\ &= \frac{1}{2} \sum_{i=1}^n I_t^{\alpha_i - \beta} I_t^{1-\alpha_i} \frac{dx_i^2(t)}{dt} \\ &= \frac{1}{2} \sum_{i=1}^n I_t^{\alpha_i - \beta} {}^C D_t^{\alpha_i} x_i^2(t) \\ &\leq \sum_{i=1}^n I_t^{\alpha_i - \beta} x_i(t) {}^C D_t^{\alpha_i} x_i(t) \\ &\leq \sum_{i=1}^n I_t^{\alpha_i - \beta} x_i(t) f_i(x_i(t)). \end{aligned}$$

Accordingly, one can notice that if

$$\sum_{i=1}^n I_t^{\alpha_i - \beta} x_i(t) f_i(t, x(t)) \leq 0,$$

then ${}^C D_t^\beta V(x(t)) \leq 0$, and then the fractional-order differential operator ${}^C D_t^\beta$ of $V(t, x(t))$ yields negative semi-definite. This implies, with the help of using the Proposition 34:

$$V(x(t)) \leq V(x(0)),$$

$\Rightarrow$

$$\|x(t)\| \leq \psi_1^{-1}(\psi_2(\|x(0)\|)).$$

Therefore, $\forall \epsilon > 0, \exists \delta > 0, \|x(0)\| < \delta \Rightarrow \|x(t)\| < \epsilon$. This leads us to deduce that system (548) is stable according to the Lyapunov scheme. On the other hand, if $\sum_{i=1}^n I_t^{\alpha_i - \beta} x_i(t) f_i(t, x(t)) < 0$ for all $x \neq 0$, then ${}^C D_t^\beta V(x(t)) < 0$ for all $x \neq 0$, and hence the negative definiteness of ${}^C D_t^\beta V$. According to [5] (relation between the class $\mathcal{K}$ and positive definite functions) and using Theorem 231, we can conclude the asymptotic stability of (548). ■

Remark 11. Using [5] (Lemma 4.1), the negative definiteness of ${}^C D_t^\beta V$ is equivalent to condition (550).

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Illustrative example

Now, we will introduce an example aimed at verifying all theoretical findings gained from the aforementioned results. Consider:

$$\begin{aligned} {}^C D_t^{\alpha_1} x_1(t) &= -x_1(t) + x_2^3(t) + x_2^3(t)x_3(t), \\ {}^C D_t^{\alpha_2} x_2(t) &= -x_1(t) - x_2(t) - x_1(t)x_3(t), \\ {}^C D_t^{\alpha_3} x_3(t) &= -x_3(t)x_1^2(t)x_2^2(t) - x_3(t), \end{aligned} \quad (555)$$

subject to the initial condition $x_0 = (0.6, 0.7, 0.8)$, where $\alpha_1 = 1/2$, $\alpha_2 = 1/3$, and $\alpha_3 = 2/3$. To confirm the stability of system (555), we propose:

$$V(x_1(t), x_2(t), x_3(t)) = \frac{1}{2}x_1^2(t) + \frac{1}{4}x_2^4(t) + \frac{1}{2}x_3^2(t). \quad (556)$$

One can easily observe that this candidate function is positive definite. Let $\beta = \min\{\alpha_1, \alpha_2\}$, then we can obtain:

$$\begin{aligned} {}^C D_t^\beta V(x_1(t), x_2(t), x_3(t)) &= \\ \frac{1}{2} {}^C D_t^\beta x_1^2(t) + \frac{1}{4} {}^C D_t^\beta x_2^4(t) + \frac{1}{2} {}^C D_t^\beta x_3^2(t). \end{aligned} \quad (557)$$

Using Proposition 33, we can gain the following equality:

$$\begin{aligned} {}^C D_t^\beta V(x_1(t), x_2(t), x_3(t)) &= \\ \frac{1}{2} I_t^{\alpha_1-\beta} {}^C D_t^{\alpha_1} x_1^2(t) + \frac{1}{4} I_t^{\alpha_1-\beta} {}^C D_t^{\alpha_1} x_2^4(t) \\ + \frac{1}{2} I_t^{\alpha_2-\beta} {}^C D_t^{\alpha_2} x_3^2(t). \end{aligned} \quad (558)$$

With the help of using Lemma 33, we can obtain:

$$\begin{aligned} {}^C D_t^\beta V(x_1(t), x_2(t), x_3(t)) &\leq \\ I_t^{\alpha_1-\beta} \left(x_1(t) {}^C D_t^{\alpha_1} x_1(t) \right. \\ + \frac{1}{2} x_2^2(t) {}^C D_t^{\alpha_1} x_2^2(t) \Big) + I_t^{\alpha_2-\beta} x_3(t) {}^C D_t^{\alpha_2} x_3(t) &\leq \\ I_t^{\alpha_1-\beta} \left(x_1(t) {}^C D_t^{\alpha_1} x_1(t) + x_2^3(t) {}^C D_t^{\alpha_1} x_2(t) \right) \\ + I_t^{\alpha_2-\beta} x_3(t) {}^C D_t^{\alpha_2} x_3(t) &\leq \\ -I_t^{\alpha_1-\beta} \left(x_1^2(t) + x_2^4(t) \right) \\ - I_t^{\alpha_2-\beta} \left(x_3^2(t)x_1^2(t)x_2^2(t) + x_3^2(t) \right). \end{aligned} \quad (559)$$

Consequently, Remark 10 can lead us to infer:

$${}^C D_t^\beta V(x_1(t), x_2(t), x_3(t)) < 0, \quad \forall x \neq 0. \quad (560)$$

In view of [5] and using Theorem 231, we can conclude the asymptotic stability of (555). However, Figure 31 confirms the stability of the system at hand.

Conclusion

In this paper, novel conditions for the stability of nonlinear incommensurate fractional-order continuous systems have been established. Specifically, the stability of these systems has been analyzed using a Lyapunov function. Several incommensurate fractional-order continuous systems

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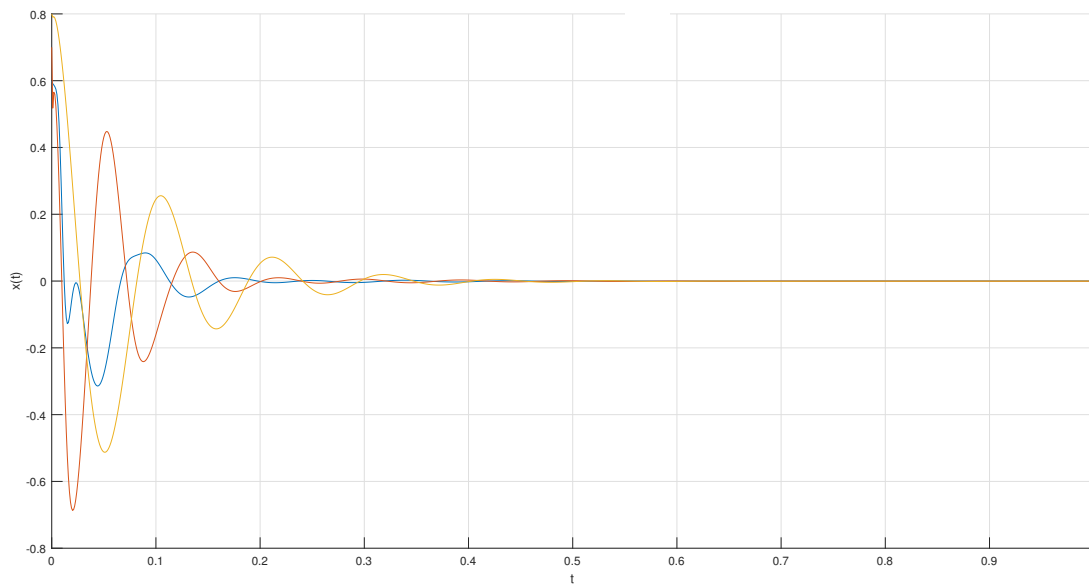


Figure 25: The numerical simulations of 555 which show the asymptotic stability of the solution: " x_1 in blue, x_2 in brown and x_3 in orange".

have been examined through numerical examples to confirm the stability of their solutions at the origin.

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On type of periodicity to a class of a competition and cooperation model for two enterprises with distributed time delay

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Abstract: In this work, oscillation for a class of a competition and cooperation delayed model is studied. By using the exponential dichotomy theory, fixed point method and applying inequality techniques, some sufficient conditions for the existence of the pseudo almost-periodic solutions for a competition and cooperation model of two enterprises with time varying coefficients and distributed time delay is obtained.

Keywords: delay-differential equations, competition and cooperation model of two enterprises, distributed time delay, pseudo almost-periodic function.

2020 Mathematics Subject Classification: Primary 34C27, 34K14, 37C25.

Introduction

Neural networks [1, 2, 4] and the competition and cooperation model of two enterprises [5, 6, 6], belong to the category of delayed differential equations, which are widely used in many fields, such as light absorption in the star substance and nonlinear dynamic systems. Therefore, the study on the dynamic behaviors of the competition and cooperation model of two enterprises has important theoretical and practical value. Many results have been obtained in the literature: in 2006, Tian and Nie [5] investigated the competition and cooperation model of two enterprises based on ecosystem

$$\begin{cases} \frac{dx_1(t)}{dt} = r_1(t)x_1(t) \left[1 - \frac{x_1(t)}{K} - \frac{\alpha(x_2(t)-c_2)}{K} \right], \\ \frac{dx_2(t)}{dt} = r_2(t)x_2(t) \left[1 - \frac{x_2(t)}{K} + \frac{\alpha(x_1(t)-c_1)}{K} \right], \end{cases} \quad (561)$$

where $x_1(t)$, $x_2(t)$ represent the output of enterprises A and B, r_1 , r_2 are the intrinsic growth rate, K denotes the carrying capacity of mark under nature unlimited conditions, α , β are the competitive coefficients of two enterprises, c_1 , c_2 are the initial production of two enterprises. Letting $a_1 = \frac{r_1}{K}$, $a_2 = \frac{r_2}{K}$, $b_1 = \frac{r_1\alpha}{K}$, $b_2 = \frac{r_2\beta}{K}$ in system (561), then system takes the form

$$\begin{cases} \frac{dx_1(t)}{dt} = x_1(t) \left[r_1 - a_1x_1(t) - b_1(x_2(t) - c_2)^2 \right], \\ \frac{dx_2(t)}{dt} = x_2(t) \left[r_2 - a_2x_2(t) + b_2(x_1(t) - c_1)^2 \right]. \end{cases} \quad (562)$$

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In this work, we give the following competition and cooperation model of two enterprises with time varying coefficients and distributed time delay:

$$\begin{cases} \frac{dx_1(t)}{dt} = x_1(t) \left[r_1(t) - a_1(t) \int_{-\infty}^t h_1(t-s)x_1(s)ds - b_1(t) \left(\int_{-\infty}^t h_2(t-s)x_2(s)ds - c_2(t) \right)^2 \right], \\ \frac{dx_2(t)}{dt} = x_2(t) \left[r_2(t) - a_2(t) \int_{-\infty}^t h_2(t-s)x_2(s)ds + b_2(t) \left(\int_{-\infty}^t h_1(t-s)x_1(s)ds - c_1(t) \right)^2 \right], \end{cases} \quad (563)$$

Here, we will use the following concepts and notations. $BC(\mathbb{R}, \mathbb{R}^n)$ denotes the set of bounded continuous functions from $\mathbb{R}$ to $\mathbb{R}^n$. Note that $(BC(\mathbb{R}, \mathbb{R}^n), \|\cdot\|_\infty)$ is a Banach space where $\|\cdot\|_\infty$ denotes the sup norm

$$\|f\|_\infty := \max_{1 \leq i \leq n} \sup_{t \in \mathbb{R}} |f_i(t)|.$$

Definition 83. [4]. Let $f \in BC(\mathbb{R}, \mathbb{R}^n)$. We say that f is almost-periodic (Bohr a.p.) or uniformly almost-periodic, when the following property is satisfied. For all $\epsilon > 0$, there exists $l_\epsilon > 0$ such that:

$$\forall \alpha \in \mathbb{R}, \exists \tau \in [\alpha, \alpha + l_\epsilon], \|f(\cdot + \tau) - f(\cdot)\|_\infty \leq \epsilon.$$

We denote by $AP(\mathbb{R}, \mathbb{R}^n)$ the set of the Bohr a.p. functions from $\mathbb{R}$ to $\mathbb{R}^n$. It is well known that the set $AP(\mathbb{R}, \mathbb{R}^n)$ is a Banach space with the supremum norm.

Define the class of functions

$$PAP_0(\mathbb{R}, \mathbb{R}^n) = \{f \in BC(\mathbb{R}, \mathbb{R}^n), \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \|f(t)\| dt = 0\}.$$

Definition 84. [4]. A function $f \in BC(\mathbb{R}, \mathbb{R}^n)$ is called pseudo almost-periodic if it can be expressed as $f = g + h$, where $g \in AP(\mathbb{R}, \mathbb{R}^n)$ and $h \in PAP_0(\mathbb{R}, \mathbb{R}^n)$. The collection of such functions will be denoted by $PAP(\mathbb{R}, \mathbb{R}^n)$.

Main results

In this section, we establish some results for the existence of the pseudo almost-periodic solution of (563). For that, we assume that the following conditions (H1) - (H2) hold.

(H1) The functions $t \mapsto a_1(t)$, $t \mapsto a_2(t)$, $t \mapsto r_1(t)$, $t \mapsto b_1(t)$, $t \mapsto b_2(t)$, $t \mapsto c_1(t)$, $t \mapsto c_2(t)$ are pseudo almost-periodic with

$$\begin{aligned} a_1^+ &= \sup_{t \in \mathbb{R}} |a_1(t)|, a_2^+ = \sup_{t \in \mathbb{R}} |a_2(t)|, b_1^+ = \sup_{t \in \mathbb{R}} |b_1(t)|, b_2^+ = \sup_{t \in \mathbb{R}} |b_2(t)|, \\ c_1^+ &= \sup_{t \in \mathbb{R}} |c_1(t)|, c_2^+ = \sup_{t \in \mathbb{R}} |c_2(t)|. \end{aligned}$$

Furthermore, we suppose that the functions $t \mapsto r_1(t)$, $t \mapsto r_2(t)$ are almost-periodic with

$$\inf_{t \in \mathbb{R}} (r_1(t)) = r_{1*} > 0, \inf_{t \in \mathbb{R}} (r_2(t)) = r_{2*} > 0.$$

(H2) The kernels $h_1, h_2 : [0, +\infty) \rightarrow [0, +\infty)$ satisfies : $\int_0^{+\infty} h_1(s)ds = h_1^+$, $\int_0^{+\infty} h_2(s)ds = h_2^+$.

In order to obtain the main result of this paper, we shall first state several lemmas which will be useful in the proving the main result.

Lemma 34. [4]. If $\varphi, \psi \in PAP(\mathbb{R}, \mathbb{R})$ and $\lambda \in \mathbb{R}$, then $\varphi + \psi$, $\varphi \times \psi$, $\varphi(\cdot - \lambda) \in PAP(\mathbb{R}, \mathbb{R})$.

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Lemma 35. [4]. Let (H2) hold. If $\varphi \in PAP(\mathbb{R}, \mathbb{R})$, then $\int_{-\infty}^t h_2(t-s)\varphi(s)ds, \int_{-\infty}^t h_2(t-s)\varphi(s)ds \in PAP(\mathbb{R}, \mathbb{R})$.

Lemma 36. Suppose that assumptions (H1)-(H2) hold. Define the nonlinear operator Γ as follows, for each $\varphi = (\varphi_1, \varphi_2) \in PAP(\mathbb{R}, \mathbb{R}^2)$, $(\Gamma\varphi)(t) := x_\varphi(t)$ where

$$x_\varphi(t) = \begin{pmatrix} \int_t^{+\infty} e^{-\int_t^s r_1(u)du} F_1(s)ds \\ \int_t^{+\infty} e^{-\int_t^s r_2(u)du} F_2(s)ds \end{pmatrix}$$

and

$$\begin{cases} F_1(s) = \varphi_1(s) \left[a_1(s) \int_{-\infty}^s h_1(s-m)\varphi_1(m)dm + b_1(s) \left(\int_{-\infty}^s h_2(s-m)\varphi_2(m)dm - c_2(s) \right)^2 \right], \\ F_2(s) = \varphi_2(s) \left[a_2(s) \int_{-\infty}^s h_2(s-m)\varphi_2(m)dm - b_2(s) \left(\int_{-\infty}^s h_1(s-m)\varphi_1(m)dm - c_1(s) \right)^2 \right], \end{cases}$$

then Γ maps $PAP(\mathbb{R}, \mathbb{R}^2)$ into itself.

Theorem 233. Let (H1)-(H2) hold. Then, there exists a unique pseudo almost-periodic solution $x(t) = (x_1(t), x_2(t))^T$ of competition and cooperation delayed model of two enterprises (??) in the region

$$\mathbb{B} = \{\varphi/\varphi \in PAP(\mathbb{R}, \mathbb{R}^2), |\varphi_1(t)| \leq R_1, |\varphi_2(t)| \leq R_2 \text{ for all } t \in \mathbb{R}\}.$$

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Chua's System Generated by Transformation: An Inquiry into Hopf Bifurcation and Attractor Basin Analysis

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Abstract: The nonlinear system's basin attraction and Hopf bifurcation have recently attracted a lot of scholarly attention due to their recent development. This work studies a Hopf bifurcation in multiscroll chaotic Chua attractors produced by parallel transformation and provides sufficient and necessary conditions for their existence. At two-parameter control values k , the critical Hopf bifurcation value has been found. We have been validated, then, an attraction basin, which is the collection of all starting conditions that will bring a system to a specific stable state or behavior, of a Chua system created parallel transformation. Since it is impossible to investigate the basins of most nonlinear dynamical systems analytically, numerical simulations are the primary instrument for the inquiry.

Keywords: Chua's system, Parallel transformation, basin attraction, multiscroll chaotic.

2020 Mathematics Subject Classification: Primary 39A21; 39A28; 39A33

Introduction

Basins of attraction are a key component of the study of multistable dynamical systems. They give information on the different asymptotic states that different sets of initial circumstances converge to. The bulk of nonlinear dynamical systems cannot be explored analytically, hence numerical simulations are the recommended way for the inquiry. It is necessary to compute the basins by comparing the trajectory of each chosen starting condition to a set of known attractors [3]. The bifurcation theory is a useful factor for explaining dynamical behavior [2]. The dynamics of bifurcations of arbitrary invariant sets of dynamical systems on changing a parameter appear to be more complex and attractive. The local birth or death of a periodic solution (self-excited oscillation) from an equilibrium as a parameter passes a critical value is referred to as a Hopf bifurcation, also known as a Poincare-Andronov-Hopf bifurcation.

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Main results

Here, to generate a $1 - D$ n scroll chaotic attractor, we present a Chua's the system from parallel transformation is as follows :

$$\begin{cases} \dot{x} = a(y - h(x)) \\ \dot{y} = x - y + z \\ \dot{z} = -by \end{cases}, \quad (564)$$

where $a = 10$, $b = 16$ and $h(x)$ is the nonlinear function, defined by one of the following form while to have an odd number n of scrolls according to the formula ($n = 2N + 3$), the function $h(x)$ is defined by:

$$h(x) = kx - pk \left[\sum_{i=0}^N \text{sgn}(x + (1 + 2i)p) - \sum_{i=0}^N \text{sgn}(x - (1 + 2i)p) \right] \quad (565)$$

where p, k are real numbers.

$$\text{sgn}(x) = \begin{cases} +1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \end{cases}$$

For $a = 10$, $b = 16$, $p = 0.5$, and $k = 0.3$, a 7 - scroll is generated as the verged chaotic attractor of system (564), with both (565), as shown in figures 26 and 27. a, b, p, k are real numbers.

Figure 26: **Graph of function Parallel transformation, when $p = 0.5$ and $k = 0.3$**

Figure 27: **One-directional 7-scroll chaotic attractors.**

Figure 28: **The result of Hopf Bifurcation**

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An effective operational matrix method for the solution of sixth and fifth-order boundary value problems

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Abstract: In this paper, we present an accurate numerical scheme for the solution of the non-linear sixth and fifth order boundary value problems with two point boundary conditions. Problems of these kinds arise in a variety of different areas of applied mathematical, physical and engineering sciences. We apply a numerical method by using the clique polynomial of a complete graph for the operational matrices to obtain solutions of sixth and fifth order boundary value problems. Numerical examples are performed to show the validity and applicability of the techniques. The proposed method is applied to the new examples and compared with the exact solutions that demonstrates the effectiveness of this method.

Keywords: The clique polynomials; Operational matrix; Collocation points; Nonlinear differential equations.

2020 Mathematics Subject Classification: 65L05, 65L10, 34B05, 34B15.

Introduction

There are many problems in applied sciences and engineering are modelled as two point boundary value problems such as the boundary layer theory in fluid mechanic. Since the boundary value problem has wide application in scientific research, therefore faster and accurate numerical solutions of boundary value problems are very importance. Many researchers developed some methods to solve boundary value problems of a different order such as [2, 3] and others. This paper is concerned with the numerical solution of solving the sixth and fifth order boundary value problems. [1] presented the theorems which listed the conditions for the existence and uniqueness of solutions of sixth order boundary value problems. there are many researchers used a family of numerical methods for the solution of special nonlinear sixth order boundary value problems such as [3] by using spline method. Also, the fifth order boundary value problems are considered. [2] used generalized Jacobi operational matrix of derivatives method to find the solution of such type of the problems. The basic motivation of this paper is to apply the operational matrix of integration method using clique polynomials of complete graphs to solve the sixth and fifth order boundary value problems. It is shown that the method provides the solution in a rapid convergent series. This method has been shown [4] to solve effectively, easily and accurately the non linear Klein Gordon equation, which converge rapidly to accurate solutions. We show that the boundary value problems of sixth and fifth order can be solved efficiently using the clique polynomials (CP). This clearly indicates that this technique may be considered as an alternative method for solving linear and nonlinear problems arising in various branches of pure and applied sciences.

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Main results

In this section, nonlinear boundary value problems of sixth and fifth order will be tested by using the proposed algorithm. Table 1 show that the numerical solutions of non linear sixth order boundary value problem (Example 2.1) by using the present method (clique polynomial method) and compared with the exact solution of this problem for $x_i = \frac{1}{10}i; i = 0, 1, \dots, 10$. Examining this table, it is clear that the absolute errors were seem to be small. Where

$$\text{Absolute error} = |\text{exact solution} - \text{numerical solution}|$$

Exemple 2.1 Consider the nonlinear sixth boundary value problem

$$y^{(6)} + (y')^2 = y^2 - (2x - 3)e^x - x^2 + 1, 0 \leq x \leq 1 \quad (566)$$

with boundary conditions

$$y(0) = 1, y'(0) = 2, y'''(0) = 1, y(1) = e + 1, y'(1) = e + 1, y'''(1) = e \quad (567)$$

The analytic solution of the above problem is

$$y = e^x + x$$

We have

$$y'(x) = V_1^T X(x) + A^T P M_5 x^5 X(x) \quad (568)$$

where

$$M_5 = \begin{bmatrix} \frac{1}{5!} & 0 & 0 & \dots & 0 \\ 0 & \frac{1}{6!} & 0 & \dots & 0 \\ 0 & 0 & \frac{1}{3 \times 4 \times 5 \times 6 \times 7} & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \dots & 0 & \frac{1}{n(n+1)\dots(n+4)} \end{bmatrix}$$

and V_1 is the $n \times 1$ matrix $V_1^T = [\alpha_2, \alpha_3, \frac{\alpha_4}{2!}, \frac{\alpha_5}{3!}, \frac{\alpha_6}{4!}, 0, 0, \dots, 0]$. $A^T = [a_0, a_1, \dots, a_{n-1}]$, $X(x) = [1, x, \dots, x^{n-1}]^T$ and P is the lower triangular $n \times n$ matrices defined by

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ 1 & 1 & 0 & 0 & \dots & 0 \\ 1 & 2 & 1 & \ddots & \dots & 0 \\ 1 & 3 & 3 & \ddots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & 1 & 0 \\ 1 & n-1 & \frac{(n-1)(n-2)}{2!} & \dots & n-1 & 1 \end{bmatrix}$$

yields

$$A^T P X(x) + (V_1^T X(x) + A^T P M_5 x^5 X(x))^2 = (V_1^T X(x) + A^T P M_6 x^6 X(x))^2 - (2x - 3)e^x - x^2 + 1$$

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x	Exact solution	Numerical solution at $n = 10$	Errors
0	1	1	0
0.1	1.20517091807565	1.20517092005764	$1.98199590073500E - 09$
0.2	1.42140275816017	1.42140276453309	$6.37292041361093E - 09$
0.3	1.64985880757600	1.64985881826001	$1.06840045699386E - 08$
0.4	1.89182469764127	1.89182471096239	$1.33211217612228E - 08$
0.5	2.14872127070013	2.14872128431904	$1.36189122201813E - 08$
0.6	2.42211880039051	2.42211881204333	$1.16528235949431E - 08$
0.7	2.71375270747048	2.71375271558513	$8.11465561412206E - 09$
0.8	3.02554092849247	3.02554093267429	$4.18182599659644E - 09$
0.9	3.35960311115695	3.35960311230540	$1.14844889154142E - 09$
1	3.71828182845905	3.71828182845914	$9.10382880192628E - 14$

Tables 1. Numerical results for Example 4.1 at $n = 10$

we get for $n = 10$

$$A = \begin{bmatrix} 369.29315 \\ -1144.96787 \\ 24.48003 \\ 4884.28290 \\ -9937.11738 \\ 10026.92127 \\ -5959.32623 \\ 2122.69535 \\ -421.14648 \\ 35.91595 \end{bmatrix}$$

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Stability, Square Integrability and Boundedness Properties of Solutions for Neutral Third Order Integro-Differential Equations With Variable Delay

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Abstract: The Lyapunov second method approach is an eigenvalue-based method for evaluating the stability of dynamical systems. It involves identifying a potential lyapunov function, which can be challenging to work with, particularly in the majority of nonlinear cases. Numerous writers in the literature have employed Lyapunov's second method to investigate certain qualitative behaviors, thereby examining the stability, boundedness, and square integrability of solutions for diverse differential equation types that differ in terms of order, linearity, autonomy, delay, or neutral case and many other characteristics . This article offers a delay integro-differential equation of neutral type (DIDEN) is presented in order to establish a new results about the stability and the boundedness and the square integrability of it's solutions. Inequalities are demonstrated using the Lyapunov second method approach. In addition a concrete example is given to illustrate the main results of this paper.

Keywords: Stability, square integrability, boundedness, integro-differential equations of third order, Lyapunov functional, delay, neutral type.

2020 Mathematics Subject Classification: 34C11, 34C25, 34D20, 34D23.

Introduction

The stability of differential equations has been the subject of extensive investigation. The majority of studies on the stability of delay differential equations make the assumption that delays and coefficients are continuous. However, in certain scenarios, such as those involving biological, ecological, or economic models, the parameters of differential equations are not constant. The stability of the system's zero solution has been investigated using a variety of techniques, including the comparison approach and the Lyapunov second method approach. This later is widely used by researchers it consists of constructing a suitable function based on an equivalent system of the differential equation.

There are many articles in the literature that deal with a large variety of differential equations from which there are delay differential equations, ones without delay and others of neutral type. for instance many interesting results were obtained in the works of Dousla and Liska; (2016)

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Ademola and Arawomo (2013); Ademola et al. (2013); Beldjerd, and Remili (2018) Burton (2005); Hara (1971); Bao and Cao (2009); Omeike (2010); Oudjedi et al. (2014); Remili and Beldjerd (2017); Remili and Oudjedi (2016); Li and Lizhi (1987); TuncÂ, (2007, 2010); Yoshizawa (1966); Zhu (1992). whereas the articles in the delay integro-differential equations of neutral type are not remarkably many we refer to the works of Baculíková, and Džurina (2010); Mihalíková and Kostíková (2009); Doušlá and Liška (2016); Oudjedi and Remili (2019); Graef et al (2022); Rahmae et al (2019); Fatmi et al (2023)

The authors in the present paper are interested to the following delay integro-differential equation

$$\begin{aligned} \left(x(t) + \rho x(t - \sigma) \right)''' + a(t)x'' + b(t)x' + c(t) \left[f(x(t)) + \rho f(x(t - r(t))) \right] &= \varphi(t, x(t), x'(t), x''(t), x'''(t)) \\ &+ \xi \int_0^t \Omega(t, s) x'(s) ds \end{aligned} \quad (569)$$

for all $t \geq t_1 = t_0 + \sigma$ where r is a bounded delay, $0 < r(t) \leq r_1, r'(t) \leq r_2, 0 < r_2 < 1$ and $\rho < 1$, ξ, r_1, r_2, ρ and σ are positive constants, $r = \max\{\sigma, r_1\}$, the primes in (569) denote differentiation with respect to t ; the functions $a, b, c : [0, \infty) \rightarrow (0, \infty)$ and $f : \mathbb{R} \rightarrow \mathbb{R}$ are continuous differentiable and $\Omega(t, s)$ is continuous for $0 \leq s \leq t < \infty$ and $f(0) = 0$. The consideration of the above equations comes from the following articles in the literature.

By solution of (569) we mean a continuous function $x : [t_x, \infty) \rightarrow \mathbb{R}$ for $t_x \geq t_1$ which satisfies the equation (569) in $[t_x, \infty)$ and such that

$$x(t) + \rho x(t - \sigma) \in \mathcal{C}^3([t_x, \infty), \mathbb{R}).$$

The motivations of this work comes from some previous articles in the literature where the authors used the second method of Lyapunov and constructed the conventable lyapunov function to obtain the results therein. The authors in this paper use the method of Lyapunov to prove to theorems on the stability, boundedness and square integrability. the results obtained her improve and inhence the previous results exist in the literature.

Main results

First, we state some assumptions on the functions that appeared in (569). Suppose that there are positive constants $a_0, c_0, a_1, b_1, \delta, A$, and η_1 , such that the following conditions are satisfied

- i) $0 < a_0 \leq a(t) \leq a_1, 0 < c_0 \leq c(t) \leq b(t) \leq b_1$, and $c'(t) \leq 0$ for $t \geq t_1$;
- ii) $\frac{f(x)}{x} \geq \delta > 0$ (for $x \neq 0$), $f'(x) \leq A$ and $f(0) = 0$;
- iii) $\int_{t_1}^{+\infty} (|a'(t)| - c'(t)) dt < \eta_1$.

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Admissibility LMI Criteria for descriptor fractional order systems with $1 \leq \alpha < 2$

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Abstract: This paper deals with the admissibility problem of singular fractional-order continuous time systems with order $1 \leq \alpha < 2$. It is based on new admissibility conditions of singular fractional-order systems expressed in a set of strict Linear Matrix Inequalities (LMIs). Then, a static output feedback controller is designed for the closed-loop system to be admissible. Numerical examples are given to illustrate the proposed methods. Singular systems referred to as descriptor systems, implicit systems, generalized state-space systems and differential– algebraic systems, have been of great interest in the control literature because of the variance of their applications. Indeed, many fundamental notions and results in control theory have been successfully extended from standard systems to descriptor ones. Singular systems include differential equations describing the dynamic characteristics of state variables and algebraic equations describing the static relationship of state variables. The studies of singular fractional-order systems have more practical significance, and the singular FOS theories have the prospect of a very broad application. Many FOS theories have been extended to singular fractional-order systems.

Keywords: singular fractional-order (SFO), Linear Matrix Inequalities (LMI), admissibility, stable.

2020 Mathematics Subject Classification: Primary 93Dxx, 93C05, 93D15.93C95.

Introduction

Fractional calculus developed slowly due to simulation difficulties. During the last two decades, with the development of computers, fractional calculus can be better described through simulations, singular fractional-order (SFO) have been gradually studied. The fundamental reason is that physical systems can be described more accurately with SFO. Therefore, SFO have been extensively developed, especially in the aspect of dynamic process modeling. For the SFO, stability and stabilization are the fundamental problems. Moreover, in the field of singular fractional-order systems, issues of interest are much more complicated than those of standard systems since the admissibility property. The paper is organized as follows. Some preliminary results are presented in Section 1. The main results are derived in Section 2 and numerical examples that illustrate the effectiveness of the proposed results.

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Main results

The Caputo definition for fractional derivative is adopted.

$$D^\alpha f(t) = \frac{d^\alpha f(t)}{dt^\alpha} = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} f^{(n)}(\tau) d\tau, n-1 < \alpha < n$$

where $\Gamma(\cdot)$ is the Euler Gamma function and is defined by $\Gamma(z) = \int_0^{+\infty} t^{z-1} e^{-t} dt, z \in \mathbb{C}, \text{Re}(z) > 0$.

Let us consider the following linear singular fractional-order (SFO) continuous system described by

$$\begin{aligned} ED^\alpha x(t) &= Ax(t) + Bu(t); \quad 1 \leq \alpha < 2 \\ y(t) &= Cx(t) \end{aligned} \quad (570)$$

where α is the time fractional derivative order, $x(t) \in \mathbb{R}^n$ is the pseudo-state, $u(t) \in \mathbb{R}^m$ is the control input and $y(t) \in \mathbb{R}^p$ is the output. $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$ and $C \in \mathbb{R}^{p \times n}$ with $\text{rank} C = p$ and matrix $E \in \mathbb{R}^{n \times n}$ is singular with $\text{rank} E = r < n$.

The unforced SFO system of (570) can be written as

$$ED^\alpha x(t) = Ax(t) \quad (571)$$

Lemma 37. The SFO system (571) or the pair (E, A) is said to be regular if and only if $\det(s^\alpha E - A)$ is not identically zero, where $s \in \mathbb{C}$.

Lemma 38. The SFO system (571) or the pair (E, A) is said to be impulse free if (571) is regular and $\deg(\det(\lambda E - A)) = \text{rank} E$, where $\lambda \in \mathbb{C}$.

Lemma 39. The SFO system (571) or the pair (E, A) is said to be stable if $|\arg(\text{sp}(E, A))| > \alpha \frac{\pi}{2}$, $\text{sp}(E, A) = \{\lambda \in \mathbb{C}, \lambda \text{ finite}, \det(\lambda E - A) = 0\}$ denotes the set of finite modes for the pair (E, A) .

Lemma 40. The SFO system (571) is admissible if it is regular, impulse free and stable.

Theorem 234. The SFO system (571) is admissible if and only if the following equivalent statements hold:

(i) There exist matrices $X > 0$ (positive semi-definite) and Y satisfying

$$\text{sym}\{\Theta \otimes A^T (XE + E_0 Y^T)\} < 0 \quad (572)$$

where $E_0 \in \mathbb{R}^{n \times (n-r)}$ is any matrix of full column rank and satisfies $E^T E_0 = 0$. $\otimes$ stands for the Kronecker product

(ii) There exist matrices $X > 0$ and Y satisfying

$$\text{sym}\{\Theta \otimes A (XE^T + E_0 Y^T)\} < 0 \quad (573)$$

where $E_0 \in \mathbb{R}^{n \times (n-r)}$ is any matrix of full column rank and satisfies $EE_0 = 0$

(iii) There exists a matrix P satisfying

$$E^T P = P^T E, E^T P \geq 0, \text{sym}\{\Theta \otimes A^T P\} < 0 \quad (574)$$

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$\otimes$ stands for the Kronecker product.

proof. in ([1]).

Numerical example : Consider the linear SFO system (571) with the following data :

$$E = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}, A = \begin{bmatrix} -1 & 2 & 0 \\ -3 & -4 & 1 \\ 0 & 0 & -2 \end{bmatrix}, \alpha = 1.5$$

The system (571) is admissible. Indeed, $\det(s^\alpha E - A)$ which is not identically zero and impulse free with $s^\alpha = \lambda$. The finite mode of the pair (E, A) lives in the stability region since it is equal to $\frac{-10}{19}$. This can also be verified by solving the linear matrix inequality (2.2) in **Theorem 2.1** which is feasible, and concludes that the singular fractional-order system (571) is admissible.

A feasible solution of (2.2) using MATLAB is as follows:

$$X = \begin{bmatrix} 4,0117 & -1,5249 & -1,3670 \\ -1,5249 & 1,5061 & -0,4056 \\ -1,3670 & -0,4056 & 1,4156 \end{bmatrix}, Y = \begin{bmatrix} 0,5736 & 0,3607 \\ 1,1342 & 1,3679 \\ 1,3225 & 1,9889 \end{bmatrix}$$

The state response of system is shown in Fig. 1 which confirms that it is asymptotically stable and its states converge to zero .

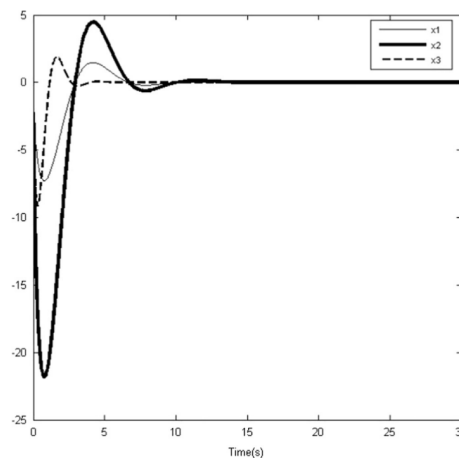


Fig. 1. Time response of $x(t)$ with initial condition $x(0) = [-1, -3, -2]^T$ and fractional order $\alpha = 1.5$.

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Operational research and Optimization

On Solving Optimal Control Problem with Bounded Constraints Using Iterative Direct Method

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Abstract: In this communication, we consider an optimal control problem with bounded variables and constraints on the accessibility set, which has numerous practical applications. The main idea consists of three stages. Firstly, we initiate the discretization of the control at regular intervals to transform the initial problem into a linear programming problem. Secondly, we aim to find the approximate solution by solving this linear programming problem using iterative direct methods, specifically the support method pioneered by R. Gabasov and F.M. Kirillova in the early 1980s. Lastly, in the third and final stage, we introduce a final procedure based on Newton's method, which provides an optimal solution to the initial problem.

Keywords: Optimal control problem, Optimality estimate, Discretization, Adaptive method, Finishing procedure.

2020 Mathematics Subject Classification: Primary 49-XX, 49N05.

Introduction

Optimal control theory has been widely used in different sectors, such as engineering, medical sciences, economics, etc[5]. Solving methods for optimal control are mainly divided into two basic categories: direct methods (optimization techniques) and indirect methods (calculus techniques) [4].

In this work, a new direct support method is proposed to find a numerical optimal solution (ε -optimal solution) for the linear optimal control problem. The method involves discretizing the initial problem and applying a method of interior points adapted from the simplex method (the support method) developed by R. Gabasov and others in the 1980s [2, 1]. Using the optimal solution of the discretized problem, another step called the final procedure is introduced based on Newton's method to provide an optimal solution to the initial problem. This procedure is described in detail using a numerical example.

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Problem statement

In the class of piecewise constant control commands, let us consider the terminal problem of optimal control:

$$\begin{cases} J_c(u(t) = c'x(t_f) \rightarrow \max_{u(t)} \\ \dot{x} = Ax(t) + Bu(t), x(0) = x_0, \\ g_* \leq Hx(t_f) \leq g^*, \\ d_* \leq u(t) \leq d^*, t \in T = [0, t_f]. \end{cases} \quad (575)$$

$J_c(u(t))$ the quality criterion, $\dot{x} = \frac{dx}{dt}$ and $x(t) = (x_j, j \in J)^T$ is an n -vector is the position of the system at the time t . A is an $(n \times n)$ -matrix with $\text{rank}(A) = m < n$, $u(t)$ is the control action (input signal) at time t limited by the scalars d_* and d^* , g_* and g^* are finite-valued n -vectors, $Hx(t_f)$ is the output signal of the system at time t_f . H is $(m \times n)$ -matrix, $B \in \mathbb{R}^m$ and $c^T \in \mathbb{R}^n$ are given constant vectors.

The objective of the problem is then to identify the best control u to maximize the performance index J_c .

An iteration of direct method

The aim of the suggested method is to find an optimal solution for the problem (575), for this we construct an algorithm based on three procedures:

1. Discretization of the initial problem.
2. Adaptive method.
3. Finishing procedure.

We discretized the initial problem using specialized techniques to turn it into an optimization problem, the problem (575) becomes the following linear programming problem:

$$\begin{cases} J_d(u_j) = \sum_{j=1}^N u_j \int_{\tau_j}^{\tau_j^+} c'F(t_f)F^{-1}(t)Bdt \longrightarrow \max \\ g_* - HF(t_f)x_0 \leq \sum_{j=1}^N u_j \int_{\tau_j}^{\tau_j^+} HF(t_f)F^{-1}(t)Bu_j dt \leq g^* - HF(t_f)x_0 \\ d_{j*} \leq u_j \leq d^{j*}, j = \overline{1, N} \end{cases} \quad (576)$$

with $u(t) = u_j, t \in T = \cup[\tau_j, \tau_j^+], j = \overline{1, N}, \tau_j = 0$, where $\tau_j^+ = \tau_j + h$ and $h = \frac{t_f}{N}$

While solving this problem using the adapted method [1, 3], we obtain the ϵ -optimal solution or optimal solution. By using the optimal solution of problem (576), we construct the support-control of the initial problem (575) (which is not the optimal solution).

Utilizing an approximate optimal solution associated with the optimal solution of the discrete problem, we continue the application of the method's steps to obtain the optimal solution. We then apply a finishing procedure that allows us to detect switching points using the Newton method for solving nonlinear equations

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Conclusion

In this work, we applied the new approach based on the techniques of the support method, concluding with a final procedure that employs the Newton method, which converges rapidly. The optimal solution of the auxiliary problem (the discretized problem) is not the optimal solution of the original problem but a solution close enough to optimality. The results show that the algorithm is capable of calculating the optimal control and its associated trajectory.

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The Estimation of Distribution Algorithm (EDA) to optimize a function over the set of efficient solutions within a Multi-objective Integer Problem (MOILP)

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Abstract:

This study focuses on employing Estimation of Distribution Algorithms (EDAs) to address multi-objective linear programming problems with discrete variables (MOILP). The proposed methodology aims to enhance the set of efficient solutions for multi-objective problems by strategically navigating the search space of integer solutions to optimize the primary objective. To showcase the effectiveness of this approach, we utilized the multi-objective resource allocation problem as a tangible example. We conducted numerical experiments to assess the performance of our approach. Moreover, this research provides an overview of fundamental concepts in multi-objective optimization, discusses tools like metaheuristics and EDAs, and presents a model for the resource allocation problem.

Keywords: MOILP, EDA, Resource Allocation Problem, Multi-objective linear mathematical programming, Multi-objective Optimization, Metaheuristics.

2020 Mathematics Subject Classification: Primary 9002, 65K05, 90C10.

Introduction

There are situations where decision-makers do not require all the efficient solutions of a multi-objective programming problem, but only those that reach the optimum for a different objective than those already defined. This involves searching for the optimal solution of a criterion among the set of efficient solutions of the multi-objective problem. The objective of our work is to study linear mathematical programming problems with multiple objectives and discrete variables, also known as MOILP (Multiple Objective Integer Linear Programming).

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The Estimation of Distribution Algorithms (EDA) were introduced in the 1990s by the German researcher Dirk Thierens, as evolutionary methods for the optimization of continuous and multi-modal functions. Their goal was to overcome the limitations of Genetic Algorithms (GA) and Evolution Strategies (ES), which often suffered from premature convergence and slow convergence towards the global optimum.

In this article, we adapt the use of EDA to solve a MOILP problem by enhancing the set of efficient solutions and intelligently exploring the solution space of integers. This is achieved by generating potential solutions according to a specified probability distribution. With each generation, we adjust the parameters of this distribution based on the significance of the solutions with respect to the primary objective ϕ .

These concepts will be implemented on a machine using Python, and a series of numerical experiments will be conducted to evaluate the performance of the developed approach. Discussions of the results will form the basis for the analysis of the effectiveness of this approach.

Mathematical modeling and experimentation results

The mathematical model used for the optimization of a function over the set of efficient solutions of a multi-objective integer problem can be formulated as follows :

$$(MOILP) \begin{cases} \min & Z_i(x) = c^i(x), i = 1, \dots, k \\ \text{s.c.} & Ax \leq b \\ & x \in \mathbb{Z} \end{cases} \quad (577)$$

where :

- A is a $(m \times n)$ -matrix and b is a $(m \times 1)$ -vector.
- c^i is a $(n \times 1)$ -vector that represents the coefficients of the objective function $Z_i(x)$ for each objective $i = 1, \dots, k$.
- $\mathbb{Z}$ is the set of integer solutions.

We denote by $\mathbb{F}$ the set of efficient solutions of the problem (577). The problem we'll be considering is $(MOILP_{\mathbb{F}})$ defined as follows:

$$(MOILP_{\mathbb{F}}) \begin{cases} \min & \Phi = \phi^T x, \\ \text{s.c.} & x \in \mathbb{F}. \end{cases} \quad (578)$$

Where ϕ represent a n -vector.

The mathematical model (578) describes a single-objective optimization problem, where the goal is to find an optimal solution while adhering to the constraints of the MOILP problem (577), which are represented by a set of efficient solutions.

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The Distribution Estimation Algorithm EDA description and fonctionnement

The EDA algorithm is a stochastic optimization method based on estimating the probability distribution of feasible solutions. This approach is used to estimate a probability distribution from an initial population of feasible solutions, and then generate a new population of solutions based on this distribution.

The algorithm starts by introducing the data of the relevant problem : $A_{m \times n}$ being the constraint matrix where n : is the number of variables and m : is the number of constraints. Similarly, $C_{k \times n}$: is the objective matrix with k as the number of objectives. b_m : is the right-hand vector and ϕ_n : is the main objective.

The parameters of EDA are initialized as follows:

npo : population size,

T : represents the number of iterations,

θ : portion of dominated solutions for generation K , is initialized to 1,

ϵ : update index where $\epsilon = \frac{2 \times \theta}{T}$,

sol_d : set of dominant solutions initialized to the empty set,

S : the size of the population generated by the uniform law and placed in the set of random solutions set where $S = \frac{1}{5}$.

And we generate our initial population using a uniform distribution $U(0.5, npo)$.

The progression of our algorithm starts with checking the constraints, followed by the dominance check. We retrieve the dominant solutions and add them to the set Sol_d . Next, we select θ solutions that give the best ϕ values among the dominated solutions, and then place them in the Dominated_Solutions set.

The final set obtained (Solution) allows us to parameterize the distribution that generates the variables for our population P^{t+1} in iteration $t + 1$. We repeat this procedure until reaching $t = T$.

In the end, we obtain the unique and optimal solution x^* , as well as the optimal value ϕ^* of the primary objective.

Some computing results

Conclusion and future works

This work involves adapting the use of EDA to solve MOILP problems by enhancing the set of efficient solutions, In order to facilitate the decision-making process by providing decision-makers with a single solution rather than choosing from a set of solutions.

A mathematical model of the multi-objective resource allocation problem has been used. To solve it, the Estimation of Distribution Algorithm (EDA) was chosen and adapted, with numerical experiments carried out using Python.

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k=50		
$m \times n$	Temps	Convergence
20×30	114.0612	0.9524
50×70	294.8201	0.7462
80×100	526.7914	1.0751
100×200	3193.4694	1.1382

Table 5: Résultats de l'exécution de l'algorithme avec différentes dimensions du problème cas: $k = 50$

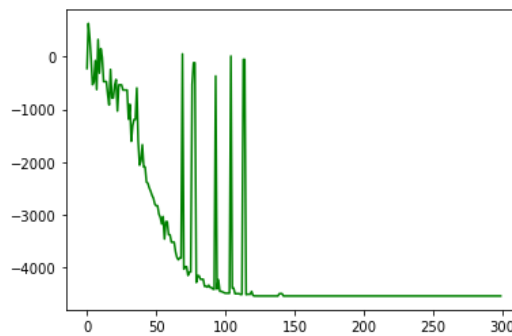


Figure 29: Le comportement des indices de performance cas $k = 50$

Through numerical experiments, we evaluated the performance of our approach. EDA proves to be effective tools for solving multi-objective integer optimization problems, providing accurate and diverse solutions.

Finally, possibilities for improving and extending this approach for more complex problems and new fields of application were considered like: Exploring variations of EDA and their application to other MOILP problems. And improving the performance of the proposed approach and comparing it with other optimization methods would constitute an interesting extension of this work.

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Exact Approach for Cardinality-constrained Portfolio Selection Problem

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Abstract: This work introduces a new resolution approach for the Cardinality-Constrained Portfolio Selection Problem (CCPSP). The problem is treated differently from the usual literature. First, the portfolio selection procedure determines the portfolio components and verifies the cardinality constraint. Second, an exact quadratic programming method -based on Merton's Analytic derivation of the efficient frontier- is used to solve the resulting mean-variance Markowitz Model from the portfolio selection procedure. This two-phase approach allows a faster and easier solving of the large-scale Mixed Integer Quadratic Programming (MIQP) portfolio selection problem.

Keywords: Portfolio selection, Cardinality Constraint, Exact method, Quadratic Programming.
2020 Mathematics Subject Classification: Primary 9090, 90C30, 90C11.

Introduction

Modern Portfolio Theory evolved considerably through the decades separating its first introduction in Markowitz's Portfolio Selection paper in 1952 from recent alternatives, improvements in resolution concepts, and hard real-world formulations appearing in literature. The limited Asset Model is the Mixed Integer Quadratic Programming formulation of portfolio selection problems mostly studied due to its complexity (NP-hard class) and applicability (consideration of restrictions on the quantity of investment and cardinality).

The resolution of this Cardinality Constrained Portfolio Selection problem explored the exact, heuristics, and hybridization of both methods. Several papers presented exact approaches giving optimal solutions but unfortunately not applicable to large-scale problems.

Specifically, this work aims to suggest a new resolution approach for large-scale cardinality-constrained portfolio selection problems based on a portfolio selection procedure to compose the portfolio components, followed by an exact quadratic programming method to determine the proportions of investment for each selected asset.

In this paper, we will first introduce the Cardinality-Constrained Portfolio Selection Model, and

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we will explain the two phases of the resolution approach: The selection procedure determines which assets will compose the portfolio in the first phase, and we will present an exact method to solve the reduced model in the second phase, and finally obtain the proportions to invest in each selected asset.

Further, we will illustrate our resolution approach with a computational example of 10 instances only. We will be viewing the main results and the resolution approach will be discussed, and we will conclude with some perspectives and future works.

Methods

In portfolio selection, investors aim to minimize the portfolio risk while keeping the expected return equal to or larger than a required value ρ . In our model, only K assets are selected in the portfolio with the proportions of investment restricted to a quantity constraint (lower and upper bound).

The mathematical formulation of the cardinality-constrained portfolio selection problem is written as follows [2]:

$$\sigma = \min \sum_{i=1}^n \sum_{j=1}^n \sigma_{ij} x_i x_j \quad (579)$$

Subject to:

$$\sum_{i=1}^n \mu_i x_i \geq \rho \quad (580)$$

$$\sum_{i=1}^n x_i = 1 \quad (581)$$

$$\sum_{i=1}^n y_i = K \quad (582)$$

$$\epsilon_i y_i \leq x_i \leq \delta_i y_i \quad (583)$$

$$y_i \in \{0, 1\} \quad i = \overline{1, n} \quad (584)$$

With ϵ_i the lower bound and δ_i the upper bound ensuring the limitation on quantity invested in each asset i .

The resolution approach presented in our work follows two separate phases to solve the problem: The portfolio selection procedure represents a classification of the n available assets according to their respective returns μ_i and risks σ_{ij} (covariance between asset i and asset j) from the most to least interesting and a selection of the K first assets classified in the total order [1].

- we calculate the ratio $\gamma_i = \frac{\sigma_{ij}}{\mu_i}$ for each asset i .
- we order the ratios γ_i for $i = \overline{1, n}$ an increasing order (create partial orders $\mathcal{O}_i$).
- we calculate the mean rank r_i for each asset.
- we construct the total order $\mathcal{O}_T$.

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- we select K first assets of the order $\mathcal{O}_T$.

Once the selection procedure is applied, the problem can be reformulated as a classical mean-variance Markowitz model [3] that could be solved using quadratic programming methods (the cardinality constraint is already verified, and the problem is reduced to a K -sized quadratic problem).

The second phase is dedicated to the determination of the proportions to invest in each asset, using appropriate methods to solve the reduced quadratic problem.

In our work, we suggest an exact approach to treat the problem based on Merton's Analytic derivation of the efficient portfolio frontier [4]. The efficient portfolio is determined according to an expected return ρ , while the proportions of investment are computed for each risky asset using Merton's constants, as well as the equation for the variance of a frontier portfolio (portfolio risk σ) written as a function of its expected return ρ .

Main results

To illustrate this resolution concept, we applied it for an instance of $n = 10$ assets of the French Stock Market of the index CAC 40. The data has been extracted from Yahoo Finance and it represents the daily prices of the respective assets. The study was conducted over 5 years (from June 2013 to June 2018). The respective returns and variances of return have been calculated, besides the matrix of variance-covariance between the assets (Find tables in [1]).

We implemented in Python, the Merton-based resolution concept and we computed the Mean-variance portfolio for increasing expected return values, starting from the required ρ . The results are presented in the Figures (30-33). The efficient portfolio has been calculated similarly, for each required value of return to permit the observation of what we could consider as the Cardinality-constrained Efficient Frontier (Figures 34-37).

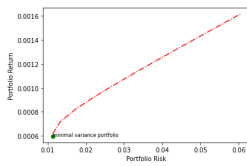


Figure 30: $N = 10$, $K = 5$ and 10 variations of ρ

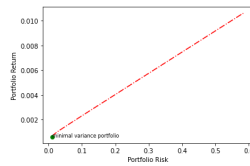


Figure 31: $N = 10$, $K = 5$ and 100 variations of ρ

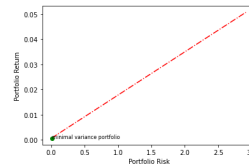


Figure 32: $N = 10$, $K = 5$ and 500 variations of ρ

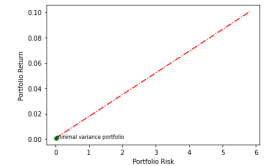


Figure 33: $N = 10$, $K = 5$ and 1000 variations of ρ

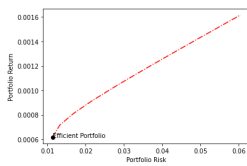


Figure 34: $N = 10$, $K = 5$ and 10 variations of ρ

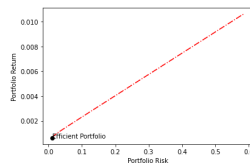


Figure 35: $N = 10$, $K = 5$ and 100 variations of ρ

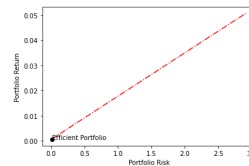


Figure 36: $N = 10$, $K = 5$ and 500 variations of ρ

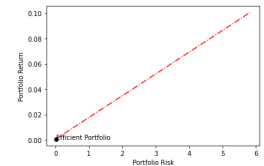


Figure 37: $N = 10$, $K = 5$ and 1000 variations of ρ

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Conclusion

Mainly, we suggested an exact approach to solve the cardinality-constrained portfolio selection problem. Our resolution concept goes through a selection procedure that permits the choice of portfolio composition, while the proportions of investments are determined by a Merton-based algorithm.

At first sight, the results obtained from our computational experiments give promising horizons. Our method gives efficient solutions and gives the ability to build a concrete cardinality-constrained Efficient Portfolio. More refined and real problem instances will be subject of computational experiments, precisely on large-scale real data sets. In future work, we are working on comparisons between metaheuristics and exact approaches for the proportions of the investments' determination phase.

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A new primal-dual interior point algorithm for convex quadratic semidefinite optimization

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Abstract: In this work, we propose a feasible primal-dual path following interior point method based on a new Darvay et al. strategy to solve a convex quadratic semidefinite optimization (CQSDO) problem. We introduce a new algebraic equivalent transformation (AET) of the non-linear equations of the central path in order to define a class of new search directions. At each iteration, the algorithm uses only full Nesterov-Todd (NT) step to obtain an approximate solution for CQSDO and therefore no line-searches are needed for generating the new iterations. The convergence of the algorithm is established and the complexity result coincides with the best-known iteration bound for these types of methods.

Keywords: Convex quadratic semidefinite optimization, interior point method, algebraic equivalent transformation, complexity of algorithm.

2020 Mathematics Subject Classification: 90C51, 90C25.

Introduction

Let $\mathbf{S}^n$ denote the space of real symmetric matrices of order n . The standard primal form of convex quadratic semidefinite optimization (CQSDO) problem is as follows

$$(\mathcal{P}) \min_X \{C \bullet X + \frac{1}{2}X \bullet Q(X) \text{ s.t. } A_i \bullet X = b_i, i = 1, \dots, m, X \geq 0\},$$

and its Lagrange dual problem

$$(\mathcal{D}) \max_{(X, y, Z)} \{b^\top y - \frac{1}{2}X \bullet Q(X) \text{ s.t. } \sum_{i=1}^m y_i A_i - Q(X) + Z = C, Z \geq 0, y \in \mathbf{R}^m\},$$

where $C, A_i \in \mathbf{S}^n$, $b \in \mathbf{R}^m$ and $Q : \mathbf{S}^n \rightarrow \mathbf{S}^n$ is a self-adjoint linear operator on $\mathbf{S}^n$, i.e., $A \bullet Q(B) = Q(A) \bullet B$, for all $A, B \in \mathbf{S}^n$. The notation $X \geq 0$ means that the matrix X is symmetric positive semidefinite. However, the symbol $A \bullet B$ denotes the trace inner-product in $\mathbf{S}^n$ defined by $A \bullet B = \text{Tr}(AB) = \sum_{i,j=1}^n A_{ij}B_{ij}$.

The CQSDO has many important applications in mathematical programming and scientific problems. For example the nearest Euclidean distance matrix problem and the nearest correlation matrix problem are models of CQSDO. Also if $Q(X) = 0$, then CQSDO reduces to a pair of primal-dual semidefinite optimization (SDO) [2]. The CQSDO includes also the standard convex quadratic optimization. It turns out that CQSDO is a special case of the so-called semidefinite linear complementarity problem.

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In the last decade, several primal-dual interior point methods (IPMs) designed for linear optimization (LO) have been successfully extended to SDO [2] and CQSDO, due to their polynomial complexity and practical efficiency. Finding the search directions plays an important role in IPMs. Recently, Kheirfam et al. [3] proposed a feasible primal-dual path-following interior point algorithm for LO. Their algorithm is based on a new search direction for LO [1] and proved that the proposed approach has the same complexity bound obtained by Darvay et al. [1].

Motivated by their work, we propose a new feasible primal-dual path-following interior point algorithm for CQSDO. This algorithm is based on an extension of the strategy presented in the work of Kheirfam et al. [3] in order to define a class of new search directions. We adopt the basic analysis used in [2] to the CQSDO case. The iteration bound for the algorithm with the small-update method is as good as the bound for the LO case [1, 3].

Main results

As a consequence, we have realized the following contributions:

1. We have successfully extended a primal-dual path-following interior-point algorithm based on a new Darvay's strategy [1] for LO to CQSDO problems.
2. We have established that the proposed algorithm is well defined and converge to an approximate solution for CQSDO.
3. We obtain the currently best known iteration bound for the associated short-step algorithm, which is the same iteration bound as in the LO, SDO and CQSDO cases.

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A generalized Newton method for solving a new general absolute value equations

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Abstract: In this paper, we consider some sufficient conditions to guarantee the unique solvability of the new general absolute value equations (NGAVE), $Ax - |Bx| = b$, ($A, B \in \mathbb{R}^{n \times n}$, $b \in \mathbb{R}^n$). Besides, Picard's iterative method for solving the NGAVE, a generalized Newton is also proposed for solving NGAVE. Moreover, under suitable assumptions, we show that the proposed methods are globally linearly convergent. We also report some numerical results of the proposed method for solving the NGAVE, which show the efficiency of our proposed methods.

Keywords: Absolute value equations, Picard's iterative method, Generalized Newton method, Global convergence.

2020 Mathematics Subject Classification: Primary 65F08, 90C33, 93C05.

Introduction

In this paper, we consider a new general absolute value equations (abbreviated as NGAVE) of type:

$$Ax - |Bx| = b, \quad (585)$$

where $A, B \in \mathbb{R}^{n \times n}$ are given, $b \in \mathbb{R}^n$, and $|Bx|$ is a vector whose i -th entry is the absolute value of the i -th entry of Bx . If $B = I$ the identity matrix, then the NGAVE (585) can be reduced to the type:

$$Ax - |x| = b. \quad (586)$$

The last one was introduced by Rohn [4] and investigated in a more general context by Mangasarian and Meyer (see [1]). The AVE (586) and NGAVE (585) have reserved much attention from the optimization community. It is currently an active research topic, due to its broad application to many areas of scientific computing and engineering application. For instance, linear complementarity, linear programming, convex quadratic programming, and bi-matrix games can be equivalently to the NGAVE (585).

The goal of this paper is twofold: First, we present some weaker sufficient conditions that guarantee the unique solvability of the NGAVE (585). Second, for its numerical solution, we propose a generalized Newton method. In particular, and under mild assumption we show that this method is always well-defined and the generated sequence converges globally and linearly to the unique solution of the NGAVE from any starting initial point. Finally, numerical results are provided to illustrate the efficiency of our proposed algorithm for solving the NGAVE. In addition, a numerical comparison is made with an available method.

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Main results

Some conditions to guarantee the unique solution of the NGAVE (585) are presented. First, for given matrices $A, B \in \mathbb{R}^{n \times n}$ and for any diagonal matrix $D \in \mathbb{R}^{n \times n}$ whose diagonal elements are ± 1 and 0, we define the matrix $(A - DB) \in \mathbb{R}^{n \times n}$. Then to achieve our main results, the following lemma is required.

Lemma 41. *Each of three conditions below implies the non singularity of $(A - DB)$.*

1. $\sigma_{\min}(A) > \sigma_{\max}(B)$,
2. $\|A^{-1}\| \|B\| < 1$, provided A is non singular,
3. the matrix $A^T A - \|B\|^2 I$ is positive definite.

The following result guarantees the unique solvability of the NGAVE.

Theorem 235. *If matrices A and B satisfy*

1. $\sigma_{\min}(A) > \sigma_{\max}(B)$,
2. $\|A^{-1}\| \|B\| < 1$, provided A is non singular,
3. The matrix $A^T A - \|B\|^2 I$ is positive definite, then the NGAVE is uniquely solvable for any b .

The generalized Newton method for the NGAVE

In this section, we propose a generalized Newton method for solving the NGAVE (585) and we show its convergence.

Let the function $F : \mathbb{R}^n \rightarrow \mathbb{R}^n$ be defined as:

$$F(x) = Ax + |Bx| - b. \quad (587)$$

The generalized Jacobian $\partial F(x)$ of $F(x)$ is given by:

$$\partial F(x) = A - D(Bx)B. \quad (588)$$

where $D(Bx) := \text{diag}(\text{sign}(Bx))$, $x \in \mathbb{R}^n$. The generalized Newton method for finding a zero of the equation $F(x) = 0$ and after some simplifications consists then of the following iteration:

$$(A - D(Bx_k)B)x_{k+1} = b, k = 0, 1, \dots$$

The details of the algorithm for solving the NGAVE (585) is described figure 1 as follows:

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Algorithm

Input

An accuracy parameter $\epsilon > 0$;
 an initial starting point $x_0 \in \mathbb{R}^n$;
 two matrices A and B and vector b ;
 set $k:=0$;
while $\|Ax_k - |Bx_k| - b\| > \epsilon$ **do**
begin
 compute x_k from the linear system $(A - D(Bx_k)B)x_{k+1} = b$;
 $k := k + 1$;
end;
end.

Fig 1. A generalized Newton algorithm for the NGAVE

Next, we will study the global convergence of the generalized Newton method.

Lemma 42. Suppose that $\|(A - DB)^{-1}\| \leq \frac{1}{2\|B\|}$, when $\|B\| \neq 0$, for any diagonal matrix D with diagonal elements of ± 1 or 0 . Then Algorithm 4.1 converges linearly from any starting point to a solution x^* of the NGAVE (585).

We are now ready to prove our main result of the global convergence. We quote first the following lemma.

Lemma 43. If assumption $\|A^{-1}\| \leq \frac{1}{\|B\|}$, when $\|B\| \neq 0$ holds, then $(A - DB)$ is nonsingular and

$$\|(A - DB)^{-1}\| \leq \frac{\|A^{-1}\|}{1 - \|A^{-1}B\|}.$$

Proposition 35. Let $\|A^{-1}\| < \frac{1}{3\|B\|}$ and $D(Bx_k) \neq 0$ for all k . Then the NGAVE is uniquely solvable for each $b \in \mathbb{R}^n$ and the generalized Newton iteration is well-defined and converges globally from any starting point to a solution x^* for any solvable NGAVE (585).

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Fuzzy Stochastic, Global Optimization (multi- Objectives), resolutions, Numerical simulations and Software

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Abstract: In many Situations of reality, one tends to model the real problems of the life with mathematical programs with deterministic variables (parameters). This hypothesis is little realistic since in reality the variables (parameters) can be impregnated with inaccuracies. With Inaccuracies of Fuzzy nature or of stochastic nature or of stochastic Fuzzy nature. This which subsequently gave rise to examples of: * Fuzzy mathematical programming (variables : L-R, Triangular, Trapezoids,...etc.). * Stochastic mathematical programming. * Fuzzy mathematical programming. * Fuzzy stochastic Global (Multi-Objective) Mathematical Programming . Fuzzy Random Variables give a better formalism to optimization problems modeled as such for example. One can quote in this sense the researchers who contributed in this meaning: Kwakernaak, Kruse, Meyer, Puri, Ralescu, recently, I . Couso, D Dubois, Shapiro, Charnes and Cooper, ...etc. Here is a case of our results to be solved including normal random variables of fuzzy type L-R with left and right deviations of positive real numbers. Subsequently, we will increase the difficulties with non-linear non-convex functions and with multi-objective mathematical problems (Global Optimization).

$$(PNLFS) \left\{ \begin{array}{l} (\tilde{c}_1(\omega) \odot x, \tilde{c}_2(\omega) \odot x, \dots, \tilde{c}_k(\omega) \odot x) \rightarrow \max \\ \sum_{j=1}^n \tilde{a}_{ij}(\omega) \odot x \leq \tilde{b}_i(\omega), i = 1, \dots, m. \\ x \in A = \{x \in \mathbb{R}^n : x_j \geq 0, j = 1, \dots, n.\} \end{array} \right.$$

Keywords: Fuzzy, Global Optimization, Mathematical Programming, multi Objectives, resolutions, Numerical simulations, Software.
bf Mathematics Subject Classification: 35R11 .

Introduction

A stochastic fuzzy multi-objectives linear program is a multi-objective linear program in the presence of:

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fuzzy random variables

Or

real random variables and fuzzy intervals.

Main results

We will therefore present our results through stochastic fuzzy multi-objective programs whose function coefficients are:

- * Determinists.
- * Blurred intervals.
- * Real Random Variables.
- * Fuzzy random variables.

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Numerical implementation of a penalty method for nonlinear complementarity problems

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Abstract: A penalty method for solving nonlinear complementarity problems (NCPs) is proposed. By using this approach, the problem (NCPs) is approximated by a sequence of nonlinear equations containing a penalty term.. The merit of this method shows that the solution of the sequence of nonlinear equations converges to that of the NCP (NCPs) when the penalty parameter tends to positive infinity under mild assumptions. An algorithm is constructed and numerical examples indicate the feasibility of our method.

Keywords: nonlinear complementarity problems, penalty method, Strongly-monotone operator.

AMS Classification: 47J20, 65K10, 65J15.

Introduction

Consider the following nonlinear complementarity problem *NCP* :

Problem

$$\begin{cases} \text{Find } x \in \mathbb{R}^n \text{ such that} \\ x \geq 0, F(x) \geq 0 \text{ and } x^t F(x) = 0, \end{cases} \quad (589)$$

where $F : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a continuous mapping from $\mathbb{R}^n$

It is worth noting that *NCP* becomes linear complementarity problem *LCP* if F is affine. So, it also includes the Linear Optimization (*LO*) and Convex Quadratic Optimization (*CQO*). *NCP* arise in many

mathematical models from engineering, physics, mechanics and economics. Various Numerical methods for solving *NCP* have been extensively investigated in the literature such as smoothing Newton methods, interior point method, and nonsmooth equation methods. In this work we present a numerical study of a penalty approach proposed by H.Chongchaoa, and W.Song the problems NCPs.

Let $C = \{x \in \mathbb{R}^n / x_i \geq 0\}$. we define the following variational inequality problem *VIP* corresponding to *NCP*:

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$$\begin{aligned} \text{Find } x \in \mathbb{R}^n \text{ such that for all } y \in C, \\ \langle F(x), y - x \rangle \geq 0 \end{aligned} \quad (590)$$

We will present and analyze a penalty method for NCP . Consider the following sequence of non-linear equations VEP_r :

$$\begin{aligned} \text{Find } x_r \in \mathbb{R}^n \text{ such that ,} \\ E(x_r) = F(x) + rB(x_r) = 0 \end{aligned} \quad (591)$$

where $r > 0$ is the penalty parameter and $B = \text{Pr}_{\mathbb{R}^n_-}$ is the penalty projection operator, the problem 3 is an approximate problem of the initial problem (589). When $r \rightarrow +\infty$, we expect that the solution x_r of problem (591) converges to that of problem (589). A detailed convergence analysis for the solution of (591) will be developed in the next section under some mild assumptions on the mapping F .

Main results

In this section, we establish some upper bounds for the distance between the solutions of problems 1 and 3 respectively. Before, we first make the following assumptions on F and C .

A1. F is continuous on $\mathbb{R}^n$.

A2. F is strongly-monotone, i.e., there exist a constant $\alpha > 0$ such that : $\langle F(x) - F(y), x - y \rangle \geq \alpha \|x - y\|^2$, $\forall x, y \in \mathbb{R}^n$.

Theorem 236. Let $\bar{x}$ and x_r be the solutions of problems 1 and 3 respectively and the assumptions A1, A2 are satisfied. There exists a constant $c > 0$, independent of x_r and r , such that:

$$\|\bar{x} - x_r\| \leq \frac{c}{\sqrt{r}} \quad (592)$$

The algorithm and its convergence

In this section, a corresponding algorithm is constructed for solving the problem LCP .

Algorithm

Begin algorithm

1. Initialization

Let $\varepsilon > 0$ be a given precision and $\theta > 1$; Let $x^0 \in \mathbb{R}^n$, $r_0 = 1$, and $k = 0$;

2. Iteration

- Find x^{k+1} solution of nonlinear equation $E(x) = F(x) + r_k \text{Pr}_{\mathbb{R}^n_-}(x) = 0$

3. If $\|\text{Pr}_{\mathbb{R}^n_-}(x^k)\| \leq \varepsilon$ or $\|x^{k+1} - x^k\| \leq \varepsilon$; stop: x^{k+1} is an approximate solution of NCP .

If not - Take:

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- - $r_{k+1} = \theta r_k$; $x^k = x^{k+1}$; $k = k + 1$, and go back to 2.

End algorithm.

Theorem 237. *Let assumption A1 holds and F strongly-coercive, then the sequence $\{x^k\}$ generated by the algorithm converges to the unique adherent value $\bar{x}$ solution of the problem (589).*

Conclusion

In this paper, we have proposed and analyzed a power penalty method for solving the nonlinear complementarity problem. The method consists of formulating the variational inequality as a sequence of penalized nonlinear equations. We have shown that the sequence of solutions of the last one converges to that of the nonlinear complementarity problem. To illustrate clearly the effectiveness of our approach and consolidate our theoretical result, we have presented some numerical simulations.

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Direct support method extension for the bounded portfolio optimization problem

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Abstract: Portfolio optimization has received increasing attention from the research community in recent years, due to its important contribution to the investment decision-making process and financial risk management. In this work, a parametric extension of the direct support method is presented to solve the Mean-Variance model encountered in the portfolio optimization problem. The studied problem includes the budget constraint as well as limits on financial assets. The proposed approach allows to iteratively identify all the extreme portfolios of the problem by calculating their associated turning points, enabling to fully derive the Markowitz efficient frontier.

Keywords: Parametric portfolio optimization, Mean-Variance model, direct support method, Markowitz efficient frontier.

Introduction

Stock markets offer excellent investment opportunities, but it's important to understand that the most profitable assets are often associated with a higher level of risk. Indeed, investors aim for an optimal allocation of their funds, in order to maximize profits while minimizing risk. This problem was first addressed in 1952 by H. Markowitz in his Mean-Variance (MV) model [1], a pioneering work in finance. This model provides a precise mathematical framework and defines a process for selecting assets to optimize a holding portfolio. Markowitz put forward the idea that a portfolio's return is the mean (expectation) of asset returns, while its risk can be measured by the variance of these returns. This formulation leads to representing the model through the following parametric quadratic optimization problem:

$$\min\left\{\frac{1}{2}x'\Sigma x - \lambda\mu'x \mid e'x = 1, l \leq x \leq u\right\}, \quad (593)$$

where, n is the problem size, x is a *portfolio* denotes the vector of proportions to be invested in each asset, μ is a n -vector of asset returns, Σ is the variance-covariance matrix assumed to be symmetric positive definite, $t \geq 0$ is a parameter to be interpreted as an investor's risk tolerance (risk aversion). e is a n -vector of ones, and $e'x = 1$ is the budget constraint. In order to further reflect real-world constraints, we consider limitations on assets by introducing lower and upper bounds represented by vectors l and u , respectively.

Parametric quadratic programming (PQP) is a commonly used framework for solving portfolio optimization problems. In this context, several approaches have been developed to solve the MV

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model [2][3] or to improve existing methods [4]. In this paper, we propose a parametric extension of the Direct Support Method (DSM) [5] for solving Problem (593), building on the work in [6]. The proposed algorithm comprises two phases, the first phase involves identifying the minimum variance portfolio, while the second phase allows to iteratively calculate all the corner portfolios of the problem by computing the associated turning points, thus providing complete information on the efficient frontier. The outline of the algorithm is described in the next section.

Support method extension for the bounded portfolio selection

KKT-optimality conditions

Before presenting the key points of the algorithm, we will introduce the Karush-Kuhn-Tucker (KKT) optimality conditions. Let $L(x, b, \nu) = \frac{1}{2}x'\Sigma x - \lambda\mu'x + y'(ex - 1) - \nu'x$, be the Lagrangian function, with $y \in \mathbb{R}$ and $\nu \in \mathbb{R}^n$ are respectively the Lagrange multipliers associated with the equality and inequality constraints. Since Problem (593) is convex (Σ is symmetric positive definite), then the following first-order KKT optimality conditions are both necessary and sufficient: souh

$$\frac{\partial L}{\partial x} = \Sigma x - \lambda\mu + ye - \nu_1 + \nu_2 = 0, \quad (594a)$$

$$\frac{\partial L}{\partial y} = ex - 1 = 0, \quad (594b)$$

$$\nu_1'(l - x) = 0, \quad \nu_2'(x - u) = 0, \quad (594c)$$

$$l \leq x \leq u, \quad \nu_1, \nu_2 \in \mathbb{R}. \quad (594d)$$

Minimum variance portfolio

In order to calculate the efficient frontier of (593), it is necessary to identify an initial portfolio. In our approach, we start with the minimum variance problem. This involves setting $t^0 = 0$ and solves the resulting quadratic problem using the support method. The optimal solution obtained serves as the starting point for the second phase of the proposed parametric support method.

Parametric approach

The second phase (parametric) involves calculating iteratively the other pivot points $0 = t^0 \leq t^1 \leq t^2 \leq \dots$, by solving the KKT system (594) for all values of t , $t^k \leq t \leq t^{k+1}$ where t^k represents the pivot point associated with the corner portfolio x^k at iteration k .

Efficient frontier

The solution of Problem (593) provides a set of optimal portfolios that form the Markowitz efficient frontier. This frontier represents the optimal trade-off between expected return and variance, characterized by parabolic segments intersecting at the extreme points (corner portfolios). Once all the corner portfolios are identified, their returns μ_p^k and risks $(\sigma_p^k)^2$ are calculated as a function of t . Then, by varying the parameter t in the interval $[t^k, t^{k+1}]$, we plot the efficient

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Figure 38: Example of the parametric efficient frontier with four corner portfolios.

frontier. Figure 38 shows an example of the parametric frontier obtained using the proposed algorithm.

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Linear programming and new search directions in interior point methods

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Abstract: In this Work, we are interested in the theoretical and numerical study of feasible path-following interior point methods of primal-dual type to solving linear programming problems which based on new directions. The latter is obtained via the application of algebraically equivalent transformations on the equations which characterizes the central-path. Under some appropriate conditions, the corresponding algorithm is well defined and converges quadratically to an optimal solution. In addition, this algorithm with short-step admits the best polynomial complexity. Finally, this work is ended by presenting some numerical results to show its efficiency .

Keywords: Linear programming, Interior point methods, Short-step algorithm, Short-step algorithm, Algebraic equivalent transformation

2020 Mathematics Subject Classification: Primary 90C05, 90C06

Introduction

Many economic and industrial phenomena can be modelled by mathematical systems of linear inequalities and equalities, which give rise to linear optimisation problems. In these linear optimisation problems, the objective is to minimise or maximise a linear function under linear constraints on the variables of the problem.

The industrial applications of linear programming are very present, for example, in the oil industry (for the extraction, refining and distribution of oil), in the food industry (optimal composition of ingredients of ready meals, etc.), in the iron and steel industry (optimal composition of steels), in the paper industry (cutting problems), in transport (flight schedule of airplanes, minimisation of transport costs, etc.) and in networks (optimisation of communication).

In 1947, Dantzig proposed the well-known simplex method for solving PL problems. The simplex algorithm has been continuously improved over the last 50 years. years and people have become convinced of the practical effectiveness of the algorithm. Although the simplex algorithm is efficient in practice, no one has been able to prove the polynomial complexity of the algorithm, i.e. the number of iterations required by the algorithm can be exponential.

In 1979, Khachiyan developed the first polynomial algorithm to solve the PL problem. Khachiyan's ellipsoid method is an important theoretical result, but its practical implementation was not a competitive alternative to the simplex method.

In 1984, Karmarkar proposed his polynomial algorithm. Karmarkar's algorithm uses the interior points of the polytope to approximate the optimal solution. The complexity of this algorithm is

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less than that of Khachiyan and the implementation of Karmarkar's algorithm has also proven to be effective in practice, especially when the problem size is large. As a result, research in the field of LP has become very active, and the field of interior point methods is still remains an important area of research.

There are three main categories of interior point methods: affine methods, potential reduction methods, and central trajectory methods. Our work consists of a theoretical and numerical study of a central trajectory type method proposed by Behrouz Kheirfam and Afsaneh Nasrollahi in 2018, where they used the integer powers of the square root function to find the search directions.

We consider the following primal-dual pair of linear problems

$$(P) \quad \left\{ \min_x \{c^t x, Ax = b, x \geq 0\} \right.$$

and

$$(D) \quad \left\{ \max_{y,z} \{b^t y, A^t y + z = c, x \geq 0, z \geq 0, y \in \mathbb{R}^m\} \right.$$

where $b, y \in \mathbb{R}^m$, $x, z, c \in \mathbb{R}^n$ and $A \in \mathbb{R}^{m \times n}$ has full row rank ($rg(A) = m \leq n$).

We note that the optimality conditions of the primal-dual pair (P) and (D) can be written in the following way:

$$\begin{cases} Ax = b, \\ A^t y + z = c, \\ xz = 0, \\ (x, z) \geq 0, \end{cases} \quad (595)$$

The basic idea of primal-dual IPMs is to replace the third equation in (595), the so-called complementary condition for (595), by the parameterized equation $xz = \mu e$, where e denotes the all-one vector and $\mu > 0$. This leads to the following system :

$$\begin{cases} Ax = b, \\ A^t y + z = c, \\ xz = \mu e, \\ (x, z) \geq 0, \end{cases} \quad (596)$$

Considering the approach introduced by Darvay [3], system (595) can be written in the following equivalent form:

$$\begin{cases} Ax = b, \\ A^t y + z = c, \\ \varphi\left(\frac{xz}{\tau\mu}\right) = \varphi(e), \\ (x, z) \geq 0, \end{cases}$$

where $\tau \in [0, 1]$ is the centering parameter and φ is a continuously differentiable function such that $\varphi(t) > 0$ for all $t > 0$. In our study, we have used the integer powers of the square root function, given by the function $\varphi(t) = t^{q/2}/t > 0, q \in \mathbb{N}$.

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Main results

- 1) We have thoroughly studied the proposed algorithm, then we performed numerical study on some problems of different sizes of linear programming.
- 2) We have improved the choice of the parameter q to obtain a good behaviour of the algorithm.

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Study of some modified Fletcher-Reeves methods for nonlinear optimization problem

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Abstract: The conjugate gradient method is one of the most efficient methods for solving unconstrained nonlinear optimization problems. In this work, a numerical comparative study was made between three hybrid methods of the conjugate gradient using Fletcher Reeves method, such as the FRM, FR-LS and FR-HS methods, using different line search rules. Numerical tests were performed on several test functions for different dimensions. The numerical tests show that the two hybrid methods FR-HS and FR-LS are very close and at the same time more efficient than the FRM method, whatever the line search carried out.

Keywords: Conjugate gradient method, Nonlinear optimization, Descent direction, Line search.

2020 Mathematics Subject Classification: 90C26, 90C30.

Introduction

Regarding the following problem:

$$(P) \begin{cases} \min f(x) \\ x \in \mathbb{R}^n \end{cases}$$

where f is a continuously differentiable function. The nonlinear conjugate gradient methods are efficient to solve the problem (P) using the following iteration form:

$$\begin{cases} x_1 & \text{initial point} \\ x_{k+1} = x_k + \alpha_k d_k & k \geq 1 \end{cases} \quad (597)$$

where the step length $\alpha_k > 0$ is determined by inexact line search, and d_k the search direction given by:

$$d_k = \begin{cases} -g_k & k = 1 \\ -g_k + \beta_k d_{k-1} & k \geq 2 \end{cases},$$

where $g_k = \nabla f(x_k)$ and β_k is a scalar conjugacy coefficient. There are some well-known formulas of β_k such as: Hestenes-Stiefel (HS), Fletcher-Reeves (FR), Polak-Ribière-Polyak (PRP), Conjugate

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Descent of Fletcher (CD), Liu-Storey (LS) and Dai - Yuan (DY). The convergence behavior of conjugate gradient methods depends on the choice of β_k . There are many convergence results with some line search conditions which has been widely studied. These methods guarantee the descent property of each direction which has provided the step length computed by carrying out a line search.

The different values assigned to β_k define the different variants of the conjugate gradient method. Among the best known of β_k , we have:

$$\beta_k^{FR} = \frac{\|g_k\|^2}{\|g_{k-1}\|^2}, \quad \beta_k^{PRP} = \frac{g_k^T y_{k-1}}{\|g_{k-1}\|^2},$$

$$\beta_k^{HS} = \frac{g_k^T y_{k-1}}{y_{k-1}^T d_{k-1}}, \quad \beta_k^{LS} = -\frac{g_k^T y_{k-1}}{g_{k-1}^T d_{k-1}},$$

where $y_{k-1} = g_k - g_{k-1}$ and $\|\cdot\|$ design the eucliden norm.

Inexact line search

The purpose of line search is to find α_k which significantly reduce the value of $f(x_k)$ using (597).

For this, many mathematicians have developed several rules such as Armijo, Wolfe, Backtracking, Wu, Fletcher, ...etc. In this work, we consider the three line search defined by:

Wolfe's rule

The Wolfe's conditions are :

$$f(x_k + \alpha d_k) \leq f(x_k) + \rho \alpha \nabla^t f(x_k) d_k$$

$$\nabla^T f(x_k + \alpha d_k) d_k \geq \sigma \nabla^t f(x_k) d_k$$

with $0 < \rho < \sigma < 1$.

Backtracking's rule

The line search method of Backtracking is given by the following procedure:

Choose $\alpha > 0$, $0 < \rho < 0.5$ and $\mu \in]0, 1[$.

While $f(x_k + \alpha d_k) > f(x_k) + \rho \alpha \nabla^T f(x_k) d_k$, take $\alpha = \alpha \mu$.

Wu's rule

Let f be a continuously differentiable function and its gradient continuously lipschitzian and strongly convex in $\mathbb{R}^n$, i.e., there are constants $\tau > 0$ and $\kappa \geq 0$ such that

$$\|\nabla f(x_{k+1}) - \nabla f(x_k)\| \leq \tau \|x_{k+1} - x_k\|, \forall x_{k+1}, x_k \in \mathbb{R}^n$$

$$[\nabla f(x_{k+1}) - \nabla f(x_k)]^T (x_{k+1} - x_k) \geq \kappa \|x_{k+1} - x_k\|^2, \forall x_{k+1}, x_k \in \mathbb{R}^n$$

The formula of α_k is

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$$\alpha_k = \frac{-\delta g_k^T d_k}{(\bar{g}_{k+1} - g_k)^T d_k + \gamma \theta_k}, \gamma > 0 \text{ and } \delta \in \left] 0, \frac{\kappa}{\rho} \right[.$$

where $\theta_k = 6(f_k - \bar{f}_{k+1}) + 3(g_k + \bar{g}_{k+1})^T d_k$

and $f_k = f(x_k)$, $\bar{f}_{k+1} = f(x_k + d_k)$, $\bar{g}_{k+1} = \nabla f(x_k + d_k)$

Main results

We are interested in three new hybrid methods of the conjugate gradient that combine the FR method with other known variants.

The first hybrid method combine the FR method with the HS method [2] given by

$$\beta_k^{FRHS} = (1 - \theta_k)\beta_k^{HS} + \theta_k\beta_k^{FR} \quad (598)$$

with $0 \leq \theta_k \leq 1$.

The second hybrid method combine the FR method with the LS method [3] given by

$$\beta_k^{FRLS} = (1 - \theta_k)\beta_k^{LS} + \theta_k\beta_k^{FR} \quad (599)$$

with $0 \leq \theta_k \leq 1$.

The third hybrid method combine the FR method with the PRP method [4] given by

$$\beta_k^{FRM} = \frac{\gamma}{\|g_{k-1}\|}\beta_k^{PRP} + \|s_{k-1}\|\beta_k^{FR} \quad (600)$$

where $s_{k-1} = x_k - x_{k-1}$ and $\gamma > 0$.

In this work, a comparative numerical study was made between these three methods of the conjugate gradient, by using different rules of inexact line search, namely: Wolfe, Wu and Backtracking, to calculate the step length α_k . Numerical tests were performed on several test functions [1] for different dimensions (n).

Conclusion

The numerical tests show that the two hybrid methods FR-HS and FR-LS are very close and at the same time more efficient than the FRM method, in terms of number of iterations and calculation time, whatever the line search carried out. For the inexact line search rules used, we noticed that some functions give better results with Wolfe, however other functions give better results with Backtracking.

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A hybrid CG algorithm for nonlinear unconstrained optimization with application in image restoration

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Abstract: This paper presents a new hybrid conjugate gradient method for solving nonlinear unconstrained optimization problems; it is based on a combination of *RMIL* (Rivaie-Mustafa-Ismail-Leong.) [2] and *hSM* (hybrid Sulaiman- Mohammed) [3] methods. The proposed algorithm enjoys the sufficient descent condition without depending on any line search; moreover, it is globally convergent under the usual and strong Wolfe line search assumptions. The performance of the algorithm is demonstrated through numerical experiments on a set of 100 test functions from [1] and four image restoration problems with two noise levels. The numerical comparisons with four existing methods show that the proposed method is promising and effective.

Keywords: Unconstrained optimization, Hybrid conjugate gradient, Global convergence, Image restoration.

2020 Mathematics Subject Classification: 90C26, 90C30.

Introduction

In this work, we consider the following unconstrained problem:

$$\min_{x \in \mathbb{R}^n} f(x), \quad (\text{P})$$

where $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is a continuously differentiable function with available gradient $\nabla f(x)$. The conjugate gradient methods (CG) are among the most effective methods for solving the problems of type (P) due to their simplicity and low storage. They take up several forms; their principle is to generate a sequence of points $\{x_k\}_{k \geq 0} \subset \mathbb{R}^n$ starting from an initial point $x_0 \in \mathbb{R}^n$ following the procedure

$$x_{k+1} = x_k + \alpha_k d_k, \quad (601)$$

where d_k is a descent direction for f at x_k and $\alpha_k \in \mathbb{R}^+$ is a step-length which determine by using a line search procedure; typically, it is chosen in such a way that it satisfies the weak Wolfe

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conditions or the strong Wolfe conditions. The search direction d_k is usually defined as

$$d_k = \begin{cases} -g_k & \text{if } k = 0, \\ -g_k + \beta_k d_{k-1} & \text{if } k \geq 1, \end{cases}$$

where the parameter β_k is a scalar which determines the different conjugate gradient methods. In this work, we propose a new hybrid conjugate gradient method, named CR (Combined method for Restoring images) method, to simultaneously solve unconstrained optimization and also deal with image restoration problems.

Main results

The parameter β_k is regarded as a convex combination as follows

$$\beta_k^{CR} = (1 - \theta_k) \beta_k^{RMIL} + \theta_k \beta_k^{hSM}, \quad (602)$$

where $\theta_k \in [0, 1]$, $\beta_k^{RMIL} = \frac{g_k^T(g_k - g_{k-1})}{\|d_{k-1}\|^2}$ and $\beta_k^{hSM} = \begin{cases} \beta_k^{RMIL} & \text{if } 0 \leq \beta_k^{RMIL} \leq \beta_k^{hSM}, \\ \beta_k^{hSM} & \text{otherwise,} \end{cases}$. The search direction d_k computed as

$$d_k = \begin{cases} -g_k & \text{if } k = 0, \\ -g_k + \beta_k^{CR}(d_{k-1} - \rho_k g_k) & \text{if } k \geq 1, \end{cases} \quad (603)$$

where $\rho_k = \frac{d_{k-1}^T g_k}{\|g_k\|^2}$. Therefore, for $k \geq 1$: $d_k = -g_k + (\beta_k^{RMIL} + \theta_k \frac{2g_k^T g_{k-1}}{\|d_{k-1}\|^2})(d_{k-1} - \rho_k g_k)$.

Now, we need to consider the parameter θ_k . It is selected such that the search direction d_k satisfies also the following conjugacy condition $y_{k-1}^T d_k = 0$. Then, we get

$$\theta_k = \frac{\zeta_k - \beta_k^{RMIL} \lambda_k}{\eta_k \lambda_k}, \quad (604)$$

where $\zeta_k = y_{k-1}^T g_k$, $\lambda_k = y_{k-1}^T d_{k-1} - \rho_k y_{k-1}^T g_k$ and $\eta_k = 2 \frac{g_k^T g_{k-1}}{\|d_{k-1}\|^2}$. During the search process, we set $\theta_k = 1$ if $\theta_k > 1$; moreover, we set $\theta_k = 0$ if for such an iteration we have $\theta_k < 0$ or $\eta_k \lambda_k = 0$.

The sufficient descent condition:

It is well known that the sufficient descent property is crucial for the global convergence to hold. The next lemma deals with this issue, moreover it shows it is independent of any line search.

Lemma 44. *Let the sequences $\{g_k\}_{k \in \mathbb{N}}$ and $\{d_k\}_{k \in \mathbb{N}}$ be given by CR method, then*

$$g_k^T d_k = -\|g_k\|^2, \quad (605)$$

i.e. the direction d_k satisfies the sufficient descent condition.

The global convergence

We require the following mild assumptions.

Assumptions 1: The level set $\Omega = \{x \in \mathbb{R}^n : f(x) \leq f(x_0)\}$ is a bounded, i.e. there exists a constant $K > 0$ such that $\|x\| \leq K$, $\forall x \in \Omega$.

Assumptions 2: In a close neighbourhood $\mathcal{N}$ of Ω , the objective function f is continuously differentiable and its gradient g is Lipschitz continuous, i.e. there exists a positive constant L such that: $\|g(x) - g(y)\| \leq L\|x - y\|$, $\forall x, y \in \mathcal{N}$.

Note that Assumptions 1 and 2 imply that there exists a positive constant $\bar{\delta}$ such that $\|g(x)\| \leq \bar{\delta}$, $\forall x \in \mathcal{N}$.

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Lemma 45. Assume that Assumptions 1 and 2 hold. If the step-length α_k satisfies the strong Wolfe conditions and d_k is a descent direction, then:

$$\alpha_k \geq \frac{(\sigma - 1)}{L} \frac{d_k^T g_k}{\|d_k\|^2}.$$

Lemma 46. Assume that Assumptions 1 and 2 hold and consider any CG method that follows the form (601) where d_k is a descent direction and α_k satisfies the strong Wolfe line search. Then the Zoutendijk condition

$$\sum_{k \geq 0} \frac{(g_k^T d_k)^2}{\|d_k\|^2} < +\infty,$$

holds.

Theorem 238. Suppose that Assumptions 1 and 2 hold. Let $\{g_k\}_{k \in \mathbb{N}}$ and $\{d_k\}_{k \in \mathbb{N}}$ be the sequences generated by CR method; then $\liminf_{k \rightarrow \infty} \|g_k\| = 0$.

Numerical experiments

In the first part of this section we compare the performance profile of Dolan and Moré, relative to the number of iterations, function evaluations, gradient evaluations and CPU-time of the proposed method with seven conjugate gradient methods that are: PRP, EPF, RMIL, hSM*, THCG+, ADHCG1 and ADHCG2. The methods PRP, EPF, RMIL, hSM* and THCG+ are implemented using the strong Wolfe conditions, whereas ADHCG1 and ADHCG2 are implemented using the weak Wolfe conditions, by setting $\delta = 10^{-4}$ and $\sigma = 10^{-3}$.

In the second part, we compare the performance of CR algorithm with the variants studied in the above part to solve image restoration problems.

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A descent nonlinear conjugate gradient method for large-scale unconstrained optimization

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Abstract:

We know many conjugate gradient algorithms (CG) for solving unconstrained optimization problems. In this paper, based on the three famous Liu-Storey (LS), Fletcher-Reeves (FR) and Polak-Ribiere-Polyak (PRP) conjugate gradient methods, a new hybrid CG method is proposed. Furthermore, the search direction satisfies the sufficient descent condition independent of the line search. Likewise, we prove, under the strong Wolfe line search, the global convergence of the new method. In this respect, numerical experiments are performed and reported, which show that the proposed method is efficient and promising.

Keywords: Unconstrained optimization, hybrid method, sufficient descent, convex combination.

2020 Mathematics Subject Classification: Primary 65K05, 90C25, 90C26, 90C27, 90C30.

Introduction

Unconstrained optimization is a branch of optimization in which we minimize an objective function that depends on real variables with the total absence of restrictions on their values of those variables. Thus, we consider the general unconstrained optimization problems, as follows:

$$\min\{f(x), x \in \mathbb{R}^n\}, \quad (606)$$

where $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is the continuously differentiable function and its first derivative is represented by $g(x) = \nabla f(x)$. Though several robust optimization algorithms with rapid convergence have shown to be available to solve the above nonlinear optimization model, many researchers still refer to the conjugate gradient algorithm (CG) as it uses low memory and good convergence properties. Besides, this method was first established by Hestenes and Stiefel [1] to solve unconstrained linear optimization problems. Then, in 1964, Fletcher and Reeves [2] extended the form of the conjugate gradient method to solve unconstrained nonlinear minimization problems. As a consequence, the results of the expansion inspired researchers to suggest a new conjugate gradient method with good computational performance and, at the same time, good convergence properties. Generally, the iterates of the CG methods are usually determined through the following recursive computational scheme:

$$x_{k+1} = x_k + \alpha_k d_k, \quad k = 0, 1, \dots, n, \quad (607)$$

and

$$d_k = \begin{cases} -g_k, & \text{if } k = 0, \\ -g_k + \beta_k d_{k-1}, & \text{if } k \geq 1, \end{cases} \quad (608)$$

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where x_k is the current iteration, g_k is the gradient of f at the point x_k , d_k is the search direction, $\beta_k \in \mathbb{R}$ is the conjugate parameter which characterizes different versions of the CG methods and $\alpha_k > 0$ is the step size many line search techniques can obtain. In the midst of which, exact line search, weak Wolfe line search, or strong Wolfe line search, but we use, in our research, strong Wolfe line search, which is defined by the following conditions:

$$f(x_k + \alpha_k d_k) - f(x_k) \leq \delta \alpha_k g_k^T d_k, \quad (609)$$

$$|g_{k+1}^T d_k| \leq -\sigma g_k^T d_k, \quad (610)$$

where scalars δ and σ satisfy $0 < \delta \leq \sigma < 1$.

As we said, the CG algorithms differ by choosing the coefficient β_k . The most common standard CG methods are the Hestenes-Stiefel (HS) method [1], the Fletcher-Reeves (FR) method [2], the Liu-Storey (LS) method [3] and the Polak-Ribière-Polyak (PRP) method [4, 5], respectively, which are defined as:

$$\beta_k^{HS} = \frac{g_k^T y_{k-1}}{y_{k-1}^T d_{k-1}} \quad \beta_k^{FR} = \frac{\|g_k\|^2}{\|g_{k-1}\|^2}$$

$$\beta_k^{LS} = \frac{g_k^T y_{k-1}}{-d_{k-1}^T g_{k-1}} \quad \beta_k^{PRP} = \frac{g_k^T y_{k-1}}{\|g_{k-1}\|^2},$$

where $y_{k-1} = g_k - g_{k-1}$ and $\|\cdot\|$ stands for the Euclidean norm. If f is a strictly convex quadratic function, all the above methods are equivalent, while they behave differently for general non-quadratic functions.

One of the essential classes of CG methods is the hybrid conjugate gradient algorithms. Further, the hybrid schemes have better computational performances and more robust convergence properties than conventional CG methods as they take advantage of the two parameters used to build them. For this reason, many researchers cared about hybrid or mixed conjugate gradient methods. Djordjevic' [6], proposed the following hybrid method: $\beta_k^{hyb} = \theta_k \beta_k^{FR} + (1 - \theta_k) \beta_k^{HS}$, Andrei [7], proposed the following hybrid method: $\beta_k^c = \theta_k \beta_k^{DY} + (1 - \theta_k) \beta_k^{HS}$. In addition, Sabrina and al [8] proposed a new hybrid CG method based on combination of FR, PRP and DY conjugate gradient algorithms in which

$$\beta_k^{hyb} = \delta_k \beta_k^{FR} + \gamma_k \beta_k^{PRP} + (1 - \delta_k - \gamma_k) \beta_k^{DY},$$

where

$$\gamma_k = -\frac{g_k^T g_{k+1} \|g_k\|^2 + \delta_k (y_k^T s_k - \|g_k\|^2) \|g_{k+1}\|^2}{(g_{k+1}^T y_k)(y_k^T s_k) - \|g_{k+1}\|^2 \|g_k\|^2}, \text{ where, } 0 < \delta_k < 1.$$

Inspired by this research, we propose in this study a new hybrid CG method based on combination of LS, FR and PRP conjugate gradient algorithms for solving unconstrained optimization problems.

Main results

In this paper, we propose another combination of LS, FR and PRP conjugate gradient algorithms. We use the following conjugate gradient parameter:

$$\beta_k^{New} = \delta_k \beta_k^{LS} + \gamma_k \beta_k^{FR} + (1 - \delta_k - \gamma_k) \beta_k^{PRP}. \quad (611)$$

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As a consequence, the direction d_k is given by:

$$d_k = \begin{cases} -g_k, & \text{if } k = 0, \\ -g_k + \beta_k^{New} d_{k-1}, & \text{if } k \geq 1, \end{cases} \quad (612)$$

Where $0 \leq \delta_k, \gamma_k \leq 1$ are named the hybridization parameters and they are given by the following form:

$$\delta_k = \frac{(g_{k+1}^T y_k (\|g_k\|^2 - d_k^T y_k) - \gamma_k (g_{k+1}^T g_k)(d_k^T y_k))}{(\|g_k\|^2 + d_k^T g_k)(-g_{k+1}^T y_k)(d_k^T y_k)} d_k^T g_k, \quad \text{where } 0 < \gamma_k < 1. \quad (613)$$

The following Theorem proved that the search direction d_k generated by the selected method satisfies the sufficient descent condition.

Theorem 239. *Let the sequences $\{g_k\}$ and $\{d_k\}$ be generated by the new method. Then, the search direction d_k satisfies the sufficient descent condition:*

$$g_k^T d_k \leq -K \|g_k\|^2, \quad \forall k \geq 0, \quad (614)$$

where $K > 0$.

Now, we give the next theorem which establishes the global convergence of our proposed method with strong Wolfe line searches.

Theorem 240. *Consider the iterative method, defined by the new method and let all conditions of Theorem (239). Then*

$$\liminf_{k \rightarrow \infty} \|g_k\| = 0. \quad (615)$$

Numerical Results:

The results are given in two Figures which are represented the performance profiles of our new hybrid method versus FR, PRP and LS based on the CPU time and number of iterations, respectively. They shown that the performance profiles of our method was higher than the other CG methods on the testing problems.

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An efficient parametric hyperbolic kernel function for linear semidefinite programming

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Abstract: In this work, we introduce an efficient parametric hyperbolic kernel function (KF). It's noteworthy that this type of KF was first proposed by Touil et al. [6]. These KFs are instrumental in the design and complexity analysis of Interior-point methods (IPMs) for solving semidefinite programming (SDP) problems.

Through simple analysis tools and by appropriately choosing parameters, we establish that the iteration bound of the developed algorithm is $O(\sqrt{n} \log n \frac{\log n}{\epsilon})$ and $O(\sqrt{n} \frac{\log n}{\epsilon})$ for large- and small-update IPMs, respectively. These bounds match the currently best-known iteration bounds.

Keywords: Linear Semidefinite Programming, Primal-Dual Interior Point Methods, Hyperbolic Kernel Functions, Iteration Bounds

2020 Mathematics Subject Classification: 90C05, 90C51.

Introduction

The primal dual interior point methods are originally developed to resolve linear programs [3, 7]. Their attractive theoretical and numerical properties have motivated researchers to elaborate extensions for more general classes of optimization: the linear complementarity problem [8], the convex programming [5], semidefinite programming [1, 2]....

The semidefinite programming is an extension of the linear programming where the vectors are replaced by the matrices and the positive orthant ($\mathbb{R}_+^n$) by the set of the symmetrical positive semidefinite matrices. This last permits to solve numerous problems, as the nonlinear programming's problems [4], quadratic programming's problems [5].

We consider the following semidefinite programming problem

$$(SDP) \begin{cases} \min[\langle C, X \rangle = \text{tr}(CX) = \sum_{i,j=1}^n C_{ij} X_{ij}] \\ AX = b \\ X \in S_+^n, \end{cases}$$

where $b \in \mathbb{R}^m$, S_+^n designates the cone of the positive semidefinite matrix on the linear space of $(n \times n)$ symmetrical matrix S^n .

A is a linear operator of S^n in $\mathbb{R}^m$ defined by

$$AX = (\langle A_1, X \rangle, \langle A_2, X \rangle, \dots, \langle A_m, X \rangle)^t.$$

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The matrices C and $A_i, i = 1, \dots, m$, are in S^n .

The dual problem associated to the problem (SDP) is defined as follows

$$(DSDP) \begin{cases} \max b^t y \\ \mathcal{A}^* y + S = C \\ S \in S_+^n, \end{cases}$$

where $\mathcal{A}^*$ is the adjoint of $\mathcal{A}$ defined of $\mathbb{R}^m$ in S^n by $\mathcal{A}^* y = \sum_{i=1}^m y_i A_i$.

Main results

Our new kernel function satisfies the following properties

Lemma 47. *Let $\psi(t)$ be our proposed kernel function. Then,*

- (i) $t\psi''(t) - \psi'(t) > 0, \forall t > 0$.
- (ii) $t\psi''(t) + \psi'(t) > 0, \forall t > 0$, i.e. $\psi(t)$ is exponentially convex for $t > 0$.
- (iii) ψ'' is monotonically decreasing on $]0, +\infty[$.

Lemma 48. *For $\psi(t)$, we have*

- $(t-1)^2 \leq \psi(t) \leq ((\psi'(t)))^2, \forall t > 0$.
- $\psi(t) \leq \frac{1}{2}\psi''(1)(t-1)^2, \forall t \geq 1$.

The two functions $\psi : [1, +\infty[\rightarrow [0, +\infty[$ and $-\frac{1}{2}\psi' :]0, 1] \rightarrow]-\infty, 0]$ are both monotonically increasing, we can define the following functions

$\rho : [0, +\infty[\rightarrow [1, +\infty[$ the inverse function of $\psi(t)$ for $t \geq 1$.

$\rho : [0, +\infty[\rightarrow]0, 1]$ the inverse function of $-\frac{1}{2}\psi'(t)$ for $0 < t \leq 1$.

Lemma 49. • $1 + \sqrt{\frac{2s}{\psi''(1)}} \leq \rho(s) \leq 1 + \sqrt{s}, \forall s \in [0, +\infty)$.

- $\coth t \leq \frac{1}{p} [\ln(2z+1) + \coth 1], z = -\frac{1}{2}\psi'(t) \geq 0, \forall t \in]0, 1]$.

Corollary 12. *Let us set $\bar{\alpha} = \frac{1}{\psi''(\rho(2\sigma))}$, as the default step size. Then*

$$f(\bar{\alpha}) \leq -\frac{\sigma^2}{\psi''(\rho(2\sigma))}. \quad (616)$$

Now, we give the upper bound for the decreasing value of the proximity in the inner iteration by the following theorem.

Theorem 241. *If $\bar{\alpha}$ is the default step size defined in Corollary 12 and $\Psi(v) \geq 1$, then*

$$f(\bar{\alpha}) \leq -\frac{(\Psi(v))^{\frac{1}{2}}}{4p \left[1 + 3 \left(\frac{\ln(2\sqrt{\Psi_0}+1)}{p} + \coth 1 \right)^2 \right]}. \quad (617)$$

Lemma 50. *Let K be the total number of inner iterations in the outer iteration. Then we have*

$$K \leq \frac{\Psi_0'}{\beta\gamma} = \left\lceil K_{\max} \Psi_0^{\frac{1}{2}} \right\rceil,$$

where $K_{\max} = \left\lceil 8p \left[1 + 3 \left(\frac{\ln(2\sqrt{\Psi_0}+1)}{p} + \coth 1 \right)^2 \right] \right\rceil$ and Ψ_0 is the value of $\Psi(V)$ after the μ -update in outer iteration.

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Finally, we derive the complexity bounds for large and small-update methods using the following theorem

Theorem 242. *The total number of iterations to obtain an approximation solution with $n\mu \leq \epsilon$ is bounded by*

$$\left[\left(8p \left[1 + 3 \left(\frac{\ln(2\sqrt{\Psi_0} + 1)}{p} + \coth 1 \right)^2 \right] \right) \Psi_0^{\frac{1}{2}} \right] \left[\frac{1}{\theta} \ln \frac{n}{\epsilon} \right].$$

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Subgradient Optimization Method in Integer Linear Programming with an Application to the TSP Problem

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Abstract: In this work, we will present a specific approach of deflected subgradient optimization. This approach aims to address both theoretical and applied aspects, with a particular focus on its application to the Traveling Salesman Problem (TSP).

Keywords: Subgradient method, TSP problem, integer programming

2020 Mathematics Subject Classification: 90C26, 90C10, 90C27, 90C06.

Introduction

We reconsider the following integer linear program:

$$(ILP) \begin{cases} z^* = \min c^T x \\ \text{s.t.} & A_1 x \leq b_1 \\ & x \in X = \{x \in \mathbb{Z}_+^n : A_2 x \leq b_2\}, \end{cases} \quad (618)$$

where x is an $n \times 1$ vector, $\mathbb{Z}_+^n$ is the set of non-negative integers, c , b_1 , b_2 , A_1 , and A_2 are $n \times 1$, $m \times 1$, $k \times 1$, $m \times n$, and $k \times n$ matrices respectively. We assume that the problem (ILP) is feasible and that X is a bounded and finite set. The problem (ILP) is the primal problem and z^* is the primal optimal value. Thus, the constraints $A_2 x \leq b_2$ are the easy constraints, in the sense that an integer linear program with only these constraints is easy to solve. Lagrangian duality is the most useful method to solve (ILP). The Lagrangian dual problem is obtained by the Lagrangian relaxation approach, where the constraints $A_1 x \leq b_1$, which are called the "complicated constraints", are relaxed by introducing a Lagrange multiplier vector $\lambda \in \mathbb{R}_+^m$, called the "Lagrange multiplier".

The formulation of the Lagrangian relaxation problem (L_λ) and the dual problem (D) are as follows:

$$(L_\lambda) \begin{cases} w(\lambda) = \min c^T x + \lambda^T (A_1 x - b_1) \\ \text{s.t.} & x \in X, \end{cases} \quad (619)$$

$$(D) \begin{cases} w^* = \max w(\lambda) \\ \lambda \geq 0. \end{cases} \quad (620)$$

With appropriate assumptions, the optimal dual value w^* is equal to z^* (strong duality). In general, w provides a lower bound for z^* . These bounds can be useful in evaluating nodes in exact methods such as the "Branch and Bound" algorithm to find the optimal solution of (ILP). The function $w(\lambda)$ is continuous and concave but non-differentiable.

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Subgradient optimization methods are frequently employed to solve non-differentiable optimization problems, particularly for efficiently maximizing the Lagrangian dual function of an integer linear programming problem, where the objective function is non-differentiable at every point. There are various subgradient procedures that differ in their performance, namely, the pure subgradient method and the deflected subgradient method.

Main results

We will present a novel approach to deflected subgradient that relies on a convex combination of the modified gradient direction d_{MGT}^k [3] and the average direction d_{ADS}^k [4]. The main theoretical result is the identification of the convex combination parameter that forces the algorithm for better deviation search than those given by pure subgradient, MGT, and ADS. The algorithm 1 provides a detailed description of the deflected subgradient method.

Algorithm 1 (deflected sub-gradient)

1. Choose an initial vector λ^1 , and let $k = 1$, $d^k = 0$.
 2. Determine a subgradient $s^k \in \partial w(u^k)$, $d^k = s^k + \Psi_k d^{k-1}$, $\lambda^{k+1} = P^+(\lambda^k + t_k d^k)$.
The rules for determining ψ_k and t_k will be provided later. Repeat the procedure from step 2 until the stopping criterion is satisfied.
 3. If a termination condition is not yet met, return to step 2. $\hookrightarrow$ Test
-

P.M. Camerini et al [3] proposed a modification of the pure subgradient method in which the subgradient direction s^k at iteration k is replaced by a deflected subgradient direction d_{TGM}^k , given by: $d_{MGT}^k = s^k + \Psi_k^{MTG} d_{MGT}^{k-1}$. With the deviation parameter Ψ_k^{MTG} given by:

$$\Psi_k^{MGT} = \begin{cases} -\eta_k \frac{s^k d_{MGT}^{k-1}}{\|d_{MGT}^{k-1}\|^2} & \text{if } s^k d_{MGT}^{k-1} < 0, \\ 0 & \text{If not,} \end{cases} \quad (621)$$

where $0 < \eta_k \leq 2$ and $d_{TGM}^{k-1} = 0$ for $k = 0$.

H.D. Sherali et al. [4] proposed this parameter as:

$$\Psi_k^{ADS} = \frac{\|s^k\|}{\|d_{ADS}^{k-1}\|}. \quad (622)$$

With this choice of the deviation parameter Ψ_k^{ADS} , the direction becomes: $d_{ADS}^k = s^k + \Psi_k^{ADS} d_{ADS}^{k-1}$, where $d_{ADS}^{k-1} = 0$ for $k = 0$.

We present a new Modified Deflected Subgradient Method (NMDS) which determines the search direction as follows:

$$d^k = (1 - \alpha_k) d_{MGT}^k + \alpha_k d_{ADS}^k, \quad \alpha_k \in (0, 1). \quad (623)$$

we get:

$$\Psi_k = \begin{cases} \frac{-\eta_k (1 - \alpha_k) s^k d^{k-1} + \alpha_k \|s^k\| \|d^{k-1}\|}{\|d^{k-1}\|^2} & \text{if } s^k d^{k-1} < 0, \\ 0 & \text{if not,} \end{cases} \quad (624)$$

Therefore, $d^k = s^k + \Psi_k d^{k-1}$.

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Numerical Results

For a numerical comparison between the four algorithms: pure, MGT, ADS, and NMDS, we chose the symmetric Traveling Salesman Problem (TSPs).

The table 6 shows the experimental results obtained by: Pure algorithm, MGT strategy, ADS strategy, and NMDS algorithm with 10 tests of symmetric instances taken from TSPLIB¹. However, the NMDS algorithm consistently outperforms the others in terms of iterations and execution time.

Table 6: Numerical Results

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¹<http://comopt.ifl.uniidelberg.de/software/TSPLIB95/>.

Solving a class of bilevel programming problems with a quadratic lower level

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Abstract: In this paper, we propose an optimization model to solve a class of bilevel programming problems in the optimistic case, in which the upper-level objective function is convex and the second-level is a quadratic problem. First, we transform the problem into a single-level optimization problem using the lower-level value function reformulation (LLVF), which is then formulated as a DC (difference of convex functions) program for which we apply DC algorithms.

Keywords: bilevel programming, DC programming, nonsmooth optimization.

2020 Mathematics Subject Classification: Primary 65K05, 90C05, 90C20.

Introduction

The most fundamental form of bilevel mathematical programming is the Stakelberg game, the Stackelberg problem is formulated as a two-level optimization problem where decision makers 1 (the leader) and 2 (the follower) correspond to the upper and lower-level problems, respectively. The general formulation of a bilevel programming problem is

$$(BP) \quad \begin{cases} \min_x F(x, y) \\ \text{s.t. } (x, y) \in \mathbb{R}^n \times \mathbb{R}^m \\ y \in \operatorname{argmin}\{f(x, y) : (x, y) \in \mathbb{R}^n \times \mathbb{R}^m\} \end{cases}$$

The variables of this problem are divided into two classes, namely the upper level variables $x \in \mathbb{R}^n$ and the lower level variables $y \in \mathbb{R}^m$. The functions $F(x, y) : \mathbb{R}^n \times \mathbb{R}^m$ and $f(x, y) : \mathbb{R}^n \times \mathbb{R}^m$ are, respectively, the upper and lower objective functions. In the present paper, we consider the following bilevel problem

$$(BPP) \quad \begin{cases} \min_x F(x, y) \\ \text{s.t. } A_1 x + B_1 y \leq b_1 \\ x \geq 0 \\ \min_y f(x, y) = \frac{1}{2} y^t Q y + c^t y \\ \text{s.t. } A_2 x + B_2 y \leq b_2 \\ y \geq 0 \end{cases}$$

where $F : \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \rightarrow \mathbb{R}$, $f : \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \rightarrow \mathbb{R}$ are the objective functions of the leader and the follower respectively, $c \in \mathbb{R}^{n_2}$, $b_1 \in \mathbb{R}^{m_1}$, $b_2 \in \mathbb{R}^{m_2}$, $A_1 \in \mathbb{R}^{m_1 \times n_1}$, $B_1 \in \mathbb{R}^{m_1 \times n_2}$, $A_2 \in \mathbb{R}^{m_2 \times n_1}$, $B_2 \in \mathbb{R}^{m_2 \times n_2}$.

Note that the upper-level objective function $F(x, y)$ is convex, The lower-level objective function is quadratic.

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The bilevel programming problem is a complex and difficult problem to solve. Many approaches have been proposed in the literature for its resolution [3].

In this paper, we develop some numerical algorithms for solving a class of bilevel programming problems based on DC programming and DCA were introduced by Pham Dinh Tao in preliminary form in 1986 and extensively developed since 1994 in the works of Tao Pham Dinh (see [4],[6]) and the references therein) used by many researchers (see [2],[7]).

DCA is a descent method without linear search for solving a general DC program, i.e. minimizing the difference of convex functions. More precisely, it is a sub-gradient primal-dual method based on local optimality and duality in DC optimization applied not to the DC function itself. same, but rather to its DC components.

The DCA algorithm has been successfully applied to a large number of non-convex optimization problems in different fields of applied science. The globality of the solutions computed by DCA is not always guaranteed. However, it has been noticed that with a good choice of the initial point, it often converges to the global solution.

Main results

We present a DC optimization approach for solving two-level optimization problems, the second level is a quadratic problem. We transform the problem into a DC program, via exact penalization, and then develop a DC algorithm to solve it. Computational tests demonstrate the efficiency of the proposed algorithm.

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Generating functions for enumerating words containing patterns of arbitrary length

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Abstract: The notion of pattern has been the subject of much study in the literature. We focus our attention on patterns that appear in language theory. The main tools used in our contribution are the algebraic generating functions of sequences of numbers and polynomials. We explain how the Fibonacci numbers and Fibonacci-type polynomials can be used to provide enumeration formulae for the number of words of arbitrary length containing patterns from a finite alphabet. We consider the case of enumerating $P\left[\begin{smallmatrix} n \\ j \end{smallmatrix}\right]$, the numbers of words of arbitrary size j containing the pattern $\underbrace{a_5 a_5 \cdots a_5}_{n \text{ times}}$ from the alphabet

set $E_5 = \{a_1, a_2, \dots, a_5\}$. In this talk we propose different steps to follow to obtain recurrence relations and explicit formulae for $P\left[\begin{smallmatrix} n \\ j \end{smallmatrix}\right]$. The results obtained can be used to make progress in language theory, such as the search for new words that convey scientific development in any type of language.

Keywords: Enumerating patterns, Generating functions, Fibonacci-type polynomials.

2020 Mathematics Subject Classification: 90C27.

Introduction

Algebraic generating functions have proved their efficiency and power in combinatorics as well as in context-free languages [1]. Generating functions can provide recurrence relations and explicit formulae for many number sequences. In this talk we are interested in the enumeration of numbers of words containing patterns of arbitrary length constructed from a given set of finite alphabets $E_5 = \{a_1, a_2, \dots, a_5\}$. Consider the set $M_j(E_5)$ of words of size j containing the pattern $\underbrace{a_5 a_5 \cdots a_5}_{n \text{ times}}$, with $n \leq j$, and denote the

numbers of these words by $P\left[\begin{smallmatrix} n \\ j \end{smallmatrix}\right]$. We prove that these numbers are related to Fibonacci and Fibonacci-type numbers of higher order. Remember that Fibonacci numbers F_n are defined by the recurrence relation $F_{j+2} = F_{j+1} + F_j$ and generated by the generating function $\frac{z}{1-z-z^2} = \sum_{j \geq 0} F_j z^j$. On the other hand, Fibonacci-type polynomials in two variables are the numbers $\mathcal{G}_j(x, y, k, u, v)$ defined by the generating function $\sum_{j \geq 0} \mathcal{G}_j(x, y, k, u, v) z^j = \frac{1}{1-x^k z - y^u z^{u+v}}$ [2, 3].

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Main results

The generating function of $P\left[\begin{smallmatrix} n & 5 \\ j \end{smallmatrix}\right]$ can be obtained by looking at the set

$$G_j = \left\{ \text{words of size } j \text{ that do not contain the pattern } \underbrace{a_5 a_5 \cdots a_5}_{n \text{ times}} \right\}.$$

Its cardinal is then

$$\#G_j = \begin{cases} 5^j & ; j < n \\ 4 \sum_{k=1}^n \#G_{j-k} & ; j \geq n. \end{cases} \quad (625)$$

Thus, we can easily derive the generating function of $\#G_j$ in the following way

$$\sum_{j \geq 0} \#G_j z^j = \frac{\sum_{k=0}^{n-1} z^k}{1 - \sum_{k=1}^n 4z^k}. \quad (626)$$

Since we know that $\overline{G_j}$ is the complementary of G_j , we deduce that $P\left[\begin{smallmatrix} n & 5 \\ j \end{smallmatrix}\right] = \# \overline{G_j} = 5^j - \#G_j$, and its generating function will be expressed as

$$\sum_{j \geq 0} P\left[\begin{smallmatrix} n & 5 \\ j \end{smallmatrix}\right] z^j = \frac{1}{1-5z} - \frac{\sum_{k=0}^{n-1} z^k}{1-4 \sum_{k=1}^n z^k}. \quad (627)$$

The latter can be reformulated in terms of the following

$$\sum_{j \geq 0} P\left[\begin{smallmatrix} n & 5 \\ j \end{smallmatrix}\right] z^j = \frac{1}{1-5z} - \frac{1-z^n}{1-5z+4z^{n+1}}. \quad (628)$$

By means of the equation (628) and the Cauchy product, we obtain the following recurrence relation

$$\begin{cases} P\left[\begin{smallmatrix} n & 5 \\ n \end{smallmatrix}\right] = 1 \\ P\left[\begin{smallmatrix} n & 5 \\ n+1 \end{smallmatrix}\right] = 9 \\ P\left[\begin{smallmatrix} n & 5 \\ j \end{smallmatrix}\right] = 10P\left[\begin{smallmatrix} n & 5 \\ j-1 \end{smallmatrix}\right] - 25P\left[\begin{smallmatrix} n & 5 \\ j-2 \end{smallmatrix}\right] - 4P\left[\begin{smallmatrix} n & 5 \\ j-n-1 \end{smallmatrix}\right] + 20P\left[\begin{smallmatrix} n & 5 \\ j-n-2 \end{smallmatrix}\right] \end{cases} ; j \geq n+2. \quad (629)$$

Given the numbers $\mathcal{G}_j$, which are generated by the function $\mathcal{G}(z) = \frac{1}{1-5z+4z^{n+1}}$, the generating function (628) becomes

$$\sum_{j \geq 0} P\left[\begin{smallmatrix} n & 5 \\ j \end{smallmatrix}\right] z^j = \frac{1}{1-5z} - (1-z^n)\mathcal{G}(z).$$

From the following development

$$(1-z^n) \sum_{j \geq 0} \mathcal{G}_j z^j = \sum_{j=0}^{n-1} \mathcal{G}_j z^j + \sum_{j \geq n} (\mathcal{G}_j - \mathcal{G}_{j-n}) z^j,$$

we can deduce that

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$$P \begin{bmatrix} n & 5 \\ j \end{bmatrix} = 5^j + \mathcal{G}_{j-n} - \mathcal{G}_j. \quad (630)$$

The numbers $\mathcal{G}_j$ are special Fibonacci polynomials. It's clear that $\mathcal{G}_j = \mathcal{G}_j(5, -4, 1, 1, n)$. So we get

$$\mathcal{G}_j = \sum_{c=0}^{\lfloor \frac{j}{n+1} \rfloor} \binom{j-cn}{c} (-4)^c 5^{j-n-c}, \quad (631)$$

and finally the explicit formula provided by the following theorem

Theorem 243. *The explicit formula of $P \begin{bmatrix} n & 5 \\ j \end{bmatrix}$ is*

$$\begin{aligned} P \begin{bmatrix} n & 5 \\ j \end{bmatrix} &= 5^j + \sum_{c=0}^{\lfloor \frac{j-n}{n+1} \rfloor} \binom{j-n(c+1)}{c} (-4)^c 5^{j-n(c+1)-c} \\ &\quad - \sum_{c=0}^{\lfloor \frac{j}{n+1} \rfloor} \binom{j-cn}{c} (-4)^c 5^{j-n-c}. \end{aligned} \quad (632)$$

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CONVERGENCE OF GENERALIZED PROXIMAL POINT ALGORITHMS

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Abstract: In this paper, we introduce and investigate a new generalization of the proximal point algorithm for finding zeros of maximal monotone operators. We also define the generalized reflection operator with its contraction properties which we used to prove convergence results.

Keywords: Monotone operator, proximal point algorithm, convergence.

2020 Mathematics Subject Classification: Primary 47H05, 47H10, 47J25.

Introduction

Consider fundamental mathematical problem :

$$\begin{cases} \text{find } y \in \mathbb{R}^n \text{ such that} \\ 0 \in T(y), \end{cases} \quad (633)$$

where T is maximal monotone operator, the inclusion problem (633) is one of the most important problems in monotone operator theory, for several reasons, but especially because of its role in many areas of mathematics and scientific computing such as optimization, fixed point problems, variational inequalities, nonlinear and convex analysis, evolution equations and semigroup theory . The major and popular method for solving problem (633) is the proximal point algorithm (PPA), which was introduced by Martinet [1]. Later Rockafellar [2, 3] showed that the PPA scheme for solving (633) can be written as

$$z_{k+1} = P_k(z_k).$$

Note that $P_k = (I + \varepsilon_k T)^{-1}$ is the proximal mapping associated with $\varepsilon_k T$. Rockafellar [3] showed the weak convergence of the sequence generated by the PPA to a zero of the maximal monotone operator. However, PPA does not necessarily converge strongly in general. The aim of this paper is to present a new generalization of the proximal point algorithm to find the zeros of the maximal monotone and study their convergence by using the properties of generalized reflection operator.

Main results

This section aims to introduce the generalized proximal point algorithm, the generalized reflection operator with some properties and we established the convergence analyses of the proposed algorithm.

Algorithm 1 (The general proximal point algorithm)

- $k = 0$; choice $y^0 \in \mathbb{R}^n$.
- $y^{k+1} = y^k - \rho_k Q_k(y^k) = (1 - \rho_k)y^k + \rho_k P_k(y^k)$.

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- $k \leftarrow k + 1$. Go to step 2.

In order to study the convergence of our algorithm, we give the definition with some propriety of the generalized reflection N_k associated $(\varepsilon_k T)$. Under certain hypotheses of T and the generalized reflection N_k of $(\varepsilon_k T)$ with supplementary propriety of contraction permit the convergence of the Algorithm 1.

Lemma 51. *Let $T : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ maximal monotone operator, and $N_k : \mathbb{R}^n \rightarrow \mathbb{R}^n$ the generalized reflection we have*

$$\begin{aligned} \|N_k(s) - N_k(s')\|^2 &\leq \|P_k(s) - P_k(s')\|^2 + \|Q_k(s) - Q_k(s')\|^2 \\ &\quad - 2\langle P_k(s) - P_k(s'), Q_k(s) - Q_k(s') \rangle, \end{aligned} \quad (634)$$

and

$$\|N_k(s) - N_k(s')\|^2 \leq \|s - s'\|^2 - 2\langle P_k(s) - P_k(s'), Q_k(s) - Q_k(s') \rangle \quad (635)$$

Proposition 36. *Let $T : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ maximal monotone operator, and $N_k : \mathbb{R}^n \rightarrow \mathbb{R}^n$ generalized reflection*

(i) If T is coercive with constant b then N_k is $\frac{1}{1 + \frac{b}{2\varepsilon_k}}$ -average.

(ii) If more, T is strongly monotone with constant a then N_k is a contractor of report $\sqrt{1 - \frac{2ab}{(a+b)\varepsilon_k}} < 1$.

Corollary 13. *Let $T : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ maximal monotone operator, and Q_k the regular operator. If T is co-coercive of constant b and :*

$$0 \leq \rho_k \leq 2 + \frac{b}{\varepsilon_k}, \quad (636)$$

and

$$\sum_{k=0}^{+\infty} \frac{\rho_k}{2 + \frac{b}{\varepsilon_k}} \left(1 - \frac{\rho_k}{2 + \frac{b}{\varepsilon_k}}\right) = +\infty \quad (637)$$

Corollary 14. *Let $T : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ maximal monotone operator, If T is co-coercive of constant b and strongly monotone with constant a then the $\{y_k\}_k$ generated by the algorithm, with a then the $\{y_k\}_k$ generated by the algorithm, with*

$$\begin{aligned} 0 &\leq \rho_k \leq 2 \\ \sum_{k=0}^{+\infty} \rho_k &= +\infty \end{aligned}$$

converges to $\bar{y}$ the unique zero of T . If further $\liminf \rho_k = \rho > 0$, the speed of convergence is linear from ρ :

$$\limsup_k \frac{\|y_{k+1} - \bar{y}\|}{\|y_k - \bar{y}\|} \leq 1 - \frac{\rho}{2} \left(1 - \sqrt{1 - \frac{2ab}{(a+b)\varepsilon_k}}\right) < 1.$$

Where $\varepsilon_k > 0$.

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Study of the numerical behavior of the EDL method towards other conjugate gradient methods of the type Dai-Liao

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Abstract: The conjugate gradient method (CG) was introduced in 1952 by Hestenes and Stiefel as an efficient approach for solving linear systems characterized by positive definite matrices, equivalent to minimizing quadratic functions. This method is highly efficient, especially for large-scale linear systems. Over time, mathematicians have extended its application to solve nonlinear optimization problems, leading to various adaptations and modifications of the original method. In this work, we are interested to the CG methods of the type Dai-Liao. We have considered the Effective Dai-Liao method (EDL) and we studied its numerical behavior with respect to the different variants of the Dai-Liao's type method of the class MHSDL, namely MHSDL1, MHSDL2, MHSDL3, MHSDL4, MHSDL5 and MHSDL6, using the Wolfe, Wu and Backtracking rules to calculate the stepsize. The numerical tests clearly show the superiority of the EDL method over the other considered methods either in number of iterations or in calculation time especially using the line search of Backtracking and Wu.

Keywords: Conjugate gradient method, Nonlinear optimization, Descent direction, Line search.

2020 Mathematics Subject Classification: 90C26, 90C30.

Introduction

We consider the unconstrained optimization problem given as follows:

$$\min_{x \in \mathbb{R}^n} f(x), \quad (638)$$

where $f : \mathbb{R}^n \rightarrow \mathbb{R}$, is a continuously differentiable function.

The iterative scheme of the conjugate gradient method is as follows:

$$\begin{cases} x_1 & \text{initial point} \\ x_{k+1} = x_k + \alpha_k d_k & k \geq 1 \end{cases}, \quad (639)$$

where x_k is the current iterate point, $\alpha_k > 0$ is the stepsize found by one of the line search methods and d_k is the search direction calculated by the formula:

$$d_k = \begin{cases} -g_k & k = 1 \\ -g_k + \beta_k d_{k-1} & k \geq 2 \end{cases}, \quad (640)$$

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where $g_k = \nabla f(x_k)$ is the gradient of f at x_k , and β_k is an appropriately defined real scalar, known as the conjugate gradient parameter.

The different variants of the CG method are extensions of the linear or the quadratic case. Several mathematicians have extended it to the nonlinear case. This was first accomplished in 1964 by Fletcher and Reeves (FR), then in 1969 by Polak-Ribiere and Ployak (PRP). Other variants have been proposed later, up to the present day, in particular variants of type Dai-Liao which was proposed for the first one by Dai and Liao (DL) in [3]. Different choices for β_k lead to distinct variations of the conjugate gradient methods. Notably, several formulas for β_k have been proposed, among the most famous are:

$$\beta_k^{HS} = \frac{g_k^T y_{k-1}}{d_{k-1}^T y_{k-1}}, \quad \beta_k^{FR} = \frac{\|g_k\|^2}{\|g_{k-1}\|^2}, \quad \beta_k^{PRP} = \frac{g_k^T y_{k-1}}{\|g_{k-1}\|^2},$$

$$\beta_k^{DL} = \frac{g_k^T y_{k-1}}{d_{k-1}^T y_{k-1}} - t \frac{g_k^T s_{k-1}}{d_{k-1}^T y_{k-1}}.$$

Where $y_{k-1} = g_k - g_{k-1}$, $s_{k-1} = x_k - x_{k-1}$, $t > 0$ and $\|\cdot\|$ denotes the Euclidean norm.

Main results

The CG methods of the type Dai-Liao are specific by their parameter t_k . Based on the various values assigned to t_k and motivated by the strong theoretical properties and numerical efficiency of Dai-Liao type methods, which have been discussed by several researchers, Branislav et al. introduced a new formula for computing t_k within the interval $[0, 1]$, given by:

$$t_k^* = \frac{\|g_k\|^2}{\max\{1, d_{k-1}^T g_k\} + (\max\{0, -\frac{d_{k-1}^T g_k}{\|g_k\|^2}\} + 1) \|g_k\|^2}, \quad (641)$$

It is easy to verify that t_k defined by (641) satisfies

$$0 < t_k^* \leq \frac{\|g_k\|^2}{1 + (0 + 1) \|g_k\|^2} = \frac{\|g_k\|^2}{1 + \|g_k\|^2} < 1. \quad (642)$$

This has led to the creation of a new variant of the Dai-Liao type CG method, named (EDL), where the new parameter β_k^{EDL} is defined as follows:

$$\beta_k^{EDL} = \frac{\|g_k\|^2 - (\frac{\|g_k\|}{\|g_{k-1}\|}) |g_k^T g_{k-1}|}{d_{k-1}^T y_{k-1}} - t_k^* \frac{g_k^T s_{k-1}}{d_{k-1}^T y_{k-1}} \quad (643)$$

Yao et al. [4] proposed a new variant of the type Dai-Liao, named MHSDL method, where the β_k is given by:

$$\beta_k^{MHSDL} = \frac{g_k^T \hat{y}_{k-1}}{d_{k-1}^T y_{k-1}} - t \frac{g_k^T s_{k-1}}{d_{k-1}^T y_{k-1}}, \quad t > 0, \quad (644)$$

where $\hat{y}_{k-1} = g_k - (\frac{\|g_k\|}{\|g_{k-1}\|}) g_{k-1}$.

In this work, we consider different variants of the type Dai-Liao, we present a comparative numerical study between the Effective Dai-Liao method (EDL) given by Branislav et al. and several variants of the MHSDL method using different values for the parameter t ($t_1, t_2, t_3, t_4, t_5, t_6$) given in [2].

We carried out numerical tests on several test functions, taken from [1] and for different dimensions. We used three inexact line search for computing the stepsize α_k , namely the rule of strong Wolfe, Backtracking and Wu.

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Conclusion

The comparative study, between the EDL method and the six variants of the MHSDL class methods of Dai-Liao type, revealed that the new EDL variant is significantly the best when using the Backtracking rule and Wu either in number of iterations or in calculation time. The behavior of all MHSDL variants were quite similar. On the other hand, the use of the Wolfe's rule did not yield good results for this new variant.

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New findings on double Roman domination number in graphs

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Abstract: This paper presents a characterization of extremal graphs regarding a Nordhaus-Gaddum bound for double Roman domination in graphs, enhancing the findings of N. Jafari Rad and H. Rahbani (2019) as well as L. Volkmann (2023). Additionally, we provide a characterization of all graphs G with a double Roman domination number twice the Roman domination number minus one, improving upon the results presented by X. Zhang et al. (2018).

Keywords: Roman domination, double Roman domination, Nordhaus-Gaddum, tree

2020 Mathematics Subject Classification: Primary 05C69.

Introduction

Given a graph $G = (V, E)$, a *Roman dominating function* (RDF) on a graph G is a function $f : V \rightarrow \{0, 1, 2\}$ satisfying the condition that every vertex u for which $f(u) = 0$ is adjacent to at least one vertex v for which $f(v) = 2$. The weight $w(f)$ of a Roman dominating function f is the value $w(f) = \sum_{u \in V} f(u)$. The minimum weight of a Roman dominating function on a graph G is called the *Roman domination number* of G , denoted by $\gamma_R(G)$. The concept of Roman domination on graphs is studied, for the first time, by Cockayne et al. in [3]. For references on Roman domination and its variations, see for example the survey [2].

A *double Roman dominating function* (DRDF) on a graph G is a function $f : V \rightarrow \{0, 1, 2, 3\}$ satisfying the condition that every vertex u for which $f(u) = 0$ is adjacent to at least one vertex v for which $f(v) = 3$ or two vertices v_1 and v_2 for which $f(v_1) = f(v_2) = 2$, and every vertex u for which $f(u) = 1$ is adjacent to at least one vertex v for which $f(v) \geq 2$. The weight $w(f)$ of an DRDF f is the value $w(f) = \sum_{u \in V} f(u)$. The double Roman domination number $\gamma_{dR}(G)$ equals the minimum weight of an DRDF on G . An DRDF of minimum weight is called a γ_{dR} -functions. For further details, we refer the reader to [1].

In this paper, we initially provide a characterization of extremal graphs concerning a Nordhaus-Gaddum bound for $\gamma_{dR}(G)$, enhancing the results presented in [4] and [5]. Furthermore, we offer a characterization of graphs G where the equality $\gamma_{dR}(G) = 2\gamma_R(G) - 1$ is satisfied.

Nordhaus-Gaddum Inequality

In the following, Jafari Rad and Rahbani [4], and Volkmann [5] introduced Nordhaus-Gaddum-type inequalities concerning the double Roman domination number with respect to the order of the graph G .

Theorem 244. [4] For any graph G of order $n \geq 2$, $\gamma_{dR}(G) + \gamma_{dR}(\overline{G}) \leq 2n + 3$, with equality if and only if $G \in \{K_n, \overline{K_n}\}$.

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Let $\mathcal{H}_1 = \{2K_2, C_4, P_4, C_5, K_n - e, \overline{K_n - e}; n \geq 3\}$, where $K_n - e$ and $K_n - \{e_1, e_2\}$ represent the complete graph minus an edge and the complete graph minus two independent edges, respectively.

Theorem 245. [4] Let G be a graph of order $n \geq 3$ such that $G \notin \{K_n, \overline{K_n}\}$. Then $\gamma_{dR}(G) + \gamma_{dR}(\overline{G}) \leq 2n + 2$, with equality if and only if $G \in \mathcal{H}_1$.

Theorem 246. [5] Let G be a graph of order $n \geq 4$ such that $G \in \{K_n, \overline{K_n}\} \cup \mathcal{H}_1$. Then $\gamma_{dR}(G) + \gamma_{dR}(\overline{G}) \leq 2n + 1$, with equality if and only if $G \in \{K_n - \{e_1, e_2\}, \overline{K_n - \{e_1, e_2\}}\}$ and $n \geq 5$ or $G \in \{P_5, 3K_2, \overline{P_5}, \overline{3K_2}\}$.

Based on Theorems 244, 245 and 246, we deduce that if G is a graph such that $G \notin \mathcal{H} = \{K_n, \overline{K_n}\} \cup \mathcal{H}_1 \cup \mathcal{H}_2$, where $\mathcal{H}_2 = \{K_n - \{e_1, e_2\}, \overline{K_n - \{e_1, e_2\}}, P_5, 3K_2, \overline{P_5}, \overline{3K_2}; n \geq 5\}$, then we get $\gamma_{dR}(G) + \gamma_{dR}(\overline{G}) \leq 2n$. In the following, we offer a characterization of graphs G of order $n \geq 4$ where $\gamma_R(G) + \gamma_R(\overline{G}) = 2n$. To achieve this, we introduce the following families of graphs:

- $\mathcal{F}_0 = \{4K_2, 2C_3, C_6, C_7\}$.
- $\mathcal{F}_1 = \{(n-6)K_1 \cup 3K_2; n \geq 7, K_2 \cup P_3, K_2 \cup C_3, K_2 \cup P_4\} \cup \{F : F \text{ is semiregular with } n(F) = 6 \text{ and } \Delta(F) = 3\}$.
- $\mathcal{F}_2 = \{(n-3)K_1 \cup P_3, (n-3)K_1 \cup C_3, (n-4)K_1 \cup P_4; n \geq 4\} \cup \{cor(P_3), cor(C_3), F_1, F_2, F_3\}$, where F_1, F_2 and F_3 are the graphs illustrated in Figure 39.

Theorem 247. Let G be a graph of order $n \geq 4$ such that $G \notin \mathcal{H}$. Then $\gamma_{dR}(G) + \gamma_{dR}(\overline{G}) \leq 2n$, with equality if and only if G or $\overline{G} \in \mathcal{F}_0 \cup \mathcal{F}_1 \cup \mathcal{F}_2$.

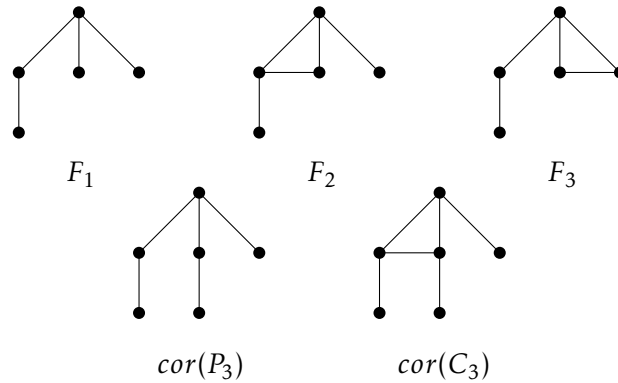


Figure 39: Graphs G in $\mathcal{F}_2$ with $\Delta(G) = 3$.

Graph with $\gamma_{dR}(G) = 2\gamma_R(G) - 1$

Now, we present a characterization of connected graphs with $\gamma_{dR}(G) = 2\gamma_R(G) - 1$. We commence by recalling some results that will be useful.

Theorem 248. [3] For any graph G , $\gamma(G) \leq \gamma_R(G)$ with equality if and only if $G = \overline{K_n}$.

Theorem 249. [1] For any graph G , $\gamma_{dR}(G) \leq 2\gamma_R(G)$ with equality if and only if $G = \overline{K_n}$.

Theorem 250. [4] If G is graph of order n with maximum degree $\Delta(G)$, then $\gamma_{dR}(G) \leq 2(n - \Delta(G)) + 1$, with equality if and only if $G \in \mathcal{F}$.

We now deduce by Theorem 249 that, if G is a nontrivial connected graph, then $\gamma_{dR}(G) \leq 2\gamma_R(G) - 1$. A characterization of the connected graphs G with γ_{dR} -functions of weight $2\gamma_R(G) - 1$ will be presented next.

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Proposition 37. *If G is a connected graph of order n with maximum degree $\Delta(G)$, then $\gamma_{dR}(G) = 2\gamma_R(G) - 1$ if and only if $\gamma_{dR}(G) = 2(n - \Delta(G)) + 1$.*

Wounded spiders are the only trees T for which $\gamma_{dR}(T) = 2\gamma_R(T) - 1$, as shown by Zhang et al. [6]. The result that follows is a direct consequence of Theorem 250 and Propositions 37.

Corollary 15. *Let G be a connected graph of order n with maximum degree $\Delta(G)$. Then the following statements are equivalent:*

- i) $\gamma_{dR}(G) = 2\gamma_R(G) - 1$.
- ii) $\gamma_{dR}(G) = 2(n - \Delta(G)) + 1$.
- ii) $G \in \mathcal{F}$.

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MOchOA/NSGA-II :A Hybrid Algorithm for solving Multi-objective optimization problems

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Abstract: This paper aims to develop a new method to solve multi-objective optimization problems. Our proposal is a hybridization of the A Chip Optimization Algorithm combined with the Non-dominated Sorting Genetic Algorithm-II. The proposed algorithm is validated on three real engineering design problems and compared with four well-known multi-objective meta-heuristics. According to the results obtained, the proposed algorithm is effective compared to the four other algorithms by providing good convergence behavior with a better diversity of solutions.

Keywords: Multi-objective optimization, A Chip Optimization Algorithm, Meta-heuristics, Engineering design problems

Introduction

Multi-objective optimization, also known as multi-criteria decision making or multi-objective decision analysis, is a branch of optimization that deals with problems involving multiple conflicting objectives [?]. In traditional optimization, the goal is to find the best solution that maximizes or minimizes a single objective function. However, in real-world scenarios, there are often multiple objectives that need to be simultaneously considered [?]. Researchers have developed numerous methods to solve multi-objective optimization problems (MOPs). However, there is no method able to guarantee the same performance for all types of optimization problems. Therefore, there is always a possibility to develop new algorithms in the hope of solving a wider range of problems with better performance than the available approaches (the No-Free-Lunch (NFL) theorem)[?]. Therefore, there is always a possibility to develop new algorithms in the hope of solving a wider range of problems with better performance than the available approaches[?]. Multi-objective problems get harder as the problem and the number of objective functions go up. This is because we have to come up with a Pareto solution set, which is a group of solutions that are not dominated by any one objective function. Meta-heuristic algorithms that can approximate the Pareto solutions set for large multi-objective problems in a reasonable computational time have received particular attention in the literature. Multi-objective meta-heuristics can be broken down into three groups: multi-objective cooperative algorithms, multi-objective evolutionary algorithms (MOEA), and multi-objective swarm intelligence algorithms. In this paper, we are interested in the cooperative algorithm, and we present MOchOA/NSGA-II, a chip optimization algorithm [?] combined with a non-dominated sorting genetic algorithm (NSGA-II)[?], to solve multi-objective engineering design problems. MOchOA/NSGA-II follows the classic scheme used for the NSGA-II algorithm, but in the step of creating a new population (binary tournament selection, recombination, and mutation operators), we replace it with the chip algorithm technique for updating the

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position or creating a new population.

Main results

The proposed multi-objective hybrid algorithm (MOchOA/NSGA-II) is implemented using the programming environment Matlab R2015a under 64-bit Windows 10. We chose four well-known algorithms for comparison: Evolutionary Algorithm based on Decomposition (MOEA/D), Multi-Objective Ant Lion Optimizer (MOALO), Multi-Objective Salp Swarm Algorithm (MSSA), and Multi-Objective Harris Hawks Optimization. We applied the MOchA/NSGA-II to three real engineering designs that are popular in the engineering design field: four-bar truss design, welded beam design, and disk brake design.

Two performance metrics—the inverted generational distance (IGD) metric for assessing convergence and the spacing (Sp) metric for measuring diversity—are used to examine the validity of the experiment's findings [?]. The statistical IGD and SP values were computed from the 31 runs in all engineering design problems, as presented in Table 1 and ??.

The results of IGD and Sp metrics obtained by the different algorithms It appears that MOchOA/NSGA-II is able to provide the best results compared to MOEA/D, MOHHO, MOALO, and MSSA. Indeed, MOchOA/NSGA-II obtained the minimum "mean" of the IGD metric on all real engineering design problems, which means that the proposed algorithm attains a good convergence to the true Pareto optimal front. Considering Sp-metric, the results show that the MOchOA/NSGA-II algorithm outperforms the others on disk brake design problems. Four-bar truss design and welded beam design show our algorithm is able to compete with other algorithms with convergent results. The obtained experimental results have proven that the proposed algorithm is efficient in solving multi-objective engineering design problems in terms of convergence and diversity.

Table 7: Results for IGD of the engineering design problems.

Bar	Best	Worst	Average	Median	STD
MOchOA/NSGA-II	0.1708	1.1449	0.3891	0.2685	0.2607
MOALO	0.56195	4.2356	1.6136	1.3997	1.0154
MOEA/D	6.3822	20.4418	17.4727	17.9605	3.1535
MSSA	0.6394	2.0213	1.01788	0.8945	0.3309
MOHHO	2.6336	19.8323	10.1678	11.0675	4.11561
WBD	Best	Worst	Average	Median	STD
MOchOA/NSGA-II	0.0079	0.3367	0.0255	0.0135	0.0582
MOALO	0.0341	0.4639	0.1922	0.1565	0.1265
MOEA/D	0.6901	0.9509	0.8349	0.8442	0.0703
MSSA	0.0241	0.3031	0.0988	0.0649	0.0785
MOHHO	0.7316	0.9765	0.8683	0.8710	0.0411
Disk	Best	Worst	Average	Median	STD
MOchOA/NSGA-II	0.0025	0.0033	0.0028	0.0028	0.0001
MOALO	0.0059	0.0738	0.0241	0.0186	0.0159
MOEA/D	0.0224	0.1289	0.0676	0.0638	0.0289
MSSA	0.0110	0.0597	0.0234	0.0216	0.0104
MOHHO	0.0436	0.2026	0.1605	0.1695	0.0355

Table 8: Results for SP of the engineering design problems.

Bar	Best	Worst	Average	Median	STD
MOchOA/NSGA-II	1.8343	36.3961	6.0401	3.4027	6.8971
MOALO	0.7648	0.8068	0.7935	0.7989	0.0121
MOEA/D	0.6423	0.7065	0.7038	0.7065	0.0118
MSSA	0.7902	0.8082	0.8052	0.8063	0.0034
MOHHO	0.5277	0.7572	0.5691	0.5327	0.0766
WBD	Best	Worst	Average	Median	STD
MOchOA/NSGA-II	0.1526	10.8459	0.6062	0.2183	1.9031
MOALO	0.5462	0.6826	0.6255	0.6321	0.0366
MOEA/D	0.4388	0.6950	0.5444	0.5434	0.0663
MSSA	0.5690	0.6769	0.6617	0.6690	0.0210
MOHHO	0.3146	0.5810	0.5078	0.5096	0.0435
Disk	Best	Worst	Average	Median	STD
MOchOA/NSGA-II	0.0526	0.1610	0.1030	0.0925	0.0273
MOALO	0.2065	0.3991	0.2607	0.2571	0.0404
MOEA/D	0.1699	0.2255	0.2052	0.2058	0.0156
MSSA	0.2147	0.3499	0.2569	0.2445	0.0345
MOHHO	0.1674	0.3043	0.2006	0.1986	0.0221

A Derivative-Free algorithm for continuous global optimization

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Abstract: In this presentation, we consider the bound constrained global optimization problem of the following form:

$$f^* = \min_{x \in D} f(x). \quad (P)$$

where $D = \prod_{i=1}^n [L_i, U_i] \subset \mathbb{R}^n$ with L_i, U_i are real numbers for $i = 1, \dots, n$ and the real objective function $f(x)$ is only continuous. The problem (P) is of interest in many real-world problems involving objective functions which are only continuous and do not possess strong mathematical properties (such as convexity, differentiability, Lipschitz or Hölderian continuity etc.). If a function f is a priori known to be only continuous, then apart from saying that f "remains near $f(x)$ in a neighbourhood of x " which translates into dealing with the modulus of continuity of f , at x nothing tractable can be inferred on the values of f away from x . One way to tackle such enormous uncertainties is to use the following result:

Theorem 251. [2] *Let f be a real function defined on a compact set $D \subset \mathbb{R}^n$. Then f is continuous if and only if for all $\varepsilon > 0$, there exists a constant $C_\varepsilon > 0$ such that for all $x, x' \in D$, we have:*

$$|f(x) - f(x')| \leq C_\varepsilon \|x - x'\| + \varepsilon. \quad (1)$$

A drawback of the above theorem is that given f and ε , there are no specific means to recover exactly the constant C_ε for example for black-box functions. Our idea is to work on a sequence $\{C_j\}_{j \in \mathbb{N}}$ of positive constants that controls the growth of C_ε . This is done by actual collecting of information while running a Lissajous curve throughout the feasible domain.

The proposed method is based on the reducing transformation technique by running in the feasible domain a single parametrized Lissajous curve, which becomes increasingly denser and progressively fills the feasible domain. By means of a one-dimensional search algorithm, we realize a mixed method which explores the feasible domain. To speed up the mixed exploration algorithm, we have incorporated a derivative-free local search algorithm to explore promising regions. This method converges in a finite number of iterations to the global minimum. The simulation results based on a set of 180 benchmark functions with diverse properties and different dimensions show the efficiency and the abilities of the proposed algorithm in finding the global optima compared with the existing methods.

Keywords: Global optimization, Local optimization, Reducing transformation method, Lissajous parametrized curve.

2010 Mathematics Subject Classification: 90C26; 90C90.

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Etude comparative et expérimentale des méthodes temps-fréquence pour l'analyse des signaux non stationnaires

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Abstract: This article is devoted to a comparative study between two methods of time-frequency analysis, their implementation, and their

biases and advantages in the analysis of multi-component, non-stationary and/or generated signals by non stationary and/or generated signals by non-linear systems.

We, at first, study and implement a RTF Time Time-Frequency Representation method: the spectrogram obtained by the Short-Term

Fourier Transform TFCT. The TFCT belonging to Cohen's class can, The TFCT belonging to Cohen's class can, in this comparative study, represent other RTFS of the

same class such as the scalogram obtained by the WT the scalogram obtained by the WT Wavelets Transform and the Distribution of Wigner istribution of Wigner-Ville.

In this context, Huang et al recently proposed another technique addressing the problem of the non non-stationary signals analysis,

it is the Empirical Modal Decomposition EMD (or called Huang ecomposition EMD (or called Huang Transformation).

Among the interesting characteristics of the EMD is that its components called IMFs (Intrinsic M ents called IMFs (Intrinsic Mode Functions), obtained

following a sifting process, fulfill the cons process, fulfill the constraints imposed by the method of estimating frequencies and instant amplitudes (FI

and AI successively) called Hilbert Transformation HT Transformation HT. And since the most desirable locality, , when we want to describe a

signal jointly in time and in frequency domain domains, is that which would give instant spectral content "instant frequency" because a

frequency, in the sense of Fourier, loses all time locality, so it is interesting loses all time locality, so it is interesting To associate the Hilbert Transformation to the

Huang Transformation to develop a time-frequency analysis me frequency analysis method called THH (Huang-Hilbert Transformation Hilbert Transformation) [2]. We have

implemented this method and made comparisons between it and the RTF cited in this paragraph (spectrogram).

Keywords: non-stationary signals, time-frequency analysis, adaptive base.

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Résumé : Cet article est consacré à une étude comparative entre deux méthodes d'analyse temps-fréquence, leur mise en œuvre, et leurs biais et avantages dans l'analyse des signaux multi-composants, non stationnaires et/ou générés par des systèmes non linéaires

Nous, dans un premier temps, étudions et mettons en œuvre une méthode de Représentation Temps de Représentation Temps-Fréquence RTF: le spectrogramme obtenu par la Transformée de Fourier à Court Terme TFCT Transformée de Fourier à Court Terme TFCT. La TFCT appartenant à la classe de Cohen . La TFCT appartenant à la classe de Cohen peut, dans cette étude comparative, représenter d'autres RTFs d'autres RTFs de la même classe tels que le scalogramme obtenu par la Transformée en ondelettes TO et la distribution de Wigner-Ville.

Huang et al ont récemment proposé une autre technique abordant la problématique de l'analyse des signaux

non stationnaires c'est la Décomposition Modale Empirique EMD (ou dite transformée de Huang). Parmi les caractéristiques intéressantes de l'EMD est que ses composantes appelées IMF (*Intrinsic Mode Functions* *Intrinsic Mode Functions*), obtenues suite à un processus de tamisage, remplissent les contraintes imposées par la méthode d'estimation des fréquences et des amplitudes instantanées (FI et AI successivement) dite Transformée de Hilbert TH. Et puisque la localité la plus souhaitable lorsqu'on désire décrire un signal conjointement en temps et en fréquence est celle qui donnerait un contenu spectral instantané « fréquence instantanée » parce qu'une fréquence au sens de Fourier perd toute localité temporelle, il est donc intéressant d'associer la TH à l'EMD pour élaborer une méthode d'analyse temps élaborer une méthode d'analyse temps-fréquence appelée THH (Transformée de Huang Hilbert) [2]. Nous avons mis en œuvre cette méthode et effectué des comparaisons entre celle-ci et la RTF citée dans ce paragraphe (spectrogramme).

Mots clés : Signaux non stationnaires, analyse temps stationnaires, analyse temps-fréquence, base adaptative

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On a new computational method for calculating the Frobenius number

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Abstract: Consider four non-negative integers $a_1 < a_2 < a_3 < a_4$ which are coprime. The Frobenius number, denoted $g(a_1, a_2, a_3, a_4)$ for these integers is known as the largest integer that cannot be expressed as a linear combination such as $a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4$, where x_1, x_2, x_3 and x_4 are also non-negative integers. In this talk, we propose our contribution to the calculation of this number in the case of four integers, where some algorithms have already been proposed. Our approach consists in using the procedure of short generating functions of a finite set that depends on a_1 and a_2 to find an algorithm that determines the Frobenius number for four integers. The results obtained are useful in several optimization problems such as number theory and automata theory.

Keywords: Frobenius number, Diophantine equation. Integer optimisation.

2020 Mathematics Subject Classification: 90-08.

Introduction

Frobenius number, also called coin problem, is a mathematical concept introduced by German mathematician Ferdinand George Frobenius in 1919. For $a_1, a_2, a_3, \dots, a_n$ positive integers with $\gcd(a_1, a_2, a_3, \dots, a_n) = 1$, Frobenius asked his students to find the largest integer m such that the Diophantine equation $a_1x_1 + a_2x_2 + a_3x_3 + \dots + a_nx_n = m$ has no solution in non-negative integers $x_1, x_2, x_3, \dots, x_n$. For this reason, the largest integer m that cannot be expressed as a linear combination of $a_1, a_2, a_3, \dots, a_n$ is called the Frobenius number, and is denoted by $g(a_1, a_2, a_3, \dots, a_n)$. Sylvester showed that $g(a_1, a_2) = a_1a_2 - a_1 - a_2$ [1]. Several formulae have been derived for all special cases of $n=3$. Some algorithms have been proposed for $n \geq 4$. For $n > 3$, the Frobenius number is a highly complex problem [3] [3]. It has applications in various fields such as number theory, combinatorics, automata theory, cryptography, optimisation and computer science [2]. In this talk, we revisit the short generating functions of the sets S and $\bar{S}$, with $S = \{a_1x_1 + a_2x_2; (x_1, x_2) \in \mathbb{N}^2\}$ to give an explicit formula of them and to obtain an algorithm that calculates the Frobenius number in the case $n=4$, whatever the integers a_1, a_2, a_3 and a_4 . This is done using the number of solutions of the Diophantine equation $a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 = m$. With the help of a program written in Python, we will calculate the values of the Frobenius number for a number of integers.

Main results

The short generating function of any discrete set S is defined by

$$P_S(x) = \sum_{n \in S} x^n. \quad (645)$$

According to this definition, we have

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$$P_S(x) = \frac{1 - x^{a_1 a_2}}{(1 - x^{a_1})(1 - x^{a_2})}, \quad (646)$$

and

$$P_{\bar{S}}(x) = \frac{1}{1 - x} - \frac{1 - x^{a_1 a_2}}{(1 - x^{a_1})(1 - x^{a_2})}. \quad (647)$$

Formal series of S and polynomial expression of $\bar{S}$ are given respectively by

$$P_S(x) = \sum_{n=0}^{g(a_1, a_2)-1} \delta_{a_1, a_2}(n) x^n + \sum_{n \geq g(a_1, a_2)+1} x^n, \quad (648)$$

and

$$P_{\bar{S}}(x) = \sum_{n=0}^{g(a_1, a_2)} (1 - \delta_{a_1, a_2}(n)) x^n \quad (649)$$

where $\delta_{a_1, a_2}(n)$ denotes the number of solutions of the Diophantine equation $a_1 x_1 + a_2 x_2 = n$. It's easy to see that

$$\deg(P_{\bar{S}}(x)) = g(a_1, a_2). \quad (650)$$

Our algorithm consists in finding all n that verify the equation $a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4 = n$ for $n = 0, \dots, g(a_1, a_2)$, such that this equation has no solution. the largest n is the Fobenius number.

the number of the Diophantine equation solutions is:

$$\delta_{a_1, a_2, a_3, a_4}(n) = \sum_{k=0}^n \left(\left\lfloor \frac{n-k}{a_4} \right\rfloor - \left\lfloor \frac{n-k-1}{a_4} \right\rfloor \right) \delta_{a_1, a_2, a_3}(k), \quad (651)$$

where $\delta_{a_1, a_2, a_3}(k)$ is the number of solutions for the Diophantine equation $a_1 x_1 + a_2 x_2 + a_3 x_3 = k$. It is expressed as:

$$\delta_{a_1, a_2, a_3}(k) = \sum_{i=0}^{\left\lfloor \frac{k}{a_3} \right\rfloor} \delta_{a_1, a_2}(k - i a_3). \quad (652)$$

Some values obtained by applying our algorithm are given in table 9.

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(a_1, a_2, a_3, a_4)	$g(a_1, a_2, a_3, a_4)$
(10, 11, 12, 13)	29
(10, 11, 12, 14)	29
(10, 11, 13, 89)	38
(10, 11, 13, 38)	29
(7, 13, 14, 17)	36
(7, 13, 21, 71)	64
(7, 13, 21, 64)	58
(13, 15, 17, 21)	61
(13, 15, 17, 61)	72

Table 9: Some values of the Frobenius number for four non-negative integers.

Physical mathematics

On the numerical aspects of the time dependent Schrödinger equation for the quantum non conservative systems

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Abstract: Quantum open i.e non conservative systems are the ones that are not isolated from an external environment and hence are subject to frictions. The temporal evolution of these systems is governed by the time dependent Schrödinger differential equation. It is first order in time and second order in space. The friction parameter is introduced phenomenologically using Caldirola-Kanai model. In This model the Hamiltonian becomes time dependent. The crank-Nicolson numerical method is generally used to solve such equations. Having at hand some analytical solutions, we intend in this study to analyze the numerical deviations that are introduced and how this is affecting the physical results.

Keywords: Schrödinger differential equations, Caldirola-Kanai frictions model, Crank-Nicolson numerical method.

2020 Mathematics Subject Classification: Primary 81Q05 , 81-08 .

Introduction

The time evolution of the wave function ψ describing the system state is given by:

$$\frac{i}{\hbar} \frac{\partial}{\partial t} \psi(x, t) + \frac{\partial^2}{\partial x^2} \psi(x, t) - V(x) \psi(x, t) = 0 \quad \forall x \in \Omega \subset \mathbb{R} \text{ and } t \in [0, T], \quad (653)$$

where Ω is some bounded domain and $\psi(x, 0) \in \mathbb{C}$ is the initial wave function. The Hamiltonian is the operator applied to $\psi(x, t)$ and it can be cast as:

$$H = -\frac{\partial^2}{\partial x^2} + V(x), \quad (654)$$

where $\frac{\partial^2}{\partial x^2}$ is representing the quantum momentum and $V(x)$ is some time independent potential. This is simply meaning that we can cast equation 653 as:

$$\frac{i}{\hbar} \frac{\partial}{\partial t} \psi(x, t) - H \psi(x, t) = 0. \quad (655)$$

The equation is given for one dimension but could easily be extended to $d \in \{1, 2, 3\}$. The equation can be resolved formally from the instant t_0 to t as:

$$\psi(t) = U(t, t_0) \psi(t_0), \quad (656)$$

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where U is the time propagator from t_0 to t and it is given by:

$$U(t, t_0) = e^{\frac{-i}{\hbar} H(t-t_0)}. \quad (657)$$

In the Cayley or the Crank-Nicolson method [1] a symmetrical form of the exponential is developed to the first order and results in:

$$\psi(t_{j+1}) = \frac{1 - \frac{i\hbar\Delta t}{2} H}{1 + \frac{i\hbar\Delta t}{2} H} \psi(t_j) \quad (658)$$

When we use Caldirola-Kanai [2, 3] phenomenological approach dealing with the friction, the Hamiltonian becomes time-dependent. Within this approach the Hamiltonian reads as:

$$H = \frac{\pi^2}{2m} e^{-\gamma t} + V(x) e^{\gamma t}, \quad (659)$$

where π (which is time dependent) is the canonical momentum conjugate of the spatial coordinate x and γ is the friction constant. We carry out the numerical integration in the following way; the discretized Schrödinger equation may be given to the n^{th} time grid by:

$$i\hbar \frac{\psi^{n+1} - \psi^n}{\Delta t} = \frac{H^{n+1} \psi^{n+1} + H^n \psi^n}{2}. \quad (660)$$

Here the Hamiltonian H depends on t . In our calculation and in order to recover the Crank-Nicolson scheme, we simply neglect this dependence and obtain:

$$i\hbar \frac{\psi^{n+1} - \psi^n}{\Delta t} = H^n \frac{\psi^{n+1} + \psi^n}{2}. \quad (661)$$

This is quite obvious first solution and our aim is to show the effect of such approximation on a non conservative system. In fact it is well known that the time iteration introduces a delay on the wave function that will likely interfere with the loss of energy introduced by the friction and hence it is interesting to highlight such a case. By proceeding to the discretization of all quantities occurring in the Hamiltonian, the whole equation can be cast as following:

$$\psi^{n+1} = D_1^{-1} D_2 \psi^n, \quad (662)$$

Where D_1 and D_2 are two tridiagonal matrices resulting from the discretization of the equation 658. D_1^{-1} being the inverse of D_1

Main results

We have at hand two exact analytical results for the time evolution of a Gaussian wave packet: the case where there is no friction and the case of a uniform friction in the whole space [4]. We intend to use these analytical solutions as bench test to analyze the consistency of the numerical approach. If we compare the analytical and the numerical results, some discrepancies could be noticed. This is actually the case when we illustrate the average position of the Gaussian wave packet through time for different x paces (figure 40). We can see clearly that whereas the case without friction is barely affected by the pace changes, the case with $\gamma = 1$ is manifesting a quite important deviation for the small illustrated time interval. This shortcoming could be remedied by a smaller x pace but this will require a greater machine time. Anyhow this deviation could interfere with the physical results mimicking an extra delay (energy loss) similar to the friction effect specially for systems where long time evolution is needed or an accumulated friction effect during time is considered [5].

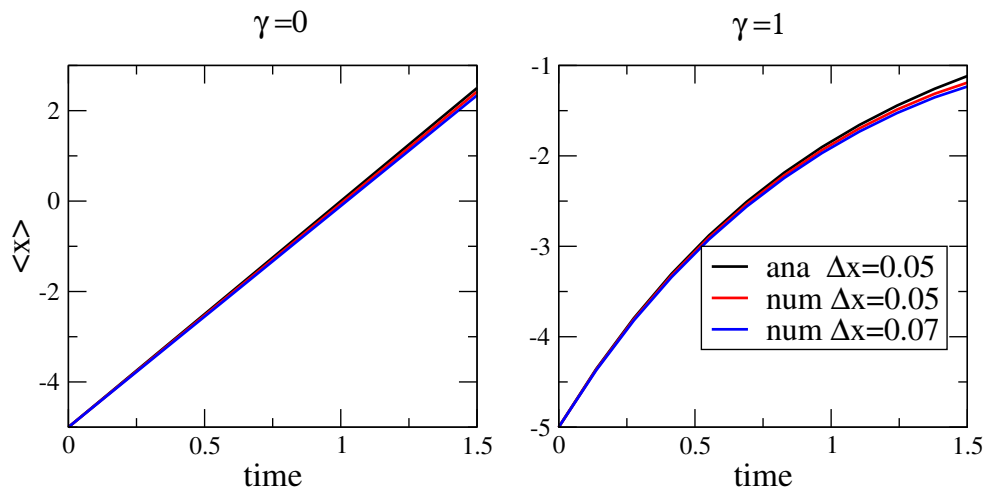


Figure 40: Comparison between analytical formulations and the numerical results for the average position $\langle x \rangle$. We are illustrating the case with no friction $\gamma = 0$ and case of a uniform friction $\gamma = 1$. This is done for different x paces as illustrated in the legend and for $\Delta t = 0.0002$.

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Analytical Modeling of total Vacancy Transfer Probabilities for elements in the atomic range $73 \leq Z \leq 92$: Statistical Analysis and Interpolation Techniques

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Abstract: In this study, we present an analytical approach to model the $\eta_{KL}(T)$ total vacancy transfer probabilities for heavy elements within the atomic range $73 \leq Z \leq 92$. The methodology involves the calculation of weighted average values for $\eta_{KL}(T)$ based on experimental data, followed by fitting these statistical values using Newton's truncated method to obtain a quadratic polynomial of the form $\eta_{KL}(T) = \sum_{i=0}^2 a_i Z^i$, where a_i are coefficients determined through the fitting process. The resulting analytical model allows for the interpolation of $\eta_{KL}(T)$ values for elements within the specified atomic range. We compare our modeled results with existing theoretical and experimental data from the literature, demonstrating good agreement overall. This work contributes to the understanding of vacancy transfer probabilities in heavy elements and provides a practical analytical framework for predicting $\eta_{KL}(T)$ values in this atomic range.

Keywords: fundamental atomic parameters, vacancy transfer probabilities, weighted average values.
2024 Mathematics Subject Classification: Physical Mathematics.

Introduction

Photon interactions with atoms play a pivotal role in numerous scientific disciplines, including atomic physics, medical research, and radiation processes. Of particular significance is the understanding of how vacancies transfer between the K-shell and L-shell in these atoms, quantified by total vacancy transfer probabilities ($\eta_{KL}(T)$), owing to its impact on nuclear phenomena such as electron capture, the photoelectric effect, and the generation of X-ray and Auger emissions. Over time, a variety of theoretical models and empirical or semi-empirical approaches have been employed to determine these transfer probabilities across different elements. For instance, Rao et al [1] focused on calculating the average number of primary L_i subshell vacancies resulting from transitions to the K-shell for elements with atomic numbers ranging from 20 to 96, while Bambynek et al [2] provided a comprehensive review of the subject. Puri et al [3] conducted calculations for the total K-shell to L_1 -, L_2 -, and L_3 -subshell vacancy transfer probabilities η_{KL_i} , covering elements with atomic numbers from 18 to 96. Additionally, Schönfeld and JanBen [4] compiled a table of semi-empirical data for K- to L-shell ($\eta_{KL}(T)$) total vacancy transfer probabilities, encompassing elements with atomic numbers ranging from 10 to 100. This study introduces a novel method to estimate $\eta_{KL}(T)$ values for elements within the atomic range of 73 to 92, leveraging a comprehensive database of experimental data obtained under various experimental conditions. These new empirical values, derived using interpolation methods, offer valuable insights into vacancy transfer probabilities across a wide range of atomic numbers. In this paper, we present an analytical approach to model the $\eta_{KL}(T)$ total vacancy transfer probabilities for heavy elements within the atomic range of 73 to 92. The methodology involves the calculation of weighted average values for $\eta_{KL}(T)$ based on experimental data, followed by fitting these statistical values using Newton's truncated method, this method is an iterative optimization algorithm used to find the minimum of a function. In this specific context, it was used to optimize the coefficients of a quadratic function to fit the given data. It's an extension of Newton's method for optimization problems. Finally, the results are presented in tabular form and compared with theoretical, fitted, semi-empirical, and experimental $\eta_{KL}(T)$ total vacancy transfer probabilities.

Main results

In this research, we introduced new parameters to compute the overall vacancy transfer probabilities for targets spanning from $_{73}\text{Ta}$ to $_{92}\text{U}$. We relied on an experimental database to derive new empirical values for $\eta_{KL}(T)$. Initially, we computed the weighted average values for all elements falling within $73 \leq Z \leq 92$ using the following formula:

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$$(\eta_{KL}(T))_W + \varepsilon = \left(\sum_i^N \frac{1}{(\Delta(\eta_{KL}(T))_{EXP-i})^2} \right)^{-1} \cdot \sum_i^N \frac{(\eta_{KL}(T))_{EXP-i}}{(\Delta(\eta_{KL}(T))_{EXP-i})^2} + \left(\left(\sum_i^N \frac{1}{(\Delta(\eta_{KL}(T))_{EXP-i})^2} \right)^{1/2} \right)^{-1} \quad (663)$$

In Eq.(663), $(\eta_{KL}(T))_{EXP-i}$ denotes the i^{th} experimental value, $\Delta(\eta_{KL}(T))_{EXP-i}$ signifies the assigned uncertainty (standard deviation) for the i th experimental value, and N represents the number of experimental data points pertaining to the element. The weighted average values $(\eta_{KL}(T))_W$ were used to determine the empirical total vacancy transfer probabilities. The $(\eta_{KL}(T))_W$ is presented as a function of Z and plotted in Figure 1. Subsequently, using Newton's truncated method, we were able to fit a quadratic polynomial. The quadratic function fitting is as follows:

$$\eta_{KL}(T) = \sum_{i=0}^2 a_i Z^i \quad (664)$$

With: $a_0 = 0.5530377526736968$, $a_1 = 0.009019518986688038$, and $a_2 = -7.25368733122199 \times 10^{-5}$.

Table 1 includes the current computation of empirical total vacancy transfer probabilities for all elements within the range $73 \leq Z \leq 92$, as per Eq. (664).

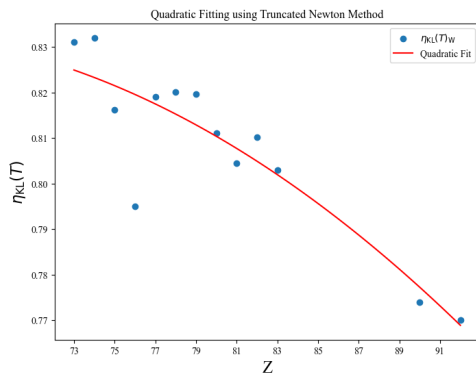


Figure 41: Distribution of $(\eta_{KL}(T))_W$ as a function of atomic number Z .

Table 1: Empirical values of $(\eta_{KL}(T))$ total vacancy transfer probability.

Z	$\eta_{KL}(T)$	Z	$\eta_{KL}(T)$
$Z = 73$	0.8249	$Z = 83$	0.8020
$Z = 74$	0.8233	$Z = 84$	0.7989
$Z = 75$	0.8215	$Z = 85$	0.7956
$Z = 76$	0.8195	$Z = 86$	0.7922
$Z = 77$	0.8175	$Z = 87$	0.7887
$Z = 78$	0.8152	$Z = 88$	0.7850
$Z = 79$	0.8129	$Z = 89$	0.7812
$Z = 80$	0.8104	$Z = 90$	0.7772
$Z = 81$	0.8077	$Z = 91$	0.7731
$Z = 82$	0.8049	$Z = 92$	0.7689

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Klein-Gordon solution with unequal mixture of Scalar and Vector potentials for harmonic interaction model:

$$V(y) = \lambda_2 y^2 + \lambda_1 y + \lambda_0$$

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Abstract: Based on the formal equivalence between the non-relativistic Schrödinger and the relativistic Klein-Gordon (K.G) equations, an analytical form of the relativistic energy spectrum for harmonic interaction model is established, by considering the more general case of uneven mixture Scalar $S(x)$ and Vector $V(x)$ potentials. It consists of transforming the Klein-Gordon wave function into the well-known first-order differential Riccati equation and by using the proper quantization rule expressed by $\int_{x_A}^{x_B} K_n(x)dx - \int_{x_{0A}}^{x_{0B}} K_0(x)dx = n\pi$. Our finding corresponds to the bound states of the particle and anti-particle, where the migration from the particle into the anti-particle and vice versa is possible.

Keywords: Proper quantization rule, exact solutions, Klein-Gordon equation, Riccati differential equation, harmonic oscillator class.

2020 Mathematics Subject Classification: 81; 81Q80; 34A34.

Introduction

Recently, non-relativistic exactly methods have received much attention from researchers to be used for resolving the relativistic Klein-Gordon (K.G.) wave function which describes the behavior of spinless bosonic particles in the presence of two types of potentials: the scalar $S(x)$ and vector $V(x)$ potentials. Among these methods we consider the hypergeometric differential equation as: Nikiforov-Uvarov (NU) [1], Asymptotic Iteration Method (AIM) [2], ..., etc), and the quantization method (Proper quantization rule [6],...) based on the WKB derived from the semi-classical Bohr-Sommerfeld quantization rules of old quantum theory and the Wentzel-Kramers-Brillouin (WKB) approximation. In the same context, we present in this study an investigation using the Proper Quantization rule proposed by Qiang and Dong in 2010 [6], in the wish to find an exact expression of a family of harmonic oscillator interaction type. It consists of reducing the relativistic K.G. wave function into a Schrödinger-like equation and exploiting the general case where the mixed unequal magnitudes Scalar and Vector potentials are related by an algebraic relation. A spatially one-dimensional, time-independent Klein-Gordon [4] equation for a spinless particle of rest mass m in the presence of mixed Vector $V(x)$ and Scalar potentials $S(x)$ is written as

$$-\frac{d^2\psi(x)}{dx^2} + 2m[(m + S(x))^2 - (E_n - V(x))^2]\psi(x) = 0. \quad (665)$$

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We keep the constant of Planck $\hbar = 1$ and the velocity of light $c = 1$ throughout the rest of this work. E_n denotes the total relativistic energy of a spinless particle for a bound state n . This equation can be rewritten as a Schrödinger equation for unequally mixed potentials $V(x)$ and $S(x)$ and will be expressed by

$$\frac{d^2\psi(x)}{dx^2} + 2m[E_{eff,n} - V_{eff}(x)]\psi(x) = 0, \quad (666)$$

where $E_{eff,n} = (E_n^2 - m^2)/2m$, and $V_{eff}(x) = [V^2 - S^2 - 2(E_n V(x) + mS(x))]/2m$, denotes respectively the effective energy of a particle and an effective solvable potential. The Schrödinger equation 665 is reduced to the nonlinear Riccati differential equation of the first order, for the ground state by performing the transformation $\varphi_0(x) = \frac{1}{\psi(x)} \frac{d\psi(x)}{dx}$ which is:

$$\frac{d\varphi_0}{dx} = -2m[E_{eff,0} - V_{eff}(x)] - \varphi_0^2, \quad (667)$$

It becomes easier to find solutions from the resolution of the Riccati equation than to find them from the radial Schrödinger equation (666)

Based on the fact that equations (665) and (666) are equivalent, we can deduce that the proper quantization rule for the relativistic Klein-Gordon is identical to the nonrelativistic proper quantization rule [6], and will be expressed as

$$\int_{x_A}^{x_B} K_n(x) dx - \int_{x_{0A}}^{x_{0B}} K_0(x) dx = n\pi, \quad (668)$$

with $K_n = \sqrt{2m[E_{eff,n} - V_{eff}(x)]}$, analogous to the classical momentum in the region of $E_{eff,n} \geq V_{eff}(x)$ between $x_A < x < x_B$. when x_A and x_B are two turning points ($x_A < x_B$) determined by calculating $E_{eff,n} = V_{eff}(x)$.

Main results

We aim in this study to obtain the bound state of the relativistic energy spectrum for the category of harmonic interaction models [5], widely used to study atomic and molecular interaction expressed as :

$$V(y) = \lambda_2 y^2 + \lambda_1 y + \lambda_0, \quad (669)$$

$\lambda_2, \lambda_1, \lambda_0 > 0, y \in \mathbb{R}$ and the potential satisfies $\frac{dy(x)}{dx} = \alpha(1 - y^2) > 0$. This set of potentials is related to the harmonic oscillator by a small linear perturbation term. The time-independent Klein-Gordon Eq.(665) wave function is investigated by considering the case when the scalar and vector potentials are coupled with unequal function, considering that their ratio is equal to a constant β in such a way $V(x) = \beta S(x)$, $-1 < \beta < 1$. On the other hand, the scalar potential is taken as $S(x) = S_0(y(x) + C)$, with S_0, C are positive constants. The effective potential can be expressed with only the scalar potential $S(x)$ such as: $V_{eff}(x) = (1 - \beta^2)S^2(x) + 2(E_n + m)S(x)/2m$, Suppose that: $(1 - \beta^2)S_0^2 = \lambda_2$, $2S_0(CS_0(1 - \beta^2) + (E_n\beta + m)) = \lambda_1$, and $((1 - \beta^2)C^2S_0^2 + 2S_0C(E_n\beta + m)) = \lambda_0$. the logarithmic derivative in the ground state takes the linear form such as $\varphi_0(x) = -ay(x) - b$, where $a, b > 0$. Substituting it into the Riccati nonlinear differential equation (667) One can obtain the algebraic formula of the constant coefficients a, b that are :

$a = (-\frac{\alpha}{2}) + (\frac{\alpha}{2})\sqrt{1 + (4(1 - \beta^2)\frac{S_0^2}{\alpha^2})}$, and $b = (CS_0^2(1 - \beta^2) + (E_n\beta + m)S_0)/a$, and the energy level for the radial function with no nodes ($n = 0$):

$$E_{eff,0} = (\alpha a + C^2 S_0^2(1 - \beta^2) + 2CS_0(E_n\beta + m) - b^2)/2m. \quad (670)$$

The resolution of the known function $E_{eff,n} - V_{eff}(x) = 0$ provides the explicit expression of the two turning points y_A and y_B which are: $y_{A,B} = \mp \frac{(CS_0(1 - \beta^2) + (E_n\beta + m))}{CS_0(1 - \beta^2)} \mp 1/2\sqrt{\Delta}$, where $\sqrt{\Delta}$ is the square root of the discriminant of the quadratic equation.

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Between the two turning points x_A and x_B , the first integral of the momentum $k_n(x)$ is calculated as

$$\int_{x_A}^{x_B} K_n(x) dx = \frac{\pi \sqrt{(1-\beta^2)S_0^2}}{\alpha} - \frac{\pi}{2\alpha} \left[\sqrt{(1-\beta^2)S_0^2(1+C)^2 + 2S_0(E_n\beta + m)(1+C) - 2mE_{eff}} \right. \\ \left. + \sqrt{(1-\beta^2)S_0^2(1-C)^2 + 2S_0(E_n\beta + m)(1-C) - 2mE_{eff}} \right]$$

Likewise, for the quantum number $n = 0$, the second integral of momentum $k_0(x)$ is calculated by replacing the effective energy $E_{eff,n}$ in the last equation, by the ground effective energy expression Eq. (670). Then, the relativistic energy spectrum for the harmonic oscillator class can be obtained algebraically by using the proper quantization rule in Eq. (668) as:

$$(E_n^2 - m^2) = - \frac{[(\alpha n - a)^2 + [S_0(E_n\beta + m) + CS_0^2(1-\beta^2)]^2]}{(\alpha n - a)^2} + [(1-\beta^2)S_0^2(1+C^2) + 2S_0C(E_n\beta + m)] \quad (671)$$

$n = 0, 1, 2, \dots$ is the principal quantum number.

To analyze the effect of the parameter β on the energy spectrum, we calculate the energy numerically using the formula Eq. (671). The results are plotted in Figure below. Part(a) of the figure pertains to the case $n = 0$ where we have a peculiar behavior. Indeed the particle/antiparticle (black and red curves respectively) states are completely indistinguishable in the interval $\beta [-1, 1]$ and come apart only in the intervals $[-1, -1.4]$ and $[1, 1.4]$. Remarkably the values of the ratio β beyond 1 and below -1 are able to suppress the degeneracy.

Parts (b) and (c) of the same figure are illustrating respectively the particle/antiparticle states for $n = 1, 2$ and 3. The curves on the two panels are quite similar but the results in the negative and positive β intervals are inverted. For β belonging to the interval $[-1, +1]$, in part (b), the particle presents bound states solutions for $\beta < 0$, as long as an attractive vector and scalar potentials are applied. For the case in which $\beta > 0$, and for high energy in the presence of a repulsive vector potential and an attractive scalar one, the particle is not confined and becomes unstable. From the part (c) of the figure, in the case $\beta < 0$, the antiparticle is unstable as long as the vector potential is repulsive and the scalar one is attractive. For $\beta > 0$ the vector and the scalar potentials are attractive and the antiparticle presents bound states solutions.

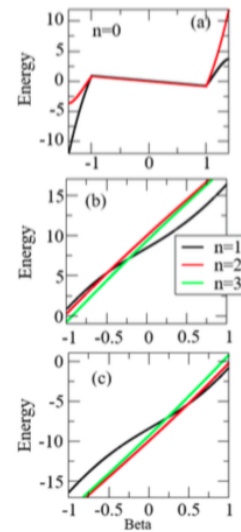


Figure 1: Energy spectrum versus the scalar/vector potential ratio for different values of the quantum number n : (a) for the ground state ($n = 0$), (b) for matter states ($n = 1, 2$ and 3) and (c) for antimatter states ($n = 1, 2$ and 3). For the numerical calculations, we consider $\alpha = 2, C = 10.0, m = 1$, and $S_0 = 1$.

Conclusion

In this work, the relativistic proper quantization rule provides exact relativistic bound states energies for a particle with zero spin, via the resolution of the relativistic Klein-Gordon equation. This is achieved by considering the formal similarity existing between the Schrödinger equation and the Klein-Gordon wave function. We studied the more general case when the scalar and vector potentials are connected with an unbalanced potential contribution. One can notify that the extension represents a perturbative term that has no effect on the closed form of energy expression. By changing the sign of the real parameter β , the migration from particle to antiparticle and vice versa is possible involving a change of the sign of the Eigenenergies. It becomes possible also to suppress the degeneracy in the ground states allowing us to locate the particle.

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Approximate solution of flow under a sluice gate with large Weber number

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Abstract: The problem of the steady two dimensional free surface flow of incompressible and inviscid fluid under a sluice gate is considered. The flow is assumed to be irrotational and steady, the effect of the surface tension is considered, but the gravity force is neglected. Schwarz–Christoffel transformation is used to map the region in the complex potential plane, onto the upper half plane. The main purpose of this work is to give an approximate solution of this problem, by using the Hilbert method and the perturbation technique for small inclination angle, the results are calculated for a large values of the Weber number and small slope of the gate.

Keywords: Free-surface flow problems, inviscid flow, Schwarz–Christoffel transformation, Hilbert method, perturbation technique.

2020 Mathematics Subject Classification: 35Q35, 76D05, 76D07, 76N10.

Introduction

The problem of the steady free surface flow of a fluid under sluice gate with large Weber number is considered. The fluid is assumed to be incompressible and inviscid. The flow is considered to be bidimensional, stationary, and irrotational. The effects of the superficial tension is considered with the neglected of gravity force. We consider the fluid region as shown in Fig 42.

This problem belongs to the class of flows for which there are intersections between the free surface and a solid, for more details related on this subject see for example Batchelor [1].

Several numerical studies have been realized on this subject; this includes the determination of the shape of the free-surface of the flow for a given solid form. Vanden-Broeck [2] and Assavanat & Vanden-Broeck [3] used a series truncation technique and the boundary-integral method, to solve the problem of flow under a gate, whilst Gasmi & Mekias [4] and Gasmi numerically studied the jet flow and flow issuing from a Funnel.

This problem is characterized by the nonlinear condition on the free surface given by the Bernoulli equation on the one hand, and the unknown form of this surface on the other. In the case where the effects of gravity and surface tension are neglected, Bounif & Gasmi [5] solve this problem analytically by using the hodograph method, but in this problem, the effects of the superficial tension are considered with the neglected of gravity force, which make it difficult to solving analytically, so in this case, we use an approximate method in order to obtain solution.

The present method of solution, where the Hilbert transformation as well as the perturbation technique are used as a basis for the approximate solution, has been successfully applied by Mina B & Abd-el-Malek [6] to give an approximate solution of gravity-affected flow from planar sluice gate at high Froude number.

The primary motivation for the present investigation is to study the effect of the inclination angle, depth ratio and the Weber number on the downstream free surface as well as to discuss the contraction coefficient as a function of the inclination angle. Attention is also given to find the curvature of the downstream free surface in general form. Next, we formulate the boundary-value problem. Then, we express the solution of the boundary-value problem in approximate form. After then, we solve the approximate equations by the

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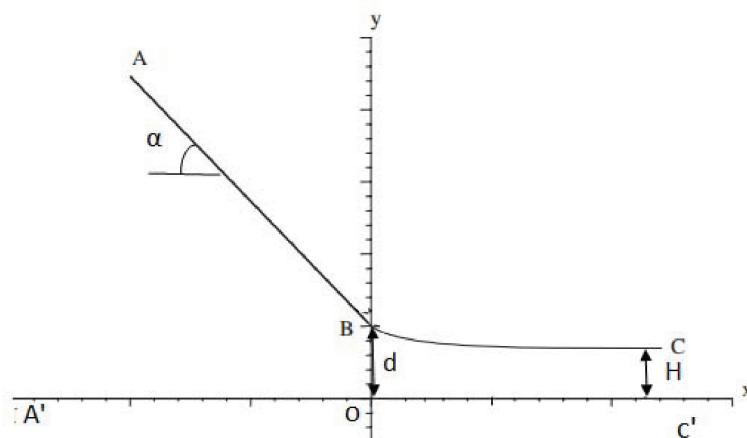


Figure 42: Sketch of the flow and of the coordinates. The width of the opening is d and the depth of the flow at infinity is H and the angle between the inclined wall AB and horizontal is α .

perturbation method. finally, we determine the solution for the boundary-value problem to first order in the small inclination angle and large weber number.

Main results

The Hilbert method is combined with the perturbation technique to find the approximate profile and some characteristics of the two-dimensional flow of an ideal fluid under a sluice gate of small slope and for large Weber number, we obtain the approximate solution of the problem considered in the first order.

The Results demonstrating the downstream free surface profiles over different sluice gate geometries are calculated in our study. These results demonstrate as expected the monotonic rise in the downstream free surface profile with increase of depth ratio and Weber number.

One quantity of physical and geometrical interest is the curvature of the downstream free surface is calculated.

Other values have also been calculated in our study, for example the length of the sluice gate.

Hilbert transformation and perturbation technique has been successfully applied to solve two-dimensional steady flow problem under sluice gate.

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Solvability of an impulsive boundary value problem on the half-line via critical point theory

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Abstract: In this paper, an impulsive boundary value problem on the half-line is considered and existence of solutions is proved using the Minimization Principal and the Mountain Pass Theorem.

Keywords: Impulsive BVPs, unbounded interval, critical point, minimization principal, mountain-pass theorem.

2010 Mathematics Subject Classification: 34B37, 35A15, 58E30.

Introduction

This paper is concerned with the following impulsive boundary value problem set on the positive half-line

$$\begin{cases} -(p(t)u'(t))' = f(t, u(t)), & \text{a.e. } t \in [0, +\infty), t \neq t_j \\ u(0) = u(+\infty) = 0, \\ \Delta(p(t_j)u'(t_j)) = h(t_j)I_j(u(t_j)), & j \in \{1, 2, \dots\}, \end{cases} \quad (672)$$

where $f : [0, +\infty) \times \mathbb{R} \rightarrow \mathbb{R}$ is a Carathéodory function, $p : [0, +\infty) \rightarrow (0, +\infty)$ satisfies $\frac{1}{p} \in L^1[0, +\infty)$, and

$$M = \int_0^{+\infty} \left(\int_t^{+\infty} \frac{1}{p(s)} ds \right) dt < +\infty.$$

One may take as examples of p , the exponential function or

$$p(t) = \begin{cases} \sqrt{t}, & \text{if } t \in [0, 1], \\ \frac{1}{2}t(1+t^2), & \text{if } t \geq 1. \end{cases}$$

Here $t_0 = 0 < t_1 < t_2 < \dots < t_j < \dots < t_m \rightarrow +\infty$ are the impulse points, while the impulsive functions $I_j : \mathbb{R} \rightarrow \mathbb{R}$ are continuous, and

$$\Delta(p(t_j)u'(t_j)) = p(t_j^+)u'(t_j^+) - p(t_j^-)u'(t_j^-),$$

where $u'(t_j^+) = \lim_{t \rightarrow t_j^+} u'(t)$ and $u'(t_j^-) = \lim_{t \rightarrow t_j^-} u'(t)$, stand for the right and the left limits of u' at t_j , respectively.

Finally $h : [0, +\infty) \rightarrow [0, +\infty)$ is such that $\sum_{j=1}^{+\infty} h(t_j) < +\infty$.

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Main results

We start with the sublinear case.

Theorem 252. Suppose that the following conditions hold:

(H₁) There exist a constant $\mu \in [0, 1)$ and positive functions $a_1, b_1 \in L^1[0, +\infty)$ such that

$$|f(t, x)| \leq a_1(t)|x|^\mu + b_1(t), \text{ for a.e. } t \in [0, +\infty) \text{ and all } x \in \mathbb{R}.$$

(I₀) There exist constants $k > 0$ and $\gamma \in [0, 1)$ such that

$$|I_j(s)| \leq k|s|^\gamma, \forall s \in \mathbb{R}, j \in \{1, 2, \dots\}.$$

Then (672) has at least one weak solution.

Next, we consider the limit case $\mu = 1$.

Theorem 253. Assume that (I₀) holds both with

(H₂) there exist positive functions $a_2, b_2 \in L^1[0, +\infty)$ with $\|a_2\|_{L^1} < \frac{1}{d^2}$ and

$$|f(t, x)| \leq a_2(t)|x| + b_2(t), \text{ for a.e. } t \in [0, +\infty) \text{ and all } x \in \mathbb{R}.$$

Then Problem (672) has at least one weak solution.

Theorem 254. Suppose that the following conditions hold:

(H₃) There exist positive functions φ, g such that $\varphi \in L^1(\mathbb{R}^+, \mathbb{R})$ and $g \in C(\mathbb{R}, \mathbb{R})$ with

$$|f(t, x)| \leq \varphi(t)g(x), \text{ for a.e. } t \in [0, +\infty) \text{ and all } x \in \mathbb{R}.$$

(H₄) $\lim_{x \rightarrow 0} \frac{f(t, x)}{|x|} = 0$ uniformly in $t \geq 0$.

(H₅) There exist positive functions $c_1, c_2 \in L^1(\mathbb{R}^+, \mathbb{R}^+)$, and $\sigma > 2$ such that

$$(a) F(t, x) \geq c_1(t)|x|^\sigma - c_2(t), \text{ for a.e. } t \in [0, +\infty), \forall x \in \mathbb{R}.$$

$$(b) \sigma F(t, x) \leq x f(t, x), \text{ for a.e. } t \geq 0, \forall x \in \mathbb{R} \setminus \{0\}.$$

(I₁) There exists $0 < \gamma \leq 2$ such that

$$\gamma \int_0^x I_j(s) ds \geq x I_j(x) > 0, \forall x \in \mathbb{R}, \forall j \in \{1, 2, \dots\}.$$

Then Problem (672) has at least one nontrivial weak solution.

Remark 12. By Theorem 254, the boundary value problem

$$\begin{cases} -(e^t u'(t))' &= \frac{u^4(t)}{(1+t)^3}, \quad t \neq t_j, \text{ a.e. } t \in [0, +\infty), \\ u(0) = u(+\infty) &= 0, \\ \Delta(e^{t_j} u'(t_j)) &= \frac{\sqrt[3]{u(t_j)}}{(1+t_j)^2}, \quad j \in \{1, 2, \dots\} \end{cases} \quad (673)$$

has at least one nontrivial solution.

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The effect of an insulating stiffener on a nonlinear thermo-elastic plate

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Abstract: This paper deals with the mathematical modeling of the behavior of a reinforced rectangular thermo-elastic plate with a thin insulating stiffener. We use a variational asymptotic analysis, with respect to the thickness of the inserted body, in order to identify limit models that reflect its effect on the plate. We carry out a mathematical modeling for a stiffener of high rigidity and a stiffener of moderate rigidity.

Keywords: Asymptotic analysis, rectangular plate, thin insulating stiffener, nonlinear partial differential equations, approximate boundary conditions.

2020 Mathematics Subject Classification: 35A15, 35B40, 35G50, 35Q74, 74K20, 74K35.

Introduction

Mathematical Modeling is a powerful tool, widely used in Science and engineering. It consists in developing mathematical descriptions of various real-world phenomena and enable investigating, understanding analysing and predicting the behavior of systems in a large variety of areas. In this context, we focus in this work on the mathematical modeling of physical phenomena arising from elasticity and mechanical structures fields. More precisely, we deal with the asymptotic modeling of the behavior of a Thermo-Elastic rectangular plate reinforced by a thin insulating stiffener on a part of its boundary. Such structures are widely used in many branches of mechanics, civil and structural engineering such as bridges or storage tanks, and a growing attention was paid to the modeling of their behavior in the last decades. Indeed, the addition of stiffeners to structures aims to increase their strength and prevent from damages or buckling. Besides, the proprieties of the constitutive material of the stiffener is of a great importance and has a significant impact on the behavior of the reinforced structure. In several situations, especially in the design of insulating structures, thermally non-conductive materials are usually chosen in order to conserve the thermal energy by reducing heat loss or heat gain.

In this work, we first transform the problem into a problem posed over a domain that doesn't depend on the small parameter δ . Thus, we establish a priori estimates that permit to pass through the limit in the variational problem and we use an asymptotic variational approach to identify limit models for the reinforced rectangular plate. We model both the effect of an insulating stiffener of high rigidity and moderate rigidity.

Main results

Statement of the problem

We consider a bi-dimensional rectangular plate occupying the set $\bar{\Omega}_+ = [0, 1] \times [0, 1]$ of boundary $\partial\Omega_+ = \bar{\Sigma} \cup \bar{\Gamma}_+$, where $\Sigma =]0, 1[\times \{0\}$. The plate is clamped on the portion Γ_+ of its boundary and is reinforced by

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a thin rigid layer on the other part Σ . The thin stiffener occupies the set $\bar{\Omega}_-^\delta = [0, 1] \times [-\delta, 0]$ of boundary $\partial\Omega_-^\delta = \bar{\Sigma}_-^\delta \cup \bar{\Sigma} \cup \bar{\Gamma}_-^\delta$, where $\Sigma_-^\delta =]0, 1[\times \{-\delta\}$. These two elastic bodies form together an heterogeneous elastic multi-structure, viewed as a rectangular plate occupying the set $\bar{\Omega}^\delta = [0, 1] \times [-\delta, 1]$, where $\Omega^\delta = \Omega_+ \cup \Sigma \cup \Omega_-^\delta$.

- **Equations of motion in $\Omega^\delta \times (0, T)$:**

$$\begin{aligned} \rho u'' - \operatorname{div} \left\{ \mathcal{C}[\epsilon(u) + f(\nabla w)] \right\} + \lambda \nabla \phi &= 0, \\ \rho [I - \Delta] w'' + D \Delta^2 w - \operatorname{div} \left\{ \mathcal{C}[\epsilon(u) + f(\nabla w)] \nabla w \right\} + \lambda \Delta \theta &= 0, \\ \rho \phi' - k \Delta \phi + \lambda \operatorname{div} u' &= 0, \quad \rho \theta' - k \Delta \theta - \lambda \Delta w' = 0. \end{aligned} \quad (674)$$

- **Free boundary conditions on $\Sigma_-^\delta \times (0, T)$:**

$$\begin{aligned} \mathcal{C}[\epsilon(u) + f(\nabla w)] n &= 0, & D[\Delta w + (1 - \nu) B_1 w] &= 0, \\ D[\partial_n \Delta w + (1 - \nu) \partial_s B_2 w] - \rho \partial_n w'' - \mathcal{C}[\epsilon(u) + f(\nabla w)] n \cdot \nabla w + \lambda \partial_n \theta &= 0, \\ k \partial_n \theta + \lambda \partial_n w' &= 0, & k \partial_n \phi - \lambda u' n &= 0. \end{aligned} \quad (675)$$

- **Clamped boundary conditions on $\Gamma_+ \cup \Gamma_-^\delta \times (0, T)$:**

$$u = 0, \quad w = \partial_n w = 0, \quad \theta = 0, \quad \phi = 0. \quad (676)$$

- **Transmission conditions on $\Sigma \times (0, T)$:**

$$\begin{aligned} [[u]] &= 0, \quad [[w]] = [[\partial_n w]] = 0, \quad [[\theta]] = [[\phi]] = 0, \\ [[\mathcal{C}[\epsilon(u) + f(\nabla w)] n]] &= 0, \quad [[D[\Delta w + (1 - \nu) B_1 w]]] = 0, \\ [[k \partial_n \theta + \lambda \partial_n w']] &= 0, \quad [[k \partial_n \phi - \lambda u' n]] = 0, \\ [[D[\partial_n \Delta w + (1 - \nu) \partial_s B_2 w] - \rho \partial_n w'' - \mathcal{C}[\epsilon(u) + f(\nabla w)] n \cdot \nabla w + \lambda \partial_n \theta]] &= 0. \end{aligned} \quad (677)$$

We associate with the equations (674)- (677) the initial conditions in Ω^δ :

$$u(0) = u^0, \quad u'(0) = u^1, \quad w(0) = w^0, \quad w'(0) = w^1, \quad \theta(0) = \theta^0, \quad \phi(0) = \phi^0, \quad (678)$$

where $u^0 \in [H^1(\Omega^\delta)]^2$, $u^1 \in [L^2(\Omega^\delta)]^2$, $w^0 \in H^2(\Omega^\delta)$, $w^1 \in H^1(\Omega^\delta)$, $\theta^0, \phi^0 \in H^1(\Omega^\delta)$.

The variables w and (u_1, u_2) represent respectively the vertical (bending) and in-plane displacement of the structure (plate-layer). θ and ϕ describe the average temperature affecting the vertical and horizontal displacement respectively. $\epsilon(u)$ is the linearised strain tensor defined by the formulae $\epsilon(u) = \frac{1}{2} (\nabla u + \nabla^T u)$ and $\mathcal{C}$ is a fourth order tensor that belongs to S , the space of 2×2 symmetric matrices, given by:

$$\mathcal{C}(\xi) = D \left[\nu (\operatorname{tr} \xi) I_S + (1 - \nu) \xi \right],$$

$\forall \xi \in S$, where I_S is the identity matrix. The function f is defined by $f(s) = (1/2)s \otimes s$, $s \in \mathcal{R}^2$ and the trace operators B_1 and B_2 are given by:

$$B_1 w \equiv 2n_1 n_2 \partial_{xy}^2 w - n_1^2 \partial_y^2 w - n_2^2 \partial_x^2 w, \quad B_2 w \equiv (n_1^2 - n_2^2) \partial_{xy}^2 w + n_1 n_2 (\partial_y^2 w - \partial_x^2 w),$$

where $n = (n_1, n_2)$ is the unit normal to Σ oriented outwardly of Ω_+ and ∂_s stands for the tangential derivative.

We denote by $D = \frac{E}{(1 - \nu^2)}$ the flexural rigidity of the structure ; E being the Young modulus, ν the Poisson's ratio, ρ is the mass density, k the coefficient of thermal conductivity and $\lambda = \frac{\alpha D(1 + \nu)}{2}$, where α denotes

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the coefficient of thermal expansion. We assume that $0 < \nu < \frac{1}{2}$ and that all these physical and thermal coefficients are piecewise positive constants: E, ρ, k, α, ν are equal to $E_+, \rho_+, k_+, \alpha_+, \nu_+$ (resp. $E_-^{\delta}, \rho_-^{\delta}, k_-^{\delta}, \alpha_-^{\delta}, \nu_-$) in Ω_+ (in Ω_-^{δ} resp.).

Accordingly, $D = D_+$, $\lambda = \lambda_+$ in Ω_+ and $D_-^{\delta}, \lambda_-^{\delta}$ in Ω_-^{δ} . All over this study, we will make the assumption that these coefficients are independent of δ in the plate Ω_+ (the Poisson's ratio ν is supposed independent of δ in both the plate and the layer).

Finally, we denote by $[[\cdot]]$ the jump through the interface Σ and g', g'' stand for the time derivatives of a function g .

An appropriate variational formulation of (674)-(678) reads:

$$(P) \begin{cases} \text{Find } u \in L^{\infty}(0, T; U(\Omega^{\delta})), w \in L^{\infty}(0, T; W(\Omega^{\delta})), w' \in L^{\infty}(0, T; V(\Omega^{\delta})), \\ u' \in L^{\infty}(0, T; [L^2(\Omega^{\delta})]^2), \phi \in L^{\infty}(0, T; L^2(\Omega^{\delta})) \cap L^2(0, T; V(\Omega^{\delta})), \\ \theta \in L^{\infty}(0, T; L^2(\Omega^{\delta})) \cap L^2(0, T; V(\Omega^{\delta})), \text{ such that:} \\ \rho \langle u', \dot{u} \rangle_{\Omega^{\delta}} + \langle \mathcal{C}[\epsilon(u) + f(\nabla w)], \epsilon(u) \rangle_{\Omega^{\delta}} + \lambda \langle \nabla \phi, \dot{u} \rangle_{\Omega^{\delta}} + \rho \langle w', \dot{w} \rangle_{\Omega^{\delta}} \\ + \langle \nabla w', \nabla \dot{w} \rangle_{\Omega^{\delta}} + a(w, \dot{w}) + \langle \mathcal{C}[\epsilon(u) + f(\nabla w)], \nabla w, \nabla \dot{w} \rangle_{\Omega^{\delta}} \\ - \lambda \langle \nabla \theta, \nabla \dot{w} \rangle_{\Omega^{\delta}} + \rho \langle \phi, \dot{\phi} \rangle_{\Omega^{\delta}} + k \langle \nabla \phi, \nabla \dot{\phi} \rangle_{\Omega^{\delta}} - \lambda \langle u', \nabla \dot{\phi} \rangle_{\Omega^{\delta}} \\ + \rho \langle \theta, \dot{\theta} \rangle_{\Omega^{\delta}} + k \langle \nabla \theta, \nabla \dot{\theta} \rangle_{\Omega^{\delta}} + \lambda \langle \nabla w', \nabla \dot{\theta} \rangle_{\Omega^{\delta}} = 0, \end{cases}$$

$\forall (\dot{u}, \dot{w}, \dot{\phi}, \dot{\theta}) \in U(\Omega^{\delta}) \times W(\Omega^{\delta}) \times V(\Omega^{\delta}) \times V(\Omega^{\delta})$, with the initial conditions (678), where:

$$a(w, \dot{w}) = \int_{\Omega^{\delta}} D \left[(\partial_x^2 w + \nu \partial_y^2 w) \partial_x^2 \dot{w} + 2(1 - \nu) \partial_{xy}^2 w \partial_{xy}^2 \dot{w} + (\partial_y^2 w + \nu \partial_x^2 w) \partial_y^2 \dot{w} \right] d\Omega^{\delta}.$$

New approximate boundary conditions on $\Sigma \times (0, T)$:

$$\begin{aligned} {}^t \tau \mathcal{C}[\epsilon(\tilde{u}_+) + f(\nabla \tilde{w}_+)] n &= E_- \partial_x \left(\partial_x \tilde{u}_{+1} + \frac{1}{2} (\partial_x \tilde{w}_+)^2 \right), \\ {}^t n \mathcal{C}[\epsilon(\tilde{u}_+) + f(\nabla \tilde{w}_+)] n &= 0, \\ D_+ [\Delta \tilde{w}_+ + (1 - \nu_+) B_1 \tilde{w}_+] &= -Q(\tilde{w}_+), \\ D_+ [\partial_n \Delta \tilde{w}_+ + (1 - \nu_+) \partial_x B_2 \tilde{w}_+] - \rho_+ \partial_n \tilde{w}_+'' - C[\epsilon(\tilde{u}_+) + f(\nabla \tilde{w}_+)] n \cdot \nabla \tilde{w}_+ + \lambda_+ \partial_n \tilde{\theta}_+ \\ &= P(\tilde{w}_+) - E_- \partial_x \left[(\partial_x \tilde{u}_{+1} + \frac{1}{2} (\partial_x \tilde{w}_+)^2) \partial_x \tilde{w}_+ \right], \\ k_+ \partial_n \tilde{\theta}_+ + \lambda_+ \partial_n \tilde{w}_+' &= 0, \quad k_+ \partial_n \tilde{\phi}_+ - \lambda_+ \tilde{u}_+' n = 0. \end{aligned}$$

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Stabilization of an Euler-Bernoulli beam with distributed disturbance

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Abstract: In this study, we examine the vibration of an Euler-Bernoulli beam induced by distributed disturbances. Through the implementation of the proposed boundary control, we successfully suppress the undesired vibrations utilizing Lyapunov's method.

Keywords: Euler-Bernoulli beam, beam vibration, boundary control, Stabilization
2020 Mathematics Subject Classification: 37N35, 35L35, 93D20.

Introduction

Here, we consider the following Euler-Bernoulli beam with distributed disturbance:

$$\begin{cases} \rho w_{tt} + EI w_{xxxx} - T w_{xx} = f(x, t), & x \in (0, L), t \geq 0, \\ w(0, t) = w_x(0, t) = w_{xx}(L, t) = 0, \\ -EI w_{xxx}(L, t) + T w_x(L, t) = U(t), & t \geq 0, \\ w(x, 0) = w^0(x), \quad w_t(x, 0) = w^1(x), & x \in (0, L), \end{cases} \quad (679)$$

where $w(x, t)$ represents the displacements transverse of the beam at the position x for time t , ρ is the uniform mass per unit length of the beam, and L is the length of the beam, T is the axial tension of the beam, EI is the bending stiffness, $f(t, x)$ is the disturbance force and , and $U(t)$ is the control force which will be specified later.

Recently, the problem of controlling slender structures, especially the problem of elastic beams, has received great attention due to its applications in various fields. The goal of controlling the structures is to suppress free vibrations or at least reduce them when they are caused by environmental disturbances. The issue of suppression or reduction of vibrations by boundary control has been discussed by several authors (see [1, 3, 5]). For related works that used internal damping, we mention them [1, 2, 4].

The main contribution of this work is to achieving the result of uniform stability in the presence of disturbance, without resorting to any internal damping, but only by boundary control.

Preliminary

New, we introduce the energy associated with (679) as

$$E(t) = \frac{\rho}{2} \int_0^L w_t^2 dx + \frac{EI}{2} \int_0^L w_{xx}^2 dx + \frac{T}{2} \int_0^L w_x^2 dx. \quad (680)$$

Observe that this is the usual classical energy. The first term represent kinetic energy, and the other terms represent potential energy.

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disturbance force and control force

The distributed disturbance can be a source of turbulence. For this reason, we assume that there exists positive constant ε such that

$$\sup_{R_+} \int_0^L |f(x, t)|^2 dx \leq \varepsilon, \quad \forall t \geq 0. \quad (681)$$

The control's objective is to reduce the free transverse vibrations of the beam. Lyapunov's direct method is used to construct a suitable boundary control at the free boundary of the beam. To stabilize system (679), we propose the following control:

$$U(t) = -kw_t(L, t), \quad (682)$$

where k is a positive constant.

Lemma 52. *The energy functional (680) satisfies*

$$E'(t) = \int_0^L f(x, t)w_t dx + U(t)w_t(L, t), \quad \forall t \geq 0. \quad (683)$$

Uniform stability

The uniform stability results given in the following theorem

Theorem 255. *The energy $E(t)$ of system (679) satisfies*

$$E(t) \leq Ce^{-\lambda t} + \alpha, \quad t \geq 0, \quad (684)$$

where C , λ and α are positive constants.

Numerical simulations

In this section, using the finite difference method, we perform a numerical simulation of problem (679) to demonstrate the effectiveness of the proposed control (682). The simulation results are shown in the following figures

Figure 43: The displacement $w(L/2, t)$ of beam: without control and with control.

Figure 44: The displacement $w(L, t)$ of beam: without control and with control.

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Solution of Schrödinger equation for the inverse square root potential plus a modified double ring-shaped potential

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Abstract: The bound state solutions and their corresponding energy eigenvalues of the non-relativistic Schrodinger equation are calculated for non central potential, a inverse square root potential plus a modified double ring-shaped potential, in the framework of exactly solvable problems. We calculate the radial and angular parts of the solutions extended Nikiforov-Uvarov method.

Keywords: extended Nikiforov-Uvarov, bound state, Schrodinger equation.

2020 Mathematics Subject Classification: 34L10, 81U15, 81-08.

Introduction

Quantum mechanics is a branch of physics that describes the behavior of matter and energy at the smallest scales, such as the atomic and subatomic levels. It provides a framework for understanding how particles like electrons, protons, and photons interact with each other and with their surroundings. Quantum mechanics is known for its unusual and often counter intuitive concepts, including wave-particle duality, superposition, and entanglement. The Schrödinger equation is one of the more important equations in quantum mechanics. This equation is a fundamental equation that describes the behavior of quantum particles [3]. It allows us to predict the probability distribution of a particle's position and energy at any given time. The Schrodinger equation considers central potentials, which are spherically symmetric and non-central potentials, these potentials depend on both the position and direction of the particle that means the potential should not have only the radial part but also the angle-dependent parts, making their analysis more intricate. By incorporating non-central potentials into the Schrodinger equation, we can investigate a wide range of physical systems, such as interactions with magnetic fields or anisotropic potentials. This research aims to extend our understanding of quantum mechanics and pave the way for new applications in various fields, including condensed matter physics, quantum chemistry, and materials science.

To solve Schrödinger equation is to determine the wave function of the system which contains all possible information about the physical properties of a system. However, the form of the wave function depends on the type of the potential.

In this talk, we deal with Schrodinger equation for a inverse square root plus a modified double ring-shaped potential. Using the extended Nikiforov-Uvarov method [1], we are able to calculate the bound states and their corresponding energy eigenvalues.

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Schrödinger equation

The time-independent Schrodinger equation is ($\hbar = M = 1$)

$$[\Delta_{\mathbf{r}} - 2(E - V(\mathbf{r}))]\Psi(\mathbf{r}) = 0. \quad (685)$$

where E is the energy of the system and $V(\mathbf{r})$ is the potential. We consider the inverse square root potential plus a modified double ring-shaped potential [2, 4], given by

$$V(\mathbf{r}) = V_0(r) + \frac{V_1(\theta)}{r^2} \quad (686)$$

where

$$V_0(r) = \frac{a}{\sqrt{r}}. \quad (687)$$

$$V_1(\theta) = \frac{b_0 + b_1 \cos^2(\theta)}{\sin^2(\theta)} + \frac{b_0 + b_1 \sin^2(\theta)}{\cos^2(\theta)} + \frac{b_2}{\sin^2(\theta) \cos^2(\theta)}. \quad (688)$$

where a and b_i , $i = 0, 1, 2$ are real constants.

Main results

In this work, we solved the Schrodinger equation with non-central potential, inverse square root potential plus a modified double ring-shaped potential. We calculate the bound state solutions and their corresponding energy eigenvalues. In the spherical coordinates, using the separation of variables, we compute angular and radial parts solution of the corresponding Schrodinger equation, via the extended Nikiforov-Uvarov [1] method. Our solutions lead to results, as special cases, for many specific potential models. We plan to use the same method applied here, to solve other equation like Dirac, Klein-Gordon equations, with other non central potentials.

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Asymptotic behavior of elastic problem with maximal monotone graph condition and friction law

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Abstract: In this paper, we consider the stationary problem in three dimensional thin domain Ω^ε with maximal monotone graph and Tresca conditions. In the first step, we present the problem statement and give the variational formulation. We then study the asymptotic behaviour when one dimension of the domain tends to zero. In the latter case a specific Reynolds limit equation is obtained and the uniqueness of the displacement of the limit problem are proved.

Keywords: Asymptotic approach. Maximal monotone graph. Tresca law. Variational problem. Weak solution.

2020 Mathematics Subject Classification: 35R35. 76F10. 78M35.35R35. 76F10. 78M35.

Introduction

The asymptotic behavior of linear elasticity has been studied by several authors. The study of the non linear problem in a stationary regime in a three dimensional thin domain with non linear friction of Tresca has been considered in [3]. The asymptotic analysis of a contact problem in a three dimensional with friction between two elastic bodies was investigated in [6]. In the last few years, some research papers have been written dealing with the asymptotic analysis of an incompressible fluid in a three-dimensional thin domain, when one dimension of the fluid domain tends to zero, (see e.g., [1, 5]) and the references cited therein. More recently, the authors in [2] have studied the asymptotic analysis of a dynamical problem of isothermal elasticity with non linear friction of Tresca type but without the intervention of the nonlinear term. The authors in [4] studied the non linear boundary value problem governed by partial differential equations which describe the evolution of linear elastic materials.

The purpose of this paper is to study the asymptotic analysis of a stationary problem for the linear elasticity represented by a thin domain Ω^ε in $\mathbb{R}^3$ with Tresca and maximal monotone graph boundary conditions. We consider the Dirichlet boundary conditions on $\bar{\Gamma}_1^\varepsilon \cup \bar{\Gamma}_L^\varepsilon$ where $\bar{\Gamma}_L^\varepsilon$ is the lateral one, the maximal monotone graph condition at the top surface $\bar{\Gamma}_1^\varepsilon$, finally, a non-linear Tresca interface condition at the bottom one ω . The asymptotic behavior of linear elasticity has been studied by several authors. The study of the non linear problem in a stationary regime in a three dimensional thin domain with non linear friction of Tresca has been considered in [3]. The asymptotic analysis of a contact problem in a three dimensional with friction between two elastic bodies was investigated in [6]. In the last few years, some research papers have been written dealing with the asymptotic analysis of an incompressible fluid in a three-dimensional thin domain, when one dimension of the fluid domain tends to zero, (see e.g., [1, 5]) and the references cited therein. More recently, the authors in [2] have studied the asymptotic analysis of a dynamical problem of isothermal elasticity with non linear friction of Tresca type but without the intervention of the nonlinear term. The authors in [4] studied the non linear boundary value problem governed by partial differential equations which describe the evolution of linear elastic materials.

The purpose of this paper is to study the asymptotic analysis of a stationary problem for the linear elasticity

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represented by a thin domain Ω^ε in $\mathbb{R}^3$ with Tresca and maximal monotone graph boundary conditions. We consider the Dirichlet boundary conditions on $\bar{\Gamma}_1^\varepsilon \cup \bar{\Gamma}_L^\varepsilon$ where $\bar{\Gamma}_L^\varepsilon$ is the lateral one, the maximal monotone graph condition at the top surface $\bar{\Gamma}_1^\varepsilon$, finally, a non-linear Tresca interface condition at the bottom one ω .

Problem statement and variational formulation

Let ω be a fixed bounded domain of $\mathbb{R}^3$ of equation $x_3 = 0$ with Lipschitz continuous boundary and is the bottom of the domain Ω^ε . We introduce a small parameter ε , which will tend to zero. The upper surface $\bar{\Gamma}_1^\varepsilon$ is defined by $x_3 = \varepsilon h(x') = \varepsilon h(x_1, x_2)$ and h a smooth bounded function such that $0 < h_* \leq h(x') \leq h^*$, $\forall (x', 0) \in \omega$. Then

$$\Omega^\varepsilon = \{(x', x_3) \in \mathbb{R}^3, (x', 0) \in \omega, 0 < x_3 < \varepsilon h(x')\},$$

The boundary of Ω^ε is Γ^ε , we have $\partial\Omega^\varepsilon = \Gamma^\varepsilon = \bar{\omega} \cup \bar{\Gamma}_1^\varepsilon \cup \bar{\Gamma}_L^\varepsilon$, where Γ_L^ε is the lateral boundary. Let σ^ε denotes the stress tensor, with $\sigma^\varepsilon = (\sigma_{ij}^\varepsilon)$, $i, j = 1, 2, 3$. Then elements σ_{ij} are given by the Hooke's law

$$\sigma_{ij}^\varepsilon(u^\varepsilon) = 2\mu d_{ij}(u^\varepsilon) + \lambda d_{kk}(u^\varepsilon) \delta_{ij},$$

where μ and λ are elasticity coefficients with $\lambda > 0$ and $\lambda + \mu \geq 0$, u^ε is the displacement field, δ_{ij} the symbol of Kröner and $d = (d_{ij})_{1 \leq i, j \leq 3}$ the symetric deformation velocity tensor

$$d_{ij}(u^\varepsilon) = \frac{1}{2} \left(\frac{\partial u_i^\varepsilon}{\partial x_j} + \frac{\partial u_j^\varepsilon}{\partial x_i} \right), 1 \leq i, j \leq 3.$$

We consider now the following mechanical problem

Problem P_1 . Find a displacement field $u^\varepsilon : \Omega^\varepsilon \rightarrow \mathbb{R}^3$ such that

$$-div \sigma^\varepsilon + (\alpha^\varepsilon)^2 u^\varepsilon = f^\varepsilon \quad \text{in } \Omega^\varepsilon, \quad (689)$$

$$u^\varepsilon = 0 \quad \text{on } \Gamma_L^\varepsilon, \quad (690)$$

$$P(u^\varepsilon) - \sigma^\varepsilon(u^\varepsilon).n = \beta_\xi(u^\varepsilon) \quad \text{on } \Gamma_1^\varepsilon, \quad (691)$$

$$u^\varepsilon.n = 0 \quad \text{on } \Gamma_1^\varepsilon \cup \omega, \quad (692)$$

$$\left. \begin{array}{l} |\sigma_T^\varepsilon| < k^\varepsilon \implies u_T^\varepsilon = 0 \\ |\sigma_T^\varepsilon| = k^\varepsilon \implies \exists \gamma \geq 0 : u_T^\varepsilon = -\gamma \sigma_T^\varepsilon \end{array} \right\} \quad \text{on } \omega \quad (693)$$

where $n = (n_1, n_2, n_3)$ is the unit outward normal to Γ^ε , and

$$u_n^\varepsilon = u^\varepsilon.n \quad u_T^\varepsilon = u^\varepsilon - (u_n^\varepsilon)n$$

$$\sigma_n^\varepsilon = (\sigma^\varepsilon.n).n \quad \sigma_T^\varepsilon = \sigma^\varepsilon.n - (\sigma_n^\varepsilon)n$$

are, respectively the normal and the tangential components of u^ε on the boundary ω and the components of the normal and the tangential stress tensor on ω . We denote by P the differential operator of the first order to coefficients lipschitzians and β_ξ defined by $\beta_\xi = \xi^{-1}(I + J_\xi)$ with $\xi > 0$, the Yosida's approach of β where $J_\xi = -(I + \xi\beta)^{-1}$ is the resolvante of β which is the maximal monotone graph such that $0 \in \beta(0)$. Condition (693) represents a Tresca friction law on ω where k^ε is the friction coefficient and $f^\varepsilon = (f_i^\varepsilon)_{1 \leq i \leq 3}$ is the body forces. Furthermore, the equation (689) represent the equilibrium equation, here $\alpha^\varepsilon \geq 0$.

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On Fractional Stochastic Differential Equations with Caputo Katugambola's Approach in Nuclear Physics

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Abstract: The field of nuclear physics is indispensable in various domains including power generation, nuclear armaments, medical diagnostics, and cosmology. Our research pursues two primary objectives. Firstly, it focuses on establishing an accurate stochastic solution. Secondly, it delves into analyzing the generalized solution of a fractional differential equation, renowned for its relevance in nuclear physics and its association with phenomena resembling Brownian motion. The solution methodology relies on the numerical application of the Caputo Katugampola approach. The theoretical exploration conducted in this paper enables the inference of the numerical solution's behavior, a validation achieved through numerical demonstrations and simulations.

Keywords: Fractional stochastic equations, brownian Motion, Caputo kutogampola approach.

2020 Mathematics Subject Classification: Primary 37A50, 35J05, 35J10, 35K05, 35L05. 42A38, 42B10

Introduction

Over the past three decades, stochastic processes have garnered increasing attention for their capacity to model complex systems across various domains. This research avenue has transcended traditional boundaries, becoming indispensable not only for mathematicians and statisticians but also for professionals in chemistry, biology, engineering, applied physics, and astronomy. Originating from R. Brown's 1827 discovery of irregular motion exhibited by a pollen particle in fluid, stochastic processes have found applications far beyond biology. Fractional models, particularly prevalent in modern physics and economics, have emerged as potent tools in addressing diverse scientific challenges. The prevalence of fractional differential equations in physics prompts the question of why physical phenomena often manifest as such equations, hinting at the intrinsic simplification they offer for localized descriptions of complex processes. For instance, neutron movement in nuclear fields is characterized by fractional differential equations, underscoring the importance for physicists and theoretical mathematicians to develop optimal numerical modelling and simulation techniques in nuclear physics. Our objective is to devise solutions for frac-stochastic equations, renowned for their relevance in nuclear physics applications.

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MAIN RESULTS

Among the most recently used approaches in the field of fractional differential equations the Caputo-Katugampola derivative

$${}^{CK}\mathbf{D}_{(t_0,t)}^{(\alpha,\varepsilon)}X(t) = \frac{\varepsilon^\alpha}{\Gamma(1-\alpha)} \int_{t_0}^t \frac{X'(s)}{(t^\varepsilon - s^\varepsilon)^\alpha} ds. \quad (694)$$

We can approximate Caputo-Katugampola derivative (CK) to the Caputo-Hadamard derivative when $\varepsilon \rightarrow 0^+$

$$\begin{aligned} \lim_{\varepsilon \rightarrow 0^+} {}^{CK}\mathbf{D}_{(t_0,t)}^{(\alpha,\varepsilon)}X(t) &= \lim_{\varepsilon \rightarrow 0^+} \frac{\varepsilon^\alpha}{\Gamma(1-\alpha)} \int_{t_0}^t \frac{X'(s)}{(t^\varepsilon - s^\varepsilon)^\alpha} ds \\ &= \lim_{\varepsilon \rightarrow 0^+} \frac{\varepsilon^\alpha}{\Gamma(1-\alpha)} \int_{t_0}^t \frac{X'(s)}{\varepsilon^\alpha (\ln t - \ln s)^\alpha} ds \\ &= \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^t \left(\ln\left(\frac{t}{s}\right)\right)^{-\alpha} X'(s) ds. \end{aligned}$$

When $\varepsilon \rightarrow 1$, we obtain Caputo's ancient approach

$$\begin{aligned} \lim_{\varepsilon \rightarrow 1} {}^{CK}\mathbf{D}_{(t_0,t)}^{(\alpha,\varepsilon)}X(t) &= \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^t \frac{X'(s)}{(t-s)^\alpha} ds \\ &= {}^C\mathbf{D}_{(t_0,t)}^{(\alpha)}X(t) \end{aligned}$$

Stochastic calculus tool

In order to introduce the stochastic process, It is necessary to present some basic conditions for the existence of a stochastic model solution. We can generalize the conditions on a stochastic differential equation with a fractional derivative operator.

In neutron scattering theory, the equations that govern the space time and neutron population dynamics are called kinetic equations, we assume the following neutron flux equation with fractional derivative of Caputo Katugampola

$$\begin{cases} {}^{CK}\mathbf{D}_{(0,1)}^{(0.5,\sqrt{2})}X(t) = 0.5X(t) + dW(t) \\ 0 = t_0 < t_1 < t_2 = 1 \end{cases}.$$

Where $X(t)$ is the neutron density computed by (neutrons/cm³). Firstly, we have the approximation expression of Brownian motion by polygonal approximation we have

$$W(t) = W_{t_i} + (W_{t_{i+1}} - W_{t_i}) \frac{t - t_i}{t_i - t_{i+1}} \quad t \in [0, 1].$$

Then the solution will become

$$\begin{aligned} X(t) &= 2 \left({}^{CK}\mathbf{D}_{(0,1)}^{(0.5,\sqrt{2})}X(t) \right) - \frac{1}{\xi_n \sqrt{\Delta t_n}} \\ &= \frac{2^{0.25}}{\Gamma(3/2)} \sum_{j=1}^2 \frac{X_j - X_{j-1}}{t_j - t_{j-1}} t_j^{1-\alpha} \left\{ \left(1 - t_{j-1}^{\sqrt{2}}\right)^{0.5} - \left(1 - t_j^{\sqrt{2}}\right)^{0.5} \right\} - \frac{1}{\xi_n \sqrt{\Delta t_n}}. \end{aligned}$$

Simulations in nuclear physics field

This section is dedicated to present some numerical applications to verify the numerical value of Caputo Katugampola's method in fractional stochastic differential space models

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$${}^{CK}\mathbf{D}_{(t_0,t)}^{(\alpha,\varepsilon)}X(t) = \{\gamma_0(t)X(t) + \gamma_1(t)\}dt + \gamma_2(t)dW(t) \quad (695)$$

$$X(0) = 0.01, 0 < q < 1$$

We keep the same values for coefficients γ_0, γ_1 and γ_2 in order to make a constructive comparison between the two situations, we use here some notations, $(\gamma_i, i = 0, \dots, 2)$ the coefficients, n number of subdivisions or sample size, NS (Numerical Solution), ES (Exact Solution), here we consider the coefficients values to be $\gamma_0 = 0.5, \gamma_1 = 0.05$ and $\gamma_2 = 0, 1$. Brownian motion example used in simulation for model proposed here is very famous in stochastic calculus defined with its following expression

$$W(t) = \varepsilon_i (\Delta t_n)^{0.5}, t \in [0, 1]$$

Where the sequence $(\varepsilon_i)_{i \in \mathbb{N}}$ is mutually independent standard Gaussian random variables. and by the polygonal approximation we have

$$w^n = w_{t_i} + (w_{t_{i+1}} - w_{t_i}) \frac{t - t_i}{t_i - t_{i+1}} \quad t \in [t_i, t_{i+1}]$$

Conclusion and future works

Increasing the number of subdivisions (sample size) enhances the convergence of numerical solutions towards the exact solution, as observed in the first table. Similarly, in the second table, maintaining a constant number of subdivisions while increasing the fractional order in the expression of Caputo Katugampola leads to improved convergence towards the exact solution. This suggests that both the number of subdivisions and the fractional order play crucial asymptotic roles in the convergence of numerical solutions using Caputo Katugampola's approach. Our future research aims to apply frac-stochastic models in modeling real physics data and develop approaches tailored to these models.

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An operational matrix based on shifted Jacobi polynomials for solving fractional integro-differential equations

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Abstract: In this intervention we implement an operational matrix of fractional integration for Shifted Legendre polynomials. We proposed an algorithm to obtain an approximation solution for linear fractional integro-differential equations, described in Riemann-Liouville sense, based on shifted Legendre polynomials. This method was applied to solve linear fractional integro-differential equation with initial conditions, and the exact solutions obtained for some illustrated examples.

Keywords: Fractional integro-differential equation, Operational matrix, Shifted Legendre polynomials, Riemann-Liouville fractional integral operator.

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Numerical Investigation into a Mathematical Model Representing Free-Surface Potential Flow

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Abstract: This study addresses the problem of a two-dimensional potential flow featuring a free surface, characterized by an incompressible, irrotational, and inviscid fluid, in the presence of a jet facing an inclined wall at an angle γ relative to the horizontal. The configuration of the free surface is determined using numerical techniques employing series truncation methods, which rely on conformal transformations and incorporate the effects of surface tension denoted by T . Boundary conditions on the free surfaces are established through Bernoulli's equation. The outcomes obtained are contingent upon parameters such as the Weber number α and the angle γ . Each value of the Weber number corresponds to a unique solution, with various shapes of the free surface of the jet being depicted.

Keywords: Free surface; Potential flow; Surface tension; Weber number; Series truncation method.

2020 Mathematics Subject Classification: Primary 76A02, 76B07, 76B10. Secondary 30C30, 30C35, 65D25.

Introduction

The focus of this investigation lies in the analysis of a two-dimensional potential flow of a jet directed towards an inclined wall at an angle γ relative to the horizontal. The fluid under consideration is presumed to be incompressible, irrotational, and inviscid. Birkoff and Zarantonello [2] introduced the hodograph transformation method to the theory of jets, leading to several generalizations of the Schwartz Christoffel formula [3]. This method transforms a region occupied by flow into one bounded by a combination of polygonal and smooth curves. Numerous studies on flow phenomena have been presented by various researchers [1, 6]. Investigations into jets impacting walls have been conducted extensively [5], with Weidong Peng and David F. Parker exploring the impact of a jet on an irregular wall [5]. In their study, they examined different wall geometries, neglecting surface tension and gravity effects, and formulated the problem as an integral equation on the free boundary, which they solved numerically. Our study addresses a similar problem but incorporates the effect of surface tension while neglecting gravity. We analyze the shape of the resulting free surface formed by the flow. Upstream, the flow velocity remains approximately constant U , with a fluid depth of $2H$. The problem is characterized by a nonlinear boundary condition described by Bernoulli's equation on unknown free surfaces. To solve this, we employ a series truncation method, previously successfully utilized by Birkoff, Zarantonello [2], and others [1, 4], to numerically compute solutions for the flow in front of a wall. It's important to note that the flow is governed by two parameters: the angle γ between the horizontal bottom and the inclined wall, and the Weber number α , defined as $\alpha = \frac{\rho U^2 H}{T}$; where T is the surface tension, and ρ is the density of the fluid.

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Figure 45: Flow in the Z -plane

Figure 46: Potential flow in the potential f -plane

Main results

The physical flow problem is described by the Figure 45

The complex potential function f and the complex velocity ξ are given by

$$f = \phi + i\psi \text{ and } \xi = \frac{df}{dz} = u - iv$$

Hence

$$u = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} \text{ and } v = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$$

Where ϕ is the potential function , ψ is the stream line, (see Figure 46)

The physical flow problem as described above can be formulated as a boundary value problem in the potential function $\phi(x,y)$, which we solve it numerically and we find the form of free-surface for some values of γ for each number of Weber α .

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Equation de Schrödinger

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Abstract: Le domaine des équations d'évolution reste en expansion rapide et s'impose de plus en plus dans divers champs d'applications, notre travail portera sur l'étude de l'équation de Schrödinger libre et forcée qui décrit le comportement de la fonction d'onde dans un système de mécanique quantique. On commencera par l'origine de l'équation puis nous allons étudier l'existence et l'unicité de la solution pour une donnée initiale $V_0 \in H^s(\mathbb{R}^N)$ du problème de Cauchy associé à l'équation puis on passera au prolongement par continuité de la solution et nous terminerons des résultats sur les estimations a priori et le comportement asymptotique de la solution et quelques remarques sur la version forcée de l'équation.

Keywords: Distribution , Opérateur unitaire , Espace de Hilbert, Transformation de Fourier.

Introduction

L'équation de Schrodinger est une equation fondamentale dans la mécanique quantique qui intervient dans des contextes différents par exemple elle décrit le mouvement d'une particule quantique soumise à un potentiel, Et c'est aussi une équation approchée pour un certain type de propagation d'ondes.

Commençons par voir les origines de l'équation, en mécanique classique les équations de mouvement pour un point matériel de masse m soumis à l'action d'un champ de forces dérivant d'un potentiel $V(x)$ sont définies par :

En notant $x(t) \in \mathbb{R}^N$ la position du point matériel à l'instant t

$$mx'' + \nabla V(x) = 0$$

Ce système d'équations du second ordre se met aussi sous la forme d'un système d'équations différentielles ordinaires d'ordre 1 et de dimension double :

$$\begin{cases} mx'' = \xi \\ \xi' = -\nabla V(x) \end{cases}$$

Ou $\xi(t)$ est l'impulsion du point à l'instant t , L'énergie E du point considéré à l'instant t est la somme de son énergie cinétique et son énergie potentielle

$$E = \frac{1}{2}m|x'| + V(x(t)) = \frac{|\xi(t)|^2}{2m} + V(x(t))$$

La formulation Hamiltonienne de la mécanique classique consiste à partir de l'énergie ci-dessus comme fonction des variables de position x et d'impulsion ξ :

$$H(x, \xi) = \frac{|\xi|^2}{2m} + V(x)$$

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Cette fonction est appelée Hamiltonien du système mécanique considéré . Après ce rappel sur les équation de mécanique classique, Voyons comment la mécanique quantique décrit l'évolution du même point matériel soumis à l'action d'un potentiel V . En mécanique quantique , le problème n'est pas de chercher la position $x(t)$ et l'impulsion $\xi(t)$ du point matériel en tout t le principe d'incertitude de Heisenberg nous dit qu'une telle description est impossible. On va donc chercher plutôt une fonction $(t, x) \rightarrow \Phi(t, x) \in \mathbb{C}$ au moins mesurable, appelée fonction d'onde ou amplitude de probabilité qui va nous donner l'état du système, d'une façon quelque peu compliquée que nous allons présenter la succinctement . La fonction d'onde doit vérifier :

$$\int_{\mathbb{R}^N} |\Phi(t, x)|^2 dx = 1 \quad \forall t \in \mathbb{R}$$

Ce qui signifie que $|\Phi(t, x)|^2$ est une densité de probabilité par rapport à la mesure de Lebesgue dx . La signification de cette densité de probabilité est la suivante : $|\Phi(t, x)|^2 dx$ représente la probabilité

(infinitésimale) de trouver le point matériel dans un volume infinitésimal dx centré en x à l'instant t ,

Plus généralement étant donné une quantité physique quelconque dont la valeur est $f(x)$ lorsque le point matériel se trouve au point $x \in \mathbb{R}^N$, On obtient la valeur observée de cette quantité à l'instant t pour un Système décrit par la fonction d'onde $\Phi(t, x)$ par la formule

$$\int_{\mathbb{R}^N} f(x) |\Phi(t, x)|^2 dx$$

Voici donc comment calculer la valeur observée à l'instant t pour une quantité

physique valant $g(\xi)$ lorsque l'impulsion du point matériel vaut ξ En appliquant le théorème de Plancherel à la fonction $x \rightarrow \Phi(t, x)$ qui est dans $L^2(\mathbb{R}^N)$ on a

$$\int_{\mathbb{R}^N} |\Phi(t, x)|^2 dx = \int_{\mathbb{R}^N} \left| \Phi(t, \frac{1}{h}\xi) \right|^2 \frac{d\xi}{(2\pi h)^N}$$

Supposons maintenant que la fonction d'onde est suffisamment régulière et décroissante à l'infini, Par exemple $\Phi(t, \cdot) \in S(\mathbb{R}^N)$ on peut vérifier que

$$\xi_j \widehat{\Phi(t, \frac{1}{h}\xi)} = \left(\frac{h}{i} \partial_{x_j} \widehat{\Phi(t, x)} \right) \left(\frac{1}{h} \xi \right)$$

Alors on a

$$\begin{aligned} \int_{\mathbb{R}^N} \frac{1}{2} \frac{|\xi|^2}{m} \left| \widehat{\Phi(t, \frac{1}{h}\xi)} \right|^2 \frac{d\xi}{(2\pi h)^N} &= \frac{1}{2m} \int_{\mathbb{R}^N} \left| \widehat{h \nabla_x \Phi} \right|^2 \left(t, \frac{1}{h}\xi \right) \frac{d\xi}{(2\pi h)^N} \\ &= \frac{1}{2m} \int_{\mathbb{R}^N} |h \nabla_x \Phi|^2(t, x) dx \end{aligned}$$

Une itégration par partie permet d'ailleurs de mettre cette dernière intégrale sous la forme

$$\frac{1}{2m} \int_{\mathbb{R}^N} |h \nabla_x \Phi|^2(t, x) dx = \int_{\mathbb{R}^N} \overline{\Phi(t, x)} \left(\frac{-h^2 \Delta_x}{2m} \right) \Phi(t, x) dx$$

Ainsi la valeur observée de l'énergie totale à l'instant t du point matériel de fonction d'onde $\Phi(t, x)$ somme de l'énergie cinétique et de l'énergie potentielle, s'écrit

$$\int_{\mathbb{R}^N} \overline{\Phi(t, x)} \left(\frac{-h^2 \Delta_x}{2m} + V(x) \right) \Phi(t, x) dx$$

La discussion ci-dessus montre que plus généralement, pour calculer la valeur observée d'une quantité physique de la forme $g(\xi) + f(x)$ pour un système décrit par la fonction d'onde $\Phi(t, x)$ ou g est une fonction polynomiale et x, ξ sont respectivement les variables de position et d'impulsion, on forme la quantité

$$\int_{\mathbb{R}^N} \overline{\Phi(t, x)} (g(hD_x) + f(x)) \Phi(t, x) dx$$

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Autrement dit, a la fonction $g(\xi) + f(x)$ de la mécanique classique qui est la valeur d'une quantité physique lorsque le point matériel considéré se trouve simultanément à la position x avec une impulsion ξ , la mécanique quantique associe l'opérateur différentiel $g(hD_x) + f(x)$.

Par exemple, au hamiltonien de la mécanique classique pour un point matériel de masse m dans un champ de potentiel $V(x)$, qui est la fonction définie par

$$H(x, \xi) = \frac{|\xi|^2}{2m} + V(x)$$

On lui associe le hamiltonien de la mécanique quantique qui est l'opérateur différentiel

$$H(x, hD_x) = \frac{-h^2 \Delta_x}{2m} + V(x)$$

Après cette évocation de quelques-unes des notions fondamentales de la mécanique quantique, qui a pour but d'en faire sentir les analogies et les différences avec la théorie plus familière qu'est la mécanique classique, nous concluons en présentant enfin l'équation de Schrodinger.

Au hamiltonien de la mécanique classique pour un point matériel de masse m dans un champ de potentiel $V(x)$, qui est une fonction des variables de position x et d'impulsion ξ , nous avons vu comment associer de façon naturelle le hamiltonien de la mécanique quantique qui est un opérateur différentiel agissant sur les fonctions de la seule variable de position, L'équation de Schrodinger s'écrit

$$ih\partial_t \Phi(t, x) = H(x, hD_x) \Phi(t, x) \quad (t, x) \in \mathbb{R} \times \mathbb{R}^N$$

C'est à dire

$$ih\partial_t \Phi(t, x) = \left(\frac{-h^2 \Delta_x}{2m} + V(x) \right) (\Phi(t, x))$$

Le travail réalisé portera sur l'existence de solutions du problème de Cauchy associé à notre équation tout en utilisant les différentes techniques de la théorie des Opérateurs et des Distribution et l'analyse de Fourier.

Main results

Nous verrons que pour toute distribution tempérée comme donnée initiale V_0 du problème de Cauchy associé à l'équation il existera une solution unique au problème, définie que pour un t strictement positif, par la suite on verra que si $V_0 \in H^s(\mathbb{R}^N)$ la solution se prolonge pour un $t \leq 0$ ce qui nous permettra de définir un groupe de solutions sur $C(\mathbb{R}, H^s(\mathbb{R}^N))$ appelé Groupe de Schrödinger libre qui sera unitaire dans $H^s(\mathbb{R}^N)$ et on terminera notre travail par des résultats sur les estimations à priori qui correspondront a une conservation de la masse dans le cas Hilbertien, et des résultats sur le comportement asymptotique de la solution et l'existence de cette dernière pour la version forcée de l'équation.

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Galerkin and Petrov-Galerkin Methods for solving a linear integral equations

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Abstract: In this work, we obtain the approximate solution of linear integral equations of the second kind by the Galerkin and Petrov-Galerkin methods. In which the approximate solution is sought in a finite dimensional space, also the Galerkin and Petrov-Galerkin equations or the variation form for those equations is solved by reducing it to an equivalent finite linear system. The convergence and error analysis of those methods are discussed. We illustrate these methods by numerical examples .

Keywords: Integral equations, Galerkin and Petrov-Galerkin methods, variational form.

Introduction

Integral equations are important class of equations along with increased interest in the applications to the scientific and technological problems, also partial differential equations can be transformed into problems of solving some approximate integral equations. Numerical methods for integral equations have attracted a lot of attention to find the approximate solution, many authors discussed for approximation methods with high efficiency such as : Galerkin and Petrov-Galerkin methods. In this work we consider Galerkin and Petrov-Galerkin methods with the variational form for the solution of such integral equations, certainly this technical leads us to one of the best of approximation .

Main results

The numerical methods for solving integral equations of the form

$$\varphi(x) - \int_0^1 k(x,t)\varphi(t)dt = f(x), \quad \forall x \in [0,1]. \quad (696)$$

with φ an unknown function in a Banach space X . The kernel $k : [a,b] \times [a,b] \rightarrow \mathbb{R}$ and the right-hand side $f : [a,b] \rightarrow \mathbb{R}$ are given functions. The equation (696) may be written

$$\varphi - k\varphi = f.$$

By Galerkin and Petrov-Galerkin methods basis on the variational form

$$\langle \varphi - k\varphi, v \rangle = \langle f, v \rangle, \quad \forall v \in X.$$

We note that the approximate solution will be measurably close to the exact solution on the entire interval $[0,1]$. The efficiency of these methods Galerkin and Petrov-Galerkin is tested by solving some examples for which the exact solution is known, this allows us to estimate the exactness with our numerical results and compare those with another results treated by another authors.

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Blowing-up solutions for time-fractional Generalized Kuramoto-Sivashinsky Equations on a bounded domain

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Abstract:

This paper is devoted to the study of initial-boundary value problems for time-fractional analogues of Generalized Kuramoto-Sivashinsky equation on a bounded domain. Sufficient conditions for the blowing-up of solutions in finite time of aforementioned equations are presented. We also discuss the maximum principle and influence of gradient non-linearity on the global solvability of initial-boundary value problems for the time-fractional Burgers equation. The main tool of our study is the Pohozaev nonlinear capacity method. We also provide some illustrative examples.

Keywords: Caputo derivative, Time-fractional Generalized Kuramoto-Sivashinsky equation, blow-up.

2020 Mathematics Subject Classification: 26A33, 34A08.

Introduction

Within the past few decades, the hypothesis of nonlinear partial differential equations (PDEs) has experienced major advancement in physics and applied science. After the incredible advancement of science in this field, the forms of nonlinear physical marvels have been effectively portrayed through models based on fractional derivatives. At the same time, the development of fractional derivatives physics and applied sciences, made the ponder more energizing and challenging within the areas of science and engineering.

Since of these fundamental reasons, conditions including fractional derivatives have ended up exceptionally intrigued from numerous analysts around the world. Particularly, we know that analyzing traveling-wave solutions for PDEs, utilizing different strategies, have long been an broad intrigued for mathematicians, physicists, engineers and other researchers. Among the numerical models depicting the flow of wave equations incorporate Korteweg-de Vries equation (KdVE), Regularized Long Wave equation (RLWE), Kawahara equation (KE) and Rosenau equation (RE).

These equations have critical applications completely different physical circumstances such as the elements of shallow-water waves, shallow water waves with surface pressure and small-amplitude long wave of water in a channel. The authors in [1] studied the blow-up (B-up) and global existence of solutions to the following time fractional nonlinear diffusion equation

$$\begin{cases} \partial_{+t_1}^{\alpha_1} U(\eta, z) - (U(\eta, z) - U(0, z)) = I_{+t_1}^{\alpha_2} (|U(\eta, z)|^{p-1} U(\eta, z)), & z \in \mathbb{R}^N, \\ U(0, z) = U_0(z), \end{cases}$$

for $\eta > 0$, where $0 < \alpha_1 < \alpha_2 < 1$, $p > 1$, $I_{+t_1}^{\alpha_2}$ and $\partial_{+t_1}^{\alpha_1} U(\eta, z)$ denote left Riemann-Liouville fractional integrals of order α_2 and the Caputo fractional derivative of order α_1 . They proved that if

$$1 < p < p^* = \max \left\{ 1 + \frac{1-\alpha_2}{\alpha_1}, 1 + \frac{2(\alpha_1+1-\alpha_2)}{\alpha_1 N} \right\},$$

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then the solutions of the equation blows up in a finite time [1]. Alsaedi *et al.* studied the initial-boundary value problems for time-fractional analogues of Korteweg-de Vries, Benjamin-Bona-Mahony, Burgers, Rosenau, Camassa-Holm, Degasperis-Procesi, Ostrovsky and time-fractional modified Korteweg-de Vries-Burgers equations on a bounded domain and presented sufficient conditions for the blowing-up of solutions in finite time of the equations [2].

This paper is given to blowing-up solutions (B-upSs) of time-fractional analogs of the above equations. The approach to the issue is based on the Pohozaev nonlinear capacity strategy; more accurately, on the choice of test functions agreeing to introductory and boundary conditions beneath thought. Here, we grant a straightforward case of the examination of a unpleasant B-up, i.e., the case where the solution tends to limitlessness as $z \rightarrow \tau^*$ on $\Xi := [0, L]$; more precisely, when for the given function ρ , the integral

$$\int_0^L U(\eta, z) \rho(\eta) d\eta \longrightarrow \infty, \quad (z \rightarrow \tau^*). \quad (697)$$

Recently, the study of B-upSs for nonlinear PDEs has attracted a large number of researchers, as many papers have emerged around this study [3, 4, 5].

The aim of this paper is to study the B-upSs of generalized Kuramoto-Sivashinsky with the form

$$\partial_{+0}^{\zeta} z + z z_v + \xi_1 z_{vv} + \xi_2 z_{vv} + \xi_3 z_{vv} = 0, \quad (698)$$

for $v \in j =: (0, x)$, $s > 0$, here $\xi_i (i = 1, 2, 3)$, non-zero constants with initial conditions

$$z(v, 0) = z_0(v), \quad v \in j =: [0, x],$$

where z_0 is a given function. This work is devoted to

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Some contributions to extended spectrum and extended eigenspaces of operators

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Abstract: In this study, we provide some new findings about extended eigenvalues and extended eigenvectors for several types of operators. We also address the cases of power bounded operator.

Keywords: Extended eigenvalues, extended eigenvectors, normal operators.

2020 Mathematics Subject Classification: Primary 47B15, 34L15, 81Q10.

Introduction

Let $\mathcal{H}$ be a separable complex Hilbert space, and denote by $B(\mathcal{H})$ the algebra of all bounded linear operators on $\mathcal{H}$.

If $T \in B(\mathcal{H})$, then a complex number λ is an extended eigenvalue of T if there is a nonzero operator X such that $TX = \lambda XT$. We denote by the symbol $\sigma_{ext}(T)$ the set of extended eigenvalues of T . The subspace generated by extended eigenvectors corresponding to λ will be denoted by $E_{ext}(T; \lambda)$.

Proposition 38. Let $T \in B(\mathcal{H})$ then:

$$\sigma_{ext}(T) \in \{\lambda \in \mathbb{C} / \sigma(T) \cap \sigma(\lambda T) \neq \emptyset\}.$$

In particular, if $\sigma(T) = \{\alpha\}$ with $\alpha \neq 0$ then: $\sigma_{ext}(T) = \{1\}$.

Corollary 16. Assume that $T \in B(\mathcal{H})$ is a quasinilpotent operator, then: $\sigma_{ext}(\alpha I + T) = \{1\}$, for all $\alpha \in \mathbb{C} \setminus \{0\}$.

Relation to the point spectrum

The following Theorem gives a relationship between the broad spectrum of an operator and its point spectrum.

Theorem 256. Let $T \in B(\mathcal{H})$ If we note by $\sigma_p(T)$ the point spectrum of T , then we have $\{\frac{\alpha}{\beta}; \alpha \in \sigma_p(T); \beta \in \sigma_p(T^*)\} \subset \sigma_{ext}(T)$

Remark 13. 1. Note that if $0 \in \sigma_p(T) \cap \sigma_p(T^*)$ then: $\sigma_{ext}(T) = \mathbb{C}$;

2. If $1 \in \sigma_p(T^*)$ then $\sigma_p(T) \subset \sigma_{ext}(T)$ an inclusion that is not generally true. Indeed, if $T = \lambda I$ then $\sigma_p(T) = \{\lambda\}$ while $\sigma_{ext}(T) = \{1\}$.

Case of invertible operators

Note first that 0 cannot be extended eigenvalue of an invertible operator, A. Lambert has shown that the wide spectrum of such an operator is located in a closed and bounded ring of center 0. In the following proposal, we give in addition the terminals of this ring.

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Proposition 39. Let $T \in B(\mathcal{H})$ be invertible, and pose $\alpha := \|T\| \|T^{-1}\|$ then:

1. $\sigma_{\text{ext}}(T) \subset \{z \in \mathbb{C}, \frac{1}{\alpha} \leq |z| \leq \alpha\} := A_{\frac{1}{\alpha}, \alpha}$.
2. If $\lambda \in \sigma_{\text{ext}}(T)$ with $|\lambda| \neq 1$. Then there is an integer $N > 1$ such that any product of N elements of $E_{\text{ext}}(\lambda)$ is null. In particular, any element of $E_{\text{ext}}(\lambda)$ is a nilpotent operator of rank N at most.

Operator powers

Here, we give the relationship between the broad spectrum of an operator and that of its powers. Let $T \in B(\mathcal{H})$, and supposonsque $\lambda \in \sigma_{\text{ext}}(T)$ with X for associated extended eigenvector, that is,

$$TX = \lambda XT.$$

So, for all natural n ,

$$T^n X = \lambda^n X T^n$$

Therefore, $\lambda \in \sigma_{\text{ext}}(T)$ implies that $\lambda^n \in \sigma_{\text{ext}}(T^n)$.

Theorem 257. Let $n \in \mathbb{N}$ and $\lambda \in \mathbb{C}$. Then $\lambda \in \sigma_{\text{ext}}(T)$ if and only if there is a n th root of λ belonging to $\sigma_{\text{ext}}(T)$.

Finite Dimension Case

The extended spectrum of an operator acting on a complex Hilbert space of finite dimension is fully described in this section.

Theorem 258. Let T be an operator on a finite-dimensional Hilbert $\mathcal{H}$ space. Then

$$\sigma_{\text{ext}}(T) = \{\lambda \in \mathbb{C} / \sigma(T) \cap \sigma(\lambda T) \neq \emptyset\}.$$

Corollary 17. Let T be a linear operator acting on a finite dimension Hilbert space, then:

1. $\sigma_{\text{ext}}(T) = \{1\}$ if and only if $\sigma(T) = \{\alpha\}$, with $\alpha \neq 0$;
2. $\sigma_{\text{ext}}(T) = \mathbb{C}$ if and only if T is not invertible;
3. If T is invertible, then $\sigma_{\text{ext}}(T) = \{\frac{\alpha}{\beta} / \alpha, \beta \in \sigma(T)\}$.

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A Hybrid Finite Element-Finite Volume Method for the generalized Stokes Problem

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Abstract: This work investigates a novel mixed finite element-finite volume scheme for solving the compressible generalized Stokes equations. The scheme leverages local jump operator to stabilize discontinuous pressure approximations. The core idea lies in strategically combining the finite volume methodology with the Q_1 - Q_0 mixed finite element method. This hybrid approach aims to achieve stability within the scheme.

Keywords: Stokes equations, Mixed methods, Finite element method, Finite volume method.

2020 Mathematics Subject Classification: Primary 65N30, 65N12, 65N15, 76D07

Introduction

Mixed finite element-finite volume methods are popular for solving incompressible flow problems numerically. These methods often rely on the Galerkin approach to approximate unknown variables like velocity and pressure. However, a crucial challenge exists: the choice of discrete velocity and pressure spaces isn't independent. A compatibility condition, known as the Babuška-Brezzi stability condition, must be satisfied for the mixed approximation to be effective.

Low-order mixed finite element methods, like the popular $Q_1 - Q_0$, $P_1 - P_0$, $P_1 - P_1$ and $Q_1 - Q_1$ formulations, offer advantages in terms of simplicity. However, a major drawback is their inherent instability in the classical Babuška-Brezzi sense. This instability has motivated research efforts to circumvent the need for this strict stability condition.

This paper proposes a jump stabilization technique, originally introduced for the finite volume method in [reference], to address the instability issue in the mixed element formulation when applied to the Stokes problem. The stabilization is achieved by incorporating an additional term into the standard generalized Stokes equations. We then theoretically demonstrate the well-posedness of the resulting stabilized discrete formulations.

Main results

Let $\Omega \subset \mathbb{R}^d$ ($d = 2$ or 3) be an open bounded domain with polygonal or polyhedral boundary $\partial\Omega$. We consider the generalized Stokes problem, which consists in finding a velocity $\mathbf{u} : \Omega \rightarrow \mathbb{R}^d$ and a pressure

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$p : \Omega \longrightarrow \mathbb{R}$ such that

$$\begin{aligned} \alpha \mathbf{u} - \mu \Delta \mathbf{u} + \nabla p &= \mathbf{f} \quad \text{in } \Omega, \\ \operatorname{div} \mathbf{u} &= 0 \quad \text{in } \Omega, \\ \mathbf{u} &= \mathbf{0} \quad \text{on } \partial\Omega, \end{aligned} \quad (699)$$

where α and μ are two positive parameters that cannot vanish simultaneously, denote the inverse of the permeability and the viscosity coefficient, respectively, and $\mathbf{f}$ is a body force.

The weak formulation of the generalized Stokes' problem (699) is given as follows:

$$\begin{aligned} \text{Find } (\mathbf{u}, p) &\in \mathbf{H}_0^1(\Omega) \times L_0^2(\Omega) \text{ such that} \\ \alpha (\mathbf{u}, \mathbf{v})_{0,\Omega} + \mu (\nabla \mathbf{u}, \nabla \mathbf{v})_{0,\Omega} - (p, \operatorname{div} \mathbf{v})_{0,\Omega} &= (\mathbf{f}, \mathbf{v})_{0,\Omega} \quad \forall \mathbf{v} \in \mathbf{H}_0^1(\Omega), \\ -(q, \operatorname{div} \mathbf{u})_{0,\Omega} &= 0 \quad \forall q \in L_0^2(\Omega). \end{aligned} \quad (700)$$

The domain Ω is subdivided into triangles or quadrilaterals such that the resulting partitioning $\mathcal{T}_h$ is regular in the usual sense (see [1]). The finite element method of interest in this paper consists of seeking approximations $(\mathbf{u}_h, p_h)$ of $(\mathbf{u}, p)$, where $\mathbf{u}_h \in \mathbf{V}_h$ and $p_h \in Q_h$, with $\mathbf{V}_h$ and Q_h are the two classical finite element spaces of piecewise linear continuous velocities and piecewise constant pressures defined by

$$\mathbf{V}_h = \left\{ \mathbf{v} \in \mathbf{V} \cap \mathbf{C}^0 : \mathbf{v}|_K \in [Q_1]^d, \forall K \in \mathcal{T}_h \right\}, \quad (701)$$

and

$$P_h = \left\{ q \in P : q|_K \in P_0(K), \forall K \in \mathcal{T}_h \right\},$$

As it was pointed out above, the combination $\mathbf{V}_h \times P_h$ is unstable in the standard Babuška-Brezzi sense.

With the above choices of the velocity-pressure finite-element spaces, we recall the global jump stabilization formulation [2, 3]

$$s_h(p_h, q_h) = \beta \sum_{K \in \mathcal{T}_h} \sum_{L \in \mathcal{N}_K} \frac{m_{K|L}}{2} (h_K + h_L) (p_K - p_L) (q_K - q_L), \quad (702)$$

with $\beta > 0$ is the stabilization parameter and $m_{K|L}$ is the measure of the edge $K|L$.

The stabilized discrete formulation of the generalized Stokes problem (700) can be stated:

$$\begin{aligned} \text{Find } (\mathbf{u}_h, p_h) &\in \mathbf{V}_h \times P_h \text{ such that} \\ \alpha (\mathbf{u}_h, \mathbf{v}_h)_{0,\Omega} + \mu (\nabla \mathbf{u}_h, \nabla \mathbf{v}_h)_{0,\Omega} - (p_h, \operatorname{div} \mathbf{v}_h)_{0,\Omega} &= (\mathbf{f}, \mathbf{v}_h)_{0,\Omega} \quad \forall \mathbf{v}_h \in \mathbf{V}_h, \\ -(q_h, \operatorname{div} \mathbf{u}_h)_{0,\Omega} - s_h(p_h, q_h) &= 0 \quad \forall q_h \in P_h, \end{aligned} \quad (703)$$

We can rewrite (703) as:

$$\begin{aligned} \text{Find } (\mathbf{u}_h, p_h) &\in \mathbf{V}_h \times P_h \text{ such that} \\ Q_h((\mathbf{u}_h, p_h); (\mathbf{v}_h, q_h)) &= (\mathbf{f}, \mathbf{v}_h)_{0,\Omega} \quad \forall (\mathbf{v}_h, q_h) \in \mathbf{V}_h \times P_h, \end{aligned} \quad (704)$$

where

$$Q_h((\mathbf{u}_h, p_h); (\mathbf{v}_h, q_h)) = Q((\mathbf{u}_h, p_h); (\mathbf{v}_h, q_h)) - s_h(p_h, q_h). \quad (705)$$

Now, we can prove the main result which ensures the well-posedness of the stabilized discrete formulation (704), i.e., the bilinear form Q_h is continuous and weakly coercive.

Theorem 259. *The finite element formulation (704) satisfies the following conditions:*

(C1) *For all $(\mathbf{u}_h, p_h), (\mathbf{v}_h, q_h) \in \mathbf{V}_h \times P_h$ we have*

$$|Q_h((\mathbf{u}_h, p_h); (\mathbf{v}_h, q_h))| \leq \delta_1 \|(\mathbf{u}_h, p_h)\|_s \|(\mathbf{v}_h, q_h)\|_s \quad (706)$$

where δ_1 is a positive constant independent of h .

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(C2) For all $(\mathbf{u}_h, p_h) \in \mathbf{V}_h \times P_h$, there exists a pair $(\mathbf{v}_h, q_h) \in \mathbf{V}_h \times P_h$ such that

$$Q_h((\mathbf{u}_h, p_h); (\mathbf{v}_h, q_h)) \geq \delta_2 \|(\mathbf{u}_h, p_h)\|_s \|(\mathbf{v}_h, q_h)\|_s \quad (707)$$

where δ_2 is a positive constant independent of h .

Proof Using Cauchy-Schwarz inequality, we obtain

$$\begin{aligned} |Q_h((\mathbf{u}_h, p_h); (\mathbf{v}_h, q_h))| &\leq c_1 \|\mathbf{u}_h\|_{1,\Omega} \|\mathbf{v}_h\|_{1,\Omega} + \|p_h\|_{0,\Omega} \|\mathbf{v}_h\|_{1,\Omega} + \|q_h\|_{0,\Omega} \|\mathbf{u}_h\|_{1,\Omega} + |s_h(p_h, q_h)| \\ &\leq \delta_1 \|(\mathbf{u}_h, p_h)\|_s^2 \|(\mathbf{v}_h, q_h)\|_s^2. \end{aligned}$$

i.e., the continuity result (706) holds.

Setting $(\mathbf{v}_h, q_h) = (\mathbf{u}_h, -p_h)$ in (705) yields

$$Q_h((\mathbf{u}_h, p_h); (\mathbf{u}_h, p_h)) \geq c_2 \|\mathbf{u}_h\|_{1,\Omega}^2 + s_h(p_h, p_h). \quad (708)$$

Let $p_h \in P_h \subset P$ be arbitrary and ξ_1 a positive constant to be determined below. Since $\xi_1 p_h \in P$, it is well known that by the surjectivity of the divergence operator [1], there exists a function $\mathbf{v}_p \in \mathbf{V}$ such that

$$\operatorname{div} \mathbf{v}_p = \xi_1 p_h \quad \text{and} \quad \|\mathbf{v}_p\|_{1,\Omega} \leq c_3 \|p_h\|_{0,\Omega}. \quad (709)$$

Now, choosing $(\mathbf{v}_h, q_h) = (-\pi_h \mathbf{v}_p, 0)$ in (705) and use the fact that

$$\xi_1 \|p_h\|_{0,\Omega}^2 - (p_h, \operatorname{div} \mathbf{v}_p) = 0,$$

by the properties of $\mathbf{v}_p$ yields

$$Q_h((\mathbf{u}_h, p_h); (-\pi_h \mathbf{v}_p, 0)) = -\alpha (\mathbf{u}_h, \pi_h \mathbf{v}_p)_{0,\Omega} - \mu (\nabla \mathbf{u}_h, \nabla (\pi_h \mathbf{v}_p))_{0,\Omega} + \xi_1 \|p_h\|_{0,\Omega}^2 + (p_h, \operatorname{div} (\pi_h \mathbf{v}_p - \mathbf{v}_p))_{0,\Omega}.$$

Estimating termwise, we have

$$\left| \alpha (\mathbf{u}_h, \pi_h \mathbf{v}_p)_{0,\Omega} \right| \leq \frac{1}{\varepsilon} \|\mathbf{u}_h\|_{0,\Omega}^2 + c_4 \varepsilon \|p_h\|_{0,\Omega}^2, \quad (710)$$

and

$$\left| \mu (\nabla \mathbf{u}_h, \nabla (\pi_h \mathbf{v}_p))_{0,\Omega} \right| \leq \frac{1}{\varepsilon} \|\nabla \mathbf{u}_h\|_{0,\Omega}^2 + c_5 \varepsilon \|p_h\|_{0,\Omega}^2, \quad (711)$$

Finally,

$$(p_h, \operatorname{div} (\pi_h \mathbf{v}_p - \mathbf{v}_p))_{0,\Omega} \leq c_6 \|p_h\|_{0,\Omega} \|\mathbf{v}_p\|_{1,\Omega} \leq c_7 \|p_h\|_{0,\Omega}^2. \quad (712)$$

Then combining (708) and (710)-(712) yields

$$Q_h((\mathbf{u}_h, p_h); (-\pi_h \mathbf{v}_p, 0)) \geq -\frac{1}{\varepsilon} \|\mathbf{u}_h\|_{1,\Omega}^2 + (\xi_1 - (c_8 \varepsilon + c_7)) \|p_h\|_{0,\Omega}^2, \quad (713)$$

with choosing $\xi_1 > c_8 \varepsilon + c_7$.

To conclude we take $(\mathbf{v}_h, q_h) = (\xi_2 \mathbf{u}_h - \pi_h \mathbf{v}_p, -\xi_2 p_h)$ with ξ_2 sufficiently large. which yields by (708) and (713)

$$\begin{aligned} Q_h((\mathbf{u}_h, p_h); (\mathbf{v}_h, q_h)) &\geq \left(\xi_2 c_2 - \frac{1}{\varepsilon} \right) \|\mathbf{u}_h\|_{1,\Omega}^2 + (\xi_1 - (c_8 \varepsilon + c_7)) \|p_h\|_{0,\Omega}^2 + \xi_2 s_h(p_h, p_h) \\ &\geq c_9 \|(\mathbf{u}_h, p_h)\|_s^2. \end{aligned} \quad (714)$$

The claim now follows by noting that $\exists c$ such that $\|(\mathbf{v}_h, q_h)\|_s \leq c \|(\mathbf{u}_h, p_h)\|_s$. ■

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Study of the structural, mechanical, and magnetic stability of a half-metal material for photovoltaic applications

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Abstract: The linearized augmented plane wave method (FP-LAPW) based on density functional theory (DFT) is employed to investigate the structural, electronic, and magnetic properties of the compound BaO. This compound is calculated in different crystalline phases: NaCl (B1), CsCl (B2), ZB (B3), NiAs (B81), and WZ (B4) using the generalized gradient approximation (GGA-PBE) and the modified Becke-Johnson approximation (mBJ-GGA-PBE) for energy and exchange-correlation potential. Lattice parameters, compressibility moduli, their first derivatives with respect to pressure, and internal parameters are calculated. Electronic band structure and density of states reveal a half-metallic and magnetic character, attributed to the presence of spin-polarized p-type orbitals in the BaO compound. The binary compound BaO exhibits a half-metallic character, with an integer magnetic moment of $1\mu_B$ per formula unit and half-metallic gaps.

Keywords: Half-metal, magnetic properties, GGA-PBE, DFT, ferromagnetic.

Introduction

We present the results of our calculation on the structural stability, electronic, and magnetic properties of the BaO compound. To determine the most stable phase, we computed the total energy for different phases, namely CsCl, zinc-blende (ZB), NaCl (RS), NiAs, and wurtzite (WZ). We considered two magnetic configurations, non-magnetic (NM) and ferromagnetic (FM). All these studies are based on Perdew et al.'s GGA-PBE (1996) [2] associated with density functional theory (DFT) [2]. The first part of this chapter involves determining the most stable structure and magnetic phase in this compound. The second part focuses on studying the electronic and magnetic properties in the most stable phase. In this work, calculations were performed using the linearized augmented plane wave method (FP-LAPW) implemented in the WIEN2K code [1]. For the treatment of exchange-correlation potential, the generalized gradient approximation (GGA-PBE) as developed by Perdew, Burke, and Ernzerhof (commonly denoted as PBE) [2] was utilized. However, for electronic properties, in addition to the mentioned approximation, the mBJ-GGA-PBE approximation, which combines the exchange part of TB-mBJ potential and the correlation part of GGA-PBE potential [2], was also applied. This novel approach to exchange and correlation potential treatment, like Tran Blaha's [4] modified Becke-Johnson exchange part [3], has been recently introduced.

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Calculation Details

In this work, calculations were performed using the WIEN2K code [1], which is a direct implementation of the FP-LAPW method [5]. This method is based on linearized augmented plane waves, associated with local orbitals, to allow for the calculation of electronic band structures, electronic density of states, electron charge densities, etc., of a periodic crystal using the Bravais lattice as the starting point as well as the space group. The unit cell is divided into two regions: the region represented by atomic spheres (referred to as muffin-tin spheres) centered on the nuclei, and the interstitial region located between the spheres. In the muffin-tin spheres, the wave function is replaced by an atomic function, while in the interstices, it is described by plane waves. Physically, this means that all electrons (core and valence) are taken into account in the calculation, and no approximation is made about the potential around the nuclei. It is therefore a full-potential all-electron method, and that's why this approach is one of the most accurate methods currently available. The basis functions, electron densities, and potentials are developed into a combination of spherical harmonics inside non-overlapping spheres surrounding the atomic sites up to $l = 10$, and Fourier series in the interstitial region with a cutoff radius $R_{MT} \times K_{MAX} = 8$. ($R_{MT} \times K_{MAX}$ determines the basis functions or the size of the matrix, with R_{MT} being the smallest muffin-tin radius and K_{MAX} being the norm of the largest wave vector used for the plane wave expansion of the eigenfunctions). We used 1500 k-points in the total Brillouin zone (BZ) for our compounds, corresponding to a grid of special k-points in the irreducible Brillouin zone (IBZ) of $11 \times 11 \times 11$ for cubic phases (CsCl, NaCl, and ZB), $14 \times 14 \times 7$ and $13 \times 13 \times 7$ for hexagonal phases NiAs and WZ, respectively. A convergence test of the various calculation parameters was performed, specifically for $R_{MT} \times K_{MAX}$ determining the basis functions or the size of the matrix, and the number of k-points in IBZ. Structural optimization involves determining the ground state, and the iteration procedure is repeated until the calculation of the total energy converges to within 0.01 mRyd. The different candidate crystalline phases CsCl, NaCl, zinc-blende (ZB), NiAs, and wurtzite (WZ) used in our calculation in the study of the structural stability of the BaO compound are reported in Table 10 (see Figure 47). For the study of structural properties at equilibrium, it is necessary to first perform a self-consistent calculation of the total energy. The second step is to determine the structural data, namely the lattice parameter of the equilibrium a , the bulk modulus B , and its derivative B' , by fitting the total energy using the Murnaghan equation of state [6] given by the following relation:

$$E(V) = E_0 + \left[\frac{B_0 V}{B'_0(B'_0 - 1)} \right] \left[B'_0 \left(1 - \frac{V_0}{V} \right) + \left(\frac{V_0}{V} \right)^{B'_0} - 1 \right] \quad (715)$$

Where E_0 , B_0 and V_0 are respectively the total energy, the bulk modulus, and the equilibrium volume. B'_0 is the derivative of the bulk modulus with respect to the equilibrium pressure. The bulk modulus is evaluated at the minimum of the $E(V)$ curve by the relation:

$$B = V \frac{\partial^2 E}{\partial V^2} \quad (716)$$

Table 10: Space group and Wyckoff coordinates of the different crystalline structures used in our calculation: CsCl, NaCl, ZB, NiAs, WZ

	N°	Space Group	Cation	Anion
CsCl(B2)	221	Pm3m	0, 0, 0	1/2, 1/2, 1/2
NaCl(B1)	225	Fm3m	0, 0, 0	1/2, 1/2, 1/2
Zb(B3)	216	F43m	0, 0, 0	1/4, 1/4, 1/4
NiAs(B81)	194	P63/mmc	0, 0, 0	1/3, 2/3, 1/4
Wz(B4)	186	P63mc	1/3, 2/3, 0	2/3, 1/3, u

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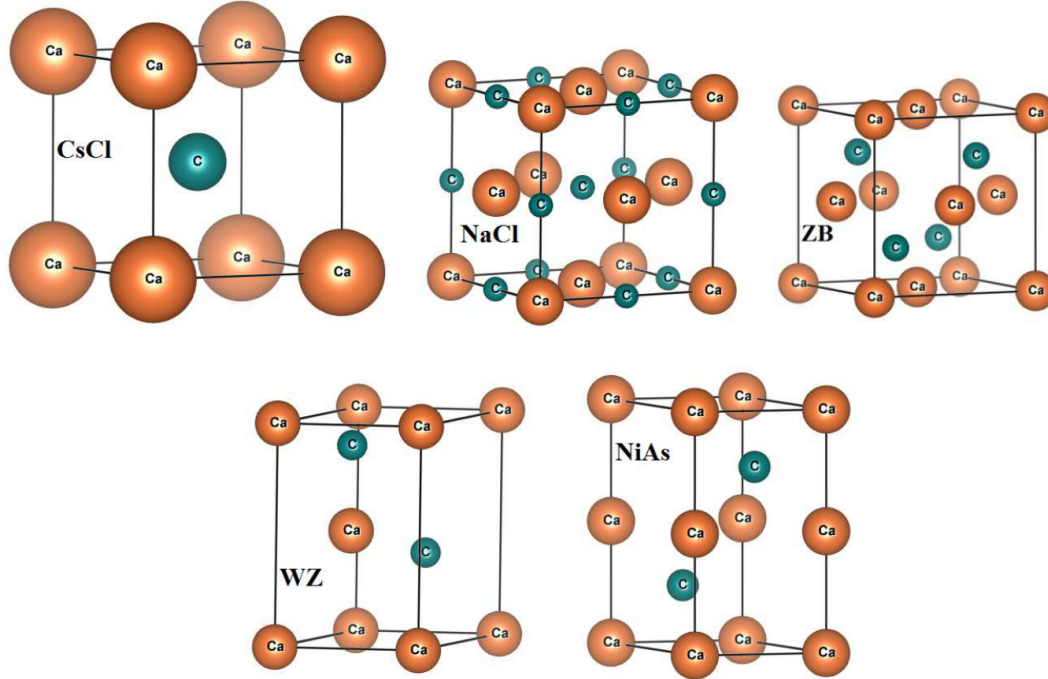


Figure 47: Representation of the structures used for the binary BaO

Results and Discussions

Structural Properties of the BaO Compound

The most stable crystalline structure and magnetic phase of the compound we studied were determined after investigating various phases. We calculated the total energy variation for the compound as a function of volume for different crystalline phases (Table 11) and in both spin configurations, namely non-magnetic (non-spin-polarized (NSP)) and ferromagnetic (spin-polarized (SP)) configurations. It is evident from this figure that the BaO compound is stable in the magnetic NaCl phase. We also calculated the cohesive energy of our BaO compound using the following relation:

$$E_c^{BaO} = E_{tot}^{BaO} - (E_{Ba} + E_O) \quad (717)$$

Where E_{tot}^{BaO} is the total energy of the BaO compound, E_{Ba} and E_O are the total energies of the isolated atoms. The cohesive energy is the energy required to break the crystal into isolated atoms, which is an indicator of the strength of bonds. For the studied BaO compound, our estimated values of the cohesive energy are shown in Table 11.

We assessed the formation energy of the compound BaO in different phases using the equation below:

$$E_{BaO}^{form} = E_{tot}^{BaO} - (E_{Ba}^{bulk} + E_O^{bulk}) \quad (718)$$

where E_{tot}^{BaO} is the total equilibrium energy of the BaO compound, and BaO, E_{Ba}^{bulk} and E_O^{bulk} are the calculated volume energies in their standard states.

In general, a negative value of the formation energy indicates that the formation of a given compound is energetically favorable from its constituent elements. The calculated formation energies for the different phases are reported in Table 11 for the BaO compound. In this study, the formation of the BaO compound in all phases is favorable.

Magnetic Properties of BaO Compound

In this work, we calculated the total and partial magnetic moment for the BaO compound using both GGA-PBE and mBJ-GGA-PBE approximations. The results are reported in Table 12. Using GGA-PBE and mBJ-

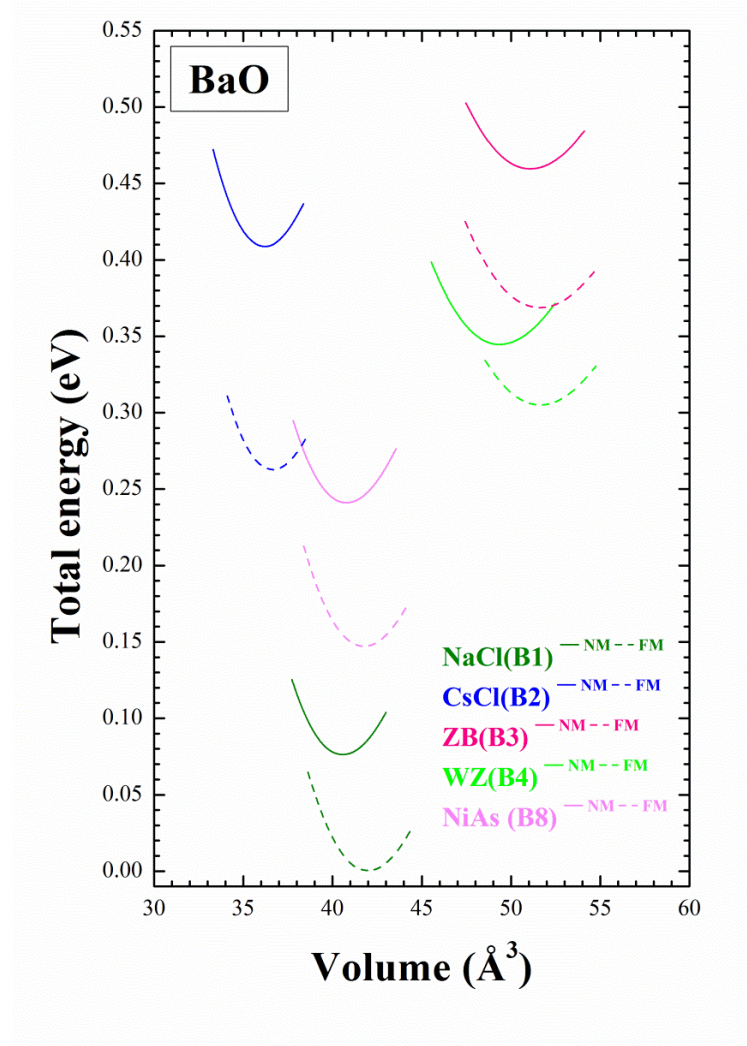


Figure 48: Total energy optimization as a function of volume using the GGA-PBE approximation.

GGA-PBE, BaO was found to be half-metallic with a total magnetic moment equal to $1\mu_B$ per molecule. The main contribution to the total magnetic moment comes from the oxygen atom. The contribution of Ba is very low compared to O. In the FP-LAPW method, the interstitial region is not referenced to any atom; therefore, we can observe a low contribution from this region. The results obtained by the GGA and GGA+mBJ approximations are very close and are not affected by the approaches used in our study.

Electronic Properties

Band Structure of BaO Compound

The band structures of the BaO compound, calculated with spin polarization in both majority and minority spin directions in the NaCl phase, are presented in Figure 32, respectively, using GGA-PBE and mBJ-GGA-PBE. We noticed that the band structures of the BaO compound calculated by GGA-PBE and mBJ-GGA-PBE are half-metallic due to the metallic character in the minority spin direction and the appearance of a fundamental gap and a half-metallic (DM) gap in the majority spin direction. This conclusion confirms the results obtained previously in the structural and magnetic properties. Table 12 shows the calculated values of the half-metallic gap (DM gap) and the fundamental gap (E_g) of the BaO compound in the NaCl phase. By comparing the results obtained with GGA-PBE and mBJ-GGA-PBE, we observed a surprising correction made by the mBJ-GGA-PBE approach. Indeed, the gap values were found to be larger than those obtained

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Table 11: Values of ferromagnetic, non-ferromagnetic, and antiferromagnetic calculations using GGA-PBE for the equilibrium lattice parameter a (Å), bulk modulus B (GPa), its derivative B' , and the energy difference between ferromagnetic and non-magnetic states $\Delta E = E_{FM} - E_{NM}$ (meV) in millielectronvolts (meV), cohesion energy E_c (Ry), and formation energy E_f (Ry)."

BaO		a (Å)	c/a	u	B (GPa)	B'	$ E_{DEFM-NM} $ (meV)	E_f (Ry)	E_c (Ry)
ZB	FM	5.90			46.15	3.95	78	-0.25	-1.09
	NM	5.88			48.74	3.04			
NaCl	FM	5.51			68.49	3.37	64	-0.26	-1.11
	NM	5.45			68.74	4.87			
NiAs	FM	3.79	1.77		67.06	4.21	77	-0.39	-1.12
	NM	3.86	1.77		69.27	4.15			
CsCl	FM	3.32			78.03	4.07	82	-0.23	-1.19
	NM	3.30			76.42	4.13			
WZ	FM	4.22	1.57	0.389	44.15	3.98	109	-0.32	-1.05
	NM	4.26	1.60	0.384	43.74	3.04			

Table 12: Values of magnetic moments, $\mu_{Total}(\mu B/cel)$, μ_{Ba} , $\mu_o(\mu B)$

	Method	$\mu_{inters}(\mu B)$	$\mu_{Ba}(\mu B)$	$\mu_o(\mu B)$	$\mu_{tot}(\mu B)$
BaO	GGA-PBE	0.13	0.05	0.82	1.00
	mBJ-GGA-PBE	0.12	0.04	0.84	1.00

with GGA-PBE. The nature of the gap in the majority spin direction is indirect in the NaCl phase with the valence band maximum at the Γ point and the conduction band minimum at the X point, as indicated in Table 13.

Table 13: Values of gap E_g (eV) and half-metallic gap (DM gap in eV) using the GGA-PBE and mBJ-GGA-PBE approximations"

	Method	gap DM (eV)	E_g (eV)	Band gap
BaO	GGA-PBE	0.42	2.16	indirect ($\Gamma - X$)
	mBJ-GGA-PBE	1.7	3.52	indirect ($\Gamma - X$)

Density of States of the BaO Compound

To better understand the band structure, it is also interesting to determine the total density of states (DOS) and partial density of states (PDOS) to understand the type of hybridization and which states are responsible for magnetism in these magnetic compounds. The calculation of the density of states requires a large number of special points (we used 5000 special points in the total Brillouin zone (BZ)). To this end, we will present the spin-polarized calculation of the total and partial density of states for the majority and minority spins of the BaO compound in the NaCl phase. The results are presented in Figure 50. The density of states (DOS) confirms the results obtained previously, using both GGA-PBE and mBJ-GGA-PBE approaches; only the BaO compound (in the NaCl phase) was found to be a half-metal. Comparing the density of states

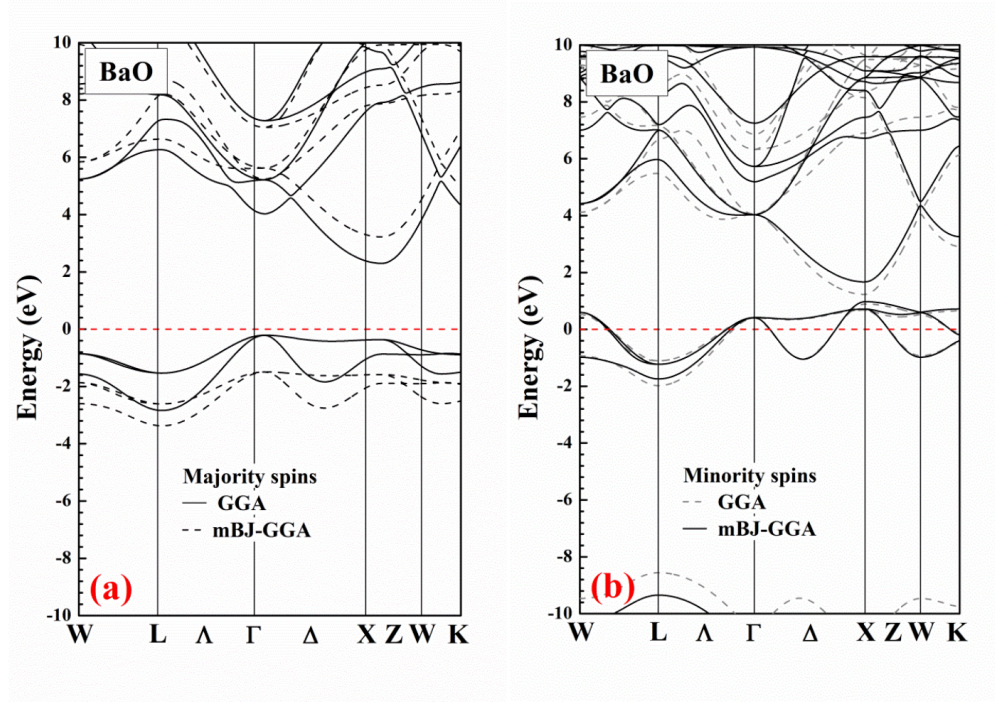


Figure 49: Spin-polarized (SP) band structure calculation of BaO: (a) Majority spin and (b) Minority spin. The horizontal line indicates the Fermi energy.

(DOS) in both majority and minority spin directions, we observed that in both approximations, there is a shift of the peaks at the Fermi level. It should be noted that this shift is larger when using mBJ-GGA-PBE. These peaks are formed by the p states of the oxygen anion. This result indicates that magnetism arises from the 2P orbitals of the oxygen atom.

Elastic Properties

The study of the elastic properties of solids plays an important role in understanding valuable information about structural stability, bonding characteristics, and hardness. The elastic behavior of a cubic single crystal is fully characterized by three independent elastic constants named C_{11} , C_{12} and C_{44} [7], as these are used to calculate the mechanical properties of solids. In our study, we calculated the elastic constants from the total energy using the method developed by Thomas CHARPIN and implemented in the WIEN2k code [1]. We reported our calculated values of the elastic constants (C_{11} , C_{12} and C_{44}) obtained by the GGA-PBE approximation for the BaO compound in Table. 5. According to Table 14, The mechanical stability was originally proposed by Born and Huang. In a cubic system, mechanical stability requires that the three independent constants obey the following Born conditions [7]

$$\begin{cases} C_{44} \geq 0 \\ C_{11} - C_{12} \geq 0 \\ C_{11} + 2C_{12} \geq 0 \end{cases} \quad (719)$$

The bulk modulus B must also satisfy a criterion: $C_{12} < B < C_{11}$. It is calculated by the following expression [8]:

$$B = \frac{1}{3}(C_{11} + 2C_{12}) \quad (720)$$

According to the results we have obtained, the conditions are met, so the BaO compound is mechanically stable. From the elastic constants C_{11} , C_{12} and C_{44} for a cubic system, the following quantities can be calculated: the anisotropy factor A , the shear modulus G , the Young's modulus E , and the Poisson's ratio σ ,

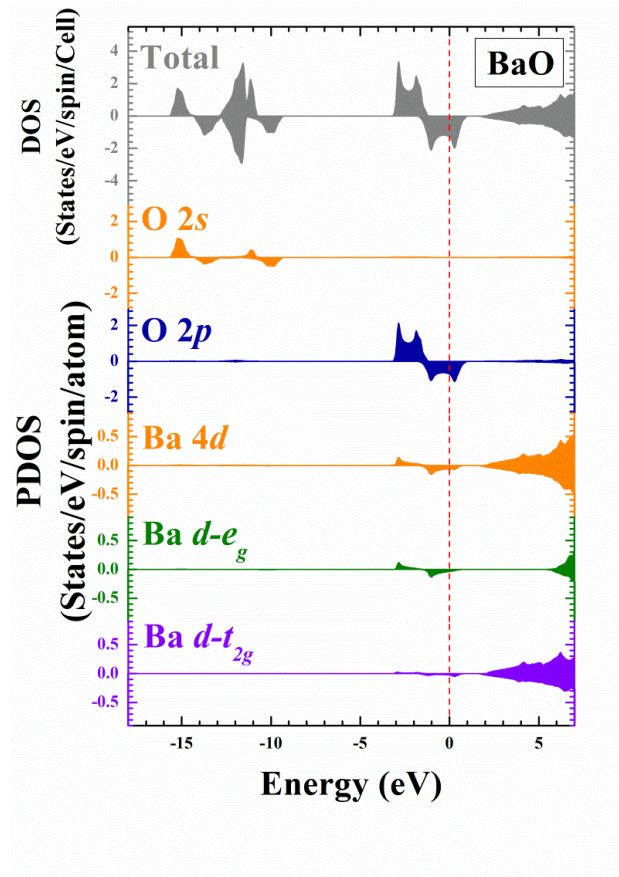


Figure 50: Spin-polarized calculation of the total and partial density of states of BaO. The vertical dashed line indicates the Fermi energy level. Positive and negative values of the density of states represent spin-up and spin-down states, respectively

according to the following expressions [8, 9]:

$$E = \frac{9BG}{3G + B} \quad (721)$$

$$G_V = \frac{C_{11} - C_{12} + 3C_{44}}{5} \quad (722)$$

$$G_R = \frac{5(C_{11} - C_{12})C_{44}}{4C_{44} + 3(C_{11} - C_{12})} \quad (723)$$

$$G_H = \frac{G_R + G_V}{2} \quad (724)$$

$$A = \frac{2C_{44}}{C_{11} - C_{12}} \quad (725)$$

$$\nu = \frac{3B - 2G}{2(3B + G)} \quad (726)$$

The results obtained are summarized in Table 14:

The bulk modulus is a very important mechanical property of solid materials; it represents the resistance of the material to isotropic compression. According to Table 14, it can be observed that the values of the bulk modulus (B) calculated from the elastic constants are in agreement with those obtained by total energy minimization (structural optimization) (Table 11).

Anisotropy is the property of a physical quantity to depend on direction; it is expressed by a factor called the anisotropy factor defined by Zener [9]. Therefore, knowledge of this factor is of great importance. If

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$A = 1$, the crystal is isotropic; if $A \neq 1$, the crystal is anisotropic. According to Table 14, the value of this factor A is different from 1, so the compound under study is anisotropic.

The shear modulus measures the resistance to shape change of materials [11]. In other words, it represents resistance to plastic deformation [10]. Young's modulus (E) is a good indicator of material rigidity [10]. When it is higher for a given material, the latter is more rigid. According to the calculated values of Young's modulus (E), BaO is rigid.

According to Frantsevich's rule [12], the Poisson's ratio (σ) allows checking the nature of the ductile or brittle behavior of a material [13]. The material is brittle if $\sigma < 0,26$ and it is ductile if $\sigma > 0,26$. According to these values of the Poisson's ratio σ , BaO is ductile.

One can also verify the ductility or brittleness of these materials from the ratio B/G , given by Pugh [13, 14]. Pugh's criterion designates a material as ductile if $B/G > 1,75$ and it is brittle $B/G < 1,75$ [13]. According to Table 14, the values of the Pugh ratio $B/G > 1,75$ also confirm what we found earlier.

Table 14: Elastic Constants and Parameters in NaCl Structure

	C_{11}	C_{12}	C_{44}	B_H	G_H	E	σ	B/G	A
BaO	303.93	81.06	68.93	155.3	81.46	208.02	0.27	1.91	0.59

Conclusion

In this chapter, we investigated the structural properties (equilibrium parameters such as lattice parameter a , B , B' , formation energy, cohesion energy), electronic properties (band structure and energy gaps), elastic constants (C_{11} , C_{12} , C_{44}) and mechanical parameters (bulk modulus, shear modulus, Young's modulus, Pugh ratio). We found that our compound is mechanically stable, anisotropic, and ductile. The band structure calculation showed that our compound exhibits a gap separating the anion and cation states for spin-up polarization, while for spin-down polarization, the valence and conduction bands overlap considerably, and there is no band gap at the Fermi level.

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On Approximate Analytical Solutions for Systems of Nonlinear Fractional Partial Differential Equations

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Abstract:

In this work, we present a new type of semi-analytical methods for solving systems of nonlinear fractional partial differential equations. This method is called Adomian decomposition method (ADM). The fractional derivative is described in the Caputo sense. Numerical examples are presented to illustrate the accuracy and effectiveness of the new technique. The result reveals that the proposed method is a powerful and applicable tool for solving systems of nonlinear fractional partial differential equations in mathematical physics and engineering.

Keywords: systems of fractional partial differential equations, Caputo fractional derivative, Adomian decomposition method, Approximate analytical solution

2020 Mathematics Subject Classification: 34A08, 26A33, 35F40, 35C10.

Introduction

In the latest years, the field of fractional calculus has played a significant role in numerous applications in physics and mathematics, which are described by fractional partial differential equations (FPDEs), for example, modeling nonlinear oscillations of earthquakes, electrodynamics, signal processing phenomena, the fractional-order fluid dynamic traffic model, fractional model of cancer chemotherapy, fractional diabetes model, and other areas [5, 6].

Finding a solution for FPDEs is very important, so many mathematicians have dedicated their work and developed numerical and analytical techniques to solve FPDEs. See. [2, 3, 4].

In this work, we use the ADM to find an approximate analytical solution for systems of nonlinear fractional partial differential equations.

Basic definition

This section presents some basic definitions and properties regarding the theory of fractional calculus

Definition 85. [5] The Riemann-Liouville fractional integral operator I^α of order α for a function $u(x, t)$ is defined as

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$$I_t^\alpha u(x, t) = \begin{cases} \frac{1}{\Gamma(\alpha)} \int_0^t (t - \xi)^{\alpha-1} u(x, \xi) d\xi, & \alpha > 0, \\ u(x, t), & \alpha = 0, \end{cases}$$

where $\Gamma(\cdot)$ is the well-known gamma function.

Definition 86. [5] The fractional derivative of $u(x, t)$ in the Caputo sense is defined as

$$\begin{aligned} D_t^\alpha u(x, t) &= I^{n-\alpha} D^n u(x, t) \\ &= \frac{1}{\Gamma(n-\alpha)} \int_0^t (t - \xi)^{n-\alpha-1} u^{(n)}(x, \xi) d\xi, \end{aligned}$$

where $n-1 < \alpha \leq n$, $n \in \mathbb{N}$.

For the Riemann-Liouville fractional integral and Caputo time-fractional derivative, we have the following relation

$$I_t^\alpha D_t^\alpha u(x, t) = u(x, t) - \sum_{k=0}^{n-1} u^{(k)}(x, 0^+) \frac{t^k}{k!}.$$

Main results

This section explains the basic idea of the ADM

Theorem 260. Consider the following general system of nonlinear fractional partial differential equations

$$\begin{cases} D_t^\alpha u(x, t) = L(u(x, t)) + N(u(x, t), v(x, t)) + f(x, t) \\ D_t^\alpha v(x, t) = P(v(x, t)) + M(u(x, t), v(x, t)) + g(x, t) \end{cases}, \quad (727)$$

with the initial conditions

$$\begin{cases} u(x, 0) = u_0(x) \\ v(x, 0) = v_0(x) \end{cases}, \quad (728)$$

where D_t^α is the Caputo time-fractional derivative operator of order $0 < \alpha \leq 1$, L, P are linear operators, N, M are nonlinear operators and $u(x, t), v(x, t)$ are known continuous functions.

Using ADM, the solution of the system (727) can be expressed as an infinite series as follows

$$\begin{cases} u(x, t) = \sum_{n=0}^{\infty} u_n(x, t) \\ v(x, t) = \sum_{n=0}^{\infty} v_n(x, t) \end{cases},$$

where

$$\begin{cases} u_n(x, t) = u_0(x) + I_t^\alpha (f(x, t)) + I_t^\alpha (L(u(x, t)) + A_{n-1}) \\ v_n(x, t) = v_0(x) + I_t^\alpha (g(x, t)) + I_t^\alpha (P(v(x, t)) + B_{n-1}) \end{cases},$$

and A_n, B_n and are Adomian polynomials [7] of the nonlinear terms and it can be calculated by formula given below

$$A_n = B_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[N \left(\sum_{i=0}^{\infty} \lambda^i u_i, \sum_{i=0}^{\infty} \lambda^i v_i \right) \right]_{\lambda=0}, \quad n = 0, 1, 2, \dots$$

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Singular Elliptic Problems for some Anisotropic Operators with Variable Exponents

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Abstract: Our main purpose of this work is to investigate the existence and regularity of non-negative solutions to some singular degenerate elliptic problems characterized by some anisotropic operators involving variable exponents. More precisely, we show that either the lower order term or the singular non-linearity term has some regularizing effects on the solutions of the Dirichlet problem. The functional framework involves anisotropic Sobolev spaces with variable exponents. as well as Lebesgue spaces with variable exponents

Keywords: Anisotropic elliptic equations, Nonlinear elliptic equations, Singular elliptic equations, Degenerate elliptic equations, Weak solutions, Variable exponents.

2020 Mathematics Subject Classification: Primary: 35J60,35J75,35J70.

Secondary: 35D30,35B65.

Introduction

This work is devoted to study the existence and regularity results of solutions for some nonlinear singular anisotropic elliptic equations with principal part characterized by degenerate coercivity:

$$-\sum_{i=1}^N D^i \left(\frac{|D^i u|^{p_i(x)-2} D^i u}{(\ln(e+|u|))^{\sigma_i(\cdot)}} \right) + u|u|^{r(\cdot)-1} = \frac{f}{u^{r(\cdot)}} + \mu \text{ in } \Omega, \quad (729)$$

$$u > 0 \quad \text{in } \Omega,$$

$$u = 0 \quad \text{on } \partial\Omega.$$

where Ω is a bounded open domain in $\mathbb{R}^N$ ($N \geq 2$) with Lipschitz boundary $\partial\Omega$, the variable exponents $p_i : \overline{\Omega} \rightarrow (1, \infty)$, $\sigma_i : \overline{\Omega} \rightarrow [0, \infty)$, $i = 1, \dots, N$, and $r : \overline{\Omega} \rightarrow (0, \infty)$ are continuous functions.

We suppose that: $\vec{p}(\cdot) \in (C_+(\overline{\Omega}))^N$,

$$\forall x \in \overline{\Omega}, p_+(x) < \vec{p}^*(x). \quad (730)$$

with

$$\vec{p}^*(x) = \begin{cases} \frac{N\vec{p}(x)}{N-\vec{p}(x)}, & \text{for } \vec{p}(x) < N, \\ +\infty, & \text{for } \vec{p}(x) \geq N. \end{cases} \quad \vec{p}(x) = \frac{N}{\sum_{i=1}^N \frac{1}{p_i(x)}},$$

and $p_+(x) = \max_{1 \leq i \leq N} p_i(x)$ while γ as in (731), $f \geq 0$ belongs to $L^1(\Omega)$, and $\mu \geq 0$ belongs to $L^1(\Omega)$.

This work is motivated by the results of [1], where the author has treated the isotropic case and has studied the existence, regularity, and uniqueness of non-negative weak solutions to the singular elliptic problem

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involving variable exponents:

$$\begin{cases} -\operatorname{div}(\widehat{a}(x, Du)) = \frac{f}{u^{\gamma(\cdot)}} + \mu, & \text{in } \Omega, \\ u > 0, & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega, \end{cases}$$

Where $0 \leq \mu \in L^1(\Omega)$, $0 \leq f \in L^{m(\cdot)}(\Omega)$, and $\widehat{a}: \Omega \times \mathbb{R}^N \rightarrow \mathbb{R}^N$ is a Leray-Lions type operator satisfying some appropriate assumptions while $\gamma: \overline{\Omega} \rightarrow (0, 1)$, $m: \overline{\Omega} \rightarrow (1, +\infty)$, and the variable exponents $p: \overline{\Omega} \rightarrow (1, +\infty)$ are continuous functions satisfying some appropriate hypotheses, with $\gamma: \overline{\Omega} \rightarrow (0, 1)$ is a continuous function satisfies:

$$0 < \gamma^- := \min_{x \in \overline{\Omega}} \gamma(x) \leq \gamma^+ := \max_{x \in \overline{\Omega}} \gamma(x) < 1, \quad |\nabla \gamma| \in L^\infty(\Omega), \text{ for all } x \in \overline{\Omega}. \quad (731)$$

In the anisotropic case when the right hand side, f is assumed to belong to $L^1(\Omega)$ with γ as in (731), we will analyse the limit case $f \in L^1(\Omega)$ showing that the lower order term $u|u|^{r(\cdot)-1}$ has still a regularizing effect on the solutions as well as the regularizing given by the singular non-linearity term $\frac{f}{u^{\gamma(\cdot)}}$.

We point out that the regularizing effect of the lower order term and the singular non-linearity term is well known for similar problems. It was pointed out in [1, 2, 4, 5, 6], for elliptic and parabolic problems.

Main results

Our aim is to prove the existence of non-negative weak solutions solutions to problem (729). Here the notions of solutions we will employ.

Definition 87. If $0 < \gamma^+ < 1$ and $\mu \geq 0$ belongs to $L^1(\Omega)$. Then, we say A non-negative function $u \in \dot{W}^{1, \vec{p}(\cdot)}(\Omega)$ is a weak solution for problem (729) if the following conditions are satisfied:

1. $u \in W_0^{1,1}(\Omega)$, $u|u|^{r(\cdot)-1} \in L^1(\Omega)$, $|D^i u|^{p_i(x)-2} D^i u \in L^1(\Omega)$, $\forall i = 1, \dots, N$, $\frac{f}{u^{\gamma(\cdot)}} \in L_{loc}^1(\Omega)$,
- 2.

$$\begin{aligned} & \sum_{i=1}^N \int_{\Omega} \left(\frac{|D^i u|^{p_i(x)-2} D^i u}{(\ln(e + |u|))^{\sigma_i(\cdot)}} \right) \cdot D^i \phi \, dx + \int_{\Omega} u|u|^{r(\cdot)-1} \cdot \phi \, dx \\ & = \int_{\Omega} \frac{f}{u^{\gamma(\cdot)}} \cdot \phi \, dx + \int_{\Omega} \mu \cdot \phi \, dx \end{aligned} \quad (732)$$

for every $\phi \in C_0^\infty(\Omega)$.

In this work we will show the following result.

Theorem 261. Under the assumptions (730)-(731). If $\mu \in L^1(\Omega)$ and $f \in L^1(\Omega)$ are non-negative functions, the variable exponents $p_i: \overline{\Omega} \rightarrow (1, +\infty)$, $\sigma_i: \overline{\Omega} \rightarrow [0, +\infty)$ $i = 1, \dots, N$, and $r: \overline{\Omega} \rightarrow (0, +\infty)$ are continuous functions such that, for all $x \in \overline{\Omega}$ and for all $i = 1, \dots, N$

$$1 + \frac{\sigma_i(x) + 1 + \gamma(x)}{r(x)} < p_i(x) < 1 + \frac{r(x)}{\sigma_i(x) + 1 + \gamma(x)}, \quad (733)$$

$$\max \left(\frac{\sigma_i(x) + 1 + \gamma(x)}{p_i(x) - 1}; (\sigma_i(x) + 1 + \gamma(x))(p_i(x) - 1) \right) < r(x), \quad (734)$$

where

$$|\nabla r| \in L^\infty(\Omega), \quad |\nabla \sigma_+| \in L^\infty(\Omega), \quad |\nabla \gamma| \in L^\infty(\Omega),$$

Then, problem (729) has at least one non-negative solution $u \in \dot{W}^{1, \vec{q}(\cdot)}(\Omega) \cap L^{r(\cdot)}(\Omega)$ satisfying (732) where $q_i(\cdot)$ are continuous functions on $\overline{\Omega}$ satisfying

$$1 < q_i(x) < \frac{p_i(x)r(x)}{r(x) + \sigma_i(x) + 1 + \gamma(x)}, \quad \forall x \in \overline{\Omega}, \quad i = 1, \dots, N, \quad (735)$$

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In order to prove these main results, we will work by approximation, truncating the singular term $\frac{f}{u^{\gamma(\cdot)}}$, so that it becomes not singular at the origin. We will get some a priori estimates on the solutions u_n of the approximating problems, which will allow us to pass to the limit and find a solution to problem (729). In order to obtain a solution in the sense described above, we will apply the Schauder's fixed point theorem and employ some fresh techniques.

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Calculation of $\left(\frac{L\gamma}{L\alpha}\right)$ X-ray intensity ratios in lanthanides: An empirical study using the least squares mathematical method

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Abstract: In this research, an analytical method is employed to model the $\left(\frac{L\gamma}{L\alpha}\right)$ X-ray intensity ratios in lanthanides with atomic number $57 \leq Z \leq 71$. Initially a database of experimental $\left(\frac{L\gamma}{L\alpha}\right)$ intensity ratio values is utilized to derive weighted average intensity ratios $\left(\frac{L\gamma}{L\alpha}\right)_W$. Subsequently, new empirical average X-ray intensity ratios $\left(\frac{L\gamma}{L\alpha}\right)$ are established using the least squares method for elements within this atomic range. This process involves applying an interpolation method based on the widely used analytical function [1] $\left[\frac{\left(\frac{L\gamma}{L\alpha}\right)_W}{1-\left(\frac{L\gamma}{L\alpha}\right)_W}\right]^{1/4} = \sum_{i=1}^n b_i Z^i$ where b_i represents coefficients obtained via the interpolation process as a function of atomic number Z . Ultimately, obtained results were compared with experimental values, showing a reasonable agreement with them.

Keywords: X-ray intensity ratios, least squares method, empirical calculation.

2024 Mathematics Subject Classification: Physical Mathematics.

Introduction

Data on the L sub-shell X-ray production cross-sections, fluorescence yields and intensity ratios are needed for many scientific, medical and engineering applications[2]. Intensity ratios play an important role in nuclear spectroscopy and atomic physics. Extensive research across various disciplines has been devoted to these quantities, resulting in them being summarized in numerous comprehensive review articles. Investigating these parameters for elements in the periodic table has been the main focus of various experiments, along with increasing theoretical interest in recent years. This interest stems from their potential applications in nondestructive element analysis in fields such as medical physics, surface chemistry, environmental science, and industry. Many efforts have been made to determine shell density ratios L across a variety of elements, either by applying theoretical models or by fitting experimental data using empirical and semi-empirical formulas. In this study we calculated new empirical $\left(\frac{L\gamma}{L\alpha}\right)$ intensity ratios then the obtained values have been compared with previously published experimental values found in the literature.

Main results

The main objective of this work is to determine the empirical mean intensity ratios of X-ray $\left(\frac{L\gamma}{L\alpha}\right)$ for elements with atomic numbers ranging from $_{57}\text{La}$ to $_{71}\text{Lu}$. To achieve this, the first step involves using the weighted average values of the intensity ratios denoted as $\left(\frac{L\gamma}{L\alpha}\right)_W$. This is motivated by the presence of several published experimental results $\left(\frac{L\gamma}{L\alpha}\right)_E$ for the same element, obtained through various methods and conditions. These values are collected in databases by [3]. The formula used to determine the weighted average values, $\left(\frac{L\gamma}{L\alpha}\right)_W$, is as follows:

$$\begin{aligned} \left(\frac{L\gamma}{L\alpha}\right)_W \pm \varepsilon &= \left(\sum_n \frac{1}{(\Delta\left(\frac{L\gamma}{L\alpha}\right)_{EXP-n})^2} \right)^{-1} \cdot \sum_n \frac{\left(\left(\frac{L\gamma}{L\alpha}\right)_{EXP-n}\right)}{(\Delta\left(\frac{L\gamma}{L\alpha}\right)_{EXP-n})^2} \\ &\pm \left(\left(\sum_n \frac{1}{(\Delta\left(\frac{L\gamma}{L\alpha}\right)_{EXP-n})^2} \right)^{1/2} \right)^{-1} \end{aligned} \quad (736)$$

Where $\left(\frac{L\gamma}{L\alpha}\right)_{EXP-n}$ is the experimental intensity ratio, $\Delta\left(\frac{L\gamma}{L\alpha}\right)_{EXP-n}$ represents the uncertainty of the n^{th} experimental value and N is the number of experimental data.

Following this, with the goal of computing the empirical average $\left(\frac{L\gamma}{L\alpha}\right)$ X-ray intensity ratios for elements within the atomic range of 57 to 71, we utilized the widely acknowledged formula:

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$$\left[\frac{\left(\frac{L\gamma}{L\alpha} \right)_W}{1 - \left(\frac{L\gamma}{L\alpha} \right)_W} \right]^{1/4} = \sum_{i=1}^n b_i Z^i = f(Z) \quad (737)$$

The method using this formula entails plotting the quantity as a function of the atomic number Z and then fitting it, as shown in Figure 1. The calculation is based on the least squares method implemented in the ORIGIN software. Therefore, the empirical $\left(\frac{L\gamma}{L\alpha} \right)_{\text{emp}}$ X-ray intensity ratios can be expressed as follows:

$$\left(\frac{L\gamma}{L\alpha} \right)_{\text{emp}} = \frac{f(Z)^4}{1 + f(Z)^4} \quad (738)$$

The fitting coefficients are: $b_0 = 3.74662349104803$, $b_1 = -1.0229605661335 \times 10^{-1}$, and $b_2 = 8.38571176798238 \times 10^{-4}$. The empirical intensity ratios $\left(\frac{L\gamma}{L\alpha} \right)_{\text{emp}}$, calculated using formula (??) for elements with $57 \leq Z \leq 71$ have been presented in Table 52.

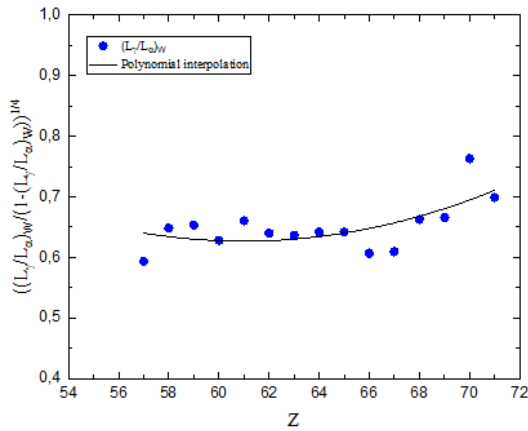


Figure 51: Distribution of $\left[\frac{\left(\frac{L\gamma}{L\alpha} \right)_W}{1 - \left(\frac{L\gamma}{L\alpha} \right)_W} \right]^{1/4}$ as a function of atomic number Z .

Z	$\left(\frac{L\gamma}{L\alpha} \right)_{\text{emp}}$
$Z = 57$	0.1439
$Z = 58$	0.1394
$Z = 59$	0.1363
$Z = 60$	0.1344
$Z = 61$	0.1338
$Z = 62$	0.1344
$Z = 63$	0.1363
$Z = 64$	0.1394
$Z = 65$	0.1439
$Z = 66$	0.1498
$Z = 67$	0.1572
$Z = 68$	0.1661
$Z = 69$	0.1767
$Z = 70$	0.1891
$Z = 71$	0.2034

Figure 52: Empirical intensity ratios $\left(\frac{L\gamma}{L\alpha} \right)_{\text{emp}}$ calculated in this study.

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Statistical Analysis and Fitting Method for Analytical Estimation of Empirical σ_{K_α} X-ray Fluorescence Cross Sections for Elements ranging from $_{30}Zn$ to $_{40}Zr$ by Photon Impact at 59.5 keV

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Abstract: The main purpose of this study is to introduce an analytical method for modeling K_α XRF cross sections within elements in the atomic number range $30 \leq Z \leq 40$. Additionally, to deduce the empirical $\sigma(K_\alpha)$ XRF cross sections, a database of experimental values are fitted using an interpolation method based on the least squares approach to obtain a model of the form $\ln \sigma(K_\alpha) = \sum_{n=0}^2 b_n Z^n$, where b_n are coefficients established throughout the fitting procedure against atomic number Z . Finally, our results were compared to experimental values from the literature, revealing a significant correlation between our findings and other research.

Keywords: XRF cross sections, weighted average values, Fitting process, least square method.

2020 Mathematics Subject Classification: Physical Mathematics.

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Introduction

A strong motivation to study K shell fluorescence cross section at various excitation energies comes from its importance in both basic and applied research. This fundamental parameter draws much attention in a several applications field, including atomic, molecular, radiation physics (transport in matter), dosimetry, space and plasma physics, medical research (cancer therapy), agriculture, forensic science and basic nuclear physics. Over time, a variety of theoretical models and empirical approaches have been employed to determine theses cross sections within a wide range of elements. An early example Krause et al.[1] determined the K-shell XRF cross section for all elements at photon energies ranging from 1 to 200 keV, using Scofield's [2] theoretical partial photoionization cross sections based on Hartree-Slater and Hartree-Fock central potential theory. Puri et al.[3] have published an extensive table of empirical K shell ($13 \leq Z \leq 92$) and L series ($35 \leq Z \leq 92$) XRF cross section at incident photon energy range 1-200 keV. In this work, we introduced an analytical method for modeling K_α XRF cross sections within elements in the atomic number $30 \leq Z \leq 40$. Besides, to deduce the empirical σ_{K_α} XRF cross sections, the experimental values are fitted using an interpolation method based on the least squares approach. This method is an iterative optimization algorithm used to find the parameters that minimize the sum of the squares of the disparities between the predicted values from a model and the experimental values, so that fitting the given data. Finally, our results are displayed in a tabular form and compared to experimental values from literature, revealing a significant correlation between our findings and other research.

Main results

To calculate the empirical average σ_{K_α} XRF cross sections for elements within the atomic range $30 \leq Z \leq 40$ at 59.5 keV, we employed the following formula:

$$\ln(\sigma_{K_\alpha}) = \sum_{n=0}^2 b_n Z^n \quad (739)$$

This method requires fitting the term $\ln(\sigma_{K_\alpha})$, after plotting it against the atomic number Z , as illustrated in . The fitting is accomplished using a least squares method. The quadratic polynomial function is given as:

$$f(Z) = \ln(\sigma_{K_\alpha}) = \sum_{n=0}^2 b_n Z^n \quad (740)$$

Where, σ_{K_α} is the experimental values, thus $b_0 = -8.53503813296$, $b_1 = 0.616465854656$, and $b_2 = -0.006529939793$. Therefore, we have derived a new empirical average σ_{K_α} Emp XRF cross sections through the following formula:

$$(\sigma_{K_\alpha})_{\text{Emp}} = e^{f(Z)} \quad (741)$$

Table 1 includes the current computation of empirical σ_{K_α} XRF cross sections for all elements within the range $30 \leq Z \leq 40$, as per Eq. (741).

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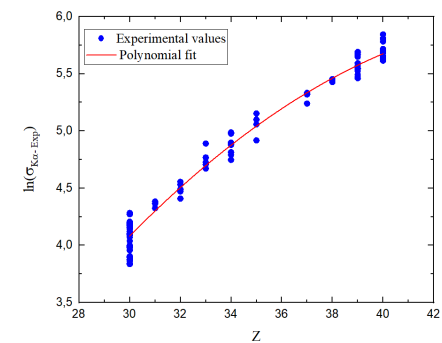


Figure 53: Distribution of $\sigma_{K_{\alpha}}$ as a function of atomic number Z .

Table 1: Empirical values of $\sigma_{K_{\alpha}}$ XRF cross sections.

Z	$\sigma_{K_{\alpha}}$ (barn/atom)	Z	$\sigma_{K_{\alpha}}$ (barn/atom)
$Z = 30$	59.26	$Z = 35$	154.79
$Z = 31$	73.71	$Z = 37$	207.41
$Z = 32$	90.49	$Z = 38$	235.43
$Z = 33$	109.64	$Z = 39$	263.77
$Z = 34$	131.13	$Z = 40$	291.69