On the Parameter Estimation of CG-LNT Radar Clutter

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Abstract *Parameter estimation of radar clutter modeled by the Compound-Gaussian with Lognormal Texture (CG-LNT) distribution is crucial for improving detection performance in high-resolution radar systems. The CG-LNT model, a heavy-tailed and non-Gaussian distribution, is widely used to characterize sea clutter variations. In paper, comparative study of the performance of the different methods of estimation is conducted. The estimators considered are higher order moment estimator (HOME), fractional order moment estimator (FOME), [zlog(z)] estimator and fractional negative order moment estimator. Furthermore, the impact of the fractional order in the FOME and FNOME estimator is also considered in his paper, where the performance of these two estimators is tested using different values of the fractional order. Through Monte Carlo simulations, the estimation accuracy of the different estimators is evaluated for both the mean and the standard deviation of the CG-LNT distribution. The Mean Squared Error (MSE) as the performance metric.*

Keywords: Sea clutter, parameters estimation, CG-LNT distribution.

I. Introduction

In radar systems, the design of constant false alarm rate (CFAR) detectors often relies on the accurate statistical modeling of background clutter [1-2]. High-resolution sea clutter, in particular, is known to exhibit non-Gaussian, heavy-tailed behavior that challenges classical detection schemes. To address this, several non-Gaussian models have been developed, including lognormal, Weibull, compound K, Pareto, Compound Inverse Gaussian (CIG), and the Compound-Gaussian with Lognormal Texture (CG-LNT) distribution [3-4].

In this paper, parameter estimation of the CG-LNT model is considered, the CG-LNT distribution is characterized by the mean and standard deviation parameters. Several estimation methods have been proposed in the literature, including the higher order moment estimator (HOME), fractional order moment estimator (FOME), [zlog(z)] estimator and fractional negative order moment estimator (FNOME) [5-8]. The performance of the different estimators is investigated using simulated dada according to the CG-LNT distribution for various values of the mean δ and standard deviation σ parameters. Also, the effect of the fractional order on the estimation performance of the FOME and the FNOME is analyzed through Monte-Carlo simulations and using the mean square error criteria (MSE).

II. CG-LNT distribution

CG-LNT distribution is heavy-tailed non-Gaussian distribution, this distribution is used to describe high-resolution sea clutter amplitude variation. CG-LNT is defined as a mixture of two components; the texture component which represents the average local level of the clutter which follow lognormal distribution and the speckle component which obeys Rayleigh distribution. The global PDF of the CG-LNT model is given as [4]

$$p(z) = \frac{z}{\sqrt{2\pi\sigma^2}} \int_0^{+\infty} \frac{2}{y^2} exp\left(-\frac{[ln(y/\delta)]^2}{2\sigma^2} - \frac{z^2}{y}\right) dy \quad (1)$$

Where σ and δ are the standard deviation and the mean respectively. For high values of σ the CG-LNT model becomes heavy-tailed and for small values of σ it tends towards a Rayleigh law [4].

Using the PDF in (1), the CCDF can be written as a function of normalized threshold T as

$$CCDF(T) = \int_0^{+\infty} exp\left(-\frac{T^2}{y}\right)\frac{1}{y\sqrt{2\pi\sigma^2}} exp\left(-\frac{(ln(y/\delta))^2}{2\sigma^2}\right) dy \quad (2)$$

The theorical expression of moment of the CG-LNT is given as [4]

$$\langle z^n\rangle = \delta^{\frac{n}{2}}\Gamma\left(1+\frac{n}{2}\right) exp\left(\frac{1}{2}\left(\frac{n\sigma}{2}\right)^2\right) \quad (3)$$

Where n represent the order and $\Gamma(.)$ denote the gamma function.

III. Estimation Methods

The HOME is based on the two first even order moments, the HOME is given as [6]

$$\begin{cases} \hat{\sigma} = \sqrt{log\left(\frac{\langle z^4\rangle}{2\langle z^2\rangle^2}\right)} \\ \hat{\delta} = \langle z^2\rangle\, exp\left(-\frac{\hat{\sigma}^2}{2}\right) \end{cases} \quad (4)$$

The FOME method is obtained by exploiting the statistical ratio $\langle z^{n+1}\rangle/\langle z\rangle\langle z^n\rangle$, this estimator is given as: [6]

$$\hat{\sigma} = 2\sqrt{\frac{1}{n} log\left(\frac{\langle z^{n+1}\rangle}{\langle z\rangle\langle z^n\rangle}\frac{0.5n\sqrt{\pi}\Gamma(0.5n)}{(n+1)\Gamma(0.5n+0.5)}\right)} \quad (5)$$

The [zlog(z)] method is obtained by the derivatives of the moment expression with respect to the order n. The expression of the [zlog(z)] estimator is: [6]

$$\hat{\sigma} = 2\sqrt{\frac{\langle z\, log(z)\rangle}{\langle z\rangle} - \langle log(z)\rangle\text{-}1 + \log(2)} \quad (6)$$

The logarithmic moments can be obtained from the data as

$$\langle log(z)\rangle = \frac{1}{M}\sum_{i=1}^{M} log(z_i) \quad (7)$$

$$\langle z\, log(z)\rangle = \frac{1}{M}\sum_{i=1}^{M} z_i\, log(z_i) \quad (8)$$

The estimate of the mean $\hat{\delta}$ is obtained for the FOME and [zlog(z)] methods as function of fractional order moment $\langle z^n\rangle$ and the estimate $\hat{\sigma}$ as

$$\hat{\delta} = \left(\frac{\langle z^n\rangle}{\Gamma(1+n/2)} exp\left(-\frac{n^2\hat{\sigma}^2}{8}\right)\right)^{\frac{2}{n}} \quad (9)$$

The FNOME estimator is developed by manipulating moments with fractional negative and positive order as

$$\langle z^n\rangle\langle z^{-n}\rangle = \Gamma\left(1+\frac{n}{2}\right)\Gamma\left(1-\frac{n}{2}\right) exp\left(\left(\frac{n\sigma}{2}\right)^2\right) \quad (10)$$

FNOME estimator is given as [7]

$$\hat{\sigma} = \frac{2}{n}\sqrt{log\left(\frac{\langle z^n\rangle\langle z^{-n}\rangle}{\Gamma(1+n/2)\Gamma(1-n/2)}\right)} \quad (11)$$

In the other hand, the closed form of $\hat{\delta}$ is given as

$$\hat{\delta} = \left(\frac{\langle z^n\rangle\Gamma(1-n/2)}{\langle z^{-n}\rangle\Gamma(1+n/2)}\right)^{\frac{1}{n}} \quad (12)$$

IV. Performance Analysis

This section evaluates the estimation performance of the HOME, FOME, [zlog(z)] and FNOME estimators. The CG-LNT clutter is generated using Matlab. The evaluation is carried out through Monte Carlo simulation. First, based on simulated data generated according to the CG-LNT distribution, the estimation accuracy is evaluated by MSE criterion with L=10000 independent trials. The samples number M is 1000

In Fig. 1 and Fig. 2, the impact of the fractional order n on the estimation accuracy of the FOME and FNOME is analyzed respectively. The results show that lower values of the fractional order n provide the minimum MSEs values.

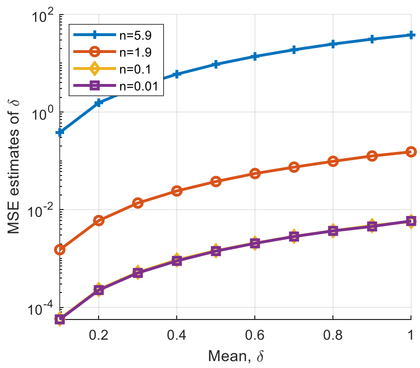

Fig.1 Impact of the fractional order on the FOME.

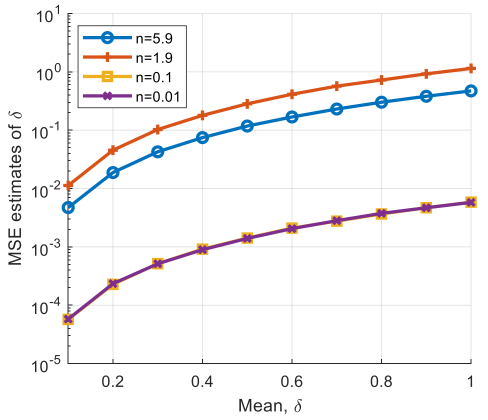

Fig.2 Impact of the fractional order on the FNOME.

Fig. 3 shows the MSEs curves of the mean δ obtained using the HOME, FOME, [zlog(z)] and FNOME methods. The fractional order n is fixed at 0.01 and the standard deviation σ is fixed at 2. The results show that MSEs curves obtained by the FOME, [zlog(z)] and FNOME methods are close together.

According to the Fig. 4, it is clear that the FNOME offers the best estimation performance especially when the clutter spikiness increases, which is the results of the increase in the standard deviation σ, for low values of the standard deviation σ the CG-LNT distribution tends to the Rayleigh distribution.

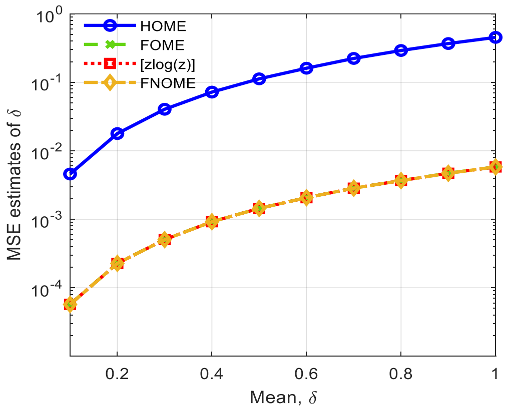

Fig.3 MSEs of the mean δ. using HOME, FOME, [zlog(z)] and FNOME methods.

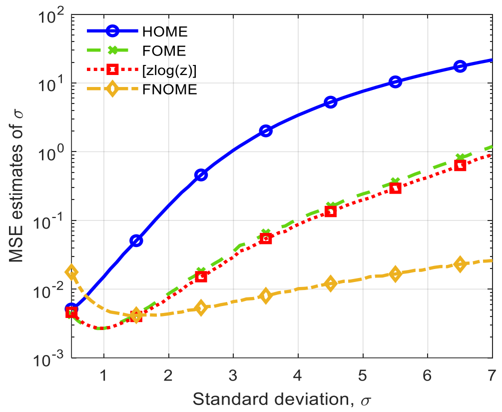

Fig.4 MSEs of the standard deviation σ using HOME, FOME, [zlog(z)] and FNOME methods.

V. Conclusion

In this paper, the performance of parameter estimation of the CGLNT clutter has been investigated. Different estimators are considered; HOME, FOME, [zlog(z)] and FNOME methods. Firstly, the impact of the fractional order on the quality of estimation of the FOME and FNOME is analyzed, the results show that the use of low values of the order conduct to accurate estimation and low MSE. Secondly, a comparison is established between HOME, FOME, [zlog(z)] and FNOME methods, the fractional order is chosen n=0.01 for the FOME and FNOME. The obtained results show the superiority of the FNOME compared to the other methods.

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