

# ABSTRACT BOOK

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# 1. FOREWORD

On behalf of the Organizing Committee, we are very pleased to welcome you to the 9th International Conference of Mathematical Sciences (ICMS 2025) to be held between 03-07 September 2025 via face-to-face and online Conference supported by Maltepe University in Istanbul. We hope that, ICMS 2025 will be one of the most beneficial scientific events, bringing together mathematicians from all over the world, and demonstrating the vital role that mathematics plays in any field of science. Welcome to our conference at Maltepe University.

**Hüseyin Çakallı**  
Chairman of the Organizing Committee

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### 3. SESSIONS

The lectures in the following parallel sessions are to be held after the plenary speakers lectures.

0. “**Plenary**” organized by Hüseyin CAKALLI,
1. “**Topology**” organized by Osman MUCUK,
2. “**Analysis and Functional Analysis**” organized by Hacer ŞENGÜL KANDEMİR and Nazlim Deniz ARAL,
3. “**Sequences, Series, Summability**” organized by İbrahim CANAK and Sefa Anil SEZER,
4. “**Fixed Point Theory**” organized by Duran TURKOGLU and Hakan ŞAHIN,
5. “**Numerical Functional Analysis**” organized by Allaberen ASHYRALYEV and Charyyar ASHYRALYEV,
6. “**Computer Science and Technology**” organized by Önder ŞAHINASLAN, and Serdar ANGUN,
7. “**Mathematical Methods in Physics**” organized by Özay Gürtuğ and Filiz CAĞATAY UÇGUN,
8. “**Mathematics Education**” organized by Vildan KATMER BAYRAKLI,
9. “**Differential Geometry**” organized by İlhan GUL,
10. “**Algebra**” organized by Leyla BUGAY,

## 4. ACKNOWLEDGMENTS

We thank firstly the founder of Maltepe University, Hüseyin ŞİMŞEK, Chairman of the Board of Trustees of Maltepe University, Ali Nevzat AYGÜN, the rector of Maltepe University, Edibe SÖZEN ÇETİNTAŞ, and the vice rector of Maltepe University, İhsan YILMAZ. We also thank the parallel session organizers, and then all scientific committee members who reviewed abstracts which made the conference better.

There are many people who spent a lot of time and effort to make this conference possible. We would like to thank especially to the following colleagues who had contributed to the success of this conference in various ways:

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Serdar ANGÜN, Maltepe University, İstanbul, Türkiye,  
Ahmet USTA, Maltepe University, İstanbul, Türkiye  
Ayşe ÇOBANKAYA, Çukurova University, Adana, Türkiye.  
**Hüseyin ÇAKALLI**  
**Chairman of the Organizing Committee**

# 5. PREFACE

**Ninth International Conference of Mathematical Sciences (ICMS 2025) Maltepe University, Istanbul-Turkey**

Huseyin Cakalli  
Maltepe University, Istanbul, Turkey

The International Conference of Mathematical Sciences 2025 (ICMS 2025) was scheduled to take place both face-to-face and online (Blackboard Maltepe University conference system) at Maltepe University, Istanbul, Turkey between September 03-07, 2025.

The aim of the conference is to bring together leading scientists of the International Mathematical Sciences community and attract researchers to present their original high quality research manuscripts.

Ten special sessions were paralelly taken place after the plenary talks in the Ninth International Conference of Mathematical Sciences (ICMS 2025) both face-to-face and via Blackboard online conference system of Maltepe University, Istanbul-Turkey. High quality papers in this proceedings have been chosen from the presentations which had been included in the following sessions

0. “**Plenary**” session organized by Hüseyin CAKALLI,
1. “**Topology**” session organized by Osman MUCUK,
2. “**Analysis and Functional Analysis**” session organized by Hacer ŞENGÜL KANDEMİR and Nazlım Deniz ARAL,
3. “**Sequences, Series, Summability**” session organized by İbrahim CANAK and Sefa Anıl SEZER,
4. “**Fixed Point Theory**” session organized by Duran TURKOGLU and Hakan ŞAHIN,
5. “**Numerical Functional Analysis**” session organized by Allaberen ASHYRALVEY and Charyyar ASHYRALYYEV,
6. “**Computer Science and Technology**” session organized by Önder ŞAHINASLAN, and Serdar AN-GUN,
7. “**Mathematical Methods in Physics**” session organized by Özay GURTUG, and Filiz ÇAĞATAY UÇGUN,
8. “**Mathematics Education**” session organized by Vildan KATMER BAYRAKLI,
9. “**Geometry**” session organized by İlhan GUL
10. “**Algebra**” session organized by Leyla BUGAY,

The selection of the papers included in this volume was based on an international peer review procedure by independently researchers, namely, each special session organizer acted as an area editor by assigning at least two referees. We would like to acknowledge to thank:

- Special thanks to Ihsan YILMAZ (Vice rector of Maltepe University)
- The scientific Committee of ICMS 2025 for their valuable suggestions and reviews (See the conference details)
- Sessions organizers for their considerable work, as well as for their valuable suggestions on the further development of the conference.

- The distinguished plenary speakers for their difficult work and for their acceptance to give invited lectures on their respective fields of expertise,
- The Organizing Committee for their difficult task,

## 6. ABSTRACTS

The abstracts are ordered by session number, and then ordered by the name of the first author of the presentation in each session.

This is not a proceedings, the abstracts in this book have been printed as submitted by the authors, after revisions if requested by the referees' and the authors are responsible for the correctness of their abstracts and defending their works during their presentations at the conference.

Selected high quality full length papers presented at the sessions of the conference can be published in the following journals "Boletim da Sociedade Paranaense de Matemática" (As a special issue named "Advances in Mathematical Science (ICMS 2025)" Web of Science, Emerging Sci. and Scopus indexed), "Proceedings of International Mathematical Sciences (PIMS)", "Maltepe Journal of Mathematics" (MJM), "e-Journal of Analysis and Applied Mathematics (e-JAAM)", Filomat and "Tampap Journal of Mathematics and Statistics" upon a request of author/ or coauthor after reviewing process.

## 6.0 PLENARY SPEAKERS

The abstracts of the plenary lectures are given in the following pages.

## On Ideal Lacunary Statistical Convergence of Order $\alpha$ in Seminormed Space

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 Haliç University, Istanbul, Turkey, rahmetsavas@gmail.com

### Abstract

In a natural way in this paper we introduce new and further general suumability methods, namely, ideal statistical convergence of order  $\alpha$  and ideal lacunary statistical convergence of order  $\alpha$  in seminormed space. In this context it should be mentioned that the concept of lacunary statistical convergence of order  $\alpha$  in seminormed space (which happens to be a special case of ideal lacunary statistical convergence of order  $\alpha$  in seminormed ) has also not studied till now. **Keywords:**Ideal, filter,  $I$ -summable of order  $\alpha$ , seminormed space .

**2020 Mathematics Subject Classification Numbers:** 40F06, 40G01

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## Quantum's Exemplary Power: Quantum Computers

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### Abstract

Significant advances have been made in quantum technologies in recent years and are expected to continue. Quantum properties such as entanglement, tunneling, and superposition underlie the advantages of quantum technologies over existing technologies. These features offer exponential speedups and significant communication security compared to existing technologies. Quantum technologies have wide-ranging applications in computing, information security, communications, finance, economics, sensitive sensors, materials science, new drug design, and the defense and space industries. This study will briefly discuss quantum computers, which are pioneers of quantum technologies.

**Keywords:** Quantum computers, communication security

**2020 Mathematics Subject Classification Numbers:** 99A

## 6.1 Topology

Session Organizer: Osman Mucuk

In the Topology Session of the conference, there are 17 contributed presentations submitted to this session. The talks concern various main topics from Topology and its relations with other mathematical disciplines, and contain a number of interesting results. The topics considered in the presentations are: Mixed neutrosophic spaces: countability aspects, Neutrosophic separation strengthened, Hausdorff and base properties of mixed neutrosophic spaces, Mixed neutrosophic separation axioms, Mixed neutrosophic topological connectedness, Theta parametric metric spaces, TD-preordered spaces, Spec( $M$ ) topology of module  $M$  over commutative ring, Lacunary statistically forward quasi Cauchyness in asymmetric metric spaces, Abel statistical convergence in a metric space, Preordered spaces, Tsc reflexive relations spaces, Ordered  $j$ -approximation spaces based on different types of  $j$ -neighborhood classes, Quasi divisor topology of modules over domains, G-convergence and G-sequential convergence in submethods, Functions preserving slowly oscillating sequences in topological vector spaces.

It is expected that full texts of some of these presentations will be published in the estimated mathematical journals. It is believed that this conference will improve the relations between the people studying in these areas to establish collaborations for future work. We hope that these topics will be interesting and attract young researchers, and therefore will be useful and helpful to enable them for further studies.

## Mixed Neutrosophic Spaces: Countability Aspects

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### Abstract

In this paper, we introduce a new type of mixed neutrosophic topological space. We define countability on mixed neutrosophic topological spaces. We investigate its different quasi type properties.

**Keywords:** Mixed neutrosophic topology, First countable mixed neutrosophic topological space, Second countable mixed neutrosophic topological space

**2020 Mathematics Subject Classification Numbers:** 54A05, 54C10, 54D30, 54D10.

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- [4] A. Acikgoz, H. Cakalli, F. Esenbel, L.D. Kocinac, A quest of G-continuity in neutrosophic spaces. *Math. Meth. Appl. Sci.*, 44, 7834–7844, (2021).
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- [6] A. Acikgoz, F. Esenbel, On A Weak Form of Semi-Open Function By Neutrosophication, *Maltepe J. Math.*, VI, 79–89, (2024).

## Neutrosophic Separation Strengthened

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### Abstract

It is common knowledge that neutrosophic topology contributes to developing techniques to address real-life applications in various areas like information systems and optimal choices. The building blocks of neutrosophic topology are neutrosophic open sets, but other extended families of neutrosophic open sets, like neutrosophic pre-open sets, can contribute to the growth of neutrosophic topology. In the present work, we create some classifications of neutrosophic topologies which enable us to obtain several desirable features and relationships. At first, we introduce and analyze stronger forms of neutrosophic pre-separation and regularity properties in neutrosophic topology called neutrosophic pre- $T_i$ ,  $i = 0, 1/2, 1, 2, 3, 4$ , neutrosophic pre-symmetric, and neutrosophic pre- $R_i$ ,  $i = 0, 1, 2, 3$  by utilizing the concepts of neutrosophic pre-open sets and quasi-coincident relation. We investigate more novel properties of these classes and uncover their unique characteristics. By presenting a wide array of related theorems and interconnections, we structure a comprehensive framework for understanding these classes and interrelationships with other separation axioms in this setting. Moreover, the relations between these classes and those in some induced topological structure.

**Keywords:** neutrosophic pre- $T_i$  spaces, neutrosophic pre- $R_i$  spaces, neutrosophic pre-symmetric spaces.  
**2020 Mathematics Subject Classification Numbers:** 54J05, 54D10.

## References

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- [2] R. Dhavaseelan, S. Jafari, C. Ozel, M.A. Al-Shumrani, Generalized neutrosophic contra-continuity, In *Infinite Study*, Volume II, pp. 365–380, (2017).
- [3] A. A. Salama, S.A. Alblowi, Neutrosophic set and neutrosophic topological spaces. *IOSR J. Math.*, 3, 31–35, (2012).
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- [6] A. Acikgoz, F. Esenbel, On A Weak Form of Semi-Open Function By Neutrosophication, *Maltepe J. Math.*, VI, 79–89, (2024).

## Hausdorff and Base Properties of Mixed Neutrosophic Spaces

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### Abstract

In this article, mixed neutrosophic topology and its topological properties have been studied. Mixed neutrosophic topology is defined with the help of quasi-coincidence and closure of a neutrosophic set in one of the neutrosophic topologies. Thus, a new neutrosophic topology is generated from the given two neutrosophic topologies. This new neutrosophic topology may or may not contain the topological properties of the parent topologies. This study identifies some topological properties that are carried to the mixed neutrosophic topology from the given parent neutrosophic topologies and some other properties which are not carried to the mixed neutrosophic topology. Here a base for mixed neutrosophic topology from the bases of the given parent topologies is constructed. Considering the regularity of one of the parent topologies mixed neutrosophic topology is investigated. Hausdorff's properties of mixed neutrosophic topological spaces are also discussed. It is now of general interest to know which properties are carried to the mixed topology and which are not. A few of these are being tried to answer here in this paper..

**Keywords:** Mixed Neutrosophic Topological Space, Base in mixed neutrosophic topological space, mixed neutrosophic Hausdorff space

**2020 Mathematics Subject Classification Numbers:** 54J05, 54D10.

## References

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- [2] R. Dhavaseelan, S. Jafari, C. Ozel, M.A. Al-Shumrani, Generalized neutrosophic contra-continuity, In Infinite Study, Volume II, pp. 365–380, (2017).
- [3] A. A. Salama, S.A. Alblowi, Neutrosophic set and neutrosophic topological spaces. *IOSR J. Math.*, 3, 31–35, (2012).
- [4] A. Acikgoz, H. Cakalli, F. Esenbel, L.D. Kocinac, A quest of G-continuity in neutrosophic spaces. *Math. Meth. Appl. Sci.*, 44, 7834–7844, (2021).
- [5] A. Acikgoz, F. Esenbel, Neutrosophic Connected topological spaces, *Fasc. Math.*, 66, 5–22, (2023).
- [6] A. Acikgoz, F. Esenbel, On A Weak Form of Semi-Open Function By Neutrosophication, *Maltepe J. Math.*, VI, 79–89, (2024).

## Mixed Neutrosophic Separation Axioms

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### Abstract

Herein, we define neutrosophic  $T_0$ -space, neutrosophic  $T_1$ -space, neutrosophic  $T_2$  (or Hausdorff), as well as neutrosophic regular and neutrosophic normal spaces in mixed neutrosophic topological spaces, and then establish relationships among these spaces. We provide some results for the abovementioned spaces in mixed neutrosophic topological spaces.

**Keywords:** Mixed neutrosophic topology, Separation Axioms in Mixed Neutrosophic Topology, mixed neutrosophic  $T_0$ -space, mixed neutrosophic  $T_1$ -space, mixed neutrosophic  $T_2$  (or Hausdorff) spaces.

**2020 Mathematics Subject Classification Numbers:** 54J05, 54D10.

## References

- [1] F. Smarandache, Neutrosophic set, A generalisation of the intuitionistic fuzzy sets, *Int. J. Pure Appl. Math.*, 24, 287–297, (2025).
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## Mixed Neutrosophic Topological Connectedness

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### Abstract

In this article, connectedness in mixed neutrosophic topology is introduced. Four notions of connectedness are investigated in mixed neutrosophic topology. From the connectedness of parent neutrosophic topology theories and concepts of connectedness in mixed neutrosophic topology are studied and established and vice versa. Many exciting examples and counterexamples are also included to give a clear picture of the concept to the reader.

**Keywords:** Mixed neutrosophic topology, Connectedness in mixed neutrosophic topology.

**2020 Mathematics Subject Classification Numbers:** 54A05, 54C10, 54D30, 54D10.

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## On Theta Parametric Metric Spaces

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### Abstract

In this presentation, we talk about on  $\theta$ -parametric metric spaces defined via  $\beta$ -actions, and we examine the concepts of ward compactness and ward continuity in  $\theta$ -parametric metric spaces.

**Keywords:** Parametric metric Spaces,  $\beta$ -action, compactness.

**2020 Mathematics Subject Classification Numbers:** 54D30, 54A10

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## $T_D$ Preordered Spaces

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### Abstract

There is a one to one relation between topological space and a transitive spatial graph (preordered set) [1, 2]. Let  $(T, \delta)$  be a topological space. For all  $m, n \in T$ , the relation

$R_\delta$  defined as  $mR_\delta n \Leftrightarrow$  every open set  $G$  containing  $m$  includes  $n$

is a specialization relation.  $(T, R_\delta)$  is a transitive spatial graph[1, 6, 5].

Let  $(B, R)$  be a transitive spatial graph and  $M \subset B$ . The upward closure of  $M$  given by

$$\uparrow_B(M) = \{b \in B : \exists a \in M, aRb\}[4].$$

The Alexandroff topology  $\delta_{alex}$  generated by the relation  $R$  and taking  $\mathcal{B} = \{\uparrow b : b \in B\}$  as a basis is the finest topology on  $B$ .

Let  $(A_1, R_1)$  and  $(A_2, R_2)$  be transitive spatial graphs. If  $f(a_1)R_2f(b_1)$  when  $a_1R_1b_1$  for all  $a_1, b_1 \in A_1$  then, the function  $f : (A_1, R_1) \rightarrow (A_2, R_2)$  is called a function preserving relation. Furthermore,  $f : (A_1, \delta_{R_1}) \rightarrow (A_2, \delta_{R_2})$  is continuous if  $f : (A_1, R_1) \rightarrow (A_2, R_2)$  is a function preserving relation.

There is a fully faithful functor from preordered sets to topological spaces that equips a preordered set with its Alexandroff topology.

A topological space  $(T, \delta)$  is called  $T_D$  if there exists an open set  $G$  containing  $t$  with  $G \setminus \{t\}$  is open for all  $t \in T$  [3]. Note that  $T_D$  is between  $T_1$  and  $T_0$ .

In this paper, we characterize  $T_D$  transitive spatial graphs as well as we examine the relationships between  $T_D$  and each of  $T_0$  and  $T_1$ .

**Keywords:** Transitive spatial graphs,  $T_D$  topological spaces, topological category .

**2020 Mathematics Subject Classification Numbers:** 54A05, 54B30, 18D15, 54D10.

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## $\widetilde{Spec}(M)$ Topology of Module $M$ Over Commutative Ring

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### Abstract

In this paper, we construct the topological space  $\widetilde{Spec}(M)$  with open sets of the form  $\widetilde{D}(S) := \{P \in \widetilde{Spec}(M) | (P : M) \cap S = \emptyset\}$  where  $S$  is a multiplicatively closed subset of a commutative ring  $R$  with unity using the prime spectrum of an  $R$ -module  $M$ . Moreover, we focus on various topological properties of this space and give equivalent conditions for  $\widetilde{Spec}(M)$  to be a locally compact and quasi-compact space. Furthermore, we also investigate equivalent conditions for it to be a discrete, Hausdorff, connected and regular space.

**Keywords:** Prime spectrum, multiplicatively closed subset, prime avoidance, connected space, discrete space.  
**2020 Mathematics Subject Classification Numbers:** 13C99, 13J99, 54B99 54D05.

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## Lacunary Statistically Forward Quasi Cauchyness in Asymmetric Metric Spaces

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### Abstract

We call a sequence  $(x_m)$  of points in an asymmetric metric space  $(X, d)$  lacunary statistically forward quasi Cauchy if

$$\lim_{r \rightarrow \infty} \frac{1}{h_r} |\{m \in I_r : d(x_m, x_{m+1}) \geq \varepsilon\}| = 0$$

for each positive  $\varepsilon$ , where  $I_r = (k_{r-1}, k_r]$  and  $k_0 = 0$ ,  $h_r = k_r - k_{r-1} \rightarrow \infty$  as  $r \rightarrow \infty$  and  $\theta = (k_r)$  is an increasing sequence of positive integers, and  $|A|$  indicates the cardinality of the set  $A$ . We prove that a subset  $E$  of  $X$  is forward totally bounded if and only if any sequence of points in  $E$  has a lacunary statistically forward quasi Cauchy subsequence. We also introduce and investigate lacunary statistically upward continuity in the sense that a function defined on  $X$  into  $Y$  is called lacunary statistically upward continuous if it preserves lacunary statistically forward quasi Cauchy sequences, i.e.  $(f(x_m))$  is lacunary statistically forward quasi Cauchy whenever  $(x_m)$  is.

**Keywords:** Keyword one, keyword two, keyword three.

**2020 Mathematics Subject Classification Numbers:** First, Second, Third.

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## On Abel Statistical Convergence in a Metric Space

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### Abstract

In this study we first consider the sequences in the sense of Abel statistical together with the functions preserving the convergence of this kind of sequences called Abel statistical continuous functions in a metric space  $X$ . Then we relate this kind of continuity with some others. A function  $f$  is Abel statistically continuous on a subset  $E$  of a metric space  $X$ , if it preserves Abel statistical convergent sequences, i.e.  $(f(p_k))$  is Abel statistically convergent whenever  $(p_k)$  is an Abel statistical convergent sequence of points in  $E$ , where a sequence  $(p_k)$  of points in  $X$  is called Abel statistically convergent to a point  $L$  in  $X$  if  $\lim_{x \rightarrow 1^-} (1-x) \sum_{k \in \mathbb{N}: d(p_k, L) \geq \varepsilon} x^k = 0$  for every  $\varepsilon > 0$ . Some other types of continuities are also studied and interesting results are obtained.

**Keywords:** Keyword one, keyword two, keyword three.

**2020 Mathematics Subject Classification Numbers:** First, Second, Third.

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## A Note on Preordered Spaces

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### Abstract

Order theory is a branch of mathematics that studies various kinds of binary relations that capture the intuitive notion of a mathematical ordering. Orders appear everywhere – at least as far as mathematics and related areas, such as computer science, are concerned [1].

The category **Prord** of preordered sets has as objects the pairs  $(B, R)$  where  $B$  is a set and  $R$  is reflexive and transitive relation on  $B$ , and as morphisms  $(B, R) \rightarrow (B_1, R_1)$  those functions  $f : B \rightarrow B_1$  such that if  $aRb$ , then  $f(a)R_1f(b)$  for all  $a, b \in B$ . Note that **Prord** is a topological category over **Set** [1, 2].

Recall that a topological space is called pre-Hausdorff (pre $T_2$ ) if whenever for any two distinct points there is a neighborhood of one missing the other, then the two points have disjoint neighborhoods [3, 5]. Two generalizations of the pre-Hausdorff objects in an arbitrary topological category were introduced [3] and the relationship between them was investigated in [4].

In this paper, we characterize pre-Hausdorff preordered spaces and investigate the relationship between them as well as the relationship between pre-Hausdorff topological spaces.

**Keywords:** Topological category, preordered spaces, pre-Hausdorff spaces.

**2020 Mathematics Subject Classification Numbers:** 54A05, 54B30, 18D15, 54D10.

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## $T_{sc}$ Reflexive Relations Spaces

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### Abstract

Let  $X$  be a nonempty set and  $R \subset X \times X$ . For all  $x \in X$  if  $xRx$ , then  $(X, R)$  is said to be a reflexive relation space. Let  $(X, R)$  and  $(Y, S)$  be reflexive relation spaces. If  $f(x)Sf(y)$  when  $xRy$  for all  $x, y \in X$  then, the function  $f : (X, R) \rightarrow (Y, S)$  is called a function preserving relation [1].

Let  $(B, R)$  be a reflexive relation space and  $M$  is nonempty subset of  $B$ .  $M$  is closed if and only if for each  $x \in B$  and there exist  $a, b \in M$  such that  $xRa$  or  $bRx$  then  $x \in M$  [3]. The closure of  $M$  is the intersection of all closed subsets of  $B$  containing  $M$ , and it is denoted by  $cl(M)$ .

Recall that a topological space  $(X, \tau)$  is said to be  $T_{sc}$  if the singleton closure property holds, i.e., for all  $x, y \in X$ ,  $x \in \overline{\{y\}} \Rightarrow \overline{\{x\}} = \overline{\{y\}}$ . Note that there is no relation between  $T_0$  and  $T_{sc}$  [2].

Let  $|X| \geq 2$  and  $\tau$  be the indiscrete topology on  $X$ . Then,  $(X, \tau)$  is not  $T_0$ . To see it is  $T_{sc}$ , let  $x, y \in X$  with  $x \in \overline{\{y\}}$  and suppose  $y \notin \overline{\{x\}}$ . Then  $X \setminus \{x\}$  is an open set other than  $\emptyset$  and  $X$ .

On the other hand, let  $X = \{a, b\}$  nad  $\tau = \{\emptyset, X, \{a\}\}$  be a topology on  $X$ . Then  $(X, \tau)$  is  $T_0$ , but not  $T_{sc}$ . Note that  $\overline{\{b\}} = \{b\}$  and  $\overline{\{a\}} = \{a, b\}$ . Hence,  $b \in \overline{\{a\}}$  but  $a \notin \overline{\{b\}}$ . Also, for a topological space  $(X, \tau)$  is  $T_1$  if and only if  $(X, \tau)$  is  $T_0$  and  $T_{sc}$  [2].

In this paper, we characterize  $T_{sc}$  reflexive relation spaces as well as we examine the relationships between  $T_{sc}$  and each of  $T_0$  and  $T_1$ .

**Keywords:** Reflexive relation spaces,  $T_{sc}$  topological spaces, topological category, closure operator.

**2020 Mathematics Subject Classification Numbers:** 54A05, 54B30, 18D15, 54D10.

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## Ordered $\gamma$ -Approximation Spaces Based on Different Types of $\gamma$ -Neighborhood Classes

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### Abstract

Rough set theory, introduced by Pawlak in 1982 based on equivalence relations, has been extended through the use of more general types of relations to address a wider range of problems. A significant contribution in this direction was made by Yao in 1998 and further developed by Atef et al. in 2020 with the introduction of  $\gamma$ -neighborhoods. Relations such as equality, inclusion, and subset have been used to define new types of  $\gamma$ -neighborhood classes, which in turn have enabled the development of approximation spaces that extend the capabilities of classical rough set theory. In 2023, the increasing and decreasing equivalence classes were introduced by jointly employing the concepts of partial order relations and equivalence relations by Shalil et al.. Then, we proposed new  $\gamma$ -neighborhood classes termed ordered  $\gamma$ -neighborhood classes, which integrate  $\gamma$ -neighborhoods with partial order relations. Subsequently, we defined generalized approximation spaces called ordered  $\gamma$ -approximation spaces, which are based on ordered  $\gamma$ -neighborhood classes. In this study, we aim to further generalize these ordered  $\gamma$ -approximation spaces by using different types of  $\gamma$ -neighborhood classes based on equality, inclusion, and subset relations. Firstly, we introduce new ordered  $\gamma$ -approximation spaces based on new ordered  $\gamma$ -neighborhood classes, which are called ordered  $A_\gamma$ ,  $C_\gamma$ , and  $S_\gamma$  approximation spaces. Their basic properties are studied, and the connections among them are illustrated. Additionally, the relationships between these approximations and the earlier ones are examined and compared. Finally, a practical example is provided to show the importance of applying these approximations.

**Keywords:** generalized rough set, ordered  $\gamma$ -neighborhood classes, ordered  $\gamma$ -approximation spaces, ordered  $A_\gamma$  approximation spaces, ordered  $C_\gamma$  approximation spaces, ordered  $S_\gamma$  approximation spaces

**2020 Mathematics Subject Classification Numbers:** 54F05, 03E20

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## Quasi Divisor Topology of Modules over Domains

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### Abstract

Let  $E$  be a module over a domain  $A$ , and  $W(E)^\# = W(E) - \text{ann}(E)$  where  $W(E) = \{a \in A : aE \neq E\}$ . We define an equivalence relation  $\sim$  on  $W(E)^\#$  as follows:  $a \sim b$  if and only if  $aE = bE$  for any  $a, b \in W(E)^\#$ . Moreover, we denote  $\text{EC}(W(E)^\#)$  to be the set of all equivalence classes  $[a]$  of  $W(E)^\#$ . In what follows, we first show that the family  $\{U_a\}_{a \in W(E)^\#}$  generates a topology which is called quasi divisor topology of  $A$ -module  $E$  denoted by  $qD_A(E)$  in the sense of Steen where  $U_a = \{[b] \in \text{EC}(W(E)^\#) : aE \subseteq bE\}$  for every  $a \in W(E)^\#$ . In this paper, we examine the connections between topological properties of the quasi divisor topology  $qD_A(E)$  and algebraic properties of  $A$ -module  $E$ . In particular, we characterize some important class of rings/modules such as simple, second and divisible modules by means of  $qD_A(E)$ .

**Keywords:** divisor topology, quasi divisor topology, divisible module

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## ***G-Convergence of Submethods***

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### **Abstract**

Classical topological notions such as openness, closedness, and convergence can be expressed in sequence form, particularly in the case of first-countable Hausdorff spaces. Motivated by this idea, a generalized convergence method known as the *G*-method, have been introduced. Different studies related to the *G*-method have been widely explored in recent literature, including studies on *G*-continuity, *G*-compactness, and other related topological properties.

In this study, we extend the framework of *G*-convergence on a set  $X$  by introducing the notion of  $G_Y$ -submethod induced on a subset  $Y$ . We examine the behavior of  $G_Y$ -open and  $G_Y$ -closed subsets under these submethods. Several characterizations are provided to determine when  $G_Y$ -closedness and  $G_Y$ -openness are preserved within submethods. In addition, we also investigate the preservation of *G*-compactness, and separation axioms such as  $G$ - $T_0$ ,  $G$ - $T_1$ , and  $G$ -Hausdorff for *G*-submethods.

**Keywords:** *G*-convergence, *G*-submethod,  $G_Y$ -closed subset,  $G_Y$ -open subset

**2020 Mathematics Subject Classification Numbers:** Primary 40J05; Secondary 54A05, 22Axx.

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## ***G-Sequential Convergence in Submethods***

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### **Abstract**

Motivated by the classical definition of the convergence in topological spaces, we define the notion of *G*-sequential convergence for a method *G* given not only on a topological space but also on a set *X*. We call a sequence to be *G*-sequentially convergent to a point *a*, if all of its terms eventually lie in each *G*-open neighborhood of the point. The collection of such *G*-sequentially open sets generates a topology, referred to be *G*-sequential topology.

Within this framework, we investigate the concept of *G*-sequential submethod, induced on a subset  $Y \subseteq X$ . In particular, we characterize *G*-sequential subspaces and explain when the subspace topology on a subset  $Y \subseteq X$  derived from the *G*-sequential topology on *X* coincides with the  $G_Y$ -sequential topology induced by the submethod. We further examine the behavior of *G*-sequentially closed and *G*-sequentially open sets within subspace. This analysis gives a foundation for understanding how *G*-sequential convergence properties are preserved in submethods, and gives new perspectives on *G*-sequential space.

**Keywords:** *G*-sequential convergence, *G*-sequential space, *G*-sequentially closed subset.

**2020 Mathematics Subject Classification Numbers:** Primary 40J05; Secondary 54A05, 22Axx.

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## **$\delta$ -Quasi Cauchy Sequences in Topological Vector Spaces**

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### **Abstract**

In this paper, we call a function defined on a subset  $E$  of a topological vector space  $X$   $\delta$ -ward continuous if it preserves  $\delta$ -quasi-Cauchy sequences where a sequence  $(x_n)$  is defined to be  $\delta$ -quasi-Cauchy if the sequence  $(\Delta x_n)$  is quasi-Cauchy. It turns out that  $\delta$ -ward continuity implies uniform continuity, but there are uniformly continuous functions which are not  $\delta$ -ward continuous. A new type of compactness in terms of  $\delta$ -quasi Cauchy sequences, namely  $\delta$ -ward compactness is also introduced, and some theorems related to  $\delta$ -ward continuity and  $\delta$ -ward compactness are obtained.

**Keywords:** Quasi cauchy sequences, ward continuity, delta ward compactness.

**2020 Mathematics Subject Classification Numbers:** 54A20.

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## 6.2 Analysis and Functional Analysis

Session Organizers: Hacer Şengül Kandemir and Nazlım Deniz Aral

The Analysis and Functional Analysis session of the 9th International Conference of Mathematical Sciences (ICMS 2025) aims to provide a platform for presenting and discussing the most recent developments in a wide variety of topics including Laplace-Hankel Transforms, Functional Autoregressive Processes, generalized Kolmogorov systems, mixed boundary conditions involving the friction, ergodic theorem, higher-order fuzzy difference equation, the periodic unfolding method, operational second-order differential equations of elliptic type, doubly anisotropic problem of Kirchhoff-type, Volterra type integral equation, semilinear differential equations, Equi-Weyl Dynamics and PyTorch Simulation, Rainville type generating functions, weighted N2-Biharmonic equations, applications of the Schwarz lemma, neutrosophic normed spaces, neutrosophic statistical analysis, polynomial approximations, modified Meyer-König and Zeller operators, fuzzy medical feature analysis, reverse Hölder Inequality and Fibonacci numbers, continuous and discontinuous quartic perturbations, Zygmund Space.

This session offers a valuable forum for early-career researchers to advance their scholarship and formulate novel open problems in their areas of specialization. We believe that a substantial proportion of the presentations merits consideration for publication in high-quality mathematical journals.

## On the strongly $p$ - summing of some non-linear operators

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### Abstract

Let  $T$  be the bounded particular non- linear operators from a Banach space  $X$  into a complete Banach lattice  $Y$ . In the present paper, we introduce to this category the concept of strongly  $p$ -summing of the parrticular non linear operators. We give an analogue to Pietsch's domination theorem and study some comparisons property .

**Keywords:** Banach lattice, Hahn Banach theorem, linear operator, non- linear operator, strongly p-summing linear operator.

**2020 Mathematics Subject Classification Numbers:** 46A40, 46B40] 46A32, 46M05,

## References

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## S-asymptotically $\omega$ Periodic Solutions of Heat Equation

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### Abstract

The continuous and bounded function  $u$  from  $\mathbb{R}_+$  to a Banach space  $E$  is called *s-asymptotically  $\omega$ -periodic* if we have:

$$\lim_{t \rightarrow \infty} (u(t + \omega) - u(t)) = 0.$$

We denote by  $SAP_\omega(E)$ , the space of such functions.

We study in this paper the existence, uniqueness and differentiable dependence on the S-asymptotically  $\omega$ -periodic mild solution of a heat equation for  $E = C_0(\Gamma, \mathbb{R})$  and  $\Gamma$  a bounded nonempty open subset of  $\mathbb{R}^n$  with a Lipschitzian boundary. The dependence of the solution concerns the initial conditions, more precisely when the initial conditions is a S-asymptotically  $\omega$ -periodic function we study the differentiable dependence of the S-asymptotically  $\omega$ -periodic solution of heat equation. To show our main result in this work we introduce the properties of the superposition operator, or also called Nemytskii operator, in the space of S-asymptotically  $\omega$ -periodic functions. The notion of derivation for the last operator will also be highlighted. We also use in this paper semi-groups which have become important tools for differential equations. In this study, our focus shifted from seeking an s-asymptotically  $\omega$ -periodic solution for our heat equation, which is a problem in dynamic systems, to one in functional analysis. Specifically, we establish an operator from  $E \times SAP_\omega(E)$  to  $SAP_\omega(E)$ . Our strategy involves employing the implicit function theorem on this operator to accomplish the goal outlined in our main theorem. In fact our theorem gives conditions to ensure that around a mild S-asymptotically  $\omega$ -periodic solution  $u_0$  of our heat equation when the initial value is  $\psi_0$ , when  $\psi$  is near to  $\psi_0$ , there exists  $u$  a mild S-asymptotically  $\omega$ -periodic solution for  $\psi$ , and  $u$  is a  $C^1$ -mapping of  $\psi$ .

**Keywords:** S-asymptotically  $\omega$ -periodic functions, Heat equation, Implicit function theorem.

**2020 Mathematics Subject Classification Numbers:** 34A12, 35K05, 26B10.

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## Numerical Study of the Navier-Stokes Equations for Film Cooling in Gas Turbine Blades

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### Abstract

This study presents a numerical investigation of film cooling performance on gas turbine blades, focusing on a NACA 6512 airfoil equipped with both cylindrical and rectangular cooling holes. The simulation is conducted using ANSYS Fluent, solving the steady-state, compressible Navier-Stokes equations coupled with the energy equation via the Finite Volume Method (FVM). A k-w SST turbulence model is employed to accurately capture the flow behavior and thermal boundary layers. Various cooling configurations are analyzed to assess the impact of hole geometry and jet injection on cooling effectiveness, flow distribution, and aerodynamic performance. The results include temperature and velocity contour plots, highlighting the interaction between coolant jets and the freestream. The study concludes with a comparative analysis of cooling effectiveness and surface coverage between cylindrical and rectangular holes, offering valuable insights into optimizing cooling strategies for high-performance turbine blade designs.

**Keywords:** Film cooling, Ansys fluent, Turbine blade, Numerical simulation.

**2020 Mathematics Subject Classification Numbers:** 46N40

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## Solvability of higher-order BVPs of Hadamard fractional differential equations on the half-line

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### Abstract

Fractional calculus began with the correspondence between Leibniz and L'Hospital in 1695. In the case of continuity, studies on fractional calculus have gained great popularity since the end of the 19th century with the contribution of many mathematicians. Although there are different definitions for fractional derivatives, the most used ones are Riemann-Liouville and Caputo (See [1, 2]). Apart from those fractional derivatives, there are also several kinds of fractional derivatives such as Hadamard, Hilfer.

Unlike Riemann-Liouville and Caputo fractional derivatives, this study will address a boundary value problem involving Hadamard fractional derivatives, which incorporate logarithmic functions in their kernel. We derive the Green's function associated with the proposed fractional boundary value problem and employ the Bai-Ge fixed point theorem [3]. It is aimed to ensure the existence and uniqueness results of solutions for the nonlinear fractional boundary value problem.

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**Keywords:** Positive solutions, Existence results.

**2020 Mathematics Subject Classification Numbers:** 34A08, 26A33.

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## Reverse Hölder Inequality and Fibonacci numbers

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### Abstract

In this talk, we introduce a sequence of inequalities for power sums, derived using reverse Hölder inequalities. We apply these results to power sums whose terms are composed of Fibonacci numbers.

**Keywords:** Hölder inequality, Fibonacci sequence, power sum.

**2020 Mathematics Subject Classification Numbers:** 26A51, 11B39.

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## Riemann–Hilbert Problem in Polydomains

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### Abstract

The Riemann–Hilbert problem for holomorphic functions in higher-dimensional polydomains is investigated, and an explicit, constructive solution is derived. Furthermore, a rigorous connection between the classical Riemann problem and the Riemann–Hilbert problem in the context of polydomains is established and demonstrated. Unlike previous approaches, our results yield explicit formulas and do not rely on restrictive or artificial assumptions. Moreover, the study adapts the Cauchy-type integral representations and a higher-dimensional Plemelj–Sokhotski formula specifically tailored to polydomains, ensuring well-posed solvability conditions and compact solution notation. This leads to a canonical and symmetry-preserving representation of both homogeneous and inhomogeneous boundary-value problems in several complex variables.

**Keywords:** Riemann–Hilbert problem, Cauchy-type integral representations, Polydomain.

**2020 Mathematics Subject Classification Numbers:** F35J40, 35C15, 35J15.

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# Classification of phase portraits for a certain class of generalized Kolmogorov systems

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## Abstract

We Consider a seven-parameter family of planar Kolmogorov systems. These systems obtained from the Lotka–Volterra systems. The polynomial differential systems can be extended to infinity and in this way we can study their dynamics in a neighborhood of infinity. Which called the closed Poincaré disc. There are many works about the global dynamics of certain Lotka–Volterra families depending on a small number of parameters. In this work, we give a distinct classification of the phase portraits in the Poincaré disc

**Keywords:** Kolmogorov system, global phase portrait, Poincaré disc.

**2020 Mathematics Subject Classification Numbers:** Primary 34C05.

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## Viability result for Volterra type integral equation

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### Abstract

Viability characterizes the connection between a dynamic and a given constraint to guarantee the existence for any initial state in the constraint, of at least one solution starting from that initial state and such that its graph lies in the constraint for some time. Here, consider the Volterra integral equation of the form:

$$y(t) = x + \int_{t_0}^t (t-s)^{q-1} f(s, y(s)) ds$$

where  $f : [a, b] \times \mathbb{R}^n \rightarrow \mathbb{R}^n$  is a given function and the constraint is defined by  $G = [a, b] \times K$  where  $K \subset \mathbb{R}^n$ , we introduce an appropriate tangency condition and provide necessary and sufficient conditions for viability. As application, we provide a theorem that insures the existence of positive solutions.

**Keywords:** Volterra equation, Viability, Tangency condition.

**2020 Mathematics Subject Classification Numbers:** 34C25, 34A60.

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## On the dynamics of a family of planar differential systems with two cycles explicitly given

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### Abstract

In this work, we study a family of multi-parameter polynomial differential systems of degree eleven. We prove that the considered family has an invariant algebraic curve, which is given explicitly. Subsequently, we demonstrate the integrability of these systems and derive an explicit expression for a first integral. Moreover, we provide sufficient conditions for the systems to possess two limit cycles explicitly given: one is algebraic, and the other is shown to be non-algebraic. The applicability of our results is illustrated by concrete examples.

**Keywords:** Dynamical system, limit cycle, first integral.

**2020 Mathematics Subject Classification Numbers:** 34C05, 34C07, 37C27.

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**A morpho-metric and ecological study of *Allium polyanthum* (Amaryllidaceae) in the region  
 -Tlemcen – Algeria**

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**Abstract**

Alliums are famous for their distinctive smell and taste, due to sulfur compounds (cysteine sulfoxides), which are converted into volatile sulfides by the alliinase enzyme during cutting or grinding [1][2].

The Allium genus comprises between 260 and 850 species, depending on classification, mainly found in temperate regions of the northern hemisphere (Eurasia, North America), but also present in Africa and South America[1][2]. Among the best-known species are garlic (*Allium sativum*), onion (*Allium cepa*), shallot (*Allium ascalonicum*), leek (*Allium porrum*), chives (*Allium schoenoprasum*) and giant garlic (*Allium giganteum*) [1][2].

This reaction notably releases allicin, responsible for the pungent aroma and antibacterial and antifungal properties

*Allium polyanthum* Schult. & Schult.f. A species of the Allium genus (family Amaryllidaceae, formerly Liliaceae), found mainly in the Mediterranean region, including western Algeria:

In this work, we studied a taxon recently reported in Algeria: *Allium polyanthum*. Through this study, the taxonomic study is essential to show the recent nomenclature, and the ecological and morpho-metric study of 48 samples of *Allium polyanthum* allowed us to know the state and the diameters of the organs of the plant where we followed the method of the regression line according to the measured parameters and compared the various types of correlation which could exist between them, then we were able to make descriptions of *Allium polyanthum* from these data obtained. Finally, for the distribution of this *Allium* in the Tlemcen-Algeria region, we chose 11 study stations well distributed throughout the region.

**Keywords:** *Allium polyanthum*, morpho-metry, taxonomy, correlation, Tlemcen, Algeria.

**2020 Mathematics Subject Classification Numbers:** 62M10, 91B82

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## A new theorem generalization on absolute matrix summability of Fourier series

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### Abstract

We generalize a main theorem dealing with weighted mean summability of Fourier series to the  $|A, \varphi_n; \delta|_k$  summability factors of Fourier series. Also, some new and known results are obtained.

**Keywords:** Weighted mean, matrix summability, summability factor, infinite series, Fourier series, Hölder inequality, Minkowski inequality, sequence space.

**2020 Mathematics Subject Classification Numbers:** 26D15, 40D15, 40F05, 40G99, 42A24, 46A45.

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## Matrix Mappings and Norms on the Absolute Cesàro and Nörlund spaces

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### Abstract

In this study, we characterize the classes of matrix mappings between the absolute Cesàro  $|C_{-1}|_k$  and the absolute Nörlund  $|N_p^u|_k$  series spaces for  $k \geq 1$ . We also obtain estimates for the norms of bounded linear operators corresponding to matrices in these classes. Thus, not only are some open problems solved, but some well-known results are also extended.

**Keywords:** Matrix mappings, Norms, Absolute summability methods.

**2020 Mathematics Subject Classification Numbers:** 40C05, 46A45, 40F05.

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## Neutrosophic Statistical Analysis of the Effect of Meteorological Elements on Earthquakes in Türkiye

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### Abstract

In this study, we examines the relationship between meteorological elements (temperature, precipitation, and pressure) and earthquakes of magnitude 5 and greater that occurred in Türkiye between 2003 and 2023. Data from 30 days before the earthquake occurred were used, and the neutrosophic standard deviation and neutrosophic coefficient of variation were calculated for the data with maximum and minimum value intervals, and the indeterminacy interval was obtained for each step. The impact of meteorological variables on earthquake magnitude was examined using both the classical multiple linear regression model and an alternative approach proposed within the scope of the this study, namely the neutrosophic regression method.

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**Keywords:** Neutrosophic sets, Linear Regression, Neutrosophic Regression

**2020 Mathematics Subject Classification Numbers:** 03B52, 62J05.

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## On the global behavior of a higher-order fuzzy difference equation

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### Abstract

The study of the qualitative behavior of fuzzy difference equations and systems has become a subject of increasing interest in recent years. In this work, we deal with the global dynamics of an abstract higher-order fuzzy difference equation. More precisely, we prove the existence of positive solutions of this equation. We obtain the criteria in order that the equation has a unique global attractor equilibrium point. Additionally, we examine the equilibrium point's asymptotic stability in the case of homogeneous functions. Finally, we present some numerical examples to illustrate our theoretical results.

**Keywords:** Fuzzy difference equation, Fuzzy number, Homogeneous function, Equilibrium point, Attractivity, Asymptotic stability.

**2020 Mathematics Subject Classification Numbers:** 39A05, 39A26, 39A30.

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## Numerical method for solving the Volterra Integro-differential forms of the Lane-Emden equations

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### Abstract

In this work, we investigate the Lane-Emden differential equations by transforming them into integro-differential equations, aiming to obtain approximate solutions. We employ the Collocation-Touchard method and compare the results with exact solutions to assess the accuracy and effectiveness of the proposed approach..

**Keywords:** integro-differential equations, Lane-Emden equation, numerical methods.

**2020 Mathematics Subject Classification Numbers:** 47Gxx, 65R20.

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## A random ergodic theorem

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### Abstract

We investigate the asymptotic behavior of a class of ergodic means obtained by taking the expectation of a randomly selected group of unitary operators. Using tools from spectral theory, we provide sufficient conditions ensuring the pointwise convergence of these averages when the sequence of selectors is a  $\mathbb{Z}^d$ -valued random walk. We also derive a rate of convergence for these means via the Rademacher-Menchoff theorem on orthogonal series

**Keywords:** Ergodic averages, unitary group action, random sampling.

**2020 Mathematics Subject Classification Numbers:** 47A35, 37A50.

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## Optimization of second-order differential inclusions of Lagrange problem with delay

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### Abstract

Optimization problems with second order differential inclusions is of interest in recent years due to their use in practice [1, 3, 4]. Many articles have been devoted to delayed optimal control problems and obtaining optimality conditions [2, 5, 6, 7]. There are numerous real-world problems, such as cancer chemotherapy, industrial engineering, management sciences and aquaculture, that find their counterparts in the optimal control problems with delay.

In this paper, optimality conditions for the Lagrange problem with second-order differential inclusions (DFIs) with delay and boundary conditions are obtained. The acceleration in the handled problem depends not only on the state and velocity at this instant, but also upon the history of the trajectory until this instant. We use the term Locally adjoint mapping(LAM)[5] to determine adjoint discrete inclusions. Some equivalence relations are applied to derive necessary and sufficient conditions for the discrete approximation problem. Passing to the limit, sufficient optimality conditions are established for the Lagrange optimal problem described by second-order DFIs with delay. Similar results for the non-convex problem are obtained by using the local tents and convex upper approximation. We give an example problem with initial conditions for which our results can be applied. We hope that with this study we will contribute to the development of optimal control theory with delayed differential inclusions (in other words functional differential inclusions).

**Keywords:** functional differential inclusion, Euler-Lagrange inclusion, Locally adjoint mapping.  
**2020 Mathematics Subject Classification Numbers:** 34A60, 34K09, 49K21.

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## Multiplication Operators on the Zygmund Space

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### Abstract

Let  $\mathbb{D}$  be the open unit disc in the complex plane  $\mathbb{C}$ . Denote by  $H(\mathbb{D})$  the class of all complex-valued functions analytic on  $\mathbb{D}$ . For a holomorphic function  $\psi$  on  $\mathbb{D}$  the linear operator  $M_\psi$  defined by  $M_\psi f = \psi f$  for all  $f \in H(\mathbb{D})$  is called multiplication operator with symbol  $\psi$ . Denote by  $\mathcal{Z}$  the class of all  $f \in H(\mathbb{D}) \cap C(\overline{\mathbb{D}})$  such that

$$\sup \frac{|f(e^{i(\theta+h)}) + f(e^{i(\theta-h)}) - 2f(e^{i\theta})|}{h} < \infty$$

where the supremum is taken over all  $e^{i\theta} \in \partial\mathbb{D}$  and  $h > 0$ . We call  $\mathcal{Z}$  the Zygmund space.

In this presentation, we establish bounds on the norm of multiplication operators on the Zygmund space of the unit disk via weighted composition operators. Also the power boundedness and mean ergodicity of multiplication operators are investigated on the Zygmund space  $\mathcal{Z}$ .

Recently, there have been numerous papers on various aspects of classes of operators on Zygmund spaces. Composition operators, weighted composition operators, and related operators acting between the Zygmund space and certain analytic function spaces have been studied in [1, 5, 3, 4, 2, 6].

**Keywords:** Weighted composition operator, Multiplication operator, Zygmund space.

**2020 Mathematics Subject Classification Numbers:** 47B38, 47A35, 30H05.

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# Ground state solutions in weighted $\frac{N}{2}$ -Biharmonic equations with critical exponential growth

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## Abstract

This paper investigates a fourth-order weighted equation represented as

$$\Delta(w(x)|\Delta u|^{\frac{N}{2}-2}\Delta u) = f(x, u) \quad \text{in } B, \quad u = \frac{\partial u}{\partial n} = 0 \quad \text{on } \partial B,$$

where  $B$  denotes the unit ball in  $\mathbb{R}^N$  and  $w(x)$  is a singular logarithmic weight. The nonlinearity is critical in view of exponential inequality of Adams' type. The proof of existence is established through constrained minimization within the Nehari set, coupled with the application of quantitative deformation lemma and degree theory results.

**Keywords:** Weighted Sobolev space, Biharmonic operator, Adam's inequality, Critical exponential growth.  
**2020 Mathematics Subject Classification Numbers:** 35J20, 35J30, 35K57, 35J60.

## References

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## The periodic unfolding method for the Stokes problem in perforated domain

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### Abstract

The aim of this work is to study the homogenization of the Stokes problem in a domain of  $\mathbb{R}^N, N \geq 3$ ,  $\varepsilon$ -periodically perforated by holes of size  $\varepsilon\delta_1$  and  $\varepsilon\delta_2$  whose diameters go to zero with  $\varepsilon \rightarrow 0$ . We use the periodic unfolding method introduced by Cioranescu, Damlamian and Griso in [1] which allows to consider a general geometric framework.

**Keywords:** homogenization, periodic unfolding, small holes, Stokes system.

**2020 Mathematics Subject Classification Numbers:** 74Q05, 34M40.

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## Stability results for Laminated Beams Subject to Multiple Damping Types

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We present a laminated beam which consist of two identical layers of uniforme thickness, which are modeled as Timoshenko beams. A thin adhesive bonds the two layers and creates a recovery force that is proportional to the amount of slip and produces damping capable of returning the system to its equilibrium state, as is used in many scientific fields. We have three motion modeling equations for this system, which have been derived from Timoshenko's theory. The third equation is coupled with the first and second equations, and it describes the dynamics of the slip, as follows:

$$\begin{cases} \rho_1 \varphi_{tt} + k(u - \varphi_x)_x = 0 \\ \rho_2(v - u)_{tt} - b(v - u)_{xx} - k(u - \varphi_x) = 0 \\ \rho_2 v_{tt} - bv_{xx} + 3k(u - \varphi_x) + 4\delta v + 4\gamma v_t = 0 \end{cases} \quad (1)$$

Where  $(x, t) \in (0, 1) \times \mathbb{R}_+$ ,  $\varphi = \varphi(x, t)$  is the transverse displacement,  $u = u(x, t)$  represents the angle of rotation, and  $v = v(x, t)$  is proportional to the amount of slip along the interface,  $\rho_1$ ,  $\rho_2$ ,  $k$ ,  $b$ ,  $\delta$ , and  $\gamma$  are positive constants.

In this presentation we will show the stability of this system with different damping.

**Keywords:** Laminated Beams, Lyapunov Function, Stability.

**2020 Mathematics Subject Classification Numbers:** 35L51, 35B35, 93D05.

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## Statistical analysis of riparian forests in the Tlemcen region, western Algeria

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### Abstract

Riparian forests play a fundamental ecological role as buffer zones, biodiversity reservoirs and ecological corridors along watercourses. However, they are increasingly threatened by human activities and climate change. The aim of this study is to characterize and map the floristic diversity of riparian forests using Geographic Information System (GIS) tools, using a section of the Oued Tafna River in the Tlemcen region as a case study.

The methodological approach combines floristic field surveys, spatial data (satellite images, DTM) and spatial analyses to assess the structure, composition and conservation status of riparian formations. The results highlight the specific richness and spatial distribution of plant groups, as well as the pressures weighing on these sensitive environments.

The use of GIS enabled us to visualize, analyze and model landscape dynamics and priority ecological zones for the sustainable management of riparian ecosystems.

**Keywords:** Statistical analysis; Biodiversity; Riparian ecology

**2020 Mathematics Subject Classification Numbers:** 62-XX, 62Hxx, 62H25.

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## Existence results for a doubly anisotropic problem of Kirchhoff-type by a monotonicity method

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### Abstract

We consider in this work the following problem

$$\begin{cases} -\sum_{i=1}^N [M(\|\partial_i u\|_{p_i}^{p_i})]^{p_i-1} \partial_i(|\partial_i u|^{p_i-2} \partial_i u) - \sum_{i=1}^N [M(\|\partial_i u\|_{q_i}^{q_i})]^{q_i-1} \partial_i(|\partial_i u|^{q_i-2} \partial_i u) = f(x, u) & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases} \quad (1)$$

where  $\Omega$  is bounded domain in  $\mathbb{R}^N (N \geq 3)$  with smooth boundary  $\partial\Omega$ ,  $M$  is continuous positive function on  $\mathbb{R}_+$ , and  $p_i, q_i > 1$  for  $1 \leq i \leq N$ . By using notion of monotone operators, we prove some existence results.

**Keywords:** Kirchhoff-type problem, anisotropic problem, monotone operator.

**2020 Mathematics Subject Classification Numbers:** 76A05, 76D50, 35Q35.

## References

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## The role of imperfect vaccination and treatment in containing an epidemic for a delayed SVITR epidemic model

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### Abstract

This research is concerned with studying the global dynamics of a delayed SVIR epidemic model with two distributed delays. The vaccination considered in this research is imperfect, which means the vaccinated person can be infected after vaccination. The basic reproduction number  $R_0$  is successfully identified and interpreted. The threshold role of this number is also determined, wherein for  $R_0 < 1$ , we established the existence of a globally stable disease-free equilibrium. Conversely, for  $R_0 > 1$ , the model demonstrates permanence, exhibiting a distinct endemic equilibrium. Furthermore, we demonstrate the global asymptotic stability of this unique endemic equilibrium. A different perspective is also cited, which is determining the effect of extensive measures, vaccination, and treatment on containing the epidemic and determining the required public health intervention force for stopping the epidemic outbreak. The findings are checked numerically using graphical representations, with a brief discussion on the biological interpretation of the results.

**Keywords:** Distributed delays, vaccination, treatment.

**2020 Mathematics Subject Classification Numbers:** 92B.

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# On a Rainville type generating functions for Ultraspherical polynomials

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## Abstract

This work contributes to the characterisation of orthogonal polynomials with Rainville-type generating functions of the form  $B(t)F(xtA(t) - R(t))$ . In this paper, we fully classify Rainville-type generating functions for (monic) ultraspherical polynomials using algebraic calculus. Consequently, we get specific bilateral generating functions that involve ultraspherical and Chebyshev polynomials.

**Keywords:** Generating functions, Ultraspherical polynomials, Chebyshev polynomials, recurrence relations.

**2020 Mathematics Subject Classification Numbers:** 33C45, 33C20, 11B37.

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## Applications of the Cauchy-Schwarz inequality for numerical radius

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### Abstract

Let  $\mathbb{B}(\mathcal{H})$  denote the  $C^*$ -algebra of all bounded linear operators on a Hilbert space  $\mathcal{H}$ . For vectors  $x$  and  $y$  in  $\mathcal{H}$ , the Cauchy-Schwarz inequality says that

$$|\langle x, y \rangle| \leq \|x\| \|y\|.$$

There are several improvements and refinements of the Cauchy-Schwarz inequality. One of the operator extension of the Cauchy-Schwarz inequality states that

$$|\langle Ax, y \rangle|^2 \leq \langle |A|^{2\alpha} x, x \rangle \langle |A^*|^{2(1-\alpha)} y, y \rangle \quad \text{for } 0 \leq \alpha \leq 1,$$

where  $A \in \mathbb{B}(\mathcal{H})$  and  $x, y \in \mathcal{H}$ . The numerical radius of  $T \in \mathbb{B}(\mathcal{H})$  is defined by

$$\omega(A) := \sup\{|\langle Ax, x \rangle| : x \in \mathcal{H}, \|x\| = 1\}.$$

It is well known that the function  $\omega(\cdot)$  defines a norm on  $\mathbb{B}(\mathcal{H})$ , which is equivalent to the standard operator norm. Specifically, we have the inequalities  $\frac{1}{2}\|\cdot\| \leq \omega(\cdot) \leq \|\cdot\|$ . Additionally, there are several extensions and refinements of these inequalities, which are outlined as follows:

$$\frac{1}{4}\| |A|^2 + |A^*|^2 \| \leq \omega^2(A) \leq \frac{1}{2}\| |A|^2 + |A^*|^2 \|,$$

$$\omega^{2r}(A) \leq \frac{1}{2}\| |A|^{2r} + |A^*|^{2r} \| \quad \text{for all } r \geq 1.$$

and

$$\omega^{2r}(A) \leq \frac{1}{2}\| |A|^{2r} + \frac{1}{4}\| |A|^{2r} + |A^*|^{2r} \| \quad \text{for all } r \geq 1.$$

In this paper, we utilized a recent refinement of the Buzano inequality to establish new upper bounds for the numerical radius.

**Keywords:** numerical radius, Cauchy-Schwarz inequality, Buzano inequality.

**2020 Mathematics Subject Classification Numbers:** 47A60.

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## Non-Uniqueness of Solutions of the Gause System : Generalization to the Kolmogorov System

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### Abstract

The aim of this article is to generalize the non-uniqueness result from [2], initially established for the Gause model with singularities, to the Kolmogorov model, which admits the origin as a singular equilibrium. Additionally, we show that this generalization applies to systems beyond Gause's, particularly to those in which both the predator's functional response and death rates depend on density-a feature absent in Gause's model [3].

**Keywords:** Gause system, Kolmogorov system, non-uniqueness of solutions, singularities at the origin.

**2020 Mathematics Subject Classification Numbers:** 34D05, 34C11, 37C75, 92D25.

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## Blow-Up Phenomena in 2D Elliptic Systems for Unveiling Sub-Quadratic Convection Dynamics

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### Abstract

We consider the existence of blow up solution for a family of nonlinear elliptic system with sub-quadratic convection term and by using the nonlinear domain decomposition method.

$$\begin{cases} -\Delta u_1 - \lambda |\nabla u_1|^q = \rho^2 e^{\gamma u_1 + (1-\gamma)u_2} & \text{in } \Omega, \\ -\Delta u_2 - \lambda |\nabla u_2|^q = \rho^2 e^{\xi u_2 + (1-\xi)u_1} & \text{in } \Omega, \\ u_1 = u_2 = 0 & \text{on } \partial\Omega, \end{cases} \quad (1)$$

$\Omega \subset \mathbb{R}^2$  be a regular bounded domain in  $\mathbb{R}^2$ ,  $q \in [1, 2)$ . Here  $\rho, \gamma, \lambda$  and  $\xi$  are positive real constants. We assume that  $\gamma, \xi \in (0, 1)$  such that  $\gamma + \xi > 1$ , in the following  $\frac{1-\xi}{\gamma}$  and  $\frac{1-\gamma}{\xi} \in (0, 1)$ . Where  $\varepsilon$  is the smallest positive parameter satisfying  $\rho^2 = \frac{2\varepsilon^2}{(1 + \varepsilon^2)^2}$ .

**Keywords:** Liouville type theorem, Blow-up, singular elliptic equations, nonlinear elliptic equation, Green's function

**2020 Mathematics Subject Classification Numbers:** 35B53, 35B44, 35J75, 35J60, 35J08.

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# A Pursuit-Evasion Model with Time-Dependent Speed

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## Abstract

Optimal control problems with incomplete information, especially in pursuit–evasion settings, pose significant challenges. This work generalizes a basic time-optimal pursuit–evasion game by introducing a time-dependent pursuer speed,  $U_{\max}(t)$ , while keeping the observer parameter  $\lambda$  constant. This models realistic situations where the pursuer’s capability changes, e.g., due to fuel limits or terrain. An optimal control law is derived from current observations, yielding the best guaranteed capture time. For piecewise constant  $U_{\max}(t)$ , an explicit formula for the capture time is obtained. The results provide an analytical framework for handling uncertainty and dynamic constraints, with applications to robotics, autonomous navigation, and search-and-rescue tasks.

**Keywords:** Pursuit-Evasion, Time-Optimal Control, Differential Games.

**2020 Mathematics Subject Classification Numbers:** 49N75, 49N70, 49K15.

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## Approximation of Fuzzy Numbers by Modified Meyer-König and Zeller Operators: Implications for Medical Diagnosis Modeling

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### Abstract

In this study, we extended the Certain Modified Meyer-König and Zeller Operators to any compact interval. In this extension, we examined their approximation rates and properties. Additionally, we demonstrated that these operators preserve monotonicity and shape properties within compact intervals. In addition, in the case when the fuzzy numbers are given in parametric form, these operators generate a sequence of fuzzy numbers that have the same support and core as the approximated fuzzy number.

As a practical application, we implement our approach in the modeling of fuzzy diagnostic criteria for diabetes. Numerical experiments, supported by illustrative graphs and tables, demonstrate that the Modified Meyer-König and Zeller Operators provide highly accurate approximations while preserving the interpretability of fuzzy medical data.

**Keywords:** linear Meyer-König and Zeller on compact interval  $[a, b]$ , Improved estimation of approximation, Fuzzy Inference, Applications.

**2020 Mathematics Subject Classification Numbers:** 41A25, 41A29, 41A35.

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## On $|A, \delta|_k$ summability of factored Fourier series

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### Abstract

In this paper, a new theorem on  $|A, \delta|_k$  summability of factored Fourier series has been obtained, which generalized a theorem of Sarigöl on  $|A|_k$  summability of factored Fourier series (see [20]).

**Keywords:** Riesz mean, matrix summability, summability factor, infinite series, Fourier series, Hölder inequality, Minkowski inequality.

**2020 Mathematics Subject Classification Numbers:** 26D15, 40D15, 40F05, 40G99, 42A24, 46A45.

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# Chebyshev Polynomial Approximations for Linear Volterra-Fredholm Integral Equations

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## Abstract

Linear Volterra-Fredholm integral equations of the second kind are fundamental in modeling phenomena across physics, engineering, and mathematical biology, where analytical solutions are often intractable. This study presents a numerical method grounded in functional analysis, utilizing Chebyshev polynomials of the first, second, third, and fourth kinds to construct highly accurate approximate solutions. By expressing the solution as a finite series in the Chebyshev basis, we transform the integral equation into a system of linear algebraic equations using the collocation technique. This approach leverages the orthogonal properties of Chebyshev polynomials in Hilbert spaces. We provide a theoretical analysis of the methods convergence and error bounds, emphasizing its roots in functional analysis. Numerical experiments validate the methods efficiency and accuracy compared to established techniques, such as the Adomian decomposition method. The findings highlight the power of Chebyshev polynomial approximations in addressing integral equations, offering a robust framework for both theoretical investigations and practical applications in mathematical sciences.

**Keywords:** Volterra-Fredholm integral equations, Chebyshev polynomials, Collocation Method.

**2020 Mathematics Subject Classification Numbers:** 45A05, 45B05, 65R20.

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## A Generalized Weighted Aggregation Bernstein Operator Based on Logarithmic Scaling for Fuzzy Medical Feature Analysis

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### Abstract

In this study, we introduce a novel fuzzy approximation operator combining the structure of Bernstein polynomials with a Gaussian-based symmetric kernel and a logarithmic Generalized Weighted Averaging (GWA) aggregation technique. This operator is specifically designed to handle intuitionistic fuzzy values (IFVs), which are crucial in modeling both membership and non-membership information in uncertain environments such as medical diagnosis. We prove key properties including positivity, linearity, shape preservation, and approximate monotonicity. As an application, we present a fuzzy classification approach for breast cancer diagnosis using normalized patient data and trapezoidal membership functions.

**Keywords:** Fuzzy approximation, Gaussian-based symmetric kernel, Bernstein polynomials.  
**2020 Mathematics Subject Classification Numbers:** 41A65.

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# A Study on Strong Convergence in Neutrosophic Normed Spaces

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## Abstract

In this study, the concept of  $\rho$ -strongly convergence of order  $\beta$  of difference sequences of fractional order in neutrosophic normed space is introduced. Some properties and some inclusion theorems related to this concept are investigated.

**Keywords:** Strong convergence, neutrosophic normed spaces, summability.

**2020 Mathematics Subject Classification Numbers:** 40A05, 40C05, 46A45.

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## Bounded Solutions for Semilinear Differential equations

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### Abstract

A pseudo almost periodic function is the sum of an almost periodic function and of an ergotic perturbation. An asymptotically a.p. (almost periodic) function is the sum of an almost periodic function and another zero at infinity. The asymptotically a.a. (almost automorphic) and pseudo almost automorphic functions generalized the one of asymptotically a.p and pseudo a.p functions. In this paper, we study the existence of asymptotically a.p., asymptotically a.a., pseudo a.p., and pseudo a.a. mild solutions for a class of semilinear differential equations with a second term which is each time of the same nature as the desired solution. We study after that the regular dependence of this solution with the initial value. more precisely when the initial conditions is a asymptotically a.p., asymptotically a.a., pseudo a.p., pseudo a.a. functions we study the differentiable dependence of asymptotically a.p., asymptotically a.a., pseudo a.p., pseudo a.a. solutions of our equation. In presenting our principal result, we delve into the proprieties of the superposition operator, alternatively referred to as the Nemytskii operator, within our defined space. Additionally, we emphasize the concept of derivation associated with this operator in the spaces of asymptotically a.p., asymptotically a.a., pseudo a.p., and pseudo a.a. functions . Furthermore, our paper incorporates the utilization of semi-groups, recognized as significant assets in the domain of differential equations. The principle of this work is to transfer our equation (dynamic system problem) to an operator between the spaces used before (functional analysis problem), and to apply the implicit function theorem of the differential of calculus in Banach spaces.

**Keywords:** asymptotically almost periodic functions, asymptotically almost automorphic functions, pseudo almost periodic functions.

**2020 Mathematics Subject Classification Numbers:** 34C27, 34C99,47J07.

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## Factorial correspondence analysis of the companion species of *Juniperus phoenicea* subsp. *turbinata* and *Juniperus oxycedrus* subsp. *macrocarpa* on the coast of the Tlemcen region (Algeria)

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### Abstract

Plants in the Cupressaceae family have stringy or scaly bark, spiral foliage and simple, sessile, petiolate evergreen leaves. These plants have a terminal or solitary cluster inflorescence with unisexual flowers that produce a fruit that can take up to two years to ripen, called cones. *Juniperus* L. is widespread in Algeria, as pointed out by Maire (1952) and Quézel & Santa (1962). There are five species, two of which are considered very rare (*J. thurifera* L. and *J. sabina* L.), one is classified as rare (*J. communis* L.), while the last two are undergoing severe degradation in semi-arid and arid regions (*J. oxycedrus* L. and *J. phoenicea* L.).

The main objective of our work is to carry out a phytoecological and phytosociological study of the species that accompany *Juniperus phoenicea* subsp *turbinata* and *Juniperus oxycedrus* subsp *macrocarpa* in the coastal region of Tlemcen.

In order to identify the species that accompany *Juniperus Phoenicea* subsp *turbinata* and *Juniperus oxycedrus* subsp *macrocarpa*, we carried out floristic surveys at coastal stations in the Tlemcen region.

Correspondence factorial analysis highlights the relationships between the various plant groupings and ecological factors such as climate and soil characteristics, and brings out groups of species linked to the two coastal *Juniperus* species in the Tlemcen region.

Results were obtained on characteristic species groupings in general, including biological and ecological aspects. *Juniperus phoenicea* subsp *turbinata* and *Juniperus oxycedrus* subsp *macrocarpa* characterize permanent communities of limestone rocks and cliffs, from the coast to areas far from it, in the semi-arid to dry bioclimatic stage, preferably in the domain of *Pistacio lentisci-Rhamnetalia alaterni: Asparago albi-Rhamnion oleoidis* and *Oleo-Ceratonion...*

**Keywords:** Factorial correspondence analysis, *Juniperus*, coast, Tlemcen, Algeria.

**2020 Mathematics Subject Classification Numbers:** 62-XX, 62Hxx, 62H25.

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## Applications of the Schwarz lemma on the boundary

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### Abstract

This paper is concerned with a boundary refinement of the Schwarz lemma within the framework of the classes  $N_a(\varphi)$ . Consider the analytic function

$$f(z) = e^{i\theta} + c_1 z + c_2 z^2 + \dots, \quad z \in E = \{z \in \mathbb{C} : |z| < 1\}, |\theta| < \pi/2,$$

and let  $\varphi(u, v)$  denote a continuous function defined on a domain  $D \subset \mathbb{C} \times \mathbb{C}$ . We derive precise estimates for the modulus of the angular derivative of  $f$  at a boundary point  $b$  satisfying  $f(b) = i \sin \theta$ . The extremal functions that attain equality are identified, establishing the sharpness of the inequalities.

**Keywords:** The Boundary Schwarz Lemma, Analytic Function, Angular Limit.

**2020 Mathematics Subject Classification Numbers:** 30A10, 30C80

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## About properties of meromorphic solutions of a class of ultrametric q-difference equations

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### Abstract

Let  $\mathbb{K}$  be an algebraically closed field, complete for an ultrametric absolute value and let  $\mathcal{M}(\mathbb{K})$  be the field of meromorphic functions in all  $\mathbb{K}$ . In this article, we consider the ultrametric functional equation of Schröder-type:  $\sum_{j=1}^n A_j(x)f(q^jx) = R(x, f(x)) = \frac{P(x, f(x))}{Q(x, f(x))}$ , where  $q$  is an element of  $\mathbb{K}$ ,  $A_1(x), \dots, A_n(x)$  are rational functions and  $P, Q$  are relatively prime polynomials in  $f$  over the field of rational functions satisfying  $p = \deg_f P$ ,  $t = \deg_f Q$ ,  $d = p - t \geq 2$ .

First we prove that, if all coefficients  $A_j(x)$   $j = (1, \dots, n)$  and all coefficients of  $R(x, f(x))$  are constants, then every meromorphic solution of our equation is a rational function.

Next, we study meromorphic solutions of the above equation in the case where the coefficients are rational and we provide some estimates on the order of growth of these solutions.

**Keywords:** Ultrametric meromorphic functions, Value distribution theory,  $q$ -Difference equation, Growth of solutions.

**2020 Mathematics Subject Classification Numbers:** 39A13, 39A22, 11F85.

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## 6.3 Sequences, Series, Summability

Session Organizers: İbrahim Çanak and Sefa Anıl Sezer

The Sequences, Series, and Summability session of the 9th International Conference of Mathematical Sciences (ICMS 2025) brings together researchers to explore both classical foundations and recent developments in the theory and applications of sequences, series, and summability. Topics include summability methods, Tauberian theory, sequence spaces and connections with functional analysis and other branches of mathematics.

This session provides a platform for exchanging ideas, fostering collaborations, and discovering new open problems in the field. It particularly encourages the participation of young researchers to enhance their understanding and broaden their perspectives. We believe that many of the contributions presented in this session will pave the way for further significant research and will be suitable for publication in prestigious mathematical journals.

## Tauberian theorems for the summability by deferred weighted means of double sequences

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### Abstract

In this paper, we examine the relation between convergence in the Pringsheim's sense and the deferred weighted mean summability of double sequences in (1,1) sense. We introduce the concepts of deferred slow decrease and deferred slow oscillation for double sequences. As a consequence, we obtain necessary and sufficient Tauberian conditions for the deferred weighted mean summability of double sequences.

**Keywords:** Deferred weighted means, Tauberian theorems, Tauberian conditions, deferred slowly decreasing sequences, deferred slowly oscillating sequences, convergence in Pringsheim's sense.

**2020 Mathematics Subject Classification Numbers:** 40G99, 40E05, 40A05.

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## Some Notes On The Spectrum Of Quadruple Band Matrix Operator Defined On The Sequence Spaces $\ell_1$ And $bv$

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### Abstract

In this work, we determine some spectrum types of quadruple band matrix operator  $G(r, s, t, u)$  defined on the sequence spaces  $\ell_1$  and  $bv$ . The matrix  $G(r, s, t, u)$  is the general form of third order difference, triple band, second order difference, double band(generalized difference) and difference matrices symbolized with  $\Delta^3$ ,  $B(r, s, t)$ ,  $\Delta^2$ ,  $B(r, s)$  and  $\Delta$ , respectively.

**Keywords:** Spectrum, Resolvent, Sequence Space.

**2020 Mathematics Subject Classification Numbers:** 47A10, 47B37, 47B39.

### References

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## On the $(\bar{N}, p)$ summability of integrals on $\mathbb{R}_{\geq 0}$

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### Abstract

Given a real-valued function  $f(x)$  which is locally integrable over  $[0, \infty)$ . In this study, we establish a characterization connecting  $(\bar{N}, p)$  summability of integrals to the convergence of an auxiliary integral. Thus, we extend certain existing results in the literature to the  $(\bar{N}, p)$  summability of integrals.

**Keywords:**  $(\bar{N}, p)$  summability, Tauberian theorems,  $(C, 1)$  summability.

**2020 Mathematics Subject Classification Numbers:** 40A10, 40C10, 40E05.

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## $\mathcal{I}_2^{rs,q}$ —Statistical Convergence of order $\varpi$

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### Abstract

Double sequences, as a natural extension of ordinary sequences, play a fundamental role in the study of two-dimensional convergence, summability methods, and their applications in functional analysis and approximation theory. Additionally, the Pringsheim limit provides a rigorous framework for convergence in two dimensions, distinguishes between genuine two-variable convergence, forms the foundation for double sequence theory and its applications. Therefore, the main goal of this paper is to present  $\mathcal{I}_2^{rs,q}$ —statistical convergence of order  $\varpi$ . Furthermore, certain precise inclusion relations between various sequence spaces are investigated.

**Keywords:** ideal, double sequences, statistical convergence.

**2020 Mathematics Subject Classification Numbers:** 40F06, 40G01.

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## Characterizing Pringsheim convergence of double sequences via weighted means in two-normed spaces

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### Abstract

This study investigates the relationship between convergence and summability for double sequences within the setting of two-normed spaces. Specifically, it addresses the problem that while a bounded double sequence convergent in Pringsheim's sense is also summable by the weighted mean method, the converse implication is not guaranteed without additional conditions. The study systematically analyzes three forms of weighted mean summability: the double weighted mean  $(\bar{N}, p, q; 1, 1)$ , and the single-index variations  $(\bar{N}, p, *; 1, 0)$  and  $(\bar{N}, *, q; 0, 1)$ . For each of these three summability methods, the study establishes necessary and sufficient conditions under which summability via weighted means implies convergence in Pringsheim's sense, thereby providing a comprehensive characterization of this relationship in two-normed spaces.

**Keywords:** Two-normed spaces, double sequences, Pringsheim convergence; weighted mean summability, Tauberian theorems.

**2020 Mathematics Subject Classification Numbers:** 40A05, 40G05, 40E05.

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## Theoretical Approaches to Set-Valued Mappings with Applications in Measure Theory

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### Abstract

In this study, we investigate the set-valued mapping  $F : X \rightarrow 2^Y$  and the potential applications of its measurable selections. Within this framework, we consider a different extended version of the classical Lusin's Theorem, which, although traditionally situated within the domains of measure theory and analysis, is shown to be applicable as a powerful tool in various fields of applied mathematics. The theorem, generalized using sequences of sets and sequences of real numbers, is anticipated to exhibit significant potential for applications in complex mathematical models. Compared to similar approaches in the literature, the proposed method stands out as a robust and effective tool from both theoretical and practical perspectives.

**Keywords:** Lusin's theorem, Sequences, Set-valued mappings.

**2020 Mathematics Subject Classification Numbers:** 28A20, 40A05, 28B20.

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## Delta p quasi Cauchy Sequences

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### Abstract

In this paper, we call a real-valued function defined on a subset  $E$  of  $\mathbb{R}$ ,  $\delta$ -  $p$ -ward continuous if it preserves  $\delta$ -  $p$ -quasi Cauchy sequences where a sequence  $\mathbf{x} = (x_n)$  is defined to be  $\delta$ -  $p$ -quasi Cauchy if the sequence  $(\Delta_p x_n) = (x_n - x_{n+p})$  is  $p$  quasi Cauchy. It turns out that  $\delta$ -  $p$ -ward continuity implies uniform continuity for any fixed positive integer  $p$ , but there are uniformly continuous functions which are not  $\delta$ -  $p$ -ward continuous. A new type compactness in terms of  $\delta$ -  $p$ -quasi Cauchy sequences, namely  $\delta$ -  $p$ -ward compactness, is also introduced, and some theorems related to  $\delta$ -  $p$ -ward continuity and  $\delta$ -  $p$ -ward compactness are obtained.

**Keywords:** Quasi cauchy sequences, ward continuity, delta ward compactness.

**2020 Mathematics Subject Classification Numbers:** 40C05, 54A20

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## 6.4 Fixed Point Theory

Session Organizers: Duran Türkoğlu and Hakan Şahin

Fixed point theory is considered one of the central areas of modern analysis. It has significant connections to topology, functional analysis, optimization, and applied mathematics. Beginning with Banach's contraction principle, the field has grown into a rich, diverse discipline providing essential tools for studying the existence, uniqueness, and stability of solutions to various mathematical models.

The Fixed Point Theory session of the 9th International Conference of Mathematical Sciences 2025 (ICMS 2025) provides a platform to researchers interested in this area to discuss and present of their ideas. The contributions presented in this session reflect the field's classical foundations and latest developments. They include new generalizations of contraction principles and extensions to multivalued and nonlinear mappings, as well as applications to differential equations, dynamical systems, and optimization problems. Such advances not only deepen our understanding of fixed point structures but also broaden their scope across pure and applied mathematics.

## Existence of extremal solutions for a class of functional fractional differential equations with maxima

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### Abstract

The purpose of this work is to study the existence and uniqueness of solutions for the following problem

$$\begin{cases} ({}^C D_{0+}^\alpha u)(t) = f(t, u(t), \max_{s \in [t-r, t]} u(s)), & t \in J = [0, T], \\ u(t) = \varphi(t), & t \in [-r, 0], \end{cases}$$

where  ${}^C D_{0+}^\alpha$  is the Caputo fractional derivative of order  $\alpha$  with  $0 < \alpha \leq 1$ ,  $T > 0$  and  $r > 0$ ,  $f : J \times \mathbb{R} \times C \rightarrow \mathbb{R}$  is continuous with  $C = C([-r, T], \mathbb{R})$  and  $\varphi : [-r, 0] \rightarrow \mathbb{R}$  are continuous.

**Keywords:** Caputo fractional derivative, maxima, upper and lower solutions, monotone iterative.  
**2020 Mathematics Subject Classification Numbers:** 34A08, 34A45, 34B15, 34B37, 34C60.

## References

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## An other version of Logistic equation

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### Abstract

In this talk we are concerned with fractional-order Logistic equation using an other approach which called a Non-conformable derivative for fractional differential equation. We study the existence, uniqueness and stability of solution. A numerical example is given to illustrate our study. .

**Keywords:** Non-conformable derivative, Logistic equation, Fractional-order.

**2020 Mathematics Subject Classification Numbers:** 97Mxx.

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## On a Class of Nonlinear Partial Differential Equations of Fractional Order with Delay

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### Abstract

A class of nonlinear fractional order partial differential equations with delay of the form

$$\frac{{}^c\partial^\alpha u(x, t)}{\partial t^\alpha} = a(t)\Delta u(x, t) + f(t, u(x, \tau(t))), \quad t \in [0, T_0] \quad (1)$$

where  $\frac{{}^c\partial^\alpha u(x, t)}{\partial t^\alpha} := {}^c\partial^\alpha$  denotes the Caputo fractional derivative of order  $\alpha$  with  $0 < \alpha < 1$ ,  $a(t)$  is the diffusion coefficient,  $f$  is a nonlinear function describing the system's interactions defined as  $f(t, u) : [0, T_0] \times \mathbb{R} \rightarrow \mathbb{R}$ ,  $x \in \Omega \subset \mathbb{R}^2$ , and  $\tau(t)$  is the delay function.

We consider the following boundary conditions:

$$u(x, t) = 0, \quad (x, t) \in \partial\Omega \times [0, T_0] \quad (2)$$

$$\frac{\partial u}{\partial N} = 0, \quad (x, t) \in \partial\Omega \times [0, T_0] \quad (3)$$

where  $N$  is the outward unit normal vector to  $\partial\Omega$ .

Using the Lebesgue dominated convergence theorem, the Leray–Schauder fixed point theorem, and the Banach contraction mapping theorem, we obtain some sufficient conditions for the existence of solutions to the above fractional order partial differential equations.

**Keywords:** Psi-Caputo fractional order Partial differential equations, Banach's fixed point theorem, existence, uniqueness, Delay .

**2020 Mathematics Subject Classification Numbers:** 34A08, 26A33, 34A12, 47H10, 34K37.

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## Fixed Point Techniques in Chebyshev Approximation of Volterra-Fredholm Equations

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### Abstract

Volterra-Fredholm integral equations of the second kind appear frequently in mathematical modeling of real-world systems across physics, engineering, and biology. Due to the challenges in finding exact solutions, this study proposes an effective numerical method based on Chebyshev polynomial approximations, interpreted within the framework of fixed point theory. The solution is constructed as a finite Chebyshev series, and the integral equation is transformed into a system of algebraic equations using the collocation method. The existence and uniqueness of the approximate solution are justified using fixed point results in Hilbert spaces, where the Chebyshev basis plays a crucial role. Convergence conditions and error estimates are established accordingly. Numerical comparisons with classical techniques, such as the Adomian decomposition method, demonstrate the robustness and accuracy of the proposed approach. This work highlights the relevance of fixed point theory in ensuring the stability and reliability of numerical solutions to integral equations.

**Keywords:** Volterra-Fredholm integral equations, Chebyshev polynomials, Fixed point theory.

**2020 Mathematics Subject Classification Numbers:** 45B05, 33C45, 47H10.

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## 6.5 Numerical Functional Analysis

The main aim of this special session is to provide impetus, motivation and to bring together researchers and scientists working in the fields of Numerical Functional Analysis, Operator Theory and Applications by providing a forum for the academic exchange of ideas and recent research works.

The selected papers examine wide-ranging and cutting edge developments in various areas of Numerical Functional Analysis, Operator Theory and their applications. The papers give a taste of current research.

## Exponential decay result for the energy associated to a flexible marine riser with vessel dynamics

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### Abstract

In this work we prove an exponential decay result for the energy associated to a flexible marine riser with vessel dynamics. By applying a control at the top boundary of the riser we shall attenuate its vibration in a fast manner. The method is based on the multiplier technique. The riser is modeled as a viscoelastic beam with a (slightly perturbed) standard relaxation kernel.

**Keywords:** Exponential stabilization; viscoelastic material; relaxation function; boundary control; multiplier technique.

**2020 Mathematics Subject Classification Numbers:** 93D20, 35L15, 35L70, 45M10, 65M60, 45N05.

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## A numerical method for solving an inverse time-dependent coefficient problem associated with a time-fractional reaction-diffusion equation

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**Abstract.** In this talk, we propose a numerical method for solving an inverse time-dependent coefficient problem associated with a time-fractional reaction-diffusion equation subject to nonlocal boundary and overdetermination conditions. The time-fractional derivative is considered in the conformable sense. By employing the Shifted Legendre collocation method, the inverse problem is transformed into a linear system of first-order differential equations, which is then solved using the Backward Euler method. Through two illustrative examples, we conduct a comparative analysis between the proposed algorithm and existing numerical methods from the literature. The results demonstrate that our approach achieves highly accurate approximations using a relatively small number of collocation points.

**Keywords:** 65N35, 35Fxx, 65Lxx.

**2020 Mathematics Subject Classification Numbers:** Legendre collocation method, Linear system of first-order differential equations, Backward Euler method.

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# Mathematical Analysis of the Fractional Stokes Equation

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## Abstract

The fractional Stokes equation represents a significant extension of classical fluid dynamics by incorporating fractional-order derivatives to model complex viscoelastic and anomalous flow behaviors. This study presents a comprehensive mathematical analysis of the fractional Stokes problem, highlighting its theoretical underpinnings and applications in non-Newtonian fluid dynamics. We investigate the well-posedness of the problem, addressing existence, uniqueness, and regularity of solutions in suitable Sobolev spaces. The analysis employs advanced tools from fractional calculus and pseudodifferential operators to establish key properties of the solution space. Furthermore, asymptotic behaviors and stability results are derived under various boundary conditions, offering insights into long-term dynamics and stability of fractional flows. The existence and uniqueness of weak solutions are rigorously demonstrated using the Lax-Milgram theorem, ensuring the formulation is well-posed under appropriate assumptions of ellipticity and boundedness.

**Keywords:** Fractional differential equations, Stokes problem, Fluid mechanics.

**2020 Mathematics Subject Classification Numbers:** 35Q35, 35R11, 76D07.

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## Solving nonlinear Volterra-Fredholm integro-differential equations using He's variational iteration method

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### Abstract

In this work, a nonlinear Volterra-Fredholm integro-differential equation is solved using He's variational iteration method. The approximate solution of this equation is calculated in the form of a sequence where its components are computed easily. Some examples are given to illustrate the effectiveness of the method. The results show that the method provides a straightforward and powerful mathematical tool for solving various a nonlinear Volterra Fredholm integro-differential equations.

**Keywords:** Integral equation, Volterra equation, Volterra-Fredholm integro-differential, variational iteration method.  
**2020 Mathematics Subject Classification Numbers:** 65R10, 65K10, 65L20.

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# Numerical Solutions of Linear Volterra-Fredholm Integral Equations Using Chebyshev Polynomials of the First Four Kinds

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## Abstract

Text of the abstract. In this work, we study a numerical method for solving linear Volterra-Fredholm integral equations of the second kind. The method uses Chebyshev polynomials of the first, second, third, and fourth kinds to build an approximate solution in the form of a series that converges to the exact solution. To test the method, we apply it to several examples and compare the results with those of other known methods. The comparisons show that this technique is accurate, efficient, and easy to apply.

In this work, we present a simple and effective numerical method for solving linear Volterra-Fredholm integral equations of the second kind. These types of equations appear in many real-world applications such as physics, engineering, and biology. Since finding exact solutions is often very difficult, we use an approximate method based on Chebyshev polynomials. We use Chebyshev polynomials of the first, second, third, and fourth kinds to build an approximate solution in the form of a finite series. These polynomials are known for their good accuracy and stability in numerical computations. By using them, we transform the integral equation into a system of algebraic equations, which can be solved easily on a computer. Our goal is to show how each kind of Chebyshev polynomial can help in solving these equations accurately. We test the method on several examples and compare the results with other known numerical methods. The results show that our method is not only simple and fast, but also very accurate. This approach is useful especially when the kernel of the integral equation is smooth and the exact solution is continuous. It gives reliable results with less computational effort. In conclusion, the use of Chebyshev polynomials provides a practical and efficient tool for solving Volterra-Fredholm integral equations. This makes the method suitable for both theoretical research and real applications in science and engineering. ..

**Keywords:** Chebyshev polynomials, numerical solution, Volterra-Fredholm integral equations, collocation technique.  
**2020 Mathematics Subject Classification Numbers:** 65R06, 65Q03.

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# A Novel Iterative Scheme for Solving Multidimensional Stochastic Differential Equations

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## Abstract

This article introduces a novel and highly efficient numerical method for solving nonlinear stochastic Itô-Volterra integral equations driven by multidimensional Brownian motion. The proposed approach combines the iterative refinement of the Picard method with the spectral accuracy of Lagrange functions, forming a robust and versatile framework for tackling these stochastic problems.

The theoretical contributions encompass a comprehensive formulation of the method, a rigorous proof of well-posedness, and an in-depth reliability analysis. A meticulous convergence study is undertaken, establishing precise accuracy conditions and demonstrating the method's capability to achieve exceptionally accurate solutions.

To validate its effectiveness, multiple numerical examples are presented, showcasing the method's remarkable accuracy even in complex scenarios. These results further reinforce the theoretical findings from the convergence analysis.

**Keywords:** Nonlinear Stochastic Volterra integral equations, Lagrange functions, Multi-dimensional Brownian motion process, Picard iteration method, Collocation method.

**2020 Mathematics Subject Classification Numbers:** 65C30, 65C20, 60H35

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## Analysis of contact problems in solid mechanics

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### Abstract

Several problems in mechanics, physics, control and those dealing with contacts, lead to the study of systems of variational inequalities. In this study we considered a deformed elastic solid with a unilateral contact of a rigid body. This model has been studied by Lions, J.L. and G. Stampacchia, [8]. In this paper, we studied the existence, uniqueness and continuity of the deformation of this solid with respect to the data.

**Keywords:** Contact, Variational Inequalities, Finite element method.

**2020 Mathematics Subject Classification Numbers:** 35B35, 74F05, 93D20.

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## On the use of spectral method for the numerical solution of Volterra-type integro-differential equations

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### Abstract

The main objective of this work is to numerically study the spectral method for to solve Volterra-type integro-differential equation, we use the finite element method combined with the orthogonal Legendre polynomials for solving this equation. We also discuss the error and convergence of the method. Furthermore, we give some numerical examples to substantiate efficiency of the proposed method.

**Keywords:** Integral equation, Volterra equation, Spectral method, Galerkin method, orthogonal polynomial.

**2020 Mathematics Subject Classification Numbers:** 65R10, 65K10, 65L20.

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## 6.6 Computer Science and Technology

Session Organizers: Önder Sahinaslan, and Serdar Angun

Welcome to the Computer Science and Technology session of the International Conference of Mathematical Sciences (ICMS 2025). This session is dedicated to exploring the latest advancements in a field where mathematical principles are the bedrock of technological innovation. The presentations here will highlight the powerful synergy between fundamental mathematical disciplines and their practical applications in areas like Artificial Intelligence (AI), Machine Learning (ML), and Natural Language Processing (NLP). The studies presented today demonstrate how mathematical concepts such as linear algebra, probability theory, optimization, and statistics drive modern technology. For instance, the algorithms used for classifying unstructured data in Robotic Process Automation (RPA) systems are rooted in mathematical principles like vector spaces and probability distributions. Similarly, new methods for boosting the performance of deep learning models aim to solve complex optimization problems built upon matrix transformations and non-linear functions. Solutions in logistics and software engineering also rely heavily on applying mathematical models to optimize operational processes. By leveraging tools like statistical methods and graph theory, these systems improve efficiency and reliability. This session offers a great platform for young researchers to see how mathematical knowledge can be applied to solve real-world technological problems. We are confident that the research presented will open new doors at the intersection of mathematics and computer science and is well-deserving of publication in prestigious journals. We wish you all a productive and inspiring conference.

# A Deep Learning Approach using ResNet-50 for Arabic Sign Language Recognition

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## Abstract

Arabic Sign Language (ArSL) serves as a vital communication medium for the deaf and hard-of-hearing community within Arabic-speaking regions. However, significant communication barriers often exist between signers and non-signers, limiting access and inclusion. Automated ArSL recognition (ArSLR) systems offer a potential solution, but their development is hindered by challenges such as ArSL's linguistic complexity, dialectal variations, and the limited availability of comprehensive datasets. Deep learning approaches, particularly Convolutional Neural Networks (CNNs), have shown significant promise in vision-based sign language recognition. This paper investigates the effectiveness of the ResNet-50 architecture, a deep CNN renowned for its powerful feature extraction capabilities and its use of residual connections to overcome the vanishing gradient problem in deep networks, for the task of ArSL recognition. We employ a transfer learning methodology, fine-tuning a ResNet-50 model pre-trained on the ImageNet dataset using publicly available ArSL alphabet datasets (such as ArSL2018 or AASL). Standard image preprocessing and data augmentation techniques are utilized to enhance model robustness. The study aims to achieve high recognition accuracy for static ArSL signs, evaluated using standard classification metrics including accuracy, precision, recall, and F1-score, thereby providing a performance benchmark comparable to state-of-the-art methods. This work contributes a rigorous evaluation of the ResNet-50 architecture within the ArSL context, reinforcing the potential of transfer learning for developing practical assistive communication technologies and promoting greater inclusivity for the Arabic deaf community.

**Keywords:** Arabic Sign Language, Deep Learning, ResNet-50, Recognition.

**2020 Mathematics Subject Classification Numbers:** 68T45.

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## A Novel Control Strategy for Capacitor Voltage Imbalance in Three- Level NPC Inverters

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### Abstract

The work in this article focuses primarily on the problem of capacitor voltage imbalance in a three-level NPC inverter. This problem is inherent in the inverter operation and significantly affects its performance, especially the harmonic distortion rate (THD), efficiency and stability of the system. Therefore, this paper proposes a novel voltage balancing control strategy based on adjusting the redundant vectors application times in the space vector modulation. Furthermore, in order to determine the appropriate adjustment direction, a control variable based on the signs of the inverter currents and the capacitor voltage deviation is incorporated into the distribution times of the redundant vectors. The proposed method is easy to implement and offers significant advantages, particularly in terms of THD values, computation time, and balancing efficiency. Finally, the proposed method was experimentally verified, and the obtained results demonstrate high capacitor voltage balancing capability and excellent overall performance.

**Keywords:** Three-level NPC inverter, Space vector modulation, capacitor voltage imbalance, Neutral point voltage, inverter modulation.

**2020 Mathematics Subject Classification Numbers:** First, Second, Third.

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# Tool-Augmented Agentic AI: A Survey on Composition, Selection and Integration

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## Abstract

Agentic AI systems augmented by tool use constitute a transformative leap in the evolution of artificial intelligence, enabling autonomous, multi-step reasoning and dynamic interaction capabilities with external environments. By precisely invoking diverse tools from their registry, which range from APIs to simulators, they are able to overcome inherent limitations to singular large language models and AI agents such as static knowledge and limited execution capabilities. This survey delves into the rapidly expanding body of research in this niche field that surrounds tool-augmentation within Agentic AI, and offers a comprehensive examination of foundational concepts, structured taxonomies, architectural designs, and methodological innovations in tool composition, selection, and integration. Its current and potential applications throughout a variety of domains are examined including research assistance, business automation, and ethical challenges, while considering key challenges such as tool invocation errors and benchmarking inadequacies. By synthesizing empirical findings and theoretical frameworks, this work attempts to provide a definitive reference that guides researchers, developers and policymakers in navigating the complex landscape of this rapidly evolving field, ultimately drawing pathways for future exploration and safe deployment of tool augmented agentic artificial intelligence.

**Keywords:** Agentic AI, Tool Augmentation, Tool Composition and Selection.

**2020 Mathematics Subject Classification Numbers:** 68T42, 68T01, 68T05.

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## Prediction of the Criticality Level of Software Bugs Using Machine Learning: A Proposal for E-Commerce Applications

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### Abstract

The software development lifecycle (SDLC) is a systematic process encompassing the planning, design, development, testing, deployment, and maintenance phases of software. Software testing, one of the most critical steps in this process, is crucial for verifying whether the developed product meets user requirements and ensuring software quality. Proper management of errors detected during the testing process is crucial to maintaining user experience and system reliability, especially in e-commerce mobile applications with high user traffic, complex payment systems, inventory management, and intensive data interaction. Currently, manually determining error criticality levels relies on the interpretations and experiences of test engineers, leading to issues such as inconsistencies, inaccurate prioritization, and resource waste. In this context, the development of a machine learning-based criticality level prediction model based on historical error records from e-commerce mobile applications is proposed. Machine learning is an artificial intelligence method based on algorithms that learn patterns from past data and make predictions based on new data. In the proposed system, textual descriptions in error reports will be processed using natural language processing (NLP) techniques. Classification can then be performed using supervised learning algorithms (Naive Bayes, Support Vector Machines, Random Forest, XGBoost, etc.). Implementing such a model will improve error management by ensuring consistency during the SDLC testing phase, offering the potential to increase software quality, customer satisfaction, and the competitiveness of e-commerce platforms.

**Keywords:** Software Testing, Bug Criticality Level, Machine Learning, Natural Language Processing (NLP)  
**2020 Mathematics Subject Classification Numbers:** 68T01, 68T42, 68U35

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## Computational Analysis of Enzyme-Ligand Interactions in *Origanum glandulosum* Desf. extract: A Molecular Docking Approach to Natural Inhibitors

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### Abstract

In this study, we present a computational investigation of enzyme-ligand interactions using molecular docking as a mathematical modeling approach to evaluate bioactive compounds from *Origanum glandulosum* Desf. Methanolic extract was tested against five key enzymes: acetylcholinesterase (AChE),  $\alpha$ -amylase,  $\alpha$ -glucosidase, lipase, and tyrosinase. Based on HPLC-DAD analysis, kaempferol, benzoic acid, and rutin were selected for in silico docking studies.

Using Auto-Dock Vina, we found that kaempferol formed strong interactions with AChE,  $\alpha$ -amylase, and  $\alpha$ -glucosidase, involving hydrogen bonds and  $\pi$ - $\pi$  interactions with key active site residues. These results support its role as a competitive inhibitor. Benzoic acid, assumed responsible for the observed lipase inhibition in methanolic extract, showed hydrogen bonding with residues Leu153 and Phe77, and  $\pi$ - $\pi$  interaction with Tyr114. Rutin was docked to tyrosinase and found to directly chelate the two copper atoms at the enzymes active site consistent with its known inhibition mechanism.

This integrated computational and biochemical study highlights the value of molecular docking in identifying and understanding natural enzyme inhibitors. The findings support the use of *Origanum glandulosum* extract as potential source of bioactive compounds and demonstrate the role of in silico modeling in modern phytochemical research.

**Keywords:** Computational biology, Molecular docking, Auto-Dock Vina

**2020 Mathematics Subject Classification Numbers:** 92C40, 92F05, 92 08

### References

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## Biomedical monitoring using bioradar signal processing

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### Abstract

Radar technology, once limited to military applications, has now found its place in civilian domains ranging from collision avoidance systems in motor vehicles to biomedical applications for non-invasive monitoring of vital signs such as respiration and heart rate.

This article begins by presenting the fundamentals of cardiology, including heart anatomy, the vascular system, and the generation of the heartbeat. It then introduces various biomedical signal acquisition techniques, such as electrocardiography (ECG), phonocardiography (PCG), and electrooculography (EOG), focusing on the processing and analysis of these signals.

A general overview of radar systems is also provided, detailing different radar types, their theoretical foundations, and operating principles. Special attention is given to bioradar, highlighting its functioning within the biomedical domain for the detection of physiological signals.

Finally, the article presents signal processing methods applied to biomedical datasets (from data.m) in order to extract vital parameters essential for monitoring patients' health status. **Keywords:** Analysis and classification, ECG signal, bio radar signal.

**2020 Mathematics Subject Classification Numbers:** 92Cxx.

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## Comparative Study Between Turning and Facing on a Lathe Through Cutting Temperature Measurement

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### Abstract

Machining generates a significant amount of heat at the tool-workpiece interface, directly affecting tool life and the quality of the machined part. Among the fundamental operations performed on a lathe, turning (longitudinal machining) and facing (transverse machining) involve distinct cutting conditions that can lead to significant thermal variations. This study aims to compare the thermal effects of these two machining operations by measuring the cutting temperature using an infrared camera. The tests are conducted on a parallel lathe with controlled cutting parameters, in order to identify the thermal differences between turning and facing, and to assess their respective impacts on the thermal behavior of both the tool and the workpiece. The design of experiments (DOE) method was employed to determine the relationships between the input parameters namely cutting speed ( $V_c$ ), feed rate ( $a$ ), and depth of cut ( $p$ ) and the output parameter, which is the temperature ( $T$ ). The results show that cutting temperature increases mainly with cutting speed during facing, and with depth of cut during turning. As for the feed rate, it primarily affects the chip shape and flow, with a less significant impact on the temperature rise.

**Keywords:** Thermal analysis, Facing, Turning, Infrared camera, Infrared thermography, Electronic component.

**2020 Mathematics Subject Classification Numbers:** 65Z05.

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## Modelling, simulation and power maximisation of a wind energy conversion system using adaptive fuzzy

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### Abstract

This paper presents a comparative analysis between classical power maximization methods (MPPT – Maximum Power Point Tracking) and a fuzzy logic-based approach, applied to a wind system. Classical techniques, such as the Perturb and Observe (PO) method, the Tip Speed Ratio (TSR) control, or the Optimal Torque Control (OTC), are evaluated in terms of speed, accuracy, and stability under variable wind speed conditions. These methods, although robust and simple to implement, have limitations due to system nonlinearities and rapid variations in environmental parameters. Conversely, fuzzy logic, based on linguistic rules and intuitive modeling, which does not require knowledge of the wind turbine characteristics, offers dynamic adaptation and better management of the uncertainties inherent in wind systems. The study demonstrates that this approach optimizes the extraction of kinetic energy from the wind with a smoother response and reduced oscillations around the maximum power point, even in high turbulence scenarios. To this end, we presented a technique based on simple and adaptive fuzzy logic. To achieve this, we presented a detailed mathematical modeling of the wind conversion system, followed by a Matlab Simulink simulation of the various components and methods studied. The experimental results highlight that fuzzy logic outperforms conventional techniques in terms of overall energy efficiency, response time, and robustness.

**Keywords:** Turbine, MPPT, Wind, Fuzzy Logic, adaptive fuzzy logic.

**2020 Mathematics Subject Classification Numbers:** 68T01

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## Simulation study of electronic and optical properties of SrSnO<sub>3</sub> perovskite

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### Abstract

We conducted a theoretical study using the linear plane wave method Augmented and linearised (FP-LAPW) within the framework of density functional theory (DFT) by the Wein2k program, to determine the structural, electronic, optical and thermal properties of the SrSnO<sub>3</sub> perovskite. Structural properties such as parameters of structures, compressibility module and its derivative are in good agreement with the available theoretical results. Calculation of the electronic band structure of the SrSnO<sub>3</sub> compound reveals that this material exhibits semiconducting character with an indirect gap, making it suitable for potential applications in optoelectronic devices. The estimated real and imaginary components of the complex dielectric function were used to determine frequency-dependent linear macroscopic optical coefficients such as absorption, reflectivity, refractive index, extinction coefficient, reflectivity, and optical conductivity. In the energy range from 6 to 30 eV, the calculated optical properties, and mainly the absorption spectrum, show very strong absorption of UV. Finally, the effect of temperature and pressure on volume, compressibility modulus, thermal expansion and thermal capacities are discussed. Thermodynamic properties were calculated using the Debye quasi-harmonic approximation implemented in code Gibbs2.

**Keywords:** DFT, Perovskite, Fermi level, band structure, absorption coefficient.

**2020 Mathematics Subject Classification Numbers:** 81V70, 82D25, 78A40.

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## New approach to calculate the closer form of false alarm probability in radar detection operating in k-distribution clutter

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### Abstract

In this study, we propose new techniques based on approximating the K-distribution using Laguerre polynomials, allowing us to calculate the false alarm probability (Pfa). To evaluate the performance of the proposed technique for different parameters of the K-distribution, two comparisons were conducted: The first comparison is between the curves of the exact K-PDF generated by the Raghavan method and the proposed technique. The second comparison is between the curves of Pfa against the detection threshold (T) for the proposed technique and Monte Carlo simulations. The results obtained in this research demonstrate that the proposed technique for approximating the K-distribution is effective and promising, opening avenues for future research.

**Keywords:** K-distribution, Laguerre polynomial, Probability of false alarm.

**2020 Mathematics Subject Classification Numbers:** First, Second, Third.

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## A Mathematical Modeling and Passive Sliding Mode FTC Method of a Manipulator Arm

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### Abstract

The objective of this paper is the application of the passive sliding mode fault tolerant control (PSMFTC) of a manipulator arm with two degrees of freedom based on the control with sliding mode. we start by mathematical modeling of this system with state space and presenting the idea of this non-linear control. We presented the theory of the variable structure, in order to use in the PSMFTC method. This last technique is applied to the mathematical model of manipulator arm with 2DOF system using MATLAB/ SIMULINK, their performance results are compared with a PID controller in order to discover who these approaches preserve system performance and ensure stability and trajectory tracking. The results of the study prove the higher performance of the PSMFTC when compared with the PID technique.

**Keywords:** Manipulator arm with 2DOF, Passive sliding mode FTC, PID control, State space system, Mathematical modeling.

**2020 Mathematics Subject Classification Numbers:** First, Second, Third.

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# Multi-Model and Assertion-Based Confidence Scoring to Improve Data Quality in Artificial Intelligence Training

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## Abstract

The performance of artificial intelligence models depends heavily on the quality of the training data. Model accuracy and generalization are weakened when randomly collected datasets contain noise, mislabels, and distribution shifts. This study proposes a practical method that combines multi-model and prompt-based confidence scoring to extract a high-quality training set from such data. In the method, the outputs of two different pre-trained models (e.g., two separate architectures for age estimation) are first obtained. These outputs are then queried with positive and negative prompts in an OpenCLIP-like text-to-image matching framework to generate a normalized confidence score for each example. Examples with scores above a specified threshold are marked as “high quality”; examples with lower scores are either discarded or placed in the review queue. The marked data is used to train a more lightweight and computationally cost-effective custom model, thereby increasing accuracy, stability, and data efficiency while reducing training time and resource consumption. In practice, the threshold value is set based on the ROC curve and the validation set; During iteration, automatic demultiplexing, sampling for imbalanced classes, and incorrect label correction steps are integrated into the process. The approach is particularly effective when working with data from weakly supervised sources and provides a scalable data preparation pipeline that can be adapted to different domains, such as image quality, text classification, and voice command recognition. In real-world scenarios, combining automatic label confidence with human supervision reduces false positives, and the model performance improvement is sustainable with small but consistent rounds of additions.

**Keywords:** Data Preprocessing, Confidence Scoring, Weakly Supervised Learning, Artificial Intelligence  
**2020 Mathematics Subject Classification Numbers:** 68T07, 68T05, 68T45

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## Modelling the SOFC Power Plant Utilizing a MISO Nonlinear Auto-Regressive Model with eXogenous Input Fuzzy Dynamic Model

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### Abstract

The solid oxide fuel cell (SOFC) is extensively utilized for decentralized and renewable energy production. The primary issue of SOFC is regulating its output voltage, which is attributed to significant nonlinearity, swift load variations, and restricted fuel supply. The SOFC system control aims to sustain the output voltage at a stable level and the fuel utilization rate within a safe range. This work employs a multiple-input multiple-output (MIMO) discrete-time Takagi-Sugeno (TS) fuzzy dynamic model with feedforward input to characterize the dynamic features of the nonlinear voltage and fuel utilization rate in a tubular solid oxide fuel cell (SOFC) system. The acquired fuzzy model will be utilized to implement constrained fuzzy model predictive control. The simulation results demonstrate the precision and efficacy of the proposed technique.

**Keywords:** Solid Oxide Fuel Cell, Nonlinear Systems, NARX Model.

**2020 Mathematics Subject Classification Numbers:** 93-10, 37N35.

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## Mathematical Modeling of Ligand-Enzyme Interactions: A Computational Study Using Molecular Docking

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### Abstract

In this study, we present a computational investigation of ligand-enzyme interactions using molecular docking techniques as a mathematical modeling approach. The target molecules include 2-(((1-methyl-1H-imidazol-2-yl)methyl)sulfanyl)methyl)-1H-benzo[d]imidazole (MIMTMB) and its zinc(II) complex, Zn(MIMTMB)Cl<sub>2</sub>. The three-dimensional structure of cytochrome c peroxidase (CCP) (PDB ID: 2X08) was recovered from the Protein Data Bank and the ligands were optimized using Chimera 1.15 tools.

A receptor grid was constructed around the active site of CCP and docking simulations were performed with AutoDock Vina, predicting binding conformations and affinities of the ligands. The coupling protocol was validated by redocking of the cocrystallized ligand, ascorbate, which produced a root mean square deviation (RMSD) value of 0.981, confirming the reliability of the method.

The docking results indicate that MIMTMB exhibits a promising binding energy of -5.7 kcal/mol, comparable to ascorbate (-6.2 kcal/mol), suggesting a strong interaction within the active site of CCP. In contrast, the zinc complex shows a weaker binding affinity (2.4 kcal/mol), correlated with a lower antioxidant activity in vitro. Specific interactions such as hydrogen bonds and hydrophobic contacts between MIMTMB and key amino acid residues were identified, including Arg184, Gly41, and Asp37, providing information on the molecular mechanisms governing antioxidant activity.

This computational approach, which integrates molecular docking as a mathematical model, demonstrates the effectiveness of quantitative structural analysis and binding affinity predictions in understanding biochemical interactions. The results contribute to the growing interdisciplinary applications of mathematical modeling in the molecular and biological sciences, providing a rational basis for further experimental and theoretical studies.

**Keywords:** Molecular docking, computational modeling, AutoDock Vina.

**2020 Mathematics Subject Classification Numbers:** 92E10, 65D18, 92E20.

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## Statistical analysis of the impact of selected ecological factors on the growth of a rupicolous species: case of *Ephedra major* Host, 1831 (= *E. nebrodensis* Tineo, 1844) in Western Algeria

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### Abstract

This study examines the impact of various ecological and anthropogenic factors on the growth of *Ephedra major* Host, 1831 (= *E. nebrodensis* Tineo, 1844) in Western Algeria, focusing on two distinct ecological sites. The factors analyzed include orientation, sampling date, and the specific ecological conditions of each site, particularly sunlight exposure and altitude. The two stations, which exhibit different environmental characteristics, were selected to compare their respective influences on the growth in length of *Ephedra major* tufts. Growth measurements, such as tuft length, density, and biomass, were collected at different sampling dates, allowing the evaluation of the impact of sampling date and season on the species' growth. Analysis of Variance (ANOVA) was used to test the effects of orientation, sampling date, as well as ecological and anthropogenic factors on tuft growth in both stations. The analyses revealed that tuft orientation (north/south exposure) had a significant impact on their development, and that sampling date influenced tuft length according to seasonal variations. The results also showed notable differences between the two stations, highlighting the importance of local environmental conditions on the growth of *Ephedra major*. This study contributes to a better understanding of the ecological and temporal requirements of *Ephedra major* and provides valuable insights for conservation strategies in the rupicolous environments of Western Algeria.

**Keywords:** *Ephedra major*, Analysis of Variance (ANOVA), Ecological factors, Rupicolous environments, Western Algeria.

**2020 Mathematics Subject Classification Numbers:** 74-10, 65Z05.

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## Modeling and artificial intelligence for the in silico study of phytotherapies: towards an innovative approach for alzheimer's disease treatment

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### Abstract

Molecular modeling and artificial intelligence play a key role in identifying new therapeutic strategies against Alzheimer's disease, a neurodegenerative disorder characterized by a progressive decline in cognitive, behavioral, and social functions. This disease is notably marked by the abnormal accumulation of  $\beta$ -amyloid (A $\beta$ ) peptide in the brain, a process linked to the activity of secretases, particularly  $\beta$ -secretase. Inhibiting this enzyme is therefore a major target for slowing disease progression.

In this context, the combined use of molecular modeling and artificial intelligence enables the exploration of the therapeutic potential of phytochemical compounds. This study focuses on molecules derived from plants of the Apiaceae family to assess their ability to inhibit  $\beta$ -secretase and provide new perspectives for the prevention and early treatment of Alzheimer's disease. The integration of artificial intelligence optimizes molecular predictions and accelerates the identification of innovative solutions from natural pharmacopoeia.

**Keywords:** Molecular modeling, Artificial intelligence, Alzheimer's disease,  $\beta$ -secretase,  $\beta$ -amyloid, phytotherapy.

**2020 Mathematics Subject Classification Numbers:** 65Z05.

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## Processor-in-the-Loop Implementation of Series Active Filter Using Seven-Level Packed U-Cell Converter with Launchpad f28379d

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### Abstract

In a low voltage electrical grid, harmonics have negative effects on electrical equipment. The objective of this work is to study the control of a PUC type multi-level converter series active filter application with the aim of improving energy quality, especially disturbed electrical voltage. This filter is based on a PUC type voltage inverter controlled by PWM, uses two methods identification of disruptive tensions. The simulation models are developed using MATLAB-Simulink software. The results of the simulation obtained demonstrates the efficiency and good performance of the filter active series especially those based on the method of powers instantaneous P-Q for compensation of harmonics and related disturbances voltage and the SRF method

**Keywords:** series active filter,PUC, power equipment control, launchpad

**2020 Mathematics Subject Classification Numbers:** First, Second, Third.

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## Transforming Unstructured Customs Data into Strategic Insights via Robotic Process Automation

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### Abstract

Strategic decision making in the logistics industry depends on timely access to large volumes of accurate data. However, many valuable trade data sources do not allow access to data via an Application Programming Interface (API). This creates significant operational bottlenecks in corporate processes and forces organizations to rely on slow, error-prone manual collection processes. This study examines a practical solution developed for using Robotic Process Automation (RPA) to address the problem of lack of access to API. The study first systematically extracted raw, unstructured import customs declaration data from Datamyne.com, which was not accessible via API, using Python and the Playwright library. This was followed by extensive cleaning and restructuring using Pandas and NumPy to ensure accuracy and usability. The curated dataset was stored in a SQL Server database, serving as a single source of truth, and integrated with Microsoft Power BI to provide dynamic, self-updating dashboards and reports for Sales and Marketing teams. The results show that RPA can effectively transform complex, unstructured web data into a strategic asset, increase operational efficiency, enable data-driven decision-making, and provide sustainable competitive advantage in the logistics field.

**Keywords:** Robotic Process Automation (RPA), Web Scraping, Business Intelligence (BI), Dynamic Reporting  
**2020 Mathematics Subject Classification Numbers:** 68-11, 68T09, 68P205

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## Mathematical Modeling and Intelligent Control of Flexible-Joint Manipulators Using Hybrid ANFIS Architecture

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### Abstract

This paper presents an intelligent control framework for robotic manipulators with flexible joints, addressing the critical challenges of nonlinear dynamics and parameter uncertainties. We propose an Adaptive Neuro-Fuzzy Inference System (ANFIS) controller and conduct a comparative study with conventional PID control for trajectory tracking applications. The system dynamics are modeled using Lagrangian mechanics with state-space formulation to account for joint elasticity. Simulation results establish ANFIS as a promising solution for high-precision robotic applications requiring adaptability to dynamic operating conditions.

**Keywords:** ANFIS, PID control, state space system, robotic manipulator with flexible joints.

**2020 Mathematics Subject Classification Numbers:** First, Second, Third.

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# Application Performance Monitoring Software Systems: An Approach to the Use of Artificial Intelligence and Machine Learning

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## Abstract

The increasing complexity of modern Java Application Servers has made Application Performance Monitoring (APM) a critical component of enterprise IT operations. This study investigates the integration of Artificial Intelligence (AI) and Machine Learning (ML) into APM software systems to enhance anomaly detection, root cause analysis, and proactive remediation. The methodology processes JSON-based performance metrics collected from two JVM instances (JVM1 and JVM2) over a 15-day observation period. Dynamic and moving threshold techniques were applied, and anomalies were identified using the Random Forest and Isolation Forest algorithms. These approaches automatically detect irregularities in performance behavior without the need for manually defined alarm thresholds. Metrics were analyzed, feature engineering was performed to optimize data relevance, and results were evaluated using accuracy, precision, recall, and F1-score. Visual analytics techniques such as histograms, kernel density estimation, and scatter plots were used to improve interpretability. Experimental results show that AI- and ML-enhanced APM systems can significantly reduce Mean Time to Detect (MTTD), Mean Time to Investigate (MTTI), and Mean Time to Resolution (MTTR), while enabling continuous operational optimization. Beyond anomaly detection, this paper proposes a forward-looking AI-driven operational decision framework for live production environments. In this approach, the AI model evaluates detected anomalies, estimates business impact, and autonomously selects IT operational responses—such as resource scaling, process restarts, or configuration adjustments—based on learned historical patterns, operational policies, and risk thresholds. This creates a closed-loop “self-healing” production environment, reducing the need for human intervention and increasing service availability.

**Keywords:** Application Performance Monitoring, Artificial Intelligence, Machine Learning, Anomaly Detection, Dynamic Thresholds, AI, ML, APM

**2020 Mathematics Subject Classification Numbers:** 68T05, 68T35, 68M20

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## Effect of cutting speed on roughness in the case of S235 steel

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### Abstract

The aim of this work is to develop an optimization algorithm for additive manufacturing parameters, such as material selection, solidification temperature, and speed (execution time), while considering surface quality constraints. To achieve this objective, the work is structured around two main axes. The first axis focuses on the numerical modeling of additive manufacturing parameters, specifically for 3D printers, based on convergence principles. The second axis is dedicated to optimizing the modeling process while meeting the requirements of design engineering.

**Keywords:** Numerical simulation Additive, Manufacturing, Surface Quality, Manufacturing Tolerances.

**2020 Mathematics Subject Classification Numbers:** 74-10, 65Z05.

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# An Artificial Intelligence-Assisted Automation System Proposal to Increase the Efficiency and Accuracy of Transit Declaration Processes in International Logistics

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## Abstract

Increasing data density, global competition, and the need for speed in the logistics sector have made time loss, human errors, and operational inefficiency caused by manual document processing critical. In international road transport, the complexity of legislation and the increased workload associated with the transition to version 5 of the New Computerized Transit System (NCTS) have transformed the preparation of transit declarations into a process that takes hours and carries a high risk of errors. This study introduces an innovative solution developed to overcome these challenges and increase efficiency in international logistics operations. Equipped with artificial intelligence (AI)-supported optical character recognition (OCR) and a digital archive management infrastructure, this system automatically scans documents, analyzes, and classifies their content with high accuracy, and automatically generates international transit declarations. Thanks to its existing integration with SGS Transitnet, it has a flexible architecture that seamlessly connects with other guarantors and official customs authorities. Following the OCR process, algorithms that mimic human intelligence identify and correct erroneous fields, minimizing errors resulting from manual data entry and improving document access speed and operational accuracy. The future goal is to create fully autonomous and customs-approved declarations. This proposed system is an innovative and applicable solution model that will contribute to the objectives of efficiency, accuracy, cost optimization, and sustainable development in international logistics.

**Keywords:** Logistics, Artificial Intelligence, Transit Declaration Automation, Efficiency

**2020 Mathematics Subject Classification Numbers:** 68T01, 68T42, 68U35

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## Study of the effect of synthetic and natural fiber orientation on the mechanical behavior of a four-ply composite material

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### Abstract

This study examines the mechanical properties of two composite plates under three-point bending, focusing on the impact of varying fiber orientations. The plates consist of four layers each, with one made from carbon fiber/epoxy and the other from jute fiber/epoxy. The ply orientations are symmetrical around the laminate's neutral axis, with 0° aligned along the x-axis, which corresponds to the largest dimension of the rectangular plate. The finite element method, implemented using ANSYS software, is used to assess von Mises stress. This analysis aims to optimize the flat plate for maximum stiffness. The article presents a simulation-based investigation of the static mechanical behavior of composite laminates, including carbon fiber fabric or jute fiber combined with epoxy resin. The laminates have stacking sequences of [0°/90°/0°/90°]s, [0°4]s, [90°4]s, and [+45°/0°/90°/-45°]s. A numerical model based on thin plate theory was developed to describe the static behavior of composite specimens. The results show that varying fiber orientations and types significantly influences bending stiffness, highlighting the importance of these factors in enhancing the stiffness of mechanical structures.

**Keywords:** Numerical simulations; Finite element; Composite; Natural and Synthetic fiber.

**2020 Mathematics Subject Classification Numbers:** 65Z05.

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## Artificial Neural Networks Based Predictive Modeling of the Mechanical Behavior of Date Palm Fiber-Reinforced Composites

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### Abstract

The artificial neural network (ANN) approach has revolutionized the evaluation of drilled holes in hybrid jute/palm/polyester bio-composite plates by enabling precise predictions of key parameters such as delamination, circularity, and cylindricity. In this study, the ANN was trained with 52 samples (70), validated with 8 samples (10), and tested with 15 samples (20) at different stages. A total of 75 holes were assessed using a coordinate measuring machine (CMM) to ensure quality, dimensional accuracy, and geometric conformity. This rigorous inspection is essential for meeting industrial standards related to circularity and cylindricity, which are critical for high-performance applications. The results demonstrate the significant influence of feed rate on the delamination factor ( $F_d$ ) ( $R^2=0.98$ ), circularity error ( $R^2=0.99$ ), and cylindricity error ( $R^2=0.98$ ). This predictive approach enhances both the reliability and efficiency of the evaluation process.

**Keywords:** Artificial Neural Network (ANN), Hybrid Bio-Composite, Jute/Palm/Polyester, Circularity and cylindricity errors, delamination factor.

**2020 Mathematics Subject Classification Numbers:** 62M45 – Neural nets and related statistical methods.

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## Towards Robust Arabic Authorship Attribution: A Transformer-Based Model for Multi-Author Imbalanced Corpora

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### Abstract

Most conventional approaches to Authorship Attribution (AA) rely on statistical methods and classification algorithms that utilize stylistic features extracted from textual document. These features may include lexical, syntactic, structural, and content-based markers that reflect an author's unique writing style. In recent years, Pre-trained Language Models (PLMs) have gained significant attention in the field of text classification. While they have demonstrated strong performance on large-scale datasets and short texts, their effectiveness in scenarios with limited data—particularly in the context of Arabic Authorship Attribution (AAA)—remains insufficiently explored. This study aims to evaluate the effectiveness of state-of-the-art Pre-trained Arabic Transformer-based Models in handling imbalanced textual datasets, with a particular focus on the under-represented domain of theological law, which has witnessed limited contributions in the context of AA. The study addresses AAA using imbalanced corpora containing texts of varying lengths, extracted from several books written by Arab authors who lived during the same historical period. We conducted several experiments involving the fine-tuning of four Pre-trained Arabic Transformer-based Models: AraBERT, AraELECTRA, ARBERT, and MARBERT. The obtained experimental results have shown that AraBERT achieved the best performance in attributing texts to their respective authors.

**Keywords:** Deep Learning; Arabic transformer; AraBERT.

**2020 Mathematics Subject Classification Numbers:** 68T05, 68T07.

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## Dynamic analysis of composite plates immersed in a fluid

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### Abstract

The analysis of vibrations of plates immersed in fluid is of particular importance due to their numerous industrial applications. Such plates are commonly used in naval engineering as structural components of hulls of ships, submarines, and other offshore structures. Their interaction with the surrounding fluid significantly affects their vibrational behavior, which can impact the performance, durability, and safety of the structures involved.

In this study, a free vibration analysis of composite plates immersed in a fluid is presented. The plate modeling is based on Reddy's higher-order shear deformation theory (HSDT). A finite element with seven degrees of freedom per node is developed and used to determine the natural frequencies of thick composite plates. The fluid pressure applied on the free surface of the plate is determined using the velocity potential function and Bernoulli's equation.

The results are compared with existing experimental and numerical data from the literature, demonstrating excellent agreement. The natural frequencies of the plate are then illustrated in graphical form, considering various fluid levels, aspect ratios, thickness-to-length ratios, and boundary conditions.

**Keywords:** Free vibration, Thick composites plates, Sandwich plate, hierarchical finite element method, C1 HSDT Fluid-structure interaction, added mass.

**2020 Mathematics Subject Classification Numbers:** 74-10, 65Z05..

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# The Impact of Unstructured Data on Human–Robot Interaction in RPA

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## Abstract

Robotic Process Automation (RPA) is a significant technology that enhances workforce efficiency by automating repetitive and rule-based tasks in business processes. However, unstructured data frequently encountered in modern workflows, such as e-mails, messaging applications, and social media content, can negatively affect human–robot interaction within RPA systems. Such data often do not conform to predefined formats and contain various ambiguities inherent to natural language, thus limiting the ability of RPA robots to understand and respond accurately. In this study, data from the RPA processes of a banking company were examined and unstructured data were classified. Artificial Intelligence (AI) is a technology aimed at enabling computers to perform human-like reasoning and decision-making; Natural Language Processing (NLP) focuses on understanding and processing human language in text and speech, while Machine Learning (ML) enables systems to learn from data to perform prediction and classification tasks. These techniques were applied to banking data in this study to demonstrate that the automatic classification of unstructured data can improve human–robot relationships and serve as a reference for similar research. A model developed using AI algorithms can classify incoming messages based on their content and enable RPA systems to take appropriate actions according to the classification results. Consequently, RPA robots can establish more natural, accurate, and contextually appropriate communication with users, thereby increasing process efficiency and user satisfaction. The results indicate that an approach based on NLP and ML plays a critical role in improving the quality of human–robot communication in RPA processes and in improving the success of digital transformation projects. This research not only provides a technical improvement but also contributes to the development of a more human-centered automation approach by strengthening the cognitive capabilities of RPA applications.

**Keywords:** Robotic Process Automation (RPA), Unstructured Data, Human–Robot Interaction, Natural Language Processing (NLP), Machine Learning (ML)

**2020 Mathematics Subject Classification Numbers:** 68T01, 68T42, 68U35

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## 6.7 Mathematical Methods in Physics

Session Organizers: Özay Gürtuğ and Filiz Çağatay Uçgun

The symposium of “Mathematical Methods in Physics” is a subbranch of the 9th International Conference of Mathematical Sciences organized in Maltepe University, Istanbul, Turkey, on 03 – 07 September, 2025. This symposium is mainly oriented towards some recent developments in quantum field theory, gravity and cosmology, nonlinear systems, special functions, boundary problems and some relevant mathematical methods.

We hope that all attending this meeting will recall it as a useful and pleasant event. We wish to thank all lecturers and other speakers for their interesting and valuable talks. We also thank all participants for their active participation. And special thanks to our sponsors for their financial supports, which were very significant for realization of this scientific activity.

Editors,

**Özay Gürtuğ and Filiz Çağatay Uçgun**

- Özay Gürtuğ is currently working as a professor at the Faculty of Engineering and Architecture in Beykoz University. At the same time he is the Acting Dean of the Faculty of Engineering and Architecture. He obtained his BSc degree in Electrical and Electronic Engineering at the Eastern Mediterranean University (EMU). He received his MSc and PhD degrees from the Physics Department of EMU. His research interests are gravitational waves, black holes and space-time singularities.

- Filiz Çağatay Uçgun is currently working as a assistant professor at the Faculty of Engineering and Natural Sciences, Maltepe University. She obtained her BSc degree in Mathematics at the Abant İzzet Baysal University. She received her MSc and PhD degrees from the Mathematics Department of İzmir Institute of Technology and Yeditepe University, respectively. Her research interests are higher order and degenerate Lagrangian theories and their Hamiltonian formalism.

## Mathematical Modeling of Wear in Rolling Contacts: A Statistical Approach

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### Abstract

The effect of solid particles on the degradation of contact surfaces has been studied extensively in recent years. The presence of these unwanted particles between two bodies in relative motion can damage the surfaces and lead to total failure of the mechanisms. This study focuses on two main aspects: firstly, the effect of solid contamination in lubrication on surfaces in rolling contact (specifically between cylindrical rollers) and, secondly, the influence of operating parameters such as load and speed on the tribological behavior of mechanical components, particularly with regard to the evolution of dimensional losses (wear). The Taguchi L9 orthogonal array was used to study the influence of three operating variables (contact type (C), speed (V) and load (Q)) on the wear response (W). The experimental results were analyzed using Response Surface Modeling (RSM) and Analysis of Variance (ANOVA). The results showed that contact type (C) was the dominant factor affecting wear in the contaminated rolling contact, contributing 71.2% of the observed wear. Furthermore, the RSM wear models developed showed a strong correlation between predicted and experimental data, with coefficients of determination greater than 80% results obtained in this study support the rationale for reducing the life of mechanisms in a contaminated environment.

**Keywords:** Solid contamination - ANOVA - Wear - MSR - Mathematical model.  
**2020 Mathematics Subject Classification Numbers:** 82-10, 82-05.

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## User of the Stabilized Error Vector Propagation Method for Solving Partial Differential Equations in Electric Discharge

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### Abstract

The modeling of many physical phenomena such as the study of plasmas or the behavior of a gas in a plasma requires the use of linear or nonlinear partial differential equations. Today, the majority of scientists, researchers and engineers use numerical simulation to complement or even replace direct experimentation. In this work, we numerically simulate the Poisson equation in two dimensions to determine the potential prevailing in an electric discharge. The mathematical formalism used is based on a system of equations: continuity equation, momentum transport equation and energy transport equation. This system is itself coupled to the Poisson equation. The numerical procedure for solving the S.E.V.P transport equations (Stabilized Error Vector Propagation) has been selected for its accuracy and relative ease towards an extension to a multidimensional geometry. The results obtained clearly show the effectiveness of the S.E.V.P method in solving the Poisson's equation compared to the relaxation method and the variable-step method.

**Keywords:** S.E.V.P Method, Partial differential equation, Poisson's equation, Transport equation.  
**2020 Mathematics Subject Classification Numbers:** 65Z05.

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## Peak Deconvolution in X-ray Diffraction Patterns Using the Pseudo-Voigt Function

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### Abstract

X-ray diffraction (XRD) remains an essential non-destructive technique for probing the crystallographic structure of materials. However, the accurate interpretation of XRD patterns—particularly in nanostructured or multi-phase systems—can be hindered by overlapping peaks, asymmetric broadening, and instrument-induced distortions. These limitations obscure critical microstructural parameters such as crystallite size and lattice strain.

In this study, we present a robust methodology for XRD peak deconvolution using the pseudo-Voigt profile, a flexible line shape that models diffraction peaks as a linear combination of Gaussian and Lorentzian components. The method allows precise fitting of complex and broadened peaks commonly observed in nanoscale and polycrystalline materials.

We applied this approach to experimental XRD data of nanostructured materials, using both commercial software (Origin, FullProf) and custom Python scripts based on nonlinear least squares fitting. The fitted peak parameters ( $2\theta$  position, full width at half maximum, and intensity) were then used to calculate crystallite sizes via the Debye–Scherrer equation and microstrain using the Williamson–Hall method.

Our results demonstrate a marked improvement in peak resolution and phase identification, enabling accurate extraction of structural parameters even in the presence of significant peak overlap. The pseudo-Voigt-based deconvolution resulted in crystallite size estimates within the nanometric range (20–40 nm) and microstrain levels consistent with expectations for defect-rich nanomaterials.

This work underscores the value of physically grounded peak profile modeling in XRD analysis and offers a reproducible workflow for researchers engaged in the structural characterization of advanced materials.

**Keywords:** XRD deconvolution, Pseudo-Voigt fitting, Crystallite size, Microstrain, Line broadening.

**2020 Mathematics Subject Classification Numbers:** 65D10, 74E15.

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## Study of X-ray diffractograms of light alloys using the Rietveld method

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### Abstract

In this study, we examined the microstructure and determined the crystal parameter of the alloys prepared. Using X-ray diffraction curves of the prepared mixtures and the Rietveld method to refines a theoretical diffraction pattern by adjusting structural parameters to best fit the observed experimental pattern. Using Rietveld's Maude software [1]; we obtained theoretical curves that closely corresponded to the experimental curves obtained by X-ray diffraction, as well as lattice dimensions that were nearly identical. We found the same phases as researchers who have previously studied these properties, despite using a different preparation method.

**Keywords:** Rietveld method, X-ray, structural parameters, lattice dimensions.

**2020 Mathematics Subject Classification Numbers:** 74Q10, 74E30.

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## Improve efficiency of Perovskite-Based Solar Cell by photon recycling

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### Abstract

In recent years, significant advancements have been made in thin-film planar heterojunction solar cells, emerging as cost-effective photovoltaic devices with high power conversion efficiency. Among the materials utilized, organometal trihalide perovskite ( $\text{CH}_3\text{NH}_3\text{PbI}_3$ ) stands out as a promising absorber material. Its appeal lies in the affordability of organic-inorganic perovskite compounds, readily available in nature, ease of fabrication, and compatibility with large-scale processing at low temperatures. In addition to its effective absorption in the ultraviolet range, this material exhibits captivating optoelectronic properties, including high crystallinity, elevated carrier mobility, and extensive carrier diffusion lengths. Despite these advantages, the highest reported power conversion efficiency for perovskite solar cells is currently at 26.1/100, as of 2022. This study introduces a thin-film organometal trihalide perovskite solar cell featuring hybrid interfaces between carefully chosen materials. These selections are the result of an in-depth study aimed at minimizing recombination and optimizing performance. Furthermore, we enhance the absorption of the incident solar spectrum by incorporating a 1D photonic crystal at the cell's bottom, facilitating the photon recycling process. The proposed solar cell parameters are numerically computed using the rigorous coupled wave algorithm through the SYNOPSYS RSOFT CAD tool. The thickness of each layer in the structure is optimized using the MOST scanning and optimization module of the RSOFT CAD tool, achieving the highest power conversion efficiency at a minimal device thickness (approximately  $2.5 \mu\text{m}$ ). Remarkably, the power conversion efficiency achieved is 27.5/100, with a fill factor of 87.4/100 at AM 1.5, showcasing great promise. This demonstrates the remarkable potential of the proposed design to achieve efficiencies exceeding 5/100, positioning it as a competitive contender in the existing crystalline silicon photovoltaic market.

**Keywords:** Recycling photon; Solar cell; R-Soft.

**2020 Mathematics Subject Classification Numbers:** 68T05, 68T07.

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## Mathematical Model of a Photovoltaic Solar System Optimized by Fuzzy Logic

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### Abstract

Solar energy, efficiently converted into electrical power by photovoltaic (PV) systems with minimal environmental impact, represents a key pillar of modern renewable energy strategies. The performance of photovoltaic cells depends largely on the properties of semiconductor materials, which convert sunlight into electrical energy through the photovoltaic effect. This article presents a simplified mathematical model for photovoltaic cells, based on a single diode equivalent circuit. This model is developed from key nominal parameters typically provided by manufacturers, such as open-circuit voltage (Voc), short-circuit current (Icc), and maximum power point voltage and current (Vmp, Imp). Particular attention is paid to the impact of environmental factors, particularly temperature and irradiance, on the electrical behaviour of the photovoltaic cell. Semiconductors are highly sensitive to these variables, and their influence alters key performance indicators such as output voltage, current and overall energy production. The model thus provides a more accurate representation of these dynamic interactions, offering a better understanding of how photovoltaic modules actually function under variable climatic conditions. After establishing the photovoltaic panel model, the study examines the application of an advanced artificial intelligence technique, fuzzy logic (FL), to improve the performance of the maximum power point tracking (MPPT) algorithm. The integration of fuzzy logic enables adaptive and intelligent control, particularly in rapidly changing environmental conditions. Simulation results obtained via Matlab/Simulink demonstrate the effectiveness of the proposed approach, showing significant improvements in terms of tracking efficiency, robustness and response speed compared to conventional MPPT methods.

**Keywords:** Photovoltaic system, Mathematical Model, Fuzzy Logic.

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## Numerical Study Based on Finite Volume Method of Natural Convection Heat Transfer in Vertical Channel with Square Ribs

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### Abstract

This study investigates the numerical analysis of natural convection heat transfer in a vertical pipe with cubic obstacles, focusing on how the position and number of these obstacles affect heat transfer. Simulations were conducted with a fixed aspect ratio of  $A=5$  and varying Rayleigh numbers ranging from 10 to 10,000, to explore the influence of obstacle configuration on heat transfer efficiency. The study uses the finite volume method (FVM) to solve the governing equations for mass, momentum, and energy, assuming steady-state, laminar flow. The discretization of the equations is completed by using the Power Low Differencing Scheme (PLDS) for the discretization of convection terms. The algorithm SIMPLE was adopted to ensure the coupling pressure velocity, and the discretized equations obtained were solved by using the method of sweeping associated with the Thomas algorithm TDMA. Results show that the placement and number of obstacles significantly affect flow patterns and heat transfer, with more obstacles enhancing turbulence and heat exchange. However, excessive obstacles may increase pressure drop, indicating a trade-off between heat transfer and flow resistance. The findings provide insights for optimizing natural convection heat transfer systems in applications like cooling and thermal management.

**Keywords:** natural convection, vertical duct, square ribs, numerical simulation, finite volume method.

**2020 Mathematics Subject Classification Numbers:** First, Second, Third.

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## Integrability Analysis of Tumor Cells Dynamics

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### Abstract

The modeling of dynamic processes across disciplines such as physics, biology, and engineering heavily relies on differential equations. Real-world phenomena typically exhibit nonlinear characteristics, posing significant challenges for analytical treatment. Although computational approaches yield approximate solutions, they often fall short of providing deep physical insight. To address this, analytical methods based on symmetry principles, first introduced by Sophus Lie in the 19th century, are employed. Among these methods, the artificial Hamiltonian approach has proven particularly effective in identifying invariants and deriving closed-form solutions for complex nonlinear systems.

Cancer is a complex disease characterized by abnormal cell growth and involves numerous variables. Mathematical modeling of tumor growth, crucial for understanding cancer dynamics, often relies on nonlinear differential equations. Due to the diversity of cancer types and treatment methods to determine analytical solutions are critical. Lie group theory and the artificial Hamiltonian approach provide effective tools for obtaining these solutions, offering valuable insights into tumor behavior and treatment strategies.

In this study, analytical solutions have been derived for the mathematical model that examines the competition between tumor cells and normal cells in terms of both volume and other resources. Graphical representations of these solutions have also been provided.

**Keywords:** First integral, artificial Hamiltonian, tumor mathematical models.

**2020 Mathematics Subject Classification Numbers:** 34A34, 34A05, 34C14.

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## A DFT+U Theoretical Study of the Effect of Cobalt Quantities on the Structural, Electronic and Optic Properties of Zinc Oxide Compound

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### Abstract

Zinc oxide (ZnO) is a multifunctional material whose surface electrical characteristics critically influence its performance in applications such as gas sensing and visible-light emission. Additionally, ZnO has attracted considerable attention in biomedical applications, particularly in oncology, owing to its biocompatibility, antimicrobial activity, and selective cytotoxicity toward cancer cells. Enhancing its functional properties through substitutional doping, particularly with transition metals like Cobalt (Co), has been shown to significantly improve its electrical conductivity and optical transparency. In this work, we employ first-principles calculations to investigate the structural, electronic and optical behavior of Co-doped ZnO, targeting scenarios where experimental validation is constrained by cost. The results reveal a bandgap reduction to 2.29 eV at a doping concentration of  $x = \text{Co}/\text{Zn} = 0.125$ , indicating improved charge carrier mobility. Notably, the optical properties remain stable under this doping regime. These findings elucidate the correlation between Co incorporation and the modified physical properties of ZnO, offering valuable insights for its deployment in advanced optoelectronic and biomedical technologies. **Keywords:** GGA+U, Gap energy, supercell.

**2020 Mathematics Subject Classification Numbers:** 82-02.

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## Mechanism and Kinetic Parameters of the Thermal Oxidation of Aluminum Slag by Thermogravimetric Analysis

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### Abstract

In this work, we studied the mechanism and the kinetic parameters of the thermal oxidation of aluminum slag using the differential thermogravimetry technique under non-isothermal conditions. The experiments were conducted between room temperature and 1300°C at heating rates of 5, 10, and 20°C/min. The obtained differential thermogravimetry curves clearly show two distinct mass gains. The first mass gain is due to the oxidation of aluminium in the solid state. Among the 32 types of differential equations describing non-isothermal kinetics, we found that the most suitable mechanism is ( $A3/2 : g(x) = [-\ln(1-x)]^{2/3}$ ), also known as the Avrami-Erofeev equation of order 2/3. The activation energy ( $E_a$ ) for the oxidation of aluminium in the solid state was determined to be 260 kJ/mol. The second mass gain corresponds to the oxidation of aluminum in the liquid state. Under the applied conditions, the oxidation process follows a second-order reaction mechanism ( $F2 : g(x) = (1-x) - 1 - 1$ ). The activation energy ( $E_a$ ) for the oxidation of aluminum in the liquid state was determined to be around 430 kJ/mol.

The parameter  $m$  is equal to 1.85 for the formation of  $\gamma$ -alumina in the aluminum slag. This value is close to 2, indicating a two-dimensional growth of  $\gamma$ -alumina crystals. On the other hand,  $m$  is equal to 2.65 for the formation of  $\alpha$ -alumina in the aluminum slag, which is close to 3, suggesting a three-dimensional growth of  $\alpha$ -alumina crystals. The morphological growth parameter  $n$  is approximately 2 for the crystallization of  $\gamma$ -alumina and around 2.5 for the crystallization of  $\alpha$ -alumina. These results indicate that bulk nucleation is the dominant mechanism in the crystallization of both  $\gamma$ -alumina and  $\alpha$ -alumina, and that crystal growth is diffusion-controlled..

**Keywords:** Differential equations, Mechanism and kinetic parameters, Thermal oxidation of aluminum slag.

**2020 Mathematics Subject Classification Numbers:** 80A23, 80A30.

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# First Principles Calculations of the Elastic, Electronic, Magnetic, and Optical Properties of CoFeYSb Quaternary Heusler Compounds

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## Abstract

This study utilizes first principles calculations, specifically Density Functional Theory (DFT), to explore the structural, elastic, electronic, magnetic and optical properties of the quaternary Heusler compounds CoFeYSb (with Y= V, Ti). The DFT simulations were performed using the CASTEP code, which solves the Kohn-Sham equations within the framework of the plane-wave pseudopotential method. The structural analysis confirms that both compounds are most stable in the YI configuration. The stability of the magnetic (I) phase for both materials is further validated through calculations of elastic constants, cohesion energy, and formation energy. These calculations were carried out by solving the Schrödinger equation for the electrons in the system, with an approximation for the exchange-correlation functional, typically the generalized gradient approximation (GGA). Band structure analysis reveals that these compounds exhibit half-metallic behavior, with a complete spin polarization at the Fermi level. The spin-down energy gaps for CoFeVSb and CoFeTiSb were calculated to be 0.55 eV and 0.61 eV, respectively. These findings are consistent with the spin-polarized DFT results, where the total magnetic moments for CoFeVSb and CoFeTiSb were found to be  $3 \mu\text{B}$  and  $2 \mu\text{B}$ , respectively. In agreement with Slater-Pauling 24- electron rule. Additionally, optical properties were investigated by computing the dielectric function, absorption coefficient, and energy loss function within the DFT framework. These calculations predict strong absorption in both the visible and ultraviolet regions, demonstrating the potential of CoFeYSb compounds for advanced optoelectronic and spintronic applications. The results open new possibilities for their integration into future electronic and photonic technologies, leveraging their unique magnetic and optical properties. This approach emphasizes the use of DFT for solving the Schrödinger equation in the context of material properties, while incorporating computational methods such as CASTEP simulations for an accurate approximation of the electronic structure.

**Keywords:** First principles calculations, Density Functional Theory (DFT), The Generalized Gradient Approximation (GGA), Half-metallicity, Spintronic devices.

**2020 Mathematics Subject Classification Numbers:** 81Q08, 81V10, 78A04.

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## Modeling the degradation of engine oil

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### Abstract

Markov chains are memory less random sequences that can be used in predictive analytics and are highly dependent on probability theory.

In this study we used Markov chains because they allow predicting the change of state of engine oils after a determined operating time.

This work aims to model the change of state of engine oils using a Markov model, which allows us to follow the deterioration of the quality of the oil by calculating the probability that the viscosity will be degraded throughout its lifetime on one side and to give an estimate of the remaining lifetime on the other side.

The Markov chains gave with precision the probabilities of the change of the quality of the engines oils which allowed us a good estimate of the state of its viscosity.

**Keywords:** Modeling, Engine oil, Markov chains.

**2020 Mathematics Subject Classification Numbers:** 60Jxx.

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## Stabilization of an Euler-Bernoulli Beam with viscoelastic term

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### Abstract

In this work, we investigate asymptotic behavior of solution of the flexible marine riser problem with vessel dynamics when a pipeline is made by the viscoelastic materials (see [1, 2]). Under a suitable control applied at the top boundary of the riser, we establish different types of decay rates for a large class of relaxation functions. In particular, exponentially and polynomially (or power type) decaying functions are included in this class. Our method is based on the multiplier technique. In this work, we consider a flexible marine riser with vessel dynamics when a pipeline is made by the viscoelastic material. Namely

$$\rho w_{tt}(x, t) + EIw_{xxxx}(x, t) - EI \int_0^t h(t-s) w_{xxxx}(s) ds - Tw_{xx}(x, t) = f(t), \quad (x, t) \in (0, L) \times [0, \infty) \quad (1)$$

with the boundary conditions and the initial data

$$\left\{ \begin{array}{l} w(0, t) = w_x(0, t) = w_{xx}(L, t) = 0, \quad t \geq 0, \\ -EIw_{xxx}(L, t) + EI \int_0^t h(t-s) w_{xxx}(L, s) ds + Tw_x(L, t) \\ = u(t) - d_S w_t(L, t) - M_S w_{tt}(L, t), \quad t \geq 0 \\ w(x, 0) = w_0(x), \quad w_t(x, 0) = w_1(x), \quad x \in [0, L]. \end{array} \right. \quad (2)$$

Here  $w(x, t)$ ,  $w(L, t)$  and  $w_t(L, t)$  are the displacement of the riser at the position  $x$  for the time  $t$ , the position and velocity of the vessel, respectively. The external forces is represented by  $f(t)$ . The positive constants  $\rho$ ,  $L$ ,  $EI$  and  $T$  represents, the uniform mass per unit length, the length, the bending stiffness and the tension of the riser respectively. The coefficients  $M_S$  and  $d_s$  denotes the mass of the surface vessel and the vessel damping, and  $u(t)$  is the top boundary control of the riser. The integral term in the Eq. 1 represents the viscoelastic damping term. The main objective of this work, is to prove uniform stability of the system (1)–(2).

**Keywords:** Stability, vibration control, arbitrary decay, viscoelasticity

**2020 Mathematics Subject Classification Numbers:** 35L20, 93D15, 93D20.

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## Application of the FP-LAPW Method to Predict and Analyze the Physical Properties of Double Perovskite Oxides

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### Abstract

In this work, we apply the Full-Potential Linearized Augmented Plane Wave (FP-LAPW) method to investigate the structural, electronic, and optical properties of  $\text{Ba}_2\text{InSbO}_6$ , a representative double perovskite oxide. Using first-principles calculations, we explore the equilibrium crystal structure, the band structure, the density of states (DOS), and the electronic charge distribution to better understand the material's potential for various applications in electronics and energy storage. Our results show that  $\text{Ba}_2\text{InSbO}_6$  exhibits an indirect band gap, confirming its potential as a semiconductor. The analysis of the optical properties reveals significant absorption in the ultraviolet and visible regions, suggesting its potential for optoelectronic applications. Additionally, the mechanical properties such as the bulk modulus and Poisson's ratio are calculated, revealing the material's mechanical stability and robustness. This comprehensive study demonstrates the effectiveness of the FP-LAPW method in predicting key physical properties of double perovskite oxides, offering valuable insights into their potential for future technological applications.

**Keywords:** FP-LAPW method, Physical properties prediction, Double Perovskite Oxides,  $\text{Ba}_2\text{InSbO}_6$ .

**2020 Mathematics Subject Classification Numbers:** 62P35, 78A10, 81V80.

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## Analysis of a High-Order Finite-Difference Scheme for Transport Equations with Discontinuous Interfaces

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### Abstract

This research presents a rigorous mathematical analysis of a new high-order finite-difference scheme designed to solve transport equations in the presence of strongly discontinuous interfaces, with theoretical application to the modeling of radiative transport at fabric-air interfaces. Our theoretical contribution revolves around constructing and analyzing a family of non-uniform numerical schemes that maintain  $O(h^4)$  order formal accuracy in homogeneous domains while efficiently handling coefficient discontinuities at interfaces. The proposed method is based on an asymptotic development of finite difference operators with coefficients optimized by minimizing the local truncation error. The mathematical analysis establishes several fundamental properties of the proposed scheme: - A formal proof of convergence with order  $O(h^4)$  in regular domains and  $O(h^2)$  in the vicinity of interfaces - A stability analysis demonstrating that the scheme satisfies a CFL condition independent of the magnitude of discontinuities at interfaces - The demonstration of discrete conservation properties that guarantee the preservation of physical invariants (energy conservation) - Complete characterization of numerical dispersion and dissipation errors Numerical tests on model problems with several orders of magnitude discontinuities confirm theoretical predictions. Overall error is significantly reduced, and the spurious oscillations typically observed with classical schemes near interfaces are eliminated. Spectral analysis of the scheme's eigenmodes also validates the method's robustness against numerical pollution phenomena. This contribution opens up new perspectives for the numerical analysis of problems with discontinuous interfaces, with potential applications in mathematical physics beyond the specific case of radiative transport.

**Keywords:** Numerical analysis, high-order finite differences, transport equations, discontinuous interfaces, stability analysis, numerical convergence, mathematical physics.

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## A Data mining Study of The production of Hydrogen from Hydrocarbons

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### Abstract

The generation of hydrogen from hydrocarbons is a vital research domain owing to its capacity to offer a more environmentally benign energy alternative to conventional fossil fuels. This study aims to examine the plasma treatment processes for waste in hydrogen production, a highly promising energy source due to rising energy demands. Hydrogen serves, for instance, as fuel for powering fuel cells in vehicles, mitigating air pollution, and addressing elevated fuel costs. Hydrogen is one of the most prevalent elements in nature and may be generated from various basic materials in diverse locations. The results are examined utilizing data mining methodologies, including principal components analysis (PCA) and partial least squares (PLS), to elucidate the existing relationships and similarities among the attributes.

**Keywords:** Hydrogen, Data mining, Principle components analysis (PCA), Partial least squares (PLS)

**2020 Mathematics Subject Classification Numbers:** 62P35.

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## A Mathematical Approach to Growth of Non-Crystalline Films Induced by Mild Energy Ion Implantation

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### Abstract

This study presents a mathematical model to describe the growth of non-crystalline films in semiconductor materials resulting from mild-energy ion implantation. The model is applicable across both lower and higher ion dose ranges, with the damage distribution depth approximated by a Gaussian function. The growth of the amorphous layer and the formation of voids are analyzed in relation to ion dose and amorphization threshold. The model employs a multi-layer framework: a 3-layer model for low ion doses and a 4-layer model for higher ion doses, accounting for the transition to void formation above a critical dose. The ion implantation process leads to the displacement of lattice atoms, with a corresponding increase in point defects that eventually relax into an amorphous state. The amorphization dose, critical energy density, and damage depth are key parameters in the model. Additionally, the model provides a comprehensive mathematical description of the energy distribution and the resulting damage zone, extending into void regions at higher ion doses. The model's predictions are useful for understanding and optimizing ion implantation processes for semiconductor surface treatments and other applications. The mathematical framework is validated against experimental data, providing insights into ion-target interactions and damage evolution.

**Keywords:** Mathematical Modelling of Ion Implantation; Amorphous Layer Growth; Semiconductor Surface Treatment; Ion Dose; Ion-Target Interactions.

**2020 Mathematics Subject Classification Numbers:** First, Second, Third.

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## Maintenance of a centrifugal pump using vibration analysis

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### Abstract

Centrifugal pumps play a strategic role in a manufacturing process, and fortunately, there are powerful tools for monitoring to prevent failures such as vibration analysis. In this work we apply vibration analysis to detect faults at an early stage in a centrifugal pump selection and conduct a thorough analysis to determine the risks they pose to better understand the failure modes of critical equipment of this pump is optimized operations predictive condition monitoring and makes a statement vibratory bench pumping and study the results of companion vibration according to the following steps:

- Organize follow -up of all centrifugal their sector to define the causes of different potential failures and to forecast their effects pumps.
- Diagnosis major vibratory pump faults and provide an improved solution.
- Educate and monitor indicators of reliability of a fleet of centrifugal pumps.
- Replacement of defective parts.

**Keywords:** Wear, vibration, preventive maintenance, centrifugal pump, bearings, failure.

**2020 Mathematics Subject Classification Numbers:** 60J10.

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## Mathematical Modeling and Finite Element Analysis of a Self-Excited Induction Generator

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### Abstract

Modeling different electromagnetic systems is essential for understanding their operation and optimizing their performance. It also makes it possible to predict their behavior under different operating conditions, which is crucial for designing more efficient and reliable systems. In addition, electromagnetic modeling can also help identify potential problems before they occur, thereby reducing maintenance costs and improving system durability. In the present work, we will model a self-excited induction generator using finite element software, enabling us to simulate its operation under different load and speed conditions. This approach will help us gain an in-depth understanding of the generator's behavior and optimize its performance according to its specific needs. In addition, this modeling will enable us to explore different improvement strategies to maximize the system's energy efficiency.

**Keywords:** Finite Element Model, Self Excited Induction Generator, Numerical modeling.

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## The Use of the DIC Method to Involve the Strain Instability Occurred in an Undergoing High Shear during the ECAE Process

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### Abstract

Equal channel angular extrusion (ECAE) is one of the most efficient methods of severe plastic deformation of materials used to modify texture and microstructure without reducing the sample cross section. The application of a single ECAE pass to polypropylene (PP) was meticulously investigated at room temperature using a 90° die-angle tooling. Mechanical properties changes of the extruded samples due to backpressure and extrusion velocity effects were analyzed via uniaxial tensile tests. The tensile samples displayed multiple strain localizations in shear banded materials, whereas quite homogeneous deformation appeared for nonbanded ones. The digital image correlation technique suitable for large deformation was used to determine the full-field strain of the tensile samples in relation to the tensile strain and ECAE conditions. DIC measurements require an artificial random speckle pattern which was generated by green dots sprayed on the surface of each sample. The random speckle pattern was applied so that the speckles did not overlap. The studied zone was illuminated by a strong white-light beam. The illuminated random speckle pattern was captured during the deformation by a digital CCD camera placed in front of the sample and at a distance of 0.5 m from the sample. The images of the sample surface were recorded at a frequency of 2 Hz. DIC method is based on comparing images of the sample surface in the undeformed (reference) and deformed states. In this way, a Lagrangian description is performed. A 10 image step was used for the image correlation. The zone of interest was divided into small square sub images of 64x64 pixels. The displacement vector was calculated using the corresponding sub-image pairs extracted from the reference and deformed states of the sample. By achieving the analysis on numerous subimages, the full-field contours of displacement were obtained. The analysis was performed with Davis software developed by Lavision.

**Keywords:** Equal channel angular extrusion, Large deformation, Digital image correlation.

**2020 Mathematics Subject Classification Numbers:** 7404, 7405, 7410.

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## Global Classical Solution to a Reaction-Diffusion System

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### Abstract

We prove in this work the existence of a unique global classical solution in the space of uniformly continuous and bounded functions to a reaction-diffusion system on an unbounded spacial domain.

**Keywords:** reaction-diffusion system, semigroups theory, global existence of solution.

**2020 Mathematics Subject Classification Numbers:** 35A01, 35K05, 35K57.

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## Numerical Functional Analysis; Density Functional Theory (DFT) Calculations in Physical Properties of Compounds

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### Abstract

The molecule of our compound comprises two fragments, namely the naphthol and benzoate rings, which are linked to the hydrazo group. The bond length of the azo unit (N=N) measures 1.3202 Å and is calculated at 1.314 Å, and for their neighboring N-C bonds lengths (N2-C7, N1-C1) are observed at 1.324(2) and 1.402(2) Å, these bonds theoretically located at 1.315 and 1.399 respectively, exhibiting typical double bond and piconjugated single bond character. The bond lengths of O1=C17 and O3=C8 are comparatively shorter than that of C17-O2 and C18-O2 due to the presence of a double-bond character in O1=C17 and O3=C8 [1.211 (2) Å, 1.245 (2), 1.340 (2) and 1.453 (3) Å, respectively]. These specific bond lengths have been precisely computed as 1.213, 1.236, 1.349, and 1.438 Å at the B3LYP/6-31G(d,p) level. The bond length for C2-C17 presents the highest distance and is calculated as 1.483 Å by DFT, while XRD obtained the values is 1.479 (3) Å. Furthermore, the bond angles in the hydrazide group exhibited the values of 121.99, 119.13, 119.88, 121.32, 116.69, and 123.48° for the N1-N2-C7, and N2-N1-C1, C6-C1-N1, N1-C1-C2, N2-C7-C12 and N2-C7-C8 through DFT while XRD determined values are 120.34 (17), 117.71 (16), 120.30 (17), 120.65 (17), 115.40 (17) and 124.44 (17)°, respectively. On the other hand, the DFT values of bond angles for O1-C17-O2, O1-C17-C2, O3-C8-C9 and O3-C8-C7 in the carbonyl groups are observed to be 122.11, 125.13, 121.52 and 121.43° while, their XRD values are examined to be 121.96 (18), 125.47 (18), 121.21 (19) and 121.90 (17)°, correspondingly. Some bond angles expressed the same values for both experimental and DFT.

Keywords: DFT calculations, numerical functional analysis, functional theory.

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## Moment-Based Inference of Cell Proliferation Rates via Stochastic Birth-Death Dynamics

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### Abstract

Biopsy samples offer crucial static snapshots of cell populations and are widely used to assess conditions such as cancer [1]. However, inferring dynamic properties—such as cell proliferation rates—from such data remains a significant inverse problem. In this work, we model tumor growth using a stochastic birth-death Markov process under a logistic growth assumption, capturing saturation effects and randomness inherent in biological systems. To analyze this system, we developed an extended Mean Field Approximation (MFA) tailored for birth-death dynamics [2], allowing us to derive theoretical statistical moments that incorporate population fluctuations. This framework enables us to address the inverse problem by matching these theoretical moments with observed data to estimate proliferation rates. Furthermore, we simulated the stochastic logistic model to generate virtual biopsy data and train machine learning algorithms on the resulting moment structures, enhancing inference accuracy. Our approach bridges stochastic dynamics and real-world biological data, providing a physics-informed framework for quantitative diagnostics.

**Keywords:** stochastic processes, statistical moments, logistic growth, inverse problem, cell proliferation, Markov dynamics

**2020 Mathematics Subject Classification Numbers:** 60J20 (Markov processes), 37N25 (Dynamical systems in biology), 92C50 (Medical applications).

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## Mathematical Modeling and Fuzzy Direct Control of Renewable Energy Systems

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### Abstract

Mathematical modelling of renewable energy systems is an essential step towards understanding their operation in view of their control and performance enhancement. This paper introduces a novel approach that emerged the field of control based on fuzzy logic for improving the performance of Wind Energy Conversion System (WECS). The fuzzy control principle is inspired from human reasoning which allows flexible and intelligent control. By exploiting its adaptability, Fuzzy Direct Control (FDC) enhances WECS performance by overcoming the limitations of traditional direct control, guaranteeing better control and greater precision in the various operating modes under random wind speed. The simulation findings obtained with the proposed control technique demonstrated satisfactory improvements in terms of ripple reduction and power quality sent to the electrical grid. The FDC's simplicity and robustness make it a revolutionary solution for improving WECS efficiency.

**Keywords:** Mathematical modeling, fuzzy logic control, wind energy conversion system

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## Finite Volume Approach for Modeling Incompressible Flow Across a Backward-Facing Step

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### Abstract

This investigation explores the numerical simulation of incompressible flow over a backward-facing step, employing the finite volume method to analyze flow dynamics and heat transfer behavior. The study examines the impact of varying geometrical parameters, specifically expansion ratios  $ER = 1.5$  and  $ER = 2$ , and flow conditions, characterized by Reynolds numbers  $Re = 150$  and  $Re = 800$ . The finite volume method discretizes the governing Navier-Stokes and energy equations on a staggered grid arrangement, ensuring accurate representation of velocity and pressure fields. The Power-Law Differencing Scheme (PLDS) is utilized for convective flux approximation, implemented within a Fortran-based program. The discretized equations are solved using an iterative line-by-line method, with the Tridiagonal Matrix Algorithm (TDMA) applied to efficiently handle the resulting linear systems. The SIMPLE algorithm facilitates pressure-velocity coupling by computing pressure corrections iteratively, ensuring mass conservation. The simulation captures detailed flow structures, including recirculation zones and boundary layer development, and quantifies heat transfer rates across the step. Results elucidate the sensitivity of flow patterns and thermal performance to geometric and flow parameters, demonstrating the robustness and precision of the finite volume method for modeling complex incompressible flow scenarios.

**Keywords:** Finite volume method, Backward-facing step, Numerical Simulation.

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## Modeling of Humidity-Induced Disturbances in the Electric Field of Insulators with Hydrophobic Surface Treatment

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### Abstract

As part of our study, we investigated the behavior of various types of high-voltage insulators, particularly those used in power transmission networks operating under harsh environmental conditions. Insulators are critical components that ensure the safety and reliability of electrical networks by preventing leakage currents and electrically isolating power lines from supporting structures, even in the presence of humidity, pollution, and extreme climatic conditions. To comprehensively evaluate their performance, we conducted both experimental and numerical tests. The practical tests were performed at the high-voltage laboratory of the University of Béjaia, where different insulators were subjected to varying voltage levels. Their performance was assessed in terms of withstand voltage and electrical discharges. In parallel, we utilized COMSOL Multiphysics 6.1 to simulate the electric field and potential distributions around the insulators, allowing for a deeper understanding of the electrical behavior under diverse environmental stresses. One focus was the effect of rain. Tests with wet silicone insulators showed that water droplets aligned and deformed along the electric field lines, promoting the initiation of partial discharges. Rain increases the surface conductivity of insulators, especially in the presence of contaminants, leading to a reduction in their dielectric strength. To mitigate this, a superhydrophobic coating was applied, effectively preventing water adhesion and maintaining a dry and clean surface. We also examined the behavior of insulators in desert environments, characterized by high temperatures, intense solar radiation, and airborne sand and dust. In such conditions, sand accumulation and surface erosion can degrade insulation performance over time. Our observations showed that desert dust can form conductive paths under humid conditions, increasing the risk of flashover. The use of hydrophobic or self-cleaning surfaces proved beneficial in reducing the impact of these contaminants. Overall, the study demonstrated a strong correlation between experimental observations and numerical simulations. These insights contribute to the design and maintenance strategies of insulators exposed to diverse and extreme environmental conditions.

**Keywords:** High-voltage insulator, rain, desert conditions, water droplets, pollution, COMSOL Multiphysics, superhydrophobic coating.

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## Sensorless Control of a Double Fed Induction Motor Drive Powered by a Renewable Source

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### Abstract

This study presents a mathematical modeling, simulation, and analysis of an efficient control method for a double fed induction motor drive (DFIM), based on sliding mode control used super twisting algorithm (SMC-ST). Hence, this work aims to ameliorate both good performance and efficiency of the closed-loop system, while also supplied by a photovoltaic generator (PV), which employs maximum power point tracking (MPPT) controller based on perturb and observer technique (P&O). Additionally, the sliding mode observer (SMO) ensures the sensorless control of the machine, where allowing for high-quality estimation of the state vector. Finally, according to the digital simulation, the obtained results show the effectiveness of the suggested method.

**Keywords:** Double fed induction motor, Sliding mode control, MPPT, Photovoltaic generator .

**2020 Mathematics Subject Classification Numbers:** 00A69, 00A79, 85-10.

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## Proposed Energy Management Control Strategy for the PV/Battery/ Fuel Cell System: Modeling and Simulation

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### Abstract

This paper investigates innovative strategies to enhance energy efficiency, reliability, and sustainability by optimizing energy management in hybrid energy systems that integrate photovoltaic (PV) technologies, battery storage, and fuel cells. When solar power is insufficient due to environmental fluctuations, fuel cells provide a flexible and sustainable power generation solution, complementing batteries to ensure a stable energy supply. This cooperative operation reduces the strain on individual components, particularly batteries, prolonging their lifespan while maintaining system performance.

A mathematical modeling approach is developed to represent the dynamic behavior of the hybrid system components, including the PV generator, battery, and fuel cell. The proposed model incorporates control algorithms designed to manage energy flow intelligently and adaptively, based on real-time energy demand and generation conditions. Through simulation, different scenarios are evaluated to assess energy efficiency, cost-effectiveness, and system reliability under varying environmental and load conditions.

The modeling of the whole system is provided, taking into account operational constraints and energy priorities. Simulation and results were performed using the Matlab/Simulink environment, demonstrating the effectiveness of the proposed strategy in optimizing energy distribution and improving overall system robustness.

**Keywords:** Photovoltaics system, Fuel cells, Battery storage, Energy management strategy.

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## 6.8 Mathematics Education

Session Organizers: Vildan KATMER BAYRAKLI

The Mathematics Education session of the 9th International Conference of Mathematical Sciences (ICMS 2025) aims to provide a platform for presenting and discussing the most recent developments in a wide variety of topics including curriculum analysis and international framework alignment, algorithms and programming integration in mathematics education, probability education through synthesized international frameworks, mathematics curriculum evaluation and development, computational thinking in mathematics curricula, digital technologies in mathematics education, teacher professional development programs, curriculum design and implementation, interdisciplinary mathematics teaching, statistics and probability education, mathematics teacher education research, educational technology tools effectiveness, and mathematical literacy development. This session of this conference is a perfect opportunity for young researchers to improve themselves and to find some new open problems in their specific area of study. We think that most of the presentations in this session deserve to be published in prestigious mathematical education journals.

## Exploring the Alignment Between the Mathematics Curriculum and the CSTA "Algorithms and Programming" Core Concepts

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### Abstract

In the 21st century, the rapid advancement of information technologies and the pervasive impact of digital transformation on education have significantly increased the importance of algorithm and programming topics in mathematics curricula. At the secondary school level in particular, these components are regarded as critical for fostering students' computational thinking skills. The K-12 Computer Science Framework, developed through a collaboration between U.S.-based organizations (ACM, Code.org, CSTA, among others) and various states and districts, emerged from the need to structure computer science education systematically and has played a pioneering role in setting international standards in this field. The framework was developed to align with and complement existing educational frameworks in mathematics, science, and language arts. Within this context, the "Algorithms and Programming" core concept is recognized as a cornerstone of computer science education and encompasses five subcomponents: algorithms, variables, control, modularity, and program development. In Türkiye, the 2024 Secondary School Mathematics Curriculum (SSMC) represents a milestone in this transformation by incorporating, for the first time, the theme of "Algorithms and Informatics" into mathematics education. This innovative approach seeks to build a bridge between mathematics learning and computational thinking skills, aiming to enhance students' algorithmic reasoning within mathematical contexts. Accordingly, the purpose of this study is to examine the extent to which the learning outcomes related to the theme of "Algorithms and Informatics" in the SSMC align with the "Algorithms and Programming" core concepts outlined in the CSTA K-12 Computer Science Framework. Adopting a qualitative research design based on document analysis. The content was analyzed through qualitative analysis software based on the indicators in the framework. To ensure validity and reliability, the analysis process was supported by the consultation of two subject-matter experts. The results reveal that secondary mathematics curriculum in Türkiye demonstrates varying degrees of alignment with components of international algorithm and programming concepts across grade levels. While 9th-grade learning outcomes show partial correspondence to foundational algorithmic concepts, the 10th-grade curriculum exhibits a shift toward traditional mathematical approaches with reduced emphasis on algorithm-related content. These results suggest that the integration of algorithm and programming concepts within the mathematics education framework requires further development to achieve comprehensive alignment with established international standards.

**Keywords:** Algorithms and programming, mathematics curriculum, curriculum evaluation.

**2020 Mathematics Subject Classification Numbers:** 97D10.

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## Evaluation the Probability Education in the Mathematics Curriculum through Synthesized International Frameworks

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### Abstract

The probability learning domain plays a pivotal role in fostering students' mathematical thinking skills as well as their reasoning processes in situations involving uncertainty in everyday life. Nevertheless, the literature indicates that students often construct only a superficial understanding of probability concepts, rely on intuitive yet misleading approaches, and therefore encounter substantial difficulties in developing probabilistic thinking [1, 2, 3]. In Türkiye, the Mathematics Curriculum, revised in 2024, was designed with the aim of supporting students' conceptual development in the probability domain. However, the curriculum has not been explicitly grounded in a particular theoretical framework. Accordingly, it is essential to evaluate the content of the curriculum in light of established theoretical approaches in the literature. In this context, this study aims to evaluate the probability education of the Turkish Mathematics Curriculum within the scope of internationally recognized probability education frameworks. A qualitative research design was adopted, employing document analysis as the primary method. To ensure validity and comprehensiveness, the synthesized framework was further refined through the contributions of three mathematics education experts. The framework was developed by integrating key perspectives from [4] probability education, [2] probability literacy-building blocks, [5] probability education standards, [3] probabilistic thinking model, and [6] probability literacy competencies. It emphasizes five core dimensions: inclusion of fundamental probability concepts, methods of probability calculation, representations and communication of probability, connections to real-life contexts, and critical reasoning processes. Guided by this synthesized framework, the curriculum documents were systematically analyzed to determine how effectively probability learning outcomes reflect essential components of probability education. The content at each grade level was analyzed through qualitative analysis software based on the indicators in the framework. The results revealed that the probability education in the curriculum substantially aligns with the five core components of the synthesized framework. At the primary level, the curriculum addresses the majority of the framework indicators either fully or partially, while at the secondary level, it meets nearly all of them at a "full" or "partial" level. Overall, the results suggest that the curriculum is largely consistent with international approaches and standards in probability education. However, the integration of technology emerges as an area that requires further strengthening.

**Keywords:** Probability education, mathematics curriculum, curriculum evaluation.

**2020 Mathematics Subject Classification Numbers:** 97C70.

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## 6.9 Geometry

Session Organizer: İlhan GUL

The Geometry session at the 9th International Conference of Mathematical Sciences (ICMS 2025) features contemporary research in differential geometry and its related fields. The works presented here explore a range of advanced topics, from the geometric properties of specialized manifolds and their submanifolds to modern developments in Weyl geometry and biconservative structures. These contributions reflect the ongoing evolution of geometric theory and its deep connections to mathematical physics. This session offers a valuable opportunity for researchers to engage with cutting-edge developments and exchange ideas in a collaborative setting.

## The Cheng-Yau Formula and Biconservativity

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### Abstract

In recent years, the study of *biconservative submanifolds* has emerged as a central topic in differential geometry, motivated by the variational analysis of bienergy functionals. Given a smooth map  $\varphi : (M^m, g) \rightarrow (N^n, h)$ , the bienergy functional is defined as

$$E_2(\varphi) = \frac{1}{2} \int_M \|\tau(\varphi)\|^2 v_g,$$

where  $\tau(\varphi)$  denotes the tension field. The associated Euler–Lagrange equation yields the *bitension field*

$$\tau_2(\varphi) = -\Delta\tau(\varphi) - \text{trace}_g R^N(d\varphi, \tau(\varphi))d\varphi,$$

and harmonic maps are trivially biharmonic. A map (or immersion) is called *biharmonic* if  $\tau_2(\varphi) = 0$ . Within this framework, *biconservative submanifolds* arise as a natural relaxation, defined by the condition that the tangential part of  $\tau_2(\varphi)$  vanishes [1], i.e.

$$(\tau_2(\varphi))^T = 0.$$

This condition has strong geometric consequences. In the hypersurface case  $\varphi : M^m \rightarrow N^{m+1}$  with shape operator  $A$ , the biconservative condition reduces to

$$A(\nabla f) = -\frac{mf}{2} \nabla f,$$

where  $f$  denotes the mean curvature.

Another useful tool in Riemannian Geometry is the Cheng–Yau formula given by

$$-\frac{1}{2} \Delta |\phi|^2 = |\nabla \phi|^2 + \langle \phi, \text{Hess trace } \phi \rangle + \sum_{i,j}^m (\lambda_i - \lambda_j)^2 R_{ijij},$$

where  $\phi$  is a symmetric, (1,1)-tensor defined on Riemann manifold  $M$  and  $\lambda_i$  are eigen values of  $\phi$  [2].

In this work, we will talk about the importance of the Cheng–Yau formula on biconservative hypersurfaces by providing a new characterization of such hypersurfaces.

**Keywords:** Rigidity, biconservative hypersurfaces, space forms.

**2020 Mathematics Subject Classification Numbers:** 53C42.

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## On some submanifolds of $\mathcal{C}$ - and $\mathcal{S}$ -manifolds

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### Abstract

The main purpose of this work is to study some geometric conditions for an invariant total geodesic submanifold of a  $\mathcal{C}$ -manifold (resp.  $\mathcal{S}$ -manifold), where the notions of  $\mathcal{C}$ - and  $\mathcal{S}$ -structures are interesting cases of the  $f$ -structure. Further, an important class of the considered invariant submanifolds called semi-parallel, pseudo-parallel and Ricci generalized pseudo-parallel submanifolds, are examined.

In addition some new results including the necessary and sufficient conditions under which the invariant submanifolds are totally geodesic, are obtained.

**Keywords:**  $\mathcal{C}$ -manifold,  $\mathcal{S}$ -manifold, totally geodesic submanifold .

**2020 Mathematics Subject Classification Numbers:** 53C25, 53C15.

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## Super Quasi-Einstein Weyl Manifolds

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### Abstract

In this work, we introduce the concept of a super quasi-Einstein Weyl manifold, which extends the notion of a quasi-Einstein Weyl manifold. We establish several results concerning super quasi-Einstein Weyl manifolds, particularly those admitting certain special vector fields.

**Keywords:** Super quasi-Einstein Weyl manifolds, quasi-Einstein Weyl manifolds, special vector fields  
**2020 Mathematics Subject Classification Numbers:** 53C18, 53C25.

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## 6.10 Algebra

Session Organizer: Leyla Bugay

Algebra, one of the main theory of mathematics, gives us the basic information we need to grasp certain problems that require more thought. In addition to the fact that the algebra theory has a rich question content, its connection with other fields of mathematics has increased the importance of algebra theory.

The main purpose of this special session is to bring together researchers and scientists working and interested in the field of algebra to share information. With this common purpose, a total of thirteen presentations (one of these is a poster presentation) were made on algebra, more specifically, on polynomial matrices, some special graphs, Fuzzy sets, finite transformation semigroups, groups, commutative rings, ideals, fields, matrix, generating sets, ranks, Jordan algebras and UP-algebras.

## Structural and algebraic properties of k-balancing-like sequences

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### Abstract

In this study, we investigate the structure of k-balancing-like sequences and establish novel algebraic identities. Additionally, we derive the generating functions associated with the k-balancing-like numbers. We further construct matrix representations corresponding to these numbers, establishing a connection between the sequences and linear algebraic structures.

**Keywords:** Balancing-like sequences, generating functions, Catalan identity.

**2020 Mathematics Subject Classification Numbers:** 11B39, 11B83, 11C20.

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## A new type of Max matrix via binomial coefficients

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### Abstract

Matrices play a key role in various branches of mathematics, including combinatorics, numerical analysis, and linear algebra. Max matrices, whose entries are determined by taking the maximum of their row and column indices, have been studied for their interesting algebraic properties and applications. This paper introduces a new type of Max matrix, called the binomial Max matrix, whose  $ij$ th entry is defined as  $\binom{n}{\max(i,j)}$ . The study explores this matrix from both structural and functional perspectives, examining fundamental properties such as its determinant, inverse, adjoint matrix, *LU* decomposition, and several matrix norms. We also provide illustrative examples to support and clarify the theoretical findings, demonstrating the practical use of the results.

**Keywords:** Max matrix, binomial coefficients, algebraic matrix properties.

**2020 Mathematics Subject Classification Numbers:** 05A10, 15A09, 15A15, 15A60, 15B36.

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## Rough intuitionistic fuzzy filters in BE-algebras: applications in artificial intelligence and medical diagnosis

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### Abstract

This paper introduces the concept of rough intuitionistic fuzzy filters in the framework of BE-algebras and investigates their fundamental properties. To extend this study, we define a set-valued homomorphism on BE-algebras, leading to the formulation of T-rough intuitionistic fuzzy filters. Furthermore, we employ the  $(\alpha, \beta)$ -cut of an intuitionistic fuzzy set in BE-algebras as a key tool for characterizing and analyzing these new structures.

The proposed framework has significant applications in various domains requiring uncertainty modeling and intelligent decision-making. In particular, rough intuitionistic fuzzy filters can enhance artificial intelligence (AI) for classification problems, medical diagnosis for handling uncertain symptomatology, decision support systems for multi-criteria decision-making, and cybersecurity for anomaly detection in uncertain environments. Through these characterizations, we develop a comprehensive theoretical foundation that facilitates the application of rough intuitionistic fuzzy filters in algebraic structures and real-world computational problems.

**Keywords:** Fuzzy set, rough set, artificial intelligence.

**2020 Mathematics Subject Classification Numbers:** 03E72, 03B52, 68T01.

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## Some methods for finding ranks of certain finite transformation semigroups

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### Abstract

Let  $S$  be a semigroup and let  $\emptyset \neq A \subseteq S$ . The smallest subsemigroup of  $S$  containing  $A$  is called the subsemigroup generated by  $A$  and denoted by  $\langle A \rangle$ . If  $S = \langle A \rangle$ , then  $A$  is called a generating set of  $S$  and the positive integer  $\text{rank}(S) = \min\{ |A| : \langle A \rangle = S \}$  is called the rank of  $S$ . A generating set with cardinality  $\text{rank}(S)$  is called minimal generating set of  $S$ . As in the other algebraic theories, it is an important problem to find the **rank** of a semigroup, and has been much studied over the last fifty years. There are some studies about different rank types, for example, idempotent rank, nilpotent rank,  $(m, r)$ -rank, quasi-idempotent rank, relative rank. Therefore, to find any kind of rank of a semigroup, explicitly we need to find at least one minimal generating set. Thus, it is an important problem to find a method which decides whether an arbitrary non-empty subset  $A$  of any semigroup  $S$  is a minimal generating set of  $S$ , or not.

Let  $T_n$  and  $P_n$  be the (full) transformations semigroup and the partial transformations semigroup on the finite set  $X_n = \{1, \dots, n\}$ , respectively. In this study we present some methods that effective for finding ranks of certain subsemigroups of  $T_n$  and  $P_n$ .

**Keywords:** Generating set, rank.

**2020 Mathematics Subject Classification Numbers:** 20M20.

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## On minimal quasi-idempotent generating sets of certain subsemigroups of symmetric inverse semigroup

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### Abstract

For any element  $a$  of a semigroup  $S$ , if  $a^2 = a$  then  $a$  is called an *idempotent*; and if  $a \neq a^2 = a^4$  then  $a$  is called a *quasi-idempotent*. The set of all quasi-idempotents in any  $U \subseteq S$  is denoted by  $Q(U)$ . For any  $\emptyset \neq A \subseteq S$ , the smallest subsemigroup of  $S$  containing  $A$  is called the subsemigroup generated by  $A$  and denoted by  $\langle A \rangle$ . It is easy to see that  $\langle A \rangle$  is the set of all finite products of elements of  $A$ . If  $S = \langle A \rangle$ , then  $A$  is called a generating set of  $S$ . If there exists a generating set  $A$  of  $S$  consists entirely of quasi-idempotents, then  $A$  is called quasi-idempotent generating set of  $S$  and the quasi-idempotent rank of  $S$  is defined by  $\text{qrank}(S) = \min\{|A| : \langle A \rangle = S, A \subseteq Q(S)\}$ . A quasi-idempotent generating set with cardinality  $\text{qrank}(S)$  is called minimal quasi-idempotent generating set of  $S$ . Now let  $I_n$  be the symmetric inverse semigroup on the finite chain  $X_n = \{1, \dots, n\}$  for any  $n \in \mathbb{Z}^+$  and let  $I_{n,r} = \{\alpha \in I_n : |\text{im}(\alpha)| \leq r\}$  which is clearly a subsemigroup of  $I_n$  for each  $1 \leq r \leq n-1$ . In this paper we construct a method to find a minimal quasi-idempotent generating set of  $I_{n,r}$  for  $n \geq 2$  and  $1 \leq r \leq n-1$ .

**Keywords:** Quasi-idempotent, symmetric inverse semigroup, minimal generating set.

**2020 Mathematics Subject Classification Numbers:** 20M20.

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## Equivalence of multivariate polynomial matrices to pencil form

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### Abstract

In this paper we give necessary and sufficient conditions under which a class of multivariate polynomial matrices are equivalent to pencil form. The class of polynomial matrices involved is one which has its determinant monic in one of the indeterminates. It turns out that the concept of Smith form plays a crucial role and the resulting state-space form involves the companion form of the determinant of the original matrix.

**Keywords:** Pencil form, equivalence, Smith form.

**2020 Mathematics Subject Classification Numbers:** 15A04, 15A20, 15A54.

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# On the index divisors and monogeneity of a certain class of polynomials defined by $x^9 + ax^7 + b$

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## Abstract

In this paper, we calculate the index of any nonic number field  $K$  generated by a root  $\alpha$  of a monic irreducible trinomial  $F(x) = x^9 + ax^7 + b \in \mathbb{Z}[x]$ . Our approach is based on Engstrom's results and the factorization of any rational prime in  $K$ . In such a way we give a complete answer of Problem 22 of Narkiewicz ([5]) for this family of nonic number fields. Also, we give generators of power integral bases in some cases where  $i(K) = 1$ . Our results are illustrated by some computational examples.

**Keywords:** Index of a number field, Monogeneity, prime ideal factorization.

**2020 Mathematics Subject Classification Numbers:** 11R04, 11R16, 11R21.

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## Extensions in UP-algebras and related properties

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### Abstract

In this paper, we introduce and investigate the concept of extensions of UP-algebras, inspired by similar constructions in KU-algebras. We begin by recalling fundamental notions related to UP-algebras and propose a formal definition of an extension in this context. Several illustrative examples are provided to demonstrate the structure and behavior of these extensions. We examine the relationship between extensions and homomorphic images, ideals, and congruences, and we establish some necessary and sufficient conditions under which a UP-algebra extension exists. Furthermore, we compare our results with those obtained in the theory of KU-algebra extensions to highlight similarities and differences. This study opens new directions in the algebraic analysis of UP-structures and lays the groundwork for further investigations, such as regular and fuzzy extensions.

**Keywords:** UP -algebras, UP -subalgebras, UP - ideals, extended UP-algebras.

**2020 Mathematics Subject Classification Numbers:** 06F35, 03G25.

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## On weakly $(2, n)$ -ideals of commutative rings

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### Abstract

Let  $R$  be a commutative ring with identity and  $I$  be a proper ideal of  $R$ . We introduce the concept of a weakly  $(2, n)$ -ideal. We call  $I$  a weakly  $(2, n)$ -ideal if for  $a, b, c \in R$  with  $0 \neq abc \in I$ , we have  $ab \in I$  or  $ac \in N(R)$  or  $bc \in N(R)$ , where  $N(R)$  denotes the nilradical of  $R$ , i.e., the intersection of all prime ideals of  $R$ . This new class of ideals serves as a generalization of both weakly  $n$ -ideals and  $(2, n)$ -ideals. In particular, we show that every  $(2, n)$ -ideal and every weakly  $n$ -ideal is a weakly  $(2, n)$ -ideal. However, by providing several counterexamples, we demonstrate that the converses of these implications do not hold in general. In this paper, we first investigate the fundamental properties of this new ideal class and establish its hierarchical relationship with other existing classes of ideals in the literature. Subsequently, we examine the behavior of this ideal structure under various algebraic constructions such as homomorphisms, quotient rings, localizations, and direct products of rings.

**Keywords:**  $n$ -Ideal, weakly  $(2, n)$ -ideal, 2-absorbing ideal.

**2020 Mathematics Subject Classification Numbers:** 13A15; 13A99.

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