Department of Electronics, University of M'Sila

Electrical & Electronic Measurements

For Second-Year Undergraduate Students Majoring in Electronics, Automation & Telecommunications

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MOHAMED BOUDIAF UNIVERSITY OF M'SILA
FACULTY OF TECHNOLOGY
DEPARTMENT OF ELECTRONICS

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شهادة إدارية

بعد الإطلاع على التقارير الايجابية الواردة من السادة الخبراء أعضاء لجنة دراسة المطبوعة الجامعية والاتية أسماؤهم:

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صادق أعضاء اللجنـــة العلميـــة على قبول المطبوعة البيداغوجية مع إمكانية إتخاذها سندا في تدريس طلبة السنة الثانية ليسانس جميع الشعب (اتصالات، الإلكترونيك و الآلية) في ميدان علوم و تكنولوجيا و أن تعتمد في أي تقييم للمسار العلمي للأستاذ المعني **صاهد محمد** (أستاذ محاضر قسم "أ"– جامعة محمد بوضياف - المسيلة) تحت عنوان :

Electrical and Electronic Measurements

رئيس القسم — رئيس اللجنة العلمية

جامعة محمد بوضياف المسيلة رئيس قسم الإلكترونيك كلية التكنولوجيا
طباخ مصطفى

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UNIVERSITY MOHAMED BOUDIAF – M'SILA

FACULTY OF TECHNOLOGY

DEPARTMENT OF ELECTRONICS

Lecture notes

ELECTRICAL AND ELECTRONIC MEASUREMENTS

Course intended for 2nd-year Bachelor's degree students (S4), Specializations: Electronics, Automation & Telecommunications

Prepared by:

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Academic year 2024/2025

بِسْمِ اللَّهِ الرَّحْمَٰنِ الرَّحِيمِ

PREAMBULE

This course is designed for second-year LMD students specializing in Electronics, Automation, and Telecommunications. It provides a comprehensive introduction to the fundamentals of electrical and electronic measurements, essential for the students' academic and professional development. The course combines a strong theoretical foundation with practical, real-world applications, ensuring a balanced learning experience. Students will explore new definitions and concepts in electrical measurement, gain insights into various measurement techniques, and learn about the operation and use of key measurement instruments.

Each chapter is structured to include:

- A concise theoretical overview, enriched with illustrative examples and practical applications.
- Exercises accompanied by detailed solutions to reinforce understanding and problem-solving skills.

OBJECTIVES

Educational Objectives: The primary educational goals of this course are to:

1. Introduce and explain the core measurement techniques for electrical and electronic quantities.
2. Develop proficiency in using both analog and digital measurement instruments and ensure proper connection and operation.
3. Provide hands-on experience through laboratory experiments spanning various domains of electricity and electronics.

Specific Objectives: Upon completing this course, students will be able to:

1. Solve basic measurement problems involving electrical and electronic quantities, such as current, voltage, resistance, impedance, and power.
2. Perform measurements and calibrations using common instruments, including ammeters, voltmeters, ohmmeters, and oscilloscopes.
3. Select the most appropriate measurement instrument for a given task, optimizing for accuracy and efficiency.
4. Analyze and interpret measurement results effectively.
5. Adhere to safety protocols to ensure secure and reliable electrical measurements.

PREREQUISITES

To succeed in this course, students are expected to have prior knowledge in the following areas:

1. **Fundamental Physics**: Understanding of basic physical laws and principles.
2. **Electrical Foundations:** Mastery of core electrical concepts, including circuit analysis and the characteristics of electrical components.
3. **Mathematical Skills**: Familiarity with key mathematical tools, such as:
   - Analysis of real functions,
   - Differential and integral calculus,
   - Complex numbers.

TARGETED SKILLS

This course aims to develop the following competencies:

1. **Knowledge and Expertise:**
   - Mastery of electrical and electronic measurement methods.
   - Understanding the limitations and applications of different measurement techniques.
2. **Practical Proficiency:**
   - Ability to perform accurate and precise electrical measurements.
   - Skill in analyzing and improving measurement procedures.
3. **Instrumentation Skills:**
   - Familiarity with the operating principles of key measurement instruments.
   - Ability to select the right instrument for specific measurement tasks.
4. **Analytical Abilities:**
   - Awareness of factors influencing electrical measurements.
   - Competence in calculating and interpreting measurement uncertainties.
5. **Safety Awareness:**
   - Adherence to safety standards and best practices during electrical measurements.

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Chapter 1

Measurements, Quantities and Uncertainties

Chapter objectives

By the end of this chapter, the student will be able to:

- Differentiate key terms and vocabulary related to measurement.
- Classify units of measurement into fundamental units, derived units, and supplementary units.
- Understand the dimensions of different physical quantities.
- Describe various systems of units and their applications.
- Identify different types of errors and uncertainties in measurements.
- Explore general methods and techniques used in measurement.

1.1 Introduction

Measuring a physical quantity means comparing it to another reference quantity of the same nature, known as a unit of measurement. Electrical measurements, in particular, are renowned for their precision and reliability, making them one of the most accurate methods for quantifying physical, mechanical, thermal, and other quantities.

To facilitate measurement, many physical quantities are first converted into electrical quantities, which can then be easily compared and analyzed. Electrical measurement technology plays a crucial role in solving significant problems, such as:

- **Telemetry:** Transmitting measurement results over long distances.
- **Mathematical operations:** Performing calculations on measured quantities.
- **Automation:** Directly controlling machines and devices using measurement data.

These capabilities not only enhance our understanding of the world but also open doors to discovering its hidden secrets and wonders. Through electrical measurements, we gain the tools to progress technologically and improve our quality of life.

1.2 Definitions

- **Physical quantity:** A physical quantity is any property of nature that can be quantified by measurement or calculation. It is expressed as a numerical value accompanied by a unit of measurement.

  Example: Time, length, mass.
- **Unit of measurement:** A unit of measurement is a specific quantity, defined by convention, used as a reference to express quantitatively other quantities of the same nature.
- **Measurement process:** This refers to the set of operations aimed at determining the value of a quantity.
- **Measurement:** It is the evaluation of a quantity by comparing it with another quantity of the same nature taken as a unit.

  Example: A length of 2 meters, a mass of 400 grams.
- **Measurement uncertainty:** It is a parameter that characterizes the dispersion of values that could be attributed to the quantity being measured.
- **Measurement standard:** A measurement standard is a reference quantity used to define or materialize a unit of measurement. It must be precise, accurate, reproducible, and universal.

  Example: The “meter” is the measurement standard for length.
- **Calibration:** It is the operation of comparing the measurement results of a standard or an instrument against a more precise reference standard or instrument [(1)]. This process determines the accuracy of a standard or an instrument, by identifying their relative measurement errors and/or quantifying their associated measurement uncertainties.

1.3 System of units

A system of units is defined by a conventional selection of fundamental (or base) quantities, each associated with a specific unit. These systems provide a standardized framework for measuring physical quantities consistently and accurately.

Examples of systems of units:

➊ CGS System:

- **Fundamental quantities:** Length, mass, time.
- **Units:** Centimeter (cm), gram (g), second (s).

[(1)] The reference instrument (or standard), often referred to as a calibrator, must itself be calibrated in a traceable manner.

❷ MKSA or GIORGI System:

- **Fundamental quantities:** Length, mass, time, electric current.
- **Units:** meter (m), kilogram (kg), second (s), ampere (A).

❸ International System of Units (SI):

- **Fundamental quantities**: Length, mass, time, electric current, thermodynamic temperature, amount of substance, luminous intensity.
- **Units:** Meter (m), kilogram (kg), second (s), ampere (A), kelvin (K), mole (mol), candela (cd).

1.3.1 International system of units (SI)

The **International System of Units** (abbreviated **SI**), inspired by the metric system, is the most widely used system of measurement worldwide. It is a **decimal-based system**, meaning that multiples and submultiples of units are related by powers of 10.

The **SI** is built on seven fundamental quantities, which are considered independent by convention. These fundamental quantities and their corresponding units serve as the foundation for all other quantities in the system.

1.3.2 Fundamental units of SI

The seven fundamental units of the SI are defined using reproducible physical phenomena or fundamental constants of nature. This means that the units are tied to unchanging, universal properties that can be observed and measured consistently anywhere in the world. For example:

- ✓ The meter is defined using the speed of light in a vacuum.
- ✓ The second is defined based on the frequency of a specific transition in a cesium atom.
- ✓ The kilogram is now defined using the Planck constant.

By linking the units to physical constants, the SI ensures that measurements are accurate, consistent, and universally reproducible. These units are summarized in Table 1-1 below.

Table 1-1: The seven fundamental units defining the SI

Base quantities	**Units**	**Symbols**
Mass	Kilogram (kg)	m
Length	Meter (m)	*l, x, r, etc.*
Time	Second (s)	t
Electric Current Intensity	Ampere (A)	I, i
Temperature	Kelvin (K)	T
Amount of substance	Mole (mol)	n
Luminous Intensity	Candela (cd)	I_v

1.3.3 Derived units of SI

The derived units in the SI are formed by combining the fundamental units through algebraic relationships that define the corresponding derived quantities. These units are essential for expressing more complex physical quantities.

Table 1-2 provides examples of derived quantities and their corresponding units. As science and technology advance, new derived quantities and units can be introduced by combining the base units in meaningful ways.

Table 1-2: Derived Units

Quantities	Units	Symbols
Area (S)	square meter	m^2
Volume (V)	cubic meter	m^3
Volume Density (ρ)		$kg.m^{-3}$
Velocity (υ)		$m.s^{-1}$
Acceleration (a)		$m{\cdot}s^{-2}$
Frequency (f)	Hertz	Hz
Force (F)	Newton	N
Pressure (P)	Pascal	Pa
Energy (E)	Joule	J
Power (P)	Watt	W
Electric Charge (q)	Coulomb	C
Electrical Voltage (U)	Volt	V
Electric Field ($\vec{E}$)		$V.m^{-1}$
Capacitance (C)	Farad	F
Resistance (R)	Ohm	Ω
Magnetic Field ($\vec{B}$)	Tesla	T
Inductance (L)	Henry	H
Molar Concentration (c)		$mol.L^{-1}$
Specific Heat Capacity (c)		$J.kg^{-1}$

1.3.4 Supplementary units of SI

The SI also includes two supplementary units for measuring angles:

- ✓ **Radian (rad):** It is the unit of plane angle, which is defined as the angle subtended at the center of a circle by an arc equal in length to the radius.
- ✓ **Steradian (sr):** It is the unit of solid angle, which is defined as the solid angle subtended at the center of a sphere by an area on its surface equal to the square of the radius.

These supplementary units are considered dimensionless and may be used in the expressions of derived units when appropriate.

1.4 Decimal multiples and submultiples of SI units

Decimal multiples and submultiples of SI units can be expressed using SI prefixes, which are listed in Table 1-3. These prefixes represent powers of 10 and are used to simplify the expression of very large or very small quantities.

Table 1-3: SI Prefixes

10^{-15}	10^{-12}	10^{-9}	10^{-6}	10^{-3}	10^{-2}	10^{-1}	1	10^{1}	10^{2}	10^{3}	10^{6}	10^{9}	10^{12}	10^{15}
femto	pico	nano	micro	milli	centi	deci	unit	Deca	Hecto	Kilo	Mega	Giga	Tera	Peta
f	p	n	μ	m	c	d	Ø	da	h	k	M	G	T	P

Note: The distinction between uppercase and lowercase is important. . For example, "m" stands for milli (10^{-3}), while "M" stands for mega (10^{6}).

1.5 Dimensional analysis

1.5.1 Dimensions of quantities

Physical quantities are organized according to a system of dimensions. Each of the seven SI fundamental quantities has its own dimension, represented symbolically by a single uppercase sans-serif Roman letter. The symbols for fundamental quantities and their dimensions are shown in Table 1-4.

Table 1-4: Fundamental quantities and dimensions

Base Quantity	**Symbol of the quantity**	**Dimension Symbol**
Mass	m	M
Length	*l, x, r, etc.*	L
Time	t	T
Electric Current Intensity	I, i	I
Temperature	T	Θ
Amount of substance	n	N
Luminous Intensity	I_v	J

Note: Some quantities, such as counting-based quantities, are considered dimensionless and have the unit 1 (Example: 1 mole of a substance contains exactly 6.02214076×10^{23} atoms (Avogadro's number). This is a counting-based quantity.).

All other quantities are derived quantities, which can be expressed in terms of the fundamental quantities using physical equations. The dimension of a derived quantity Q is written as follows:

$$dim\,(Q) = [Q] = L^{\alpha}M^{\beta}T^{\gamma}I^{\delta}\Theta^{\varepsilon}N^{\zeta}J^{\eta}$$

where α, β, γ, δ, ε, ζ and η are dimensional exponents (typically small integers, positive, negative, or zero).

The dimension of a derived quantity provides the same information as its SI unit, which is expressed as a product of powers of the base units.

✓ Additional Notes:

- **Symbols for Quantities and Dimensions:** Symbols representing physical quantities are always written in italics. However, symbols representing dimensions are written in uppercase sans-serif Roman letters (sometimes enclosed in square brackets, e.g., [L], [M], [T]).
- **Recommended vs. Mandatory Symbols:** The symbols provided for physical quantities are recommended but not strictly mandatory. However, the symbols for units, as well as their style and form (e.g., uppercase/lowercase, italics), must be used as specified in this course.
- **Algebraic Treatment of Dimensions:** Dimension symbols and exponents follow the ordinary rules of algebra.

  Examples:

  - The dimension for area is written as L^2.
  - The dimension for velocity is written as LT^{-1}.
  - The dimension for force is written as MLT^{-2}.

1.5.2 Dimensional equation

A dimensional equation symbolically expresses the relationship between physical quantities. It is used to check the consistency of formulas or equations. For an equation to be valid, both sides must have the same dimension.

To ensure that an equation is homogeneous, it is sufficient to verify that both sides of the equation have the same dimension.

Example: $A = B \implies [A] = [B]$. This equation is therefore homogeneous.

If the dimensions in both sides differ, the equation is **invalid**.

A few simple rules:

1. **Sum and difference:** The dimension of the sum (or difference) of two physical quantities A and B is equal to the dimension of A (and to the dimension of B):

$$[A + B] = [A] = [B]$$

2. **Product:** The dimension of the product of two physical quantities A and B is equal to the product of the dimensions of A and B:

$$[A \times B] = [A] \times [B]$$

- The operator [] is therefore multiplicative.

3. **Division:** The dimension of the quotient of two physical quantities A and B is equal to the quotient of the dimensions of A and B:

$$\left[\frac{A}{B}\right] = \frac{[A]}{[B]} = [A] \times [B]^{-1}$$

4. **The dimension of the *n*-th power:** The dimension of the *n*-th power of a physical quantity A is equal to the *n*-th power of the dimension of A:

$$[A^n] = [A]^n$$

5. **The dimension of the *n*-th derivative:** The dimension of the *n*-th derivative of a physical quantity A with respect to B has the dimension:

$$\left[\frac{d^n A}{dB^n}\right] = \frac{[A]}{[B]^n} = [A] \times [B]^{-n}$$

6. **The dimension of the integral:** The dimension of the integral of a physical quantity A with respect to B has the dimension:

$$\left[\int A\, dB\right] = [A] \times [B]$$

7. **Transcendental functions:** The arguments of transcendental functions cosine, sine, tangent, exponential, logarithm, ... have no dimension (they are said to be dimensionless). In these cases, $[A] = 1$. The same is true for a constant.

Example: Determine the dimension of velocity, acceleration, force, energy, power, potential U and resistance R.

- **Velocity:**

$$v = \frac{dx}{dt} \quad \Longrightarrow \quad [v] = \frac{[x]}{[t]} = \mathrm{LT^{-1}}$$

- **Acceleration:** In the case of rectilinear motion

$$a = \frac{d^2x}{dt^2} \quad \Longrightarrow \quad [a] = \frac{[x]}{[t]^2} = \mathrm{LT^{-2}}$$

- **Force:**

$$\vec{F} = m \times \vec{a} \quad \Longrightarrow \quad [F] = [m] \times [a] = \mathrm{MLT^{-2}}$$

- **Energy:**

$$E = \frac{1}{2} m v^2 \quad \Longrightarrow \quad [E] = \frac{1}{2}[m] \times [v]^2 = \mathrm{ML^2T^{-2}}$$

- **Power:** is energy per unit of time

$$P = \frac{E}{t} \quad \Longrightarrow \quad [P] = \frac{[E]}{[t]} = \mathrm{ML^2T^{-3}}$$

- **Potential:**

$$P = U \times I \quad \Longrightarrow \quad U = \frac{P}{I} \quad \Longrightarrow \quad [U] = \frac{[P]}{[I]} = \mathrm{ML^2T^{-3}I^{-1}}$$

- **Resistance:** According to Ohm's law

$$U = R \times I \quad \Longrightarrow \quad R = \frac{U}{I} \quad \Longrightarrow \quad [R] = \frac{[U]}{[I]} = \mathrm{ML^2T^{-3}I^{-2}}$$

1.6 Measuring Instruments

1.6.1 Definition

A measuring instrument (or device) is a device that transforms a physical quantity [(2)] into usable information that can be estimated and/or visualized.

1.6.2 Types of Measuring Instruments

In general, two different types of instruments are distinguished:

- **Analog instruments:** Also called needle or deflection instruments, they indicate a value exactly proportional to the value of the quantity to be measured. Their principle is to give a needle deflection on a graduated scale.
- **Digital instruments:** they give a value representing the quantity to be measured to the nearest quantization step. This value is given in the form of a number (digital display).

These two types will be discussed in detail in the last chapter.

In the electrical and electronic field, the measuring instruments used are:

- the voltmeter to measure voltages,
- the ammeter to measure currents,
- the wattmeter to measure power,
- the ohmmeter to measure resistances,
- the frequency meter for measuring frequency, period and time
- the oscilloscope to visualize the shape of a wave and to obtain various information (amplitude, period...).

The voltmeter, ammeter, and ohmmeter are often combined into a single device called a "*multimeter*".

1.6.3 Structure of a Measuring Instrument

The basic structure includes at least three stages (See Figure 1-1):

- **A sensor:** it is sensitive to variations in a physical quantity and translates its value to be measured into an electrical, optical or mechanical signal that is easier to manipulate and quantify.
- **A signal conditioner:** transforms the output signal from the sensor (by amplifying its amplitude or filtering it) to make it compatible with the display and/or utilization module.
- **A display and/or utilization unit:** allows the measurement to be displayed and/or used.

This basic structure is found in all instruments and measurement chains, regardless of their complexity and nature. Nowadays, almost all measuring instruments are electronic chains.

[(2)] This quantity (or this magnitude) concerns a physical phenomenon inaccessible (or difficultly accessible) to our senses.

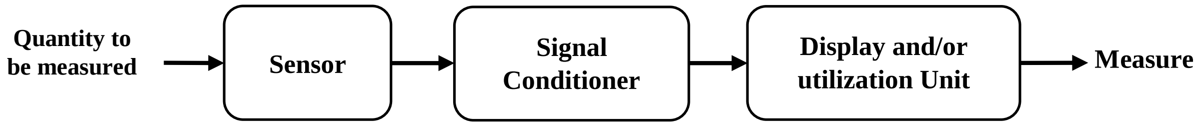


Figure 1-1: Structure of a measuring instrument

1.6.4 Characteristics of a Measuring Instrument

In reality, there is no perfect measuring instrument. Therefore, the applicability and quality of the measurements taken depend mainly on the characteristics of the measuring instrument used. The main characteristics of measuring instruments are:

- **Accuracy or precision:** This is the set of values of the measurand for which a measuring instrument is supposed to provide a correct measurement.
- **Fidelity:** This is the quality of the device to provide the same indication when several measurements are made for the same measured quantity (even if these measurements are distant in time).
- **Measurement Range:** This is the set of values of the measurand for which a measuring instrument is supposed to provide a correct measurement.
- **Measurement Span:** corresponds to the difference between the maximum and minimum values of the measurement range. For devices with an adjustable measurement range, the maximum value of the measurement span is called "full scale" (See Figure 1-2).

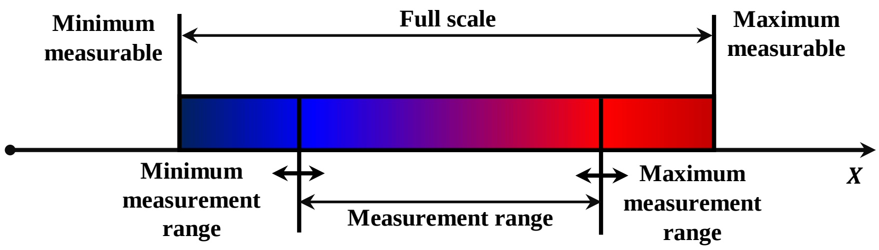


Figure 1-2: Measurement scale

- **Calibration Curve:** It is specific to each device (sensor or measuring instrument). It allows the raw measurement to be transformed into a corrected measurement. Calibrating an instrument consists of applying a known value to the input of the measurement system to verify that the output corresponds to the expected value; if this is not the case, the device's setting is corrected. **Example:** a standard weight is weighed, and the position of the needle is corrected so that it indicates the correct value. This is the so-called *single-point* calibration.
- **Sensitivity:** This is the smallest variation of a physical quantity that can be measured by a measuring instrument. Let x be the quantity to be measured and y the signal provided by the

measuring device. To all values of x, belonging to the measurement span, corresponds a value of $y = f(x)$. The sensitivity around a value of x is the quotient K:

$$K(x) = \frac{dx}{dy}$$

If the function is linear, the sensitivity of the device is constant:

$$K(x) = \frac{\Delta x}{\Delta y}$$

- **Repeatability:** A measurement is repeatable when the closeness of agreement between the results of successive measurements of the same measurand, carried out under the same conditions of measurement, is verified: same measurement procedure, same observer, same instrument, same location, and repetition over a short period of time.
  The dispersion of the results allows the repeatability to be quantified.
- **Reproducibility:** A measurement is reproducible when the closeness of agreement between the results of measurements of the same measurand, carried out under different measurement conditions – to be defined on a case-by-case basis – is verified.
- **Accuracy Class:** Value as a percentage of the ratio between the largest possible error and the measurement range. For an analog measuring instrument, the accuracy class (Cl) is defined as follows:

$$\text{Cl}(\%) = \frac{\text{Largest possible error}}{\text{Measurement range}} \times 100$$

  The C42-100 standard defines the following class values:
  - ✓ Calibration instruments: class 0.1, 0.2, and 0.5 (used in laboratories).
  - ✓ Control instruments: class 0.5 and 1 (used for inspection and verification).
  - ✓ Industrial instruments: class 1.5 and 2.5.
  - ✓ Indicators: class 5.
- **Resolution:** For digital measuring instruments, the resolution is defined by:

$$\text{Resolution} = \frac{\text{Measurement range}}{\text{Number of measurement points}}$$

- **Sensitivity:** It qualifies the influence of the measuring instrument on the phenomenon being measured. It is high when the device minimally disturbs the quantity being measured.

- **Speed, Response Time:** This is the ability of an instrument to follow sudden variations in the measurand. The main criterion for evaluating the speed of a measuring instrument is its N% response time, denoted $Tr_{N\%}$ (often the 5% response time).

  Knowing this response time is essential when performing measurements. It allows one to determine how long, after a sudden change in the measurand, the value provided by the instrument is actually representative of the measurand.

  Example: For a step change in the sensed quantity leading to an increase in the measurement, the 5% response time ($Tr_{5\%}$) is defined as the time required for the measurement to reach its equilibrium value, which is the exact value, from its initial value to remain between 95% and 105% of its total variation.

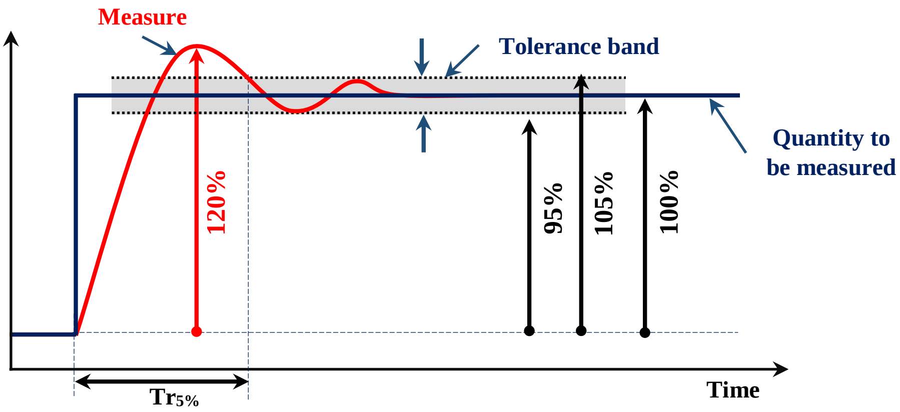


Figure 1-3: Step response

- **Bandwidth:** Frequency band for which the instrument's gain is greater than or equal to the maximum gain.
- **Influence Quantity and Compensation:** An influence quantity is any physical quantity other than the quantity being measured. Generally, temperature is the most frequently encountered influence quantity.

1.7 Measurement Methods

The main measurement methods are:

- **Direct Methods:** The measurement method is said to be direct when the measuring instrument directly provides the value of the measurand.

  Example: measurement of current intensity with an ammeter.

- **Indirect methods:** The measurement method is said to be indirect if the measurement is obtained from the values of other quantities by applying physical laws.

  Example: Calculation of the value of a resistance, by applying Ohm's law, by measuring the values of the current flowing through it and the voltage across its terminals.

Measurement methods can also be classified according to the procedures used in the measurement:

- **Deflection methods:** in this type of method, the value read, during the deflection of the measuring instrument, is used in the calculations (for example, the determination of a resistance using an ohmmeter).
- **Null methods:** when the adjustment is complete, no current flows in the measuring instrument, and the measured quantity can then be determined by an appropriate relationship (the typical example is the measurement of resistance using a Wheatstone bridge).
- **False null methods:** when the adjustment is complete, the same current flows in the measuring instrument when the circuit of a branch of the circuit is opened or closed. We simply observe that the deflection of the measuring instrument does not change, without measuring it.
- **Opposition methods:** an e. m.f. is opposed. m. or a potential difference to that existing across a variable resistor through which a current flows (precise measurement of voltages).
- **Potentiometric methods:** these are opposition methods for which the numerical value of the adjustable resistance is a multiple of the voltage to be measured.

1.8 Measurement Errors and Uncertainties

A measurement operation is, in general, inaccurate, limited, and has imperfections that cause errors in the measurement result.

1.8.1 Concept of Error

When measuring any quantity, an exact value can never be obtained. Error is the difference between the measured value and the exact value of the quantity being measured.

An error is the inaccuracy due to the imperfection of the measuring instruments and possibly the reading of the measurements. Errors can be minimized and/or compensated for by making a good choice of instruments and measurement methods.

1.8.2 Classification of Errors

There are three types of errors:

- **Random errors:** These are errors that likely arise from unpredictable temporal and spatial variations in influencing quantities.

  They must be treated statistically or probabilistically. They cannot therefore be eliminated, but can be minimized by increasing the number of measurements or observations.
- **Systematic errors:** These are reproducible errors in each measurement that generally arise from defects in the construction of the measuring instruments. They can be eliminated by appropriate corrections.

  There are many sources of systematic errors:
  - Accuracy errors of the measuring instrument (poor calibration, zero offset...).

- Disturbance due to the presence of a measuring instrument (connecting a measuring instrument modifies the quantity that one wishes to measure).
- Effect of influence quantities (temperature, pressure...).

A systematic error can be considered as a constant error (a bias) that affects each of the measurements.

It cannot be reduced by increasing the number of measurements but can be compensated for by applying a correction.

It can be detected and quantified by measuring the same quantity with different instruments or methods.

- **Accidental errors:** These result from an incorrect procedure, misuse, or malfunction of the measuring instrument. They are generally not taken into account in determining the measurement.

But since the exact value is unknown, the error made cannot be known. The result is therefore always uncertain. This is referred to as measurement uncertainty.

Note: The concept of error is theoretical and errors cannot be known exactly.

1.8.3 Concept of Uncertainty

Measurement uncertainty, often denoted as ΔX, is a parameter, associated with the result of the measurement, which characterizes the dispersion of the values that could reasonably be attributed to the measurand. It provides access to an interval around the measured value in which the exact value is assumed to belong. In practice, uncertainty can only be estimated.

The three causes of uncertainties are:

- The imperfection of the measuring instrument.
- Defect in the measurement method.
- The operator's limitations when reading analog devices.

Note: It is necessary to distinguish the terms "error" and "uncertainty". They are not synonymous but represent completely different concepts. They should not be confused or misused for one another.

There are two types of uncertainty:

- **Absolute uncertainty** ΔX, which has the same unit as the measured quantity.

  The quantity X being measured directly, we denote X_i the value found at the i-th measurement. After repeating the measurement n times, the average value of X is:

$$\bar{X} = \frac{X_1 + X_2 + \cdots + X_n}{n}$$

  The absolute uncertainty on X is the largest of the differences between X and its average $\bar{X}$:

$$\Delta X = sup|X - \bar{X}|$$

- **Relative uncertainty** $\frac{\Delta X}{X}$, which is expressed as a percentage.

The result of a measurement is never a single value; it is always given as a range of probable values of the measurand $X = x \pm \Delta X$ associated with a confidence level. The exact value necessarily lies within this interval $[x - \Delta X, x + \Delta X]$. The result can therefore be expressed in two ways:

$$X = x \pm \Delta X\ [unit]$$

Either

$$X = x\ [unit] \pm \frac{\Delta X}{X} [\%]$$

Example: The measured value of a resistance $R = 10\Omega \pm 5[\%]$ or$R = (10 \pm 0.5)\Omega$.

1.8.4 Calculation of Uncertainties in Direct Measurements

- **Measurement Uncertainties of Analog Devices:** The value measured by an analog measuring device is determined by a direct reading of the device's deflection and is given by the following relationship:

$$X = \frac{R \times M_R}{S} [unit]$$

where R is the reading (number of graduations read on the scale), M_R is the measurement range (caliber), and S is the scale (total number of graduations on the scale).

Analog measuring devices generally have two types of uncertainties. One is instrumental uncertainty (or class) related to the device itself, and the other is reading uncertainty, related to the operator.

❶ **Class uncertainty:** It is a function of the device's precision and is expressed as follows:

$$\Delta X_C = \frac{\text{Class} \times M_R}{100}$$

❷ **Reading uncertainty:** This is due either to poor eyesight or to poor reading conditions. If we denote by ΔR the fraction of a graduation error made (also called the estimated fraction of division during measurement or reading error), the reading uncertainty will be given by the following relationship:

$$\Delta X_R = \frac{\Delta R \times M_R}{S}$$

The total uncertainty in a measurement using an analog device will be the sum of the class uncertainty and the reading uncertainty:

$$\Delta X = \Delta X_C + \Delta X_R$$

If the measurement method is also a source of uncertainty to be evaluated (denoted ΔX_{Method}), the total uncertainty is then given by:

$$\Delta X = \Delta X_C + \Delta X_R + \Delta X_{Method}$$

Example: We want to measure the voltage across a resistor using an analog voltmeter with the following characteristics: Accuracy Class: $Cl = 1.5$, Total number of divisions: $N = 100$. For a measurement range of $M_R = 30V$, the reading is $R = 80$, and for $M_R = 300V$, the value read on the same scale is $R = 8$. Assuming the operator makes a reading error of $\Delta R = 0.25$, calculate for each range:

1. The voltage U.
2. The absolute uncertainty ΔU.
3. Which range is appropriate?

Solution:

a) For $M_R = 30V$, we have $R = 80$

1. Voltage Calculation U

$$U = \frac{R \times M_R}{N} = \frac{80 \times 30}{100} = 24V$$

2. Calculation of the absolute uncertainty ΔU

By definition, we have: $\Delta U = \Delta U_C + \Delta U_R$

where $\Delta U_C = \frac{\text{Cl} \times M_R}{100}$ is the instrumental (or class) uncertainty

and $\Delta U_R = \frac{\Delta R \times M_R}{N}$ is the reading uncertainty.

$$\Delta U = \frac{Cl \times M_R}{100} + \frac{\Delta R \times M_R}{N} = \frac{1.5 \times 30}{100} + \frac{0.25 \times 30}{100} = 0.45 + 0.075 = 0.525V.$$

Given a relative uncertainty: $\frac{\Delta U}{U} = \frac{0.525}{24} = 0.021875 \approx 2.19\%$.

b) For $M_R = 300V$ the value read is $R = 8$

1. Voltage Calculation U

$$U = \frac{R \times M_R}{N} = \frac{8 \times 300}{100} = 24V$$

2. Calculation of the absolute uncertainty ΔU

$$\Delta U = \frac{Cl \times M_R}{100} + \frac{\Delta R \times M_R}{N} = \frac{1.5 \times 300}{100} + \frac{0.25 \times 300}{100} = 4.5 + 0.75 = 5.25V.$$

Given a relative uncertainty: $\frac{\Delta U}{U} = \frac{5.25}{24} = 0.21875 \approx 21.87\%$.

3. Therefore, for the voltage measurement U, we must choose the range $M_R = 30V$ for which the absolute and relative uncertainties are less than those obtained for the range $M_R = 300V$.

- **Measurement Uncertainties of Digital Devices:** For digital display devices, the concept of class is not defined, but manufacturers provide an indication under the name of accuracy that allows the calculation of the total uncertainty on the measurement.

  This accuracy is very often given as a percentage of the reading plus or minus a constant expressed in units or points (digits) in the following way:

$$\Delta X = \pm(x\%\ of\ reading + n\ counts)$$

where $x\%$ is given by the manufacturer and n is the number of error points committed by the device.

In this case, the uncertainty is calculated according to the following relationship:

$$\Delta X = x\%R + \frac{n.G}{N} = \frac{x.L}{100} + \frac{n.G}{N}$$

With G being the range used [unit] and R being the reading displayed directly on the device's display and N being the total number of points of the device.

The accuracy of digital devices can be expressed in another way, as follows:

$$\Delta X = \pm(x\%\ of\ reading + y\%\ of\ range)$$

The uncertainty is calculated, this time, by:

$$\Delta X = \frac{x.R}{100} + \frac{y.G}{100}$$

Example 1: We want to measure the voltage U across an ohmic conductor, connected in a circuit, using a digital voltmeter with three displays and having 300 points and presenting an accuracy of $(0.2\% \pm 1\text{unité})$. The measurement is made on the $30V$ range, the reading taken is $R = 24V$.

Solution: On range $30V$, the resolution of the device is:1 unit $\Rightarrow \frac{G}{N} = \frac{30}{300} = 0.1V$.

The absolute uncertainty ΔU due to the device error is given by:

$$\Delta U = \frac{x.R}{100} + \frac{n.G}{N} = \frac{0.2 \times 24V}{100} + \frac{1 \times 30}{300} \approx 0.15V$$

The corresponding relative uncertainty is therefore:

$$\frac{\Delta U}{U} = \frac{0.15}{24} \approx 0.006 \approx 0.6\%$$

In the expression of ΔU, the second term is generally the most important and as it is in the form $(\frac{G}{N})$ it is advantageous, when making a measurement, to choose the smallest possible range.

Example 2: We measured a voltage $(U = 1V)$ in a circuit with a digital voltmeter with a resolution of 1mV on its 2V range. The indicated accuracy is: $\Delta U = \pm 0{,}1\% R \pm 2d$, (With: R: reading; d: digit or unit). Calculate the absolute uncertainty for a reading of $R = 1V$.

Solution: The absolute uncertainty ΔU due to the device error is given by:

$$\Delta X = \frac{x.R}{100} + \frac{n.G}{N} = \frac{0.1 \times 1V}{100} + 2 \times 1mV = 3mV$$

Example 3: We measured a current ($I = 2.5A$) in an electrical circuit with a 300-point digital ammeter, on its $4A$ range, whose accuracy indicated on the device's instructions is $\pm(0.1\%\ of\ reading + 0.01\%\ of\ range)$. Calculate the absolute uncertainty for this measurement.

Solution: The absolute uncertainty ΔU due to the device error is given by:

$$\Delta X = \frac{x.R}{100} + \frac{y.G}{100} = \frac{0.1 \times 2.5}{100} + \frac{0.01 \times 4}{100} = 0.0025 + 0.0004 = 0.0029A$$

Note: For digital display devices, it is not required to calculate the uncertainty on the reading due to the operator, this uncertainty is already taken into account in the device's accuracy.

1.8.5 Calculation of Uncertainties on Indirect Measurements

Suppose that the value of the quantity to be measured X_e is obtained by the following mathematical relationship: $X_e = f(a_1, a_2, \dots, a_n)$, with f being a function of n independent variables. Differential calculus can then be used to determine the uncertainties.

The absolute uncertainty is expressed as follows:

$$\Delta X = \left|\frac{\partial f}{\partial a_1}\right| \Delta a_1 + \left|\frac{\partial f}{\partial a_2}\right| \Delta a_2 + \cdots + \left|\frac{\partial f}{\partial a_n}\right| \Delta a_n$$

The corresponding relative uncertainty is given as follows:

$$\frac{\Delta X}{X} = \left|\frac{\partial f}{\partial a_1}\right| \frac{\Delta a_1}{X} + \left|\frac{\partial f}{\partial a_2}\right| \frac{\Delta a_2}{X} + \cdots + \left|\frac{\partial f}{\partial a_n}\right| \frac{\Delta a_n}{X}$$

Example 1: Determine the absolute and relative uncertainties when measuring a combination of two resistors in series: $R = R_1 + R_2$.

Solution: By applying the preceding formulas, we obtain:

$$\Delta R = \Delta R_1 + \Delta R_2 \quad \text{or} \quad \frac{\Delta R}{R} = \frac{\Delta R_1 + \Delta R_2}{R_1 + R_2}$$

Example 2: Determine the absolute and relative uncertainties when measuring an electric current intensity given by: $I = I_1 - I_2$ (the difference between two currents I_1 and I_2).

Solution: Similarly, by applying the preceding formulas, we obtain:

$$\Delta I = \Delta I_1 + \Delta I_2 \quad \text{or even} \quad \frac{\Delta I}{I} = \frac{\Delta I_1 + \Delta I_2}{R_1 - R_2}$$

Example 3: Determine the absolute and relative uncertainties in the case of energy given by: $W = U \times I \times t$ (product of three variables)

Solution: $\Delta W = It\Delta U + Ut\Delta I + UI\Delta t$ or $\frac{\Delta W}{W} = \frac{\Delta U}{U} + \frac{\Delta I}{I} + \frac{\Delta t}{t}$

Example 4: Determine the absolute and relative uncertainties of the following quotient: $= \frac{P}{I}$.

Solution: $\Delta U = \frac{1}{I}\Delta P + \frac{P}{I^2}\Delta I$ or $\frac{\Delta U}{U} = \frac{\Delta P}{P} + \frac{\Delta I}{I}$

We can then conclude that:

- In the case of a sum or a difference, the absolute uncertainties are added.
- In the case of a product or a quotient, the relative uncertainties are added.

1.9 Application Exercises

Exercise 1.1 Dimensional Analysis

An ideal electromotive force generator E supplies a series circuit consisting of a capacitor with capacitance C and a resistance R. The temporal evolution of the charge $q(t)$ of the initially discharged capacitor is

$$q(t) = CE\left(1 - e^{-t/\tau}\right) \qquad \text{avec} \qquad \tau = RC$$

1. Determine the dimension of the parameter τ.
2. Verify the homogeneity of the relationship.

Exercise 1.2 Measurement Method Error

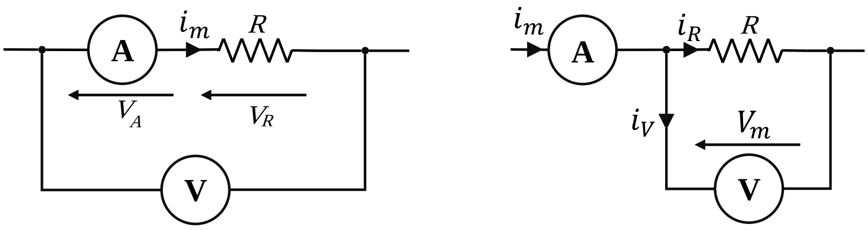
We want to measure the value of a resistance using an ammeter and a voltmeter, using a long shunt (upstream arrangement) or a short shunt (downstream arrangement). These two arrangements are represented in the figure below.

We will denote r_A the internal resistance of the ammeter and r_V the internal resistance of the voltmeter.

1. Which of these two arrangements gives the true value of the resistance R? Explain.
2. Calculate the corrections to be made to the results $\left(\frac{V_m}{i_m}\right)$ in the upstream arrangement and $\left(\frac{V_m}{i_R}\right)$ in the downstream arrangement to obtain the resistance value R.

   Numerical Application: ❶Upstream arrangement: $V_m = 10V$, $i_m = 10mA$, $r_A = 124\Omega$.

   ❷Downstream arrangement: $V_m = 8.72V$, $i_R = 10mA$, $r_V = 200\text{k}\Omega$.



Upstream Assembly **Downstream Assembly**

Exercise 1.3 Calculation of Uncertainties on Indirect Measurements

We have two resistances with respective values $R_1 = (10.7 \pm 0.2)\ \Omega$ and $R_2 = (26.5 \pm 0.5)\ \Omega$.

1. Give the value of the equivalent resistance R and its absolute uncertainty ΔR when these two resistances are connected in series.
2. Same question as before in the case where they are connected in parallel.

Exercise 1.4 Calculation of Uncertainties on Indirect Measurements

A circuit element subjected to a voltage U is traversed by a current I. The experimental study gave $U = (120 \pm 2)\ V$ and $I = 24.2A \pm 1.65\%$.

1. Calculate the absolute uncertainty on the power consumed by this circuit element.
2. What is the corresponding relative error?
3. Express the result in two ways.

Solution of exercises

Exercise 1.1

1. Using the voltage-current relationships for a resistor and a capacitor, we have

$$u_R = Ri \quad \text{and} \quad i = C\frac{du_c}{dt}$$

Thus

$$[RC] = [R][C] = \left[\frac{u_R}{i}\right]\left[\frac{i}{\frac{du_c}{dt}}\right] = \frac{[u_R]}{[i]}\frac{[i][t]}{[u_t]} = [t] = \mathrm{T}$$

Therefore, the constant τ is homogeneous to a time.

2. By definition, the current intensity i is equal to the derivative of the electric charge q with respect to time t. Then we have

$$i = \frac{dq}{dt} \quad \Longrightarrow \quad [i] = \left[\frac{dq}{dt}\right] = \frac{[q]}{[t]} = \frac{[q]}{\mathrm{T}} = \mathrm{I}$$

Therefore

$$[q] = \mathrm{IT}$$

Similarly, and taking into account that the argument of the exponential function is dimensionless, we have

$$[CE(1 - e^{-t/\tau})] = [C][E][(1 - e^{-t/\tau})] = [C][E][1] = [C][E]$$

From the previous question, we saw that the intensity of the current flowing through a capacitor is equal to the product of the capacitance C and the time derivative of the voltage u_c. Therefore

$$[i] = \mathrm{I} = \left[C\frac{du_c}{dt}\right] = [C]\frac{[u_c]}{[t]} = [C]\frac{[u_c]}{\mathrm{T}} \quad \Longrightarrow \quad [C][u_c] = \mathrm{IT} = [C][E]$$

We then deduce that

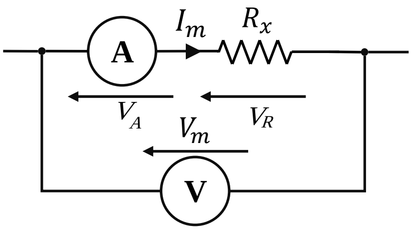
$$[CE(1 - e^{-t/\tau})] = \mathrm{IT}$$

Both sides have the same dimension. The equation is therefore homogeneous.

Exercise 1.2

1. Upstream assembly case:

1.1 Expression of the exact value R_x as a function of R_A and R_m :



Upstream Assembly

The measured potential difference V_m is the sum of the potential difference V_R across the resistor R_x and the potential difference V_A across the ammeter:

$$V_m = V_R + V_A$$

From the figure opposite, we can find: $V_m = R_m I_m$, $V_R = R_x I_m$ and $V_A = R_A I_m$.

Therefore,

$$R_m I_m = R_x I_m + V_A I_m \Longrightarrow R_m = R_x + R_A$$

This will give:

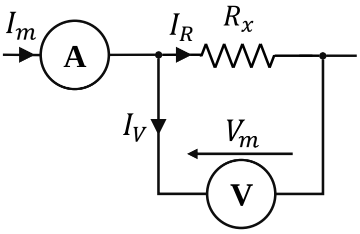
$$\boldsymbol{R_x = R_m - R_A}.$$

1.2 Determination of the absolute uncertainty of the upstream method:

The absolute uncertainty of this method is the difference between the measured value R_m and the exact value R_x assumed to be known. According to the previous result, the absolute uncertainty is then given by:

$$\Delta R_x = |R_m - R_x| \Longrightarrow \boldsymbol{\Delta R_x = R_A}$$

1.3 Derivation of the Relative Uncertainty of the Upstream Method:



Downstream Assembly

The relative uncertainty of this method is expressed for the case of an upstream setup by:

$$\boldsymbol{\frac{\Delta R_x}{R_x} = \frac{R_A}{R_x}}$$

Interpretation of Result: The relative uncertainty of the upstream method is lower if the resistance to be measured is much larger than the internal resistance of the ammeter ($R_x \gg R_A$). Thus, since the internal resistance of the ammeter is very small, the upstream setup is better suited for measuring large resistances.

2. Downstream Setup Case:

2.1 Expression of the Exact Value R_x as a Function of R_V and R_m :

In this case, the measured current I_m is the sum of the current I_R flowing through the resistance R_x and the current I_V flowing through the voltmeter:

$$I_m = I_R + I_V$$

According to the figure opposite, we have: $I_m = \frac{V_m}{R_m}$, $I_R = \frac{V_m}{R_x}$ and $I_V = \frac{V_m}{R_V}$.

Substituting into the previous expression, we have:

$$\frac{V_m}{R_m} = \frac{V_m}{R_x} + \frac{V_m}{R_V} \Longrightarrow \frac{1}{R_m} = \frac{1}{R_x} + \frac{1}{R_V} \Longrightarrow R_m = \frac{R_x R_V}{R_x + R_V}$$

It then follows that

$$\boldsymbol{R_x = \frac{R_x R_V}{R_V - R_x}}$$

2.2 Determination of the Absolute Uncertainty of the Downstream Method:
The absolute uncertainty of this method is given by:

$$\Delta R_x = |R_m - R_x| = \left|\frac{R_x R_V}{R_x + R_V} - R_x\right| = \left|\frac{R_x R_V - R_x^2 - R_x R_V}{R_x + R_V}\right| \Longrightarrow \boldsymbol{\Delta R_x = \frac{R_x^2}{R_x + R_V}}$$

2.3 Derivation of the Relative Uncertainty of the Downstream Method for $R_x \ll R_V$:
The relative uncertainty is expressed in this case by:

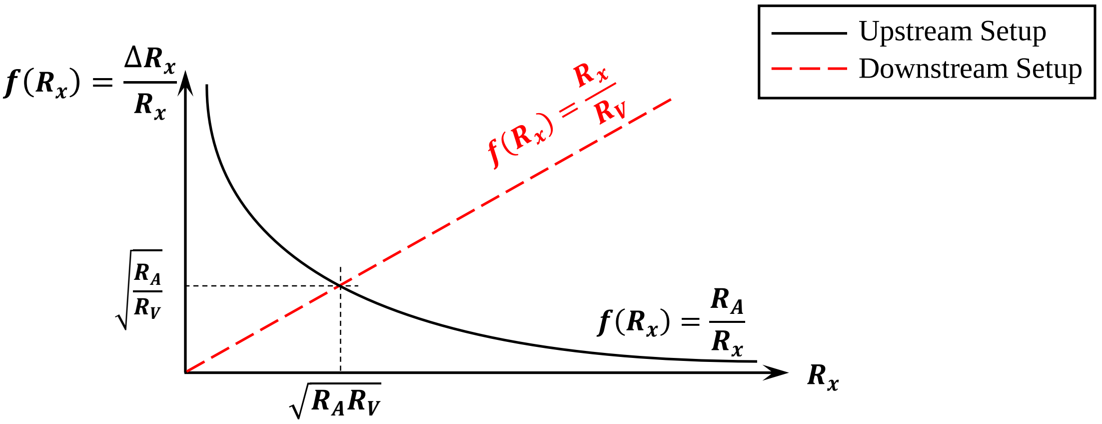
$$\frac{\Delta R_x}{R_x} = \frac{R_x}{R_x + R_V} = \frac{R_x}{R_x\left(1 + \frac{R_V}{R_x}\right)} = \frac{1}{1 + \frac{R_V}{R_x}}$$

Since $R_x \ll R_V \Leftrightarrow \frac{R_V}{R_x} \gg 1 \Longrightarrow 1 + \frac{R_V}{R_x} \approx \frac{R_V}{R_x}$, it follows that:

$$\boldsymbol{\frac{\Delta R_x}{R_x} = \frac{R_x}{R_V}}$$

Interpretation of Result: The relative uncertainty of the downstream method is lower if the resistance to be measured is much smaller than the internal resistance of the voltmeter ($R_x \ll R_V$). Thus, since the internal resistance of the voltmeter is very large, this setup is better suited for measuring small resistances.

3. Plotting the Curves of the Function $f(R_x) = \frac{\Delta R_x}{R_x}$ for Each Setup:



Curve of the Relative Uncertainty $f(R_x) = \frac{\Delta R_x}{R_x}$ of Both Upstream and Downstream Setups.

4. Choosing the Appropriate Setup to Measure Each of the Three R_{m1}, R_{m2} and **R_{m3}**:

From the curves in the previous figure, we can draw the following conclusions:

- If $R_x < \sqrt{R_A R_V} \Longrightarrow$ we use the downstream configuration;
- If $R_x > \sqrt{R_A R_V} \Longrightarrow$ we use the upstream configuration;
- If $R_x = \sqrt{R_A R_V} \Longrightarrow$ both configurations are equivalent in terms of precision;

In our case, $\sqrt{R_A R_V} = 2\text{k}\Omega$, it follows from the previous statements that:

- We use the downstream configuration to measure R_{m1} since $\boldsymbol{R_{m1} = 25\Omega} < \sqrt{\boldsymbol{R_A R_V}}$;
- We can use both configurations to measure R_{m2} since $\boldsymbol{R_{m2} = 2k\Omega} = \sqrt{\boldsymbol{R_A R_V}}$;
- We use the upstream configuration to measure R_{m3} since $\boldsymbol{R_{m3} = 15k\Omega} > \sqrt{\boldsymbol{R_A R_V}}$.

Exercise 1.3

1. **Series Configuration:**

$$R = R_1 + R_2 = 10.7 + 26.5 = 37.2\Omega$$

$$\Delta R = \Delta R_1 + \Delta R_2 = 0.2 + 0.5 = 0.7\Omega$$

Therefore, $R = 37.2 \pm 0.7\Omega$

2. **Parallel Configuration:**

$$R = \frac{R_1 R_2}{R_1 + R_2} = 7.62231\Omega$$

$$\Delta R = \left|\frac{\partial R}{\partial R_1}\right| \Delta R_1 + \left|\frac{\partial R}{\partial R_2}\right| \Delta R_2 = \frac{R_2(R_1 + R_2) - R_1 R_2}{(R_1 + R_2)^2} \Delta R_1 + \frac{R_1(R_1 + R_2) - R_1 R_2}{(R_1 + R_2)^2} \Delta R_2$$

After simplifications, we will have

$$\Delta R = R\left(\frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} - \frac{\Delta R_1 + \Delta R_2}{R_1 + R_2}\right)$$

$$\Delta R = 7.62231 \times (0.018691 + 0.018868 - 0.018817) = 7.62231 \times 0.018742 = 0.143$$

Therefore, $\Delta R = 0.143 = 0.1$ and $R = 7.6 \pm 0.1\Omega$.

Exercise 1.4

We have $U = (120 \pm 2)[V]$ and $I = 24.2[A] \pm 1.65\%$.

1. Calculation of the absolute uncertainty on the power consumed

$$P = U.I = 120 \times 24.2 = 2904\text{W}$$

Power is a function of U and I, $P = f(U, I)$, therefore

$$dP = \frac{\partial P}{\partial U} dU + \frac{\partial P}{\partial I} dI$$

Or

$$\Delta P = \left|\frac{\partial P}{\partial U}\right| \Delta U + \left|\frac{\partial P}{\partial I}\right| \Delta I$$

with $\frac{\partial P}{\partial U} = I$ and $\frac{\partial P}{\partial I} = U \Rightarrow \Delta P = I\Delta U + U\Delta I$

where $\Delta U = 2V$ and $\frac{\Delta I}{I} = 0.0165 \Rightarrow \Delta I = I \times 0.0165 = 24.2 \times 0.0165$

$$\Rightarrow \Delta I \approx 0.4A$$

Then

$$\Delta P = I\Delta U + U\Delta I = 24.2 \times 2 + 120 \times 0.4 = 48.4 + 48 \Rightarrow \Delta P = 96.4\text{W}$$

2. Calculation of the corresponding relative error

$$\left(\frac{\Delta P}{P}\right)_{\%} = 100 \times \frac{\Delta P}{P} = 100 \times \frac{96.4}{2904} \Rightarrow \left(\frac{\Delta P}{P}\right)_{\%} \approx 3.32\%$$

3. Result Expressions

1st way: $P = (P_m \pm \Delta P)[\mathrm{W}] = (2904 \pm 96.4)[\mathrm{W}]$

2nd way: $P = P_m[\mathrm{W}] \pm \left(\frac{\Delta P}{P}\right)_{\%} = 2904[\mathrm{W}] \pm 3.32\%.$

Chapter

Measurement Methods

2

Chapter objectives

At the end of Chapter 2, the student will be able to:

- Classify measurement methods, such as direct methods and indirect methods.
- Understand the different methods of measuring current, voltage, and power in direct current (DC) and alternating current (AC) circuits.
- Knowing how to measure resistance, capacitance, inductance, phase, and frequency using different methods.
- Knowing how to calculate the uncertainty of different measurement techniques.

2.1 Introduction

In this chapter, we will explore the different basic classical measurement methods: determination of voltages, current intensities, resistances, inductances, capacitances, phase shifts, and frequencies.

2.2 Voltage Measurement

2.2.1 Direct Method of Voltage Measurement

A direct or alternating voltage can be measured directly using an indicating device: deflection voltmeter, digital display voltmeter, oscilloscope, etc. The accuracy of the results depends on the class of the devices used and the systematic error due to the respective resistances of the circuit to be measured and the voltmeter.

Example: Let's measure the potential difference (p.d.) between the two points *A* and *B* of the circuit in Figure 2-1. Thevenin's theorem allows us to represent the equivalent diagram with:

$$U_{AB} = \frac{R_2}{R_1 + R_2} E$$

$$R_i = \frac{R_1 R_2}{R_1 + R_2}$$

where U_{AB} is the open-circuit voltage and R_i is the resistance measured from terminals *A* and *B* when the source is short-circuited.

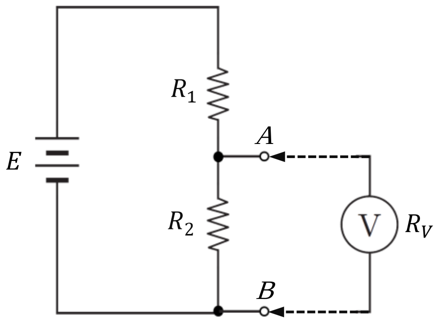


Figure 2-1: Direct voltage measurement

In this case, the voltage measured U_m by the voltmeter is given by

$$U_m = \frac{R_V}{R_i + R_V} U_{AB}$$

with R_V being the internal resistance of the voltmeter.

For U_m to be approximately equal to U_{AB}, R_V must be much greater than R_i.

The direct method is used in direct current and alternating current in all cases where high measurement accuracy is not required. For laboratory measurements, more refined methods are used.

2.2.2 AC Voltage Measurements

In alternating current, the effective value of the measured signal is often needed, also called the RMS value (*Root Mean Square*). For this, a ferromagnetic or moving-coil voltmeter with a rectifier can be used when the measured signal is sinusoidal, or a digital voltmeter can be used.

For TRMS (*True Root Mean Square*) digital voltmeters, there are two coupling modes:

➊**DC Mode:** In this mode, the voltmeter displays the RMS value of the measured signal according to the algorithm shown in Figure 2-2.

➋**AC Mode:** In this mode, the voltmeter indicates the RMS value of the AC component of the signal to be measured. That is, it first eliminates the DC component of the signal, then displays the RMS value of the AC component according to the algorithm presented in Figure 2-3.

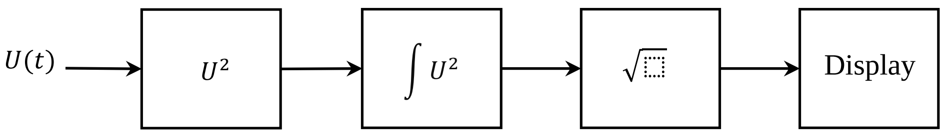


Figure 2-2: Diagram of a TRMS voltmeter in DC mode

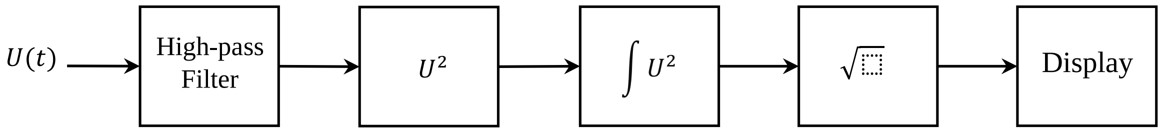


Figure 2-3: Diagram of a TRMS voltmeter in AC mode

2.2.3 Indirect Voltage Measurement by the Opposition Method

This method, also called the potentiometric method, is used in direct current and allows for the determination of a potential difference or an electromotive force (emf) with very high precision. The setup for this method is shown in Figure 2-4. There are three main circuits:

- A calibrated circuit, also called a potentiometer, consisting of two resistors R_1 and R_2 whose sum remains constant and a calibration resistor R_T.
- A calibration circuit consisting of a standard cell E_0.
- A voltage measurement circuit U_x.

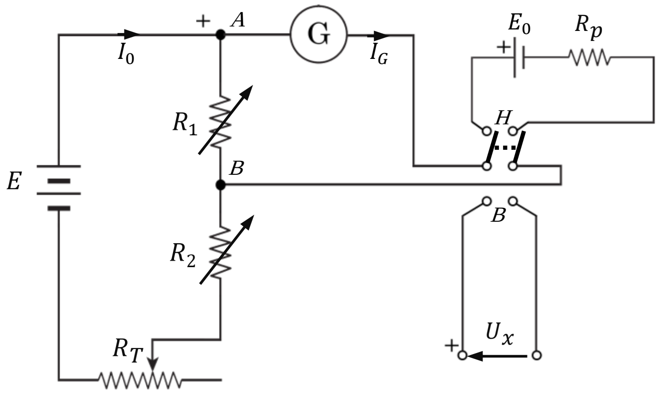


Figure 2-4: Voltage Measurement Circuit by Opposition Method

- **Practical Analysis of the Method:** The two resistors R_1 and R_2 are adjustable resistance boxes whose sum always remains constant.
  The switch in position H allows the current I_0 in the circuit to be adjusted using the resistor R_T.
  At galvanometer equilibrium:

$$E_0 = I_0 \cdot R_1 \Longrightarrow I_0 = \frac{E_0}{R_1}$$

  After calibration, R_T should no longer be adjusted.
  The switch in position B allows the measurement of U_x by adjusting R_1 and R_2. At galvanometer equilibrium, let $R_1^{'}$ and $R_2^{'}$ be the new values of R_1 and R_2 with $R_1 + R_2 = R_1^{'} + R_2^{'}$.
  Since the current I_0 has not changed:

$$U_x = R_1^{'} \cdot I_0 = \frac{R_1^{'}}{R_1} E_0$$

Generally, the current I_0 is chosen such that it has a simple value, a power of 10 for example. It should not be too high (difficulty of adjustment), it should not be too low (appearance of parasitic contact resistances). It is customary to set I_0 such that

$$10^{-5}\text{A} \leq I_0 \leq 10^{-3}\text{A}$$

The source resistance R_p is a protective resistor limiting the battery's current flow. It is short-circuited when close to equilibrium. $R_p = 1\text{M}\Omega$.

2.3 Current Measurement

2.3.1 Direct Current Measurement Method

A direct or alternating current intensity can be measured directly using an indicating instrument. As with voltage measurement, the accuracy depends on the class error of the instruments and the systematic error. The latter appears because the insertion of the ammeter into the circuit modifies its resistance (Figure 2-5).

The systematic error is negligible if $R_A \leq \frac{\sum|R|}{100}$A. This condition is met in most cases. Indeed, an ammeter generally has a very low internal resistance.

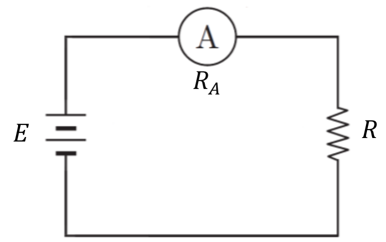


Figure 2-5: Direct Current Measurement

2.3.2 Indirect Current Measurement by the Opposition Method

The opposition method is used in direct current and amounts to determining the voltage developed across a standard resistor R by the current to be measured I_x. The resistor R is of the shunt type and is equipped with four terminals: two current terminals, used to bring in the current; the other two terminals, called voltage terminals, are used to sample the voltage and define the terminals of the standard itself (This type of resistor will be discussed in detail in sub-paragraph 2.4.3). The two other standard resistors R_1 and R_2 of the setup maintain their constant sum during the measurements.

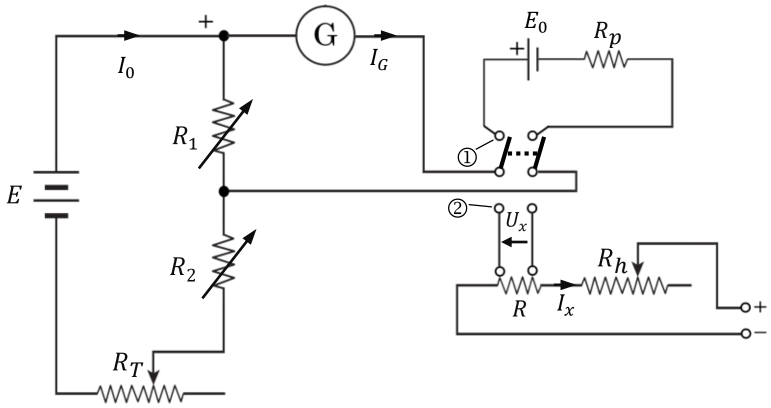


Figure 2-6: Current measurement circuit by opposition method

The switch is in position 1, at equilibrium, we will have

$$I_0 = \frac{E_0}{R_1}$$

The switch is in position 2, at equilibrium,

$$U_x = R_1^{'} \cdot I_0 = \frac{R_1^{'}}{R_1} E_0$$

And since

$$U_x = R \cdot I_x$$

We then obtain

$$I_x = \frac{R_1^{'}}{R \cdot R_1} E_0$$

2.3.3 Using a Simple Shunt

A shunt is a calibrated resistor designed for current measurement. To do this, the voltage across it is measured using a voltmeter connected in parallel. Using Ohm's law, the current flowing through the shunt can be deduced. It can measure currents of several kiloamperes (kA).

In order to limit the voltage drop caused by its use and to limit losses due to the Joule effect in the shunt, it must have a very low resistance value, on the order of a few $m\Omega$. However, it should not be too small so that the voltage can be transmitted and measured without auxiliary disturbance.

Shunts are used for measuring direct and alternating currents, whether low or high frequency.

2.4 Resistance Measurement

The degree of opposition to the flow of electric current in a circuit defines the electrical resistance of that circuit. Their values vary over a very wide range from $10^{-8}\Omega$ up to $10^{15}\Omega$. From a measurement perspective, there are three main types of resistances according to their values.

2.4.1 Classification of Resistances

To allow for the selection of measurement methods, resistances can be classified, according to their values, into three main categories:

❶ **Low Resistances:** all resistances less than or equal to 1Ω.

❷ **Average Resistances:** all resistances ranging from 1Ωup to $100\text{k}\Omega$.

❸ **High Resistances:** all resistances greater than $100\text{k}\Omega$.

Practically, it is impossible to have a single method or a single instrument that can measure all these ranges of values.

2.4.2 Methods for Measuring Medium Resistances

There are several methods for measuring medium resistances. In this course, we will discuss the following methods: Volt-ammeter method, Substitution method, and Wheatstone Bridge method.

❶ **Voltammetric Method:** This is the simplest method for measuring medium resistances in laboratories. It consists of measuring the value of an unknown resistance by applying Ohm's law. R_x

$$R_x = \frac{V}{I}$$

where V is the measured voltage and I is the measured current.

To measure these two electrical quantities, we simply use an ammeter and a voltmeter.

Depending on the position of the voltmeter (before or after the ammeter), there are two possible setups: the upstream setup (long shunt) and the downstream setup (short shunt).

- **Upstream setup:** The diagram of this setup is shown in Figure 2-7. Here, the ammeter measures the true value of the current flowing through the resistance to be measured R_x. On the other hand, the voltmeter measures the voltage across the resistance and ammeter assembly.

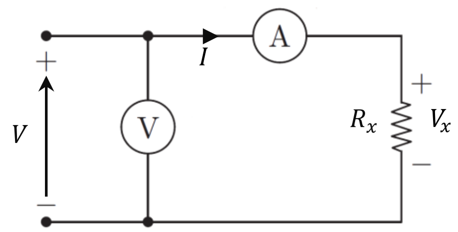


Figure 2-7: Upstream setup

According to this setup, the measured resistance, R_m, is given by

$$R_m = \frac{V}{I} = \frac{V_x + V_A}{I} = \frac{I \cdot R_x + I \cdot R_A}{I} = R_x + R_A$$

Therefore

$$R_x = R_m - R_A$$

where R_m is the measured resistance, V_x is the potential difference across the resistance R_x, V_A is the voltage across the ammeter, and R_A is the internal resistance of the ammeter.

The absolute uncertainty associated with the upstream setup is $\Delta R_x = |R_m - R_x| = R_A$.

The relative uncertainty will therefore be $\frac{\Delta R_x}{R_x} = \frac{R_A}{R_x}$.

It is clear that the relative uncertainty is all the lower if the value of the resistance to be measured is sufficiently large compared to the internal resistance of the ammeter ($R_x \gg R_A$).
Consequently, the upstream setup is better suited for measuring high-value resistances.

- **Downstream setup:** In this setup, the voltmeter measures the true value of the voltage across the resistance to be measured R_x, and the ammeter measures, this time, the sum of the currents flowing through the resistance R_x and through the voltmeter. The diagram of this setup is presented in the following figure.

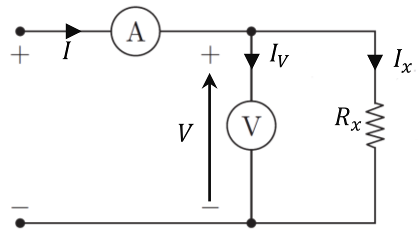


Figure 2-8: Downstream Setup

According to the circuit above, we have

$$I = I_x + I_V = \frac{V}{R_x} + \frac{V}{R_V}$$

Where I_x is the current flowing through the resistor R_x, I_V is the current flowing in the voltmeter branch, and R_V is the internal resistance of the voltmeter.
The measured resistance, R_m, is therefore given by

$$R_m = \frac{V}{I} = \frac{V}{\frac{V}{R_x} + \frac{V}{R_V}} = \frac{R_x R_V}{R_x + R_V}$$

Therefore

$$R_x + R_V = \frac{R_x R_V}{R_m} \Leftrightarrow R_x \left[1 - \frac{R_V}{R_m}\right] = -R_V$$

$$R_x = \frac{-R_m R_V}{R_m - R_V} = R_m \left(\frac{1}{1 - \frac{R_m}{R_V}} \right)$$

The absolute uncertainty related to the downstream setup is

$$\Delta R_x = |R_m - R_x| = \left| \frac{R_x R_V}{R_x + R_V} - R_x \right| = \left| \frac{R_x R_V - R_x^2 - R_x R_V}{R_x + R_V} \right| = \frac{R_x^2}{R_x + R_V}$$

The corresponding relative uncertainty will therefore be

$$\frac{\Delta R_x}{R_x} = \frac{R_x}{R_x + R_V} = \frac{1}{1 + \frac{R_V}{R_x}}$$

This time, the relative uncertainty is even lower if the value of the resistance to be measured is sufficiently smaller than the internal resistance of the voltmeter ($R_x \ll R_V$).
Consequently, the downstream setup is better suited for measuring low-value resistances.

Example 1:

A resistance R is measured with the downstream setup of Figure 2-2. The measured current is $0.5A$ and the value read on the voltmeter is $500V$. If the internal resistance of the ammeter is $R_a = 10\Omega$ and the sensitivity of the voltmeter for the voltage range $1000V$ is $10\text{k}\Omega/V$. Calculate the value of the resistance R.

❷ **Substitution Method:** The setup for this method is shown in Figure 2-9. It consists of a known standard resistor , a rheostat to adjust the current intensity in the circuit, and the resistance to be measured. The setup is powered by an electromotive force (emf) generator . RR_h R_xE

Initially, the switch is set to position 1 (it is then connected to the resistance R_x). In this case, the current I_1 measured by the ammeter is given by

$$I_1 = \frac{E}{R_x}$$

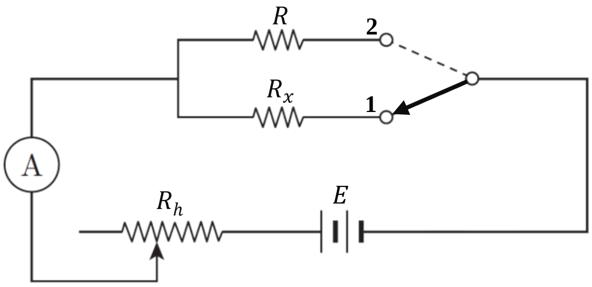


Figure 2-9: Substitution Method Setup

Subsequently, the switch is set to position 2 (it is now connected to the known resistance R). This time, the current, denoted I_2 , is given as follows

$$I_2 = \frac{E}{R}$$

By dividing the two equations, we obtain

$$R_x = R \cdot \frac{I_2}{I_1}$$

❸ **Wheatstone Bridge Method:** The Wheatstone bridge is the most suitable circuit for measuring medium resistances. It consists of two fixed, calibrated, and known resistances R_1 and R_3, a known variable (adjustable) resistance R_2, the resistance to be measured R_x, a detector which is generally a galvanometer G and a DC power source (see Figure 2-10).

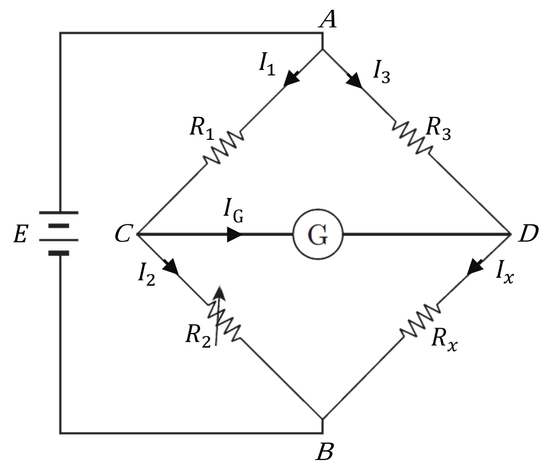


Figure 2-10: Wheatstone Bridge

The bridge is balanced when the potential difference between the two nodes *C* and *D* is zero (i.e., $V_C = V_D$). This can be achieved by simply adjusting the resistances R_1, R_2 and R_3 so as to cancel the current I_G in the branch *CD* (the galvanometer needle does not deflect).

Indeed, at bridge equilibrium (i.e., if $I_G = 0$), we can write

$$I_1 = I_2 \quad \text{and} \quad I_3 = I_x$$

$$U_{AC} = U_{AD} \quad \text{and} \quad U_{CB} = U_{DB}$$

$$U_{CB} = V_C = U_{DB} = V_D$$

With $U_{CB} = V_C - V_B = V_C$ and $U_{DB} = V_D - V_B = V_D$ (The reference potential V_B is considered zero).

The potentials of the two points *C* and *D* are obtained using the voltage divider bridge and are equal to

$$V_C = \frac{R_2}{R_1+R_2}E \quad \text{and} \quad V_D = \frac{R_x}{R_x+R_3}E$$

By equating the two potentials V_C and V_D calculated previously, we find

$$V_C = V_D \Leftrightarrow \frac{R_2}{R_1 + R_2} = \frac{R_x}{R_x + R_3}$$

After simplification, the value of R_x is therefore

$$R_x = R_2 \cdot \frac{R_3}{R_1}$$

Practically, finding the balance in a Wheatstone bridge consists of giving the ratio, $\frac{R_3}{R_1}$, a certain value and then varying R_2 until the detector remains at zero.

The sensitivity of the bridge is defined by the smallest variation of the adjustment branch R_2 that causes a perceptible imbalance of the bridge. We define the voltage that appears across the galvanometer at equilibrium by:

$$U = V_C - V_D = \left[\frac{R_2}{R_1+R_2} - \frac{R_x}{R_x+R_3}\right]E = \left[\frac{R_2R_3 - R_1R_x}{(R_1+R_2)(R_x+R_3)}\right]E = 0$$

For a small variation ΔR_2 of the adjustment branch, an imbalance voltage ΔU is detected. It follows:

$$\Delta U = \left[\frac{(R_2 + \Delta R_2)R_3 - R_1 R_x}{(R_1 + R_2 + \Delta R_2)(R_x + R_3)}\right] E$$

If we neglect ΔR_2 compared to $(R_1 + R_2)$, and considering that $R_2 R_3 = R_1 R_x$, we obtain:

$$\Delta U = \left[\frac{R_3 \Delta R_2}{(R_1 + R_2)(R_x + R_3)}\right] E = \left[\frac{\frac{\Delta R_2}{R_2}}{\left(\frac{R_1}{R_2} + 1\right)\left(\frac{R_x}{R_3} + 1\right)}\right] E$$

By calling x the two ratios $\frac{R_1}{R_2}$ and $\frac{R_3}{R_x}$, this last relationship is then written as follows:

$$\Delta U = \left[\frac{x \frac{\Delta R_2}{R_2}}{(x+1)^2}\right] E$$

The sensitivity of the bridge is then defined by

$$\sigma = \frac{\Delta U}{\frac{\Delta R_2}{R_2}} = \frac{x}{(x+1)^2} E$$

This function goes through a maximum for $x = \frac{R_3}{R_x} = 1$. We can conclude by emphasizing that the sensitivity of a bridge increases with the supply voltage and is maximum when the equality of resistances R_x and R_3 is achieved.

The sensitivity error, denoted ε, is defined by the ratio $\frac{\Delta R_2}{R_2}$ destroying the equilibrium.

$$\varepsilon_s\% = 100 \times \frac{\Delta R_2}{R_2}$$

Experimentally, it is determined by varying R_2 by ΔR_2 to cause a perceptible deflection $\Delta\alpha$ around zero on the galvanometer. This error is considered negligible if it obeys the inequality:

$$\varepsilon_s \leq \frac{\varepsilon_e}{10}$$

where ε_e is the construction error of the resistance R_2.

Example 1:

Consider a Wheatstone bridge (like the one in Figure 2-4), with $R_1 = 7\text{k}\Omega$ and $R_3 = 3.5\text{k}\Omega$. The bridge is balanced when $R_2 = 5.51\text{k}\Omega$.

1. Calculate the value of the unknown resistance R_x.
2. Determine the resistance measurement range for this bridge if R_2 is adjustable from 1kΩ up to8kΩ.

Solution:

1. Calculation of the unknown resistance R_x :

We have

$$R_x = R_2\frac{R_3}{R_1} = 5.51\text{k}\Omega \times \frac{3.5\text{k}\Omega}{7\text{k}\Omega} \Rightarrow R_x = 2.755\text{k}\Omega$$

2. If $R_2 = 1\text{k}\Omega$:

$$R_x = R_2\frac{R_3}{R_1} = 1\text{k}\Omega \times \frac{3.5\text{k}\Omega}{7\text{k}\Omega} \Rightarrow R_x = 500\Omega$$

If $R_2 = 8\text{k}\Omega$:

$$R_x = R_2\frac{R_3}{R_1} = 8\text{k}\Omega \times \frac{3.5\text{k}\Omega}{7\text{k}\Omega} \Rightarrow R_x = 4\text{k}\Omega$$

Then the measurement range of the bridge is $[500\Omega,\ 4\text{k}\Omega\ ]$

2.4.3 Methods for Measuring Low Resistances

When the resistance to be measured is less than 1Ω, the previous methods often give inaccurate results, because of the contact resistance that establishes the connection of the resistance to be measured with the others. These contacts can introduce errors of the same order of magnitude as the quantity to be measured. It is then necessary to use special methods designed to eliminate these contacts, or at least to make them negligible. We will mention here the following three methods: Volt-ampere method, Kelvin bridge method and Kelvin double bridge method.

Low ohmic value resistors or “shunts” are very common in practice for measuring and controlling currents (ammeter shunt, dynamo armature sections, connection cables or wires, etc.). The equivalent diagram in Figure 2-11 (a) shows that the electrical connection wire can modify the value of a resistance R_x. The actual resistance between the two connection points *N* and *M* is $R_x + 2r$ and not R_x. It is therefore important to make the contact resistance r negligible compared to R_x.

Indeed, measuring a small resistance with proper accuracy involves the use of four-terminal resistors, in order to eliminate the influence of contact resistances (Figure 2-11 (b)). The two terminals *N-M*, called the current terminals of the resistor, are used to supply the current; the other two terminals, called voltage terminals, always located between the *N-M* terminals, are used to measure the voltage and define the terminals of the standard itself. A device for measuring small resistances must therefore also have four terminals: two terminals supplying the current I and two others allowing the measurement of the potential difference U. The resistance R_x is then calibrated between the two voltage terminals with $R_x = \frac{U}{I}$ and the contact resistances are then eliminated.

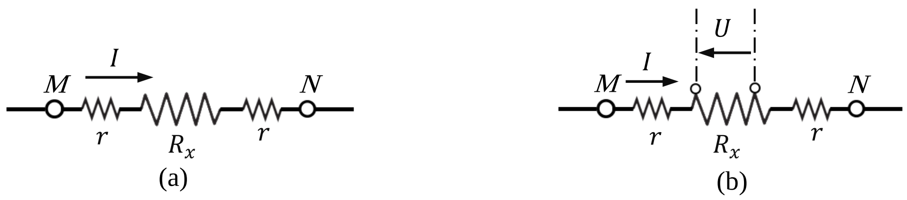


Figure 2-11: (a) Equivalent diagram of a real resistor (b) Four-terminal resistor

We will now discuss all these methods in detail in the following pages.

❶ **Voltammetric Method:** This method has already been discussed in Section 2.4.2. It can also be used to measure resistances of low ohmic values.

❷ **Kelvin Bridge Method:** This method offers high precision in the measurement of low resistances. The Wheatstone bridge cannot be used because the resistances of contacts and connecting cables are no longer negligible compared to the resistances to be measured, which introduces errors in the measurement of small resistance values. To overcome this drawback, it is necessary to use the Kelvin bridge, which is just a modified version of the Wheatstone bridge (Figure 2-12).

The resistance r represents the resistance of the connecting lead connecting the two resistances R_3 and R_x. The resistance R_x being the unknown resistance to be measured. The galvanometer can be connected either at point a or at point c. When it is connected at point a, the resistance r of the connecting lead is added to the unknown resistance R_x. The measured value of the resistance R_x is therefore too high compared to the true value. And if it is connected at point c, the resistance r of the connecting lead is added to the known resistance R_3. The value of R_3 therefore becomes greater than its true value and the measured value of R_x becomes lower than the desired value.

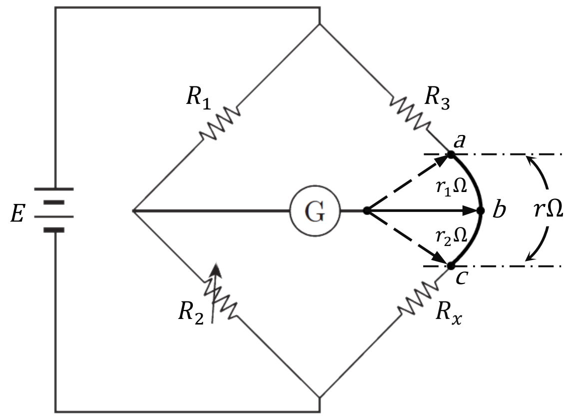


Figure 2-12: Kelvin Bridge

Now, instead of using one of the two connection points a and c which introduce errors in the measurement result, the galvanometer is connected, this time, to any intermediate point b, between the two points a and c, so that the following condition is verified,

$$\frac{r_1}{r_2} = \frac{R_1}{R_2}$$

In this case, the bridge balance equation is then given by the relation,

$$\frac{R_x + r_1}{R_3 + r_2} = \frac{R_1}{R_2} \Leftrightarrow R_x + r_1 = (R_3 + r_2)\frac{R_1}{R_2}$$

Taking into account that $r = r_1 + r_2$ and $\frac{r_1}{r_2} = \frac{R_1}{R_2}$, then, the two resistances r_1 and r_2 can be calculated as follows,

$$\frac{r_1}{r_2} + \mathbf{1} = \frac{R_1}{R_2} + \mathbf{1} \Leftrightarrow \frac{r_1 + r_2}{r_2} = \frac{R_1 + R_2}{R_2}$$

$$\frac{r}{r_2} = \frac{R_1 + R_2}{R_2}$$

The resistance r_2 is therefore

$$r_2 = r\frac{R_2}{R_1 + R_2}$$

Similarly, the resistance r_1 is given by

$$r_1 = r - r_2 = r - r\frac{R_2}{R_1 + R_2}$$

$$r_1 = r\frac{R_1}{R_1 + R_2}$$

By substituting the two expressions of r_1 and r_2 in the bridge balance equation, we will have

$$R_x + r_1 = (R_3 + r_2)\frac{R_1}{R_2} \Leftrightarrow R_x + r\frac{R_1}{R_1 + R_2} = \left(R_3 + r\frac{R_2}{R_1 + R_2}\right)\frac{R_1}{R_2}$$

$$\Leftrightarrow R_x + r\frac{R_1}{R_1 + R_2} = R_3\frac{R_1}{R_2} + r\frac{R_2}{R_1 + R_2}\frac{R_1}{R_2}$$

$$R_x = R_3\frac{R_1}{R_2}$$

Then, the standard bridge balance equation does not depend on the resistance r. The effect of this resistance is completely eliminated by connecting the galvanometer to the intermediate position *b*. The process described above is obviously not a practical way to achieve the desired measurement result, as it is certainly difficult to determine the proper bridge balance point.

❸ **Kelvin Double Bridge Method**[(3)]**:** The purpose of this bridge arrangement is to eliminate the influence of contact resistances by allowing them to be neglected. The method consists of implementing the circuit shown in Figure 2-13. The unknown resistance R_x and the comparison resistance R_3 are four-terminal resistors and are connected in series using an intermediate resistance r. The four other resistances R_1, R_2, R_1' and R_2' are arranged as shown in the figure and are connected by a galvanometer G.

[(3)]The Kelvin double bridge is also called the Thomson bridge.

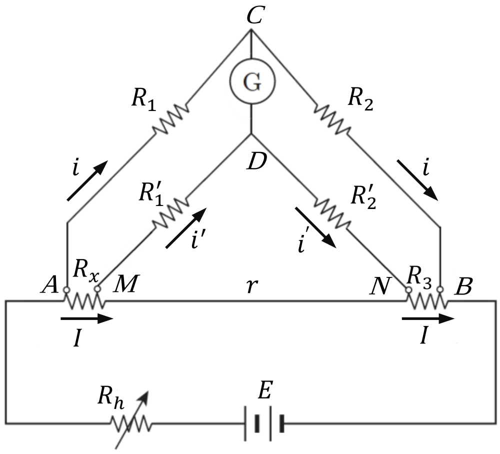


Figure 2-13: Kelvin Double Bridge

The setup and operating principle of the Kelvin double bridge are similar to those of the Wheatstone bridge, except that the former has additional resistances. The Kelvin double bridge uses a second pair of ratio arms, hence the name *double bridge*.

Bridge balance can be achieved by a suitable adjustment of the resistances R_1, R_2, $R_1^{'}$ and $R_2^{'}$, such that the ratio of the resistances of the first arm $\frac{R_1}{R_2}$ is equal to that of the resistances of the second arm $\frac{R_1^{'}}{R_2^{'}}$.

Under balanced conditions, no current flows in the galvanometer, which means that the voltage drop between *A* and *C*, U_{AC}, is equal to the voltage drop U_{AMD} between *A* and *D*.

Now,

$$U_{AC} = \frac{R_1}{R_1 + R_2} U_{AB}$$

with

$$U_{AB} = \left(R_x + R_3 + \frac{\left(R_1^{'} + R_2^{'} \right) r}{R_1^{'} + R_2^{'} + r} \right) I$$

and

$$U_{AMD} = R_x I + R_1^{'} i^{'} = R_x I + R_1^{'} \left(\frac{r}{R_1^{'} + R_2^{'} + r} \right) I$$

or

$$U_{AMD} = \left(R_x + \frac{R_1^{'} r}{R_1^{'} + R_2^{'} + r} \right) I = \left(R_x + \frac{R_1^{'}}{R_1^{'} + R_2^{'}} \times \frac{\left(R_1^{'} + R_2^{'} \right) r}{R_1^{'} + R_2^{'} + r} \right) I$$

At equilibrium, we have$U_{AC} = U_{AMD}$

$$\frac{R_1}{R_1 + R_2} \left(R_x + R_3 + \frac{\left(R_1^{'} + R_2^{'} \right) r}{R_1^{'} + R_2^{'} + r} \right) I = \left(R_x + \frac{R_1^{'}}{R_1^{'} + R_2^{'}} \times \frac{\left(R_1^{'} + R_2^{'} \right) r}{R_1^{'} + R_2^{'} + r} \right) I$$

After some algebraic simplifications, we will have

$$R_x = \frac{R_1}{R_2}R_3 + \frac{R_2' r}{R_1' + R_2' + r}\left(\frac{R_1}{R_2} - \frac{R_1'}{R_2'}\right)$$

Now, if $\frac{R_1}{R_2} = \frac{R_1'}{R_2'}$, the value of R_x is then

$$R_x = \frac{R_1}{R_2}R_3$$

2.4.4 Methods of Measuring High Resistances

There are several methods for measuring high resistances. Here, we will only discuss the voltage drop method.

❶ **Voltage Drop Method:** The electrical circuit for this method is shown in Figure 2-14. The unknown resistanceis connected in parallel with a capacitor and an electrostatic voltmeter. The capacitor is initially charged by means of a battery, and then it discharges through the resistance following an exponential law. R_x C R_x

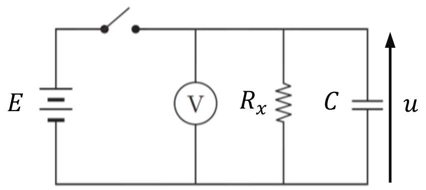


Figure 2-14: Voltage Drop Method

The voltage across the capacitor, at time *t*, is

$$u = Ee^{-\frac{t}{CR_x}}$$

The unknown resistance R_x is then given by

$$R_x = \frac{t}{C\ln\left(\frac{E}{u}\right)}$$

Knowing E, u, C and t and the unknown resistance R_x can then be easily calculated.

❷ **Series Voltmeter Method:** This method is also used for the measurement of high resistances. The setup for this method is shown in Figure 2-15. It consists of a voltmeter with internal resistance, R_V the resistance to be measured R_x, and a DC voltage generator E.

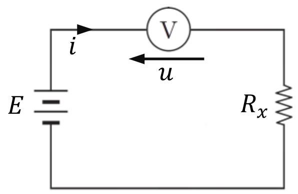


Figure 2-15: Series Voltmeter Method

By applying Ohm's law, the resistance R_x is then given by

$$R_x = \frac{E - u}{i}$$

And since, $u = R_V i$, then the resistance R_x is expressed as follows:

$$R_x = \left(\frac{E}{u} - 1\right) R_V$$

2.5 Impedance Measurement

2.5.1 Capacitance Measurement

In general, the voltammetric method (upstream and downstream setups) allows for the measurement of impedance Z at the industrial frequency.

An electrical capacitor is defined by its capacitance, that is, the ratio of its electric charge to its potential difference. The unit of capacitance is the farad. A capacitor is an open switch in direct current. In alternating current, it presents a complex impedance

$$\overline{Z_C} = \frac{1}{j\omega C}$$

and shifts the current by $\frac{\pi}{2}$ ahead of the voltage.

with $\omega = 2\pi f$ and f the frequency given in Hz.

In reality, a real capacitor is never perfect, and capacitor losses are referred to as the active energy dissipated as a result of insulation leakage and losses due to dielectric hysteresis.

Two electrical equivalences allow for the representation of a real capacitor.

- **Parallel Equivalence:** The parallel resistance represents the losses (Figure 2-16 ①). The current diagram allows for the definition of the loss angle α of the capacitor (Figure 2-16 ③).

$$\tan(\alpha) = \frac{I_R}{I_C} = \frac{\frac{U}{R}}{UC\omega} \Rightarrow \tan(\alpha) = \frac{1}{RC\omega}$$

The current-voltage phase shift is not $\frac{\pi}{2}$ but rather $\left(\frac{\pi}{2} - \alpha\right)$.

- **Series Equivalence:** The losses are represented here by a series resistance (Figure 2-16 ②). The loss angle α of the capacitor in this case is defined by

$$\tan(\alpha) = rC\omega$$

Remark: These two equivalences will find application in the study of AC bridges.

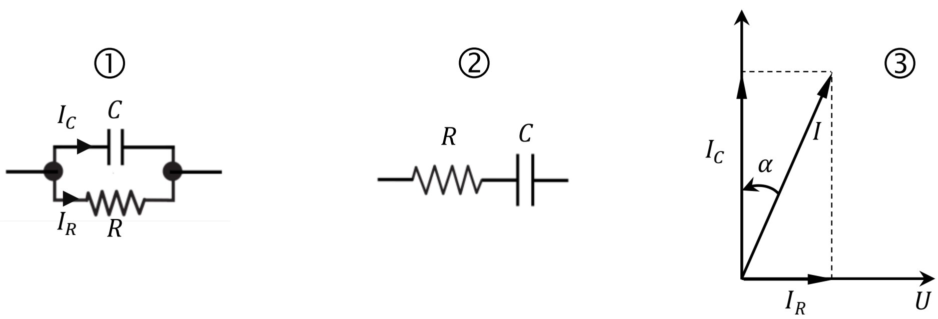


Figure 2-16: Equivalent Diagrams of a Capacitor

In most cases, the impedance of the capacitor is quite high ($Z_C \gg Z_V$). The upstream setup is therefore the most suitable.

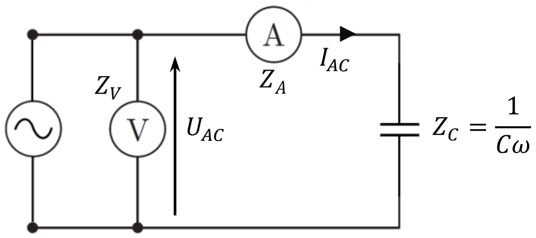


Figure 2-17: Impedance Measurement of a Capacitor by Upstream Setup

This method only allows for the determination of the impedance of a capacitor at low frequencies (LF) and the value of its capacitance. The loss angle, very small at industrial frequencies, cannot be measured; therefore, it only requires a single measurement in alternating current.
The impedance of the capacitor is measured by:

$$Z_C = \frac{U_{AC}}{I_{AC}} = \frac{1}{C\omega} \Rightarrow C = \frac{1}{Z_C\omega}$$

This method is used, in the low frequency (LF) domain, for its speed and ease of implementation. It only applies to non-polarized capacitors. The accuracy of the results is approximately 1%.
Remark: The capacitance of a capacitor can be measured directly using a capacitance meter.

2.5.2 Inductance Measurements

A coil traversed by a variable current is the seat of a self-induced electromotive force (emf). This emf is all the higher as the coefficient of self-induction, or inductance Z_L, is large. Depending on whether the coil has an iron core or not, the electrical equivalent is different (See Figure 2-18 ① and ②).

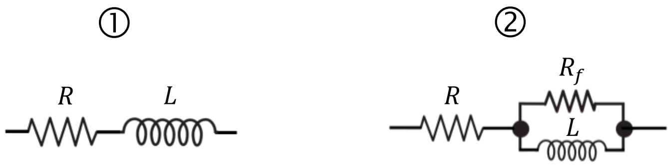


Figure 2-18: Equivalent diagrams of a coil

Here, we assume a coil without an iron core, so the equivalent diagram that represents a series arrangement (the ohmic resistance R and the inductance L of the winding) will be adopted. The complex impedance of such a coil is defined by the relationship:

$$\overline{Z_L} = R + j\omega L$$

For the case of a coil, two measurements are necessary.

❶ **DC Measurement:** This allows for the definition of the resistance value.R The methods used have already been discussed in the previous section 2.4.

❷ **AC Measurement:** This allows for defining the impedance Z_L magnitude of the coil. The value of Z_L determines the type of circuit to use. Generally, the impedance of a coil Z_L is low ($Z_L \ll Z_V$). The downstream circuit is therefore the most suitable.

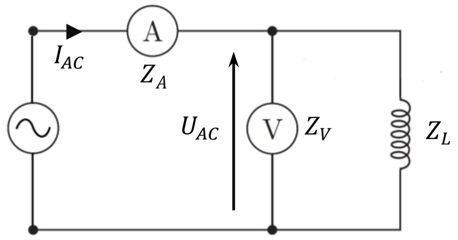


Figure 2-19: Impedance measurement of a capacitor using a downstream setup

Depending on the setup, the real impedance Z_L of the coil is given by

$$Z_L = \frac{U_{AC}}{I_{AC}}$$

Its modulus is therefore

$$Z_L = \sqrt{R^2 + \omega^2 L^2}$$

The inductance of the coil is then given by

$$L = \frac{1}{\omega}\sqrt{Z_L^2 - R^2}$$

This method is very commonly used in industry because it only requires widely available measuring instruments and its operating procedure is simple and quick to implement.

It leads to sufficiently accurate results in many cases (accuracy of 4 to 5%). However, it cannot be used for iron-core coils in saturated mode.

Note: the inductance of a coil can be measured directly using an henrymeter.

2.5.3 AC Bridges

The precise measurement of an impedance requires the use of a null method. The setups used are derived from the Wheatstone bridge.

The general form of an AC bridge under equilibrium conditions is presented in the following figure. The four branches of the bridge Z_1, Z_2, Z_3 and Z_4 are impedances. The diagonal AB is powered from a BF generator. The diagonal CD is connected to a detector which can be a telephone receiver or an oscilloscope.

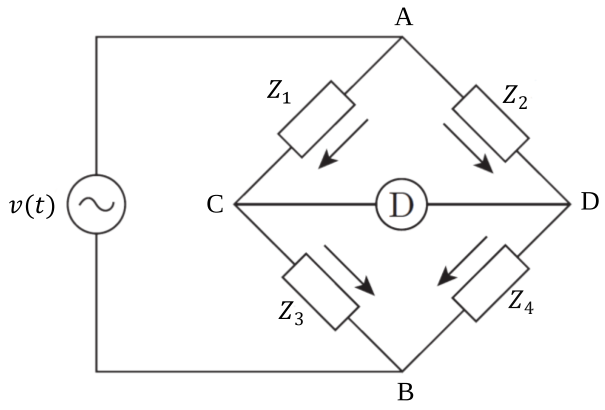


Figure 2-20: AC bridge under equilibrium conditions

At bridge equilibrium, we have

$$E_{\mathrm{AC}} = E_{\mathrm{AD}}$$
$$I_1 Z_1 = I_2 Z_2$$

where

$$I_1 = \frac{V}{Z_1 + Z_3}$$

and

$$I_2 = \frac{V}{Z_2 + Z_4}$$

The equilibrium conditions in complex notation and in sinusoidal alternating current are:

$$\overline{Z_1} \times \overline{Z_4} = \overline{Z_2} \times \overline{Z_3} \Leftrightarrow Z_1 Z_4 \angle\theta_1 + \angle\theta_4 = Z_2 Z_3 \angle\theta_2 + \angle\theta_3$$

It follows : $Z_1 Z_4 = Z_2 Z_3$ and $\theta_1 + \theta_4 = \theta_2 + \theta_3$

This equilibrium can be achieved in an infinite number of ways, but for convenient manipulation, the number of parameters is reduced by constituting two of the bridge branches with pure resistances identified as P and Q.

P/Q bridges allow the measurement of capacitive impedances, while P.Q bridges allow the measurement of inductive impedances.

➊ **P/Q Bridges:** When the unknown impedance Z_4 is capacitive, the expression $Z_4 = Z_3 \frac{P}{Q}$ shows that balance can be achieved if P and Q are pure resistances and Z_3, the adjustment element, is a capacitive impedance. Indeed, both sides of the equation have negative arguments.

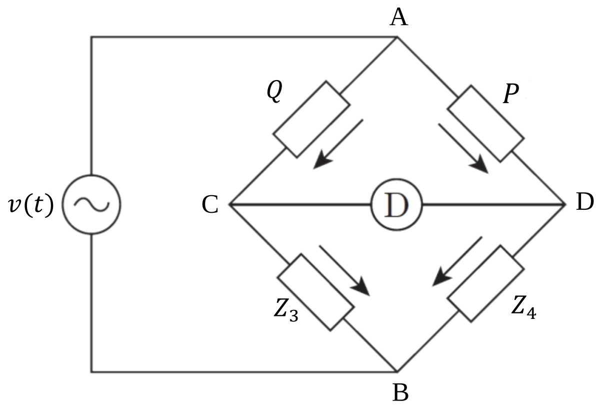


Figure 2-21: P/Q AC bridge

❶ **P.Q Bridges:** If the unknown impedance $Z_4$is inductive, the expression $Z_4 = \frac{P.Q}{Z_1}$ shows that balance can be achieved if P and Q are pure resistances and ,Z_1 the adjusting element, is a capacitive impedance. Indeed, the argument of is negative and the argument of the term is positive Z_1 $\frac{P.Q}{Z_1}$.
Regardless of the impedance to be measured, the adjustment branch is a capacitive impedance (C, R). It is technologically easier to construct capacitors with sufficient precision than impedances.

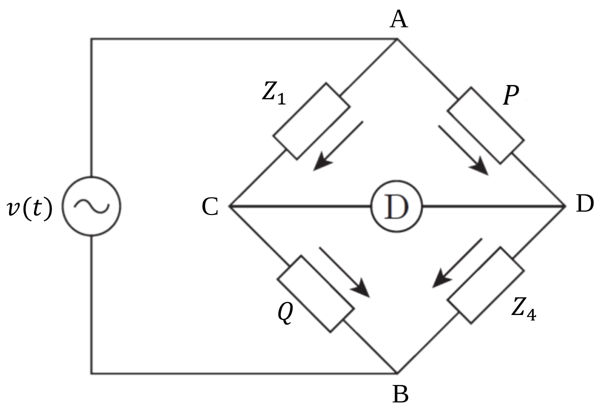


Figure 2-22: AC P.Q Bridge

With rare exceptions, all AC bridges are of the P.Q or P/Q type. For a systematic approach to manipulation, the terms P and Q will no longer be used, but will be noted in the order
Z_1, Z_2, Z_3 and Z_4 the constituent impedances of the bridge. The impedance Z_4 will always be the unknown branch. If Z_4 is formed by an inductance and a resistance, the notations will be L_4 and R_4. The two branches $Z_1$and Z_3 are most often the adjustment branches. When Z_1 is constituted of a capacitance and a resistance, the notations will be C_1 and R_1.
Now, we will present two examples of bridges used to measure capacitances and inductances, respectively.

2.6.2.1 De Sauty Bridges

This bridge is suitable for the measurement of very high-quality capacitance. At equilibrium, the two points C and D have the same potential. So

$$I_1R_1 = I_2R_2$$

and

$$-\frac{j}{\omega C_4}I_1 = -\frac{j}{\omega C_3}I_2$$

The conditions for balancing this bridge are therefore

$$C_4 = C_3\frac{R_1}{R_2}$$

The bridge has maximum sensitivity when $C_4 = C_3$. The method based on the De Sauty bridge is simple to implement and use, but perfect balance is difficult to achieve if the capacitors have dielectric losses.

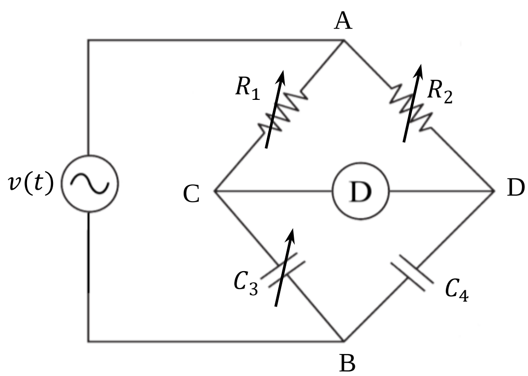


Figure 2-23: De Sauty Bridge

2.6.2.2Maxwell Bridges

This bridge is used to measure unknown inductances with a small argument ($L\omega < R$) in terms of adjustable resistances and capacitances.

The balance conditions for this bridge are:

$$(R_4 + j\omega L_4)\left(\frac{R_1}{1 + j\omega R_1C_1}\right) = R_2R_3$$

Which gives after simplification

$$R_4 = R_3\frac{R_2}{R_1}$$

And

$$L_4 = R_2R_3C_1$$

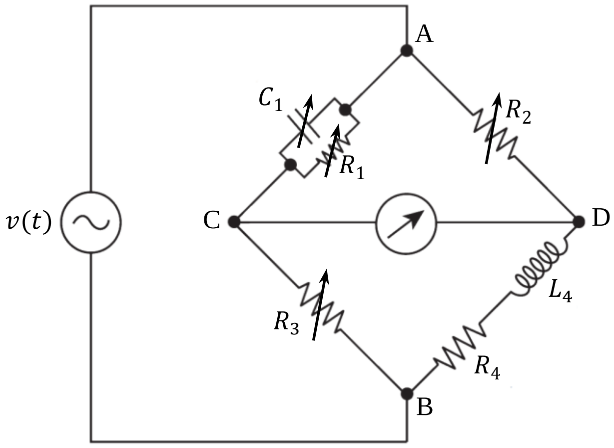


Figure 2-24: Maxwell Bridge

2.6 Power Measurement

2.6.1 DC Power Measurement

The power absorbed by any receiver is given by:

$$P = U \cdot I$$

There are two methods for measuring DC power: the voltammeter method and the electrodynamic wattmeter method.

❶ **Voltammetric Method:** The measurement of voltage U and current I allows the calculation of power P. To measure these two electrical quantities, we simply use an ammeter and a voltmeter. Depending on the voltmeter's position (before or after the ammeter), there are two possible setups: the upstream setup (long shunt) and the downstream setup (short shunt).

- **Upstream configuration:** The diagram of this configuration is shown in Figure 2-25. The ammeter here measures the true value of the current flowing through the resistive receiver R. On the other hand, the voltmeter measures the voltage across the receiver and ammeter assembly.

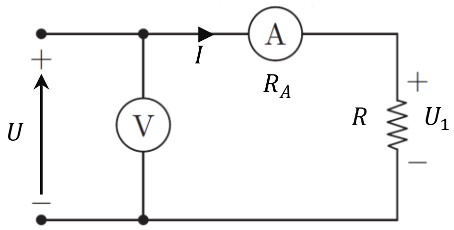


Figure 2-25: Upstream Configuration

We can easily have

$$U_1 = U - R_A \cdot I$$

By multiplying both sides of the equation by I, the power absorbed by the receiver R can be expressed as follows

$$P = U_1 \times I = U \times I - R_A \times I^2$$

The measurement introduces a systematic error by excess, $R_A I^2$, which represents the consumption of the ammeter.

- **Downstream configuration:** In this configuration, the voltmeter measures the true voltage value across the receiver R and the ammeter measures, this time, the sum of the currents flowing through the resistor R and the voltmeter (see Figure 2-26).

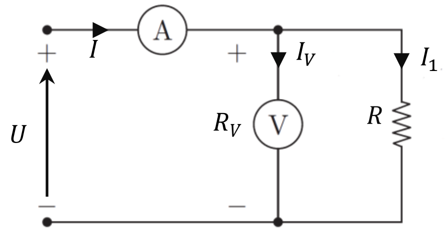


Figure 2-26: Downstream Configuration

The current flowing in R is

$$I_1 = I - I_V$$

Now, by multiplying both sides of this equation by the voltage U, the power absorbed by the receiver R can be given this time by

$$P = U \times I_1 = U \times I - U \times I_V$$

With$I_V = \frac{U}{R_V}$

The absorbed power becomes:

$$P = UI - \frac{U^2}{R_V}$$

The measurement introduces a systematic error by excess, $\frac{U^2}{R_V}$, which represents the voltmeter's consumption.

❷ **Electrodynamic Wattmeter Method:** The measurement of power uses an electrodynamic instrument, which is the wattmeter.

A Wattmeter Is a Measuring Device Intended to Indicate the Electrical Power (Expressed in Watts) Consumed by the Receiver. ①

- One coil, used for current measurement, called the "current circuit" or thick wire, comparable to an ammeter with internal resistance R_A ;
- One coil, used for voltage measurement, called the "voltage circuit" or thin wire, comparable to a voltmeter with internal resistance R_V.

On the dial of a wattmeter, one finds:

- The accuracy class,
- The type of current: AC or DC.
- The scale (in the general case, a single scale usable in AC and DC).
- The current consumption of the voltage circuit.

The wattmeter is an astatic device (insensitive to external fields); it essentially consists of:

- A current circuit: there are two direct ranges in a ratio of 1 to 2 (example: 0.5A and 1A or 1.25A and 2.5A).
- A voltage circuit: there are many ranges (from 15 V to 600 V).
- A scale with regular graduations.

The consumption of the voltage circuit allows the determination of the resistance of the voltage circuit. The wattmeter constant is defined by

$$K = \frac{\text{Range } U \times \text{Range } I}{\text{Scale}}$$

This constant represents the power per division of the scale.

The Current Circuit Is Connected in Series and the Voltage Circuit Is Connected in Parallel in Two Different Modes: Upstream Connection and Downstream Connection (See Figure 2.27 and 2.28 Respectively). ②③

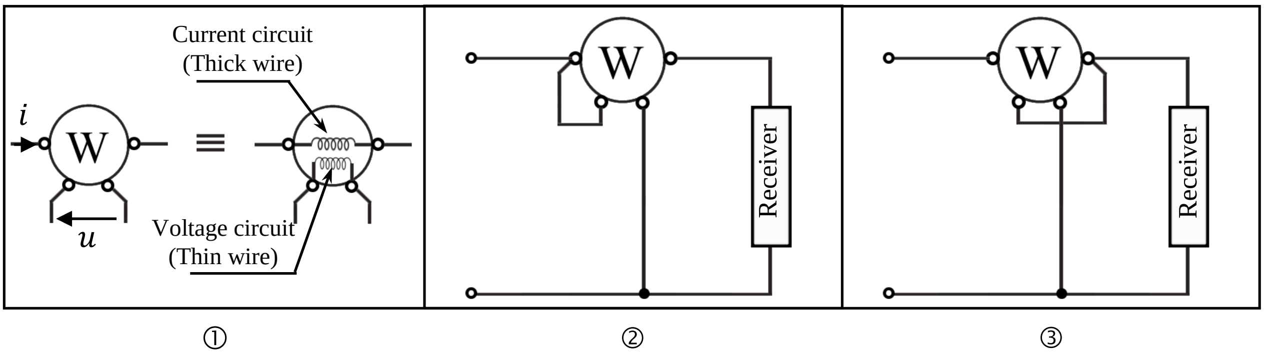


Figure 2-27: Connection diagram of a wattmeter.
①Schematic diagram of a wattmeter ② Upstream connection ③ Downstream connection

The Downstream Connection Is Chosen Here (See Figure 2.28). The Voltmeter and Ammeter Only Have an Indicative Role to Avoid Exceeding the Range Values of the Wattmeter.

In this case, the measured power is expressed by

$$P_{\text{Exact}} = P_{\text{Lue}} - \left(\frac{U^2}{R_W} + \frac{U^2}{R_V}\right)$$

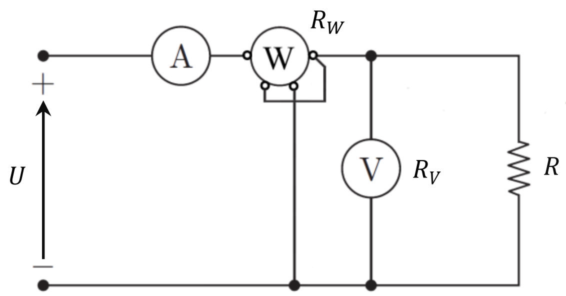


Figure 2-28: Downstream connection of the wattmeter method

2.6.2 AC Power Measurement

2.6.2.1 Single-Phase Active Power Measurement

The average power absorbed by a receiver subjected to a periodic voltage is:

$$P = \frac{1}{T}\int_0^T u \cdot i \, dt$$

If u and i are instantaneous sinusoidal quantities:

$$u(t) = \sqrt{2}\, U \cos(\omega t)$$

$$i(t) = \sqrt{2}\, I \cos(\omega t + \varphi)$$

Note: U and I are effective (RMS) values.

Instantaneous Power: is the voltage-current product at any instant:

$$P(t) = u(t) \cdot \; i(t) = 2\, U\, I \;\cos(\omega t)\cos(\omega t + \varphi)$$

After simplifying the product, we find:

$$P(t) = U\, I \cos(\varphi) + U\, I \cos(2\omega t + \varphi)$$

Fluctuating Power: is the variable part of the instantaneous power:

$$P_f(t) = U\, I \cos(2\omega t + \varphi)$$

Active Power or Average Power: is the average value of the instantaneous power over a period $T = 2\pi$ and is expressed as follows

$$P = U\, I \cos(\varphi)$$

This is the power that corresponds to actual physical work; its unit is the *Watt* (W).

Apparent Power: is the product of the effective (RMS) values:

$$S = U\, I$$

This power is often called "*sizing power*"; it is the characteristic quantity for the insulation and cross-section of conductors, i.e., the dimensions of the equipment. Its unit is the *Volt-Ampere* (VA).

Reactive Power: is the power with no physical effect in terms of work, corresponding to the "reactive" part of the current. It is only defined in sinusoidal conditions and is written as:

$$Q = U\, I \sin(\varphi)$$

Its unit is the *Volt-Ampere-Reactive* (VAR).

Once these powers are defined, it is imperative to memorize the definitions and relationships summarized in Figure 2-29.

Complex Apparent Power: to analytically determine the various powers, we form the complex apparent power:

$$\underline{S} = \underline{U}\, \underline{I}^*$$

where $\underline{I}^*$ is the complex conjugate of $\underline{I}$.

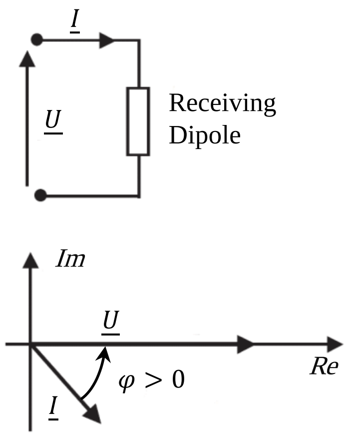
It can be shown that and that:

$$\underline{S} = P + j \cdot Q$$

and that

$$|\underline{S}| = S = \sqrt{P^2 + Q^2}$$

This power is only a computational expression intended for the raw determination of the various powers by identifying the real and imaginary parts.



Active Power: $P = U\,I\cos(\varphi)$ (W)

Reactive Power: $Q = U\,I\sin(\varphi)$ (VAR)

Apparent Power: $S = U\,I$ (VA)

$U = U_{eff} = \frac{U_{max}}{\sqrt{2}}, I = I_{eff} = \frac{I_{max}}{\sqrt{2}}$

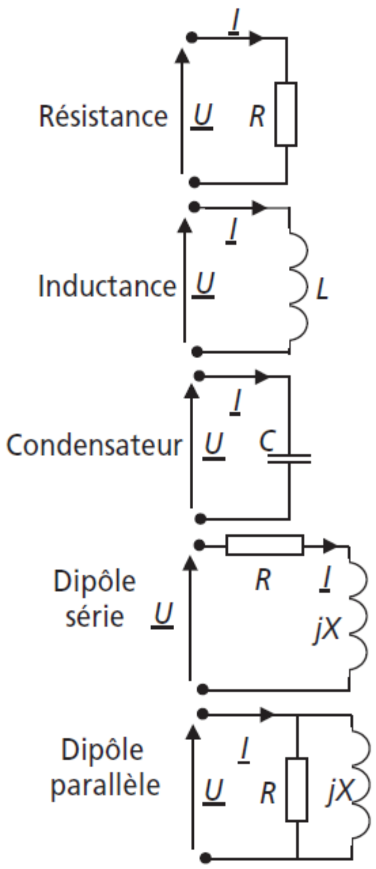
Relationship: $S^2 = P^2 + Q^2$

Power factor :$k = \frac{P}{S} = \cos(\varphi)$

$\tan(\varphi) = \frac{Q}{P}$

Figure 2-29: Power in sinusoidal conditions

As an example, the complex apparent power is used in Figure 2-30, which synthetically shows the expressions of the active and reactive powers of the most common dipoles encountered in electrical engineering. It is imperative to perfectly master the data in this box and, at worst, to know how to find it easily.



$\underline{S}$	P	Q
$\underline{S} = R\cdot\underline{I}\cdot I^*$ $= RI^2$ $= U^2/R$	$RI^2 = U^2/R$	0
$\underline{S} = jL\omega\cdot I\cdot I^*$ $= jL\omega I^2$ $= jU^2/L\omega$	0	$jL\omega I^2 = jU^2/L\omega$
$\underline{S} = -j/C\omega\cdot I\cdot I^*$ $= -j/C\omega I^2$ $= jU^2/L\omega$	0	$-j/C\omega I^2 = -jC\omega U^2$
$\underline{S} = (R + jX)I\cdot I^*$ $= RI^2 + jX\cdot I^2$	$R\cdot I^2$	$X\cdot I^2$
$\underline{S} = \underline{U}\cdot I^*$ $\underline{U} = \underline{I}/(R\,//\,jX)$	U^2/R	U^2/X

Figure 2-30: Powers associated with common dipoles

There are several methods for measuring active power in single-phase systems. In this paragraph, we will discuss the following methods: Three-ammeter method, three-voltmeter method, and electrodynamic wattmeter method.

❶Three-Ammeter Method: The setup for this method is shown in Figure 2.31. We can write in instantaneous values.

$$p = u\cdot i$$

and

$$i_2 = \frac{u}{R_2}$$

where R_2 is a standard resistor that includes the resistance of the ammeter A_2.

According to Kirchhoff's current law (KCL):

$$i_1 = i + i_2$$

We can also have

$$\begin{aligned} i_1^2 &= i^2 + i_2^2 + 2\cdot i\cdot i_2 \\ &= i^2 + i_2^2 + 2\cdot i\cdot\frac{u}{R_2} \\ &= i^2 + i_2^2 + \frac{2}{R_2}\cdot p \end{aligned}$$

Then, the instantaneous power is expressed as follows

$$p = \frac{R_2}{2}(i_1^2 - i_2^2 - i^2)$$

By considering the RMS values of the currents, the average power is given by:

$$P = \frac{R_2}{2}(I_1^2 - I_2^2 - I^2)$$

An excess error (consumption of the ammeter A) is introduced: it can be taken into account for the calculation of the exact absorbed power.

$$P_{\text{Exact}} = \frac{R_2}{2}\left(I_1^2 - I_2^2 - I^2\right) - R_A \cdot I_2^2$$

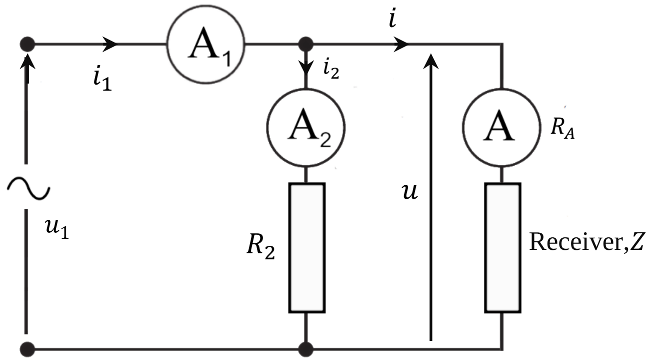


Figure 2-31: Three-ammeter method setup

❷**Three-Voltmeter Method:** The setup for this method is shown in Figure 2.32. Assuming negligible voltmeter consumption, if is a standard resistor, we can write in instantaneous values$R$

$$u_1 = u_2 + u$$

With$u_2 = R \cdot i$

We can also have:

$$u_1^2 = u_2^2 + u^2 + 2 \cdot u_2 \cdot u \Leftrightarrow u_1^2 = u_2^2 + u^2 + 2 \cdot R \cdot i \cdot u$$

Then

$$p = u \cdot i = \frac{u_1^2 - u_2^2 - u^2}{2\,R}$$

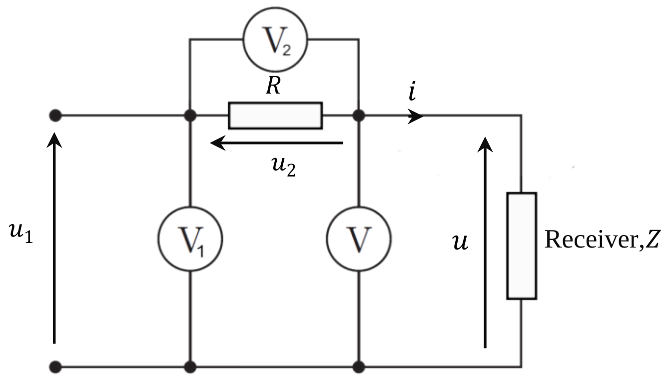


Figure 2-32: Three-voltmeter method setup

By considering the RMS values of the voltages, the average power is given by:

$$P = \frac{U_1^2 - U_2^2 - U^2}{2\,R}$$

❸**Electrodynamic Wattmeter Method:** Neglecting the self-inductance of the wattmeter's fine wire, the measured power is:

- **Upstream assembly:**$P_1 = U\, I \cos(\varphi) + r_A \cdot I^2$
  where r_A is the resistance of the ammeter and wattmeter current circuit.
  If P is the power absorbed by the receiver, then:

$$P = P_1 - r_A \cdot I^2$$

- **Downstream Setup**33. In this case,

$$P_1 = U\, I \cos(\varphi) + \frac{U^2}{R_V} + \frac{U^2}{R_W}$$

  $R_V$and R_W being the respective resistances of the voltmeter and the voltage circuit of the wattmeter.
  If P is the power absorbed by the receiver, then:

$$P = P_1 - \left(\frac{U^2}{R_V} + \frac{U^2}{R_W}\right)$$

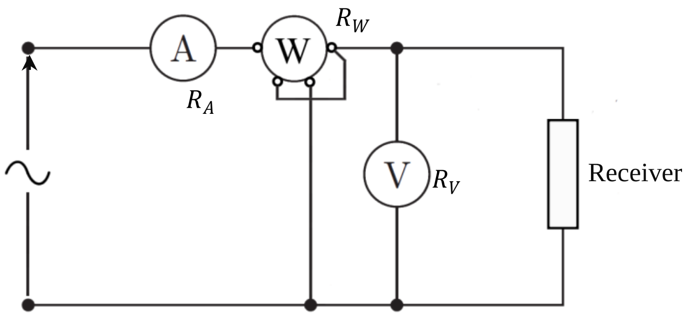


Figure 2-33: Downstream connection of the wattmeter method in AC

2.6.2.2 Active Power Measurement in Three-Phase

The Following Analysis Concerns Only Networks Operating in Sinusoidal Mode and Constituting a Balanced Voltage System (Equal Component Voltages and Phase-Shifted by $2\pi/3$, See the Diagram in Figure 2.34).

The voltages supplied by a three-phase generator whose neutral point is called **point 0** are:

Single-phase voltages: $\overrightarrow{V_{10}}, \overrightarrow{V_{20}}$ and $\overrightarrow{V_{30}}$.

Compound Stresses : $\overrightarrow{U_{12}} = \overrightarrow{V_{10}} - \overrightarrow{V_{20}}, \overrightarrow{U_{23}} = \overrightarrow{V_{20}} - \overrightarrow{V_{30}}$ and$\overrightarrow{U_{31}} = \overrightarrow{V_{30}} - \overrightarrow{V_{10}}$

With $\overrightarrow{V_{10}} + \overrightarrow{V_{20}} + \overrightarrow{V_{30}} = 0$ and$\overrightarrow{U_{12}} + \overrightarrow{U_{23}} + \overrightarrow{U_{31}} = 0$

The single-phase voltages across the terminals of the receiver whose neutral point is called **point N** are: $\overrightarrow{V_{1N}}, \overrightarrow{V_{2N}}$ and $\overrightarrow{V_{3N}}$.

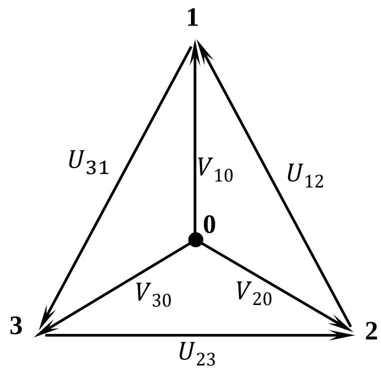


Figure 2-34: Voltage diagram

Conventions: They allow, without risk of error, to consider all the artifices of wattmeter connection.

- Measuring the voltage V_{10} amounts to defining the potential difference (p.d.) of phase 1 with respect to point 0.
- Measuring the current I_2 specifies that the ammeter is placed in series on phase 2.
- Measuring the power, P_{10-2}, defines the power considered, and explains the connection of the wattmeter. The first two indices, 1 and 0, specify the connection of the voltage circuit: Input at 1, and output at 0. The last index, 2, specifies that the current circuit is in series on phase 2.

  Example: See the Example in Figure 2.35. The Wattmeter Measures the Power. P_{13-1}

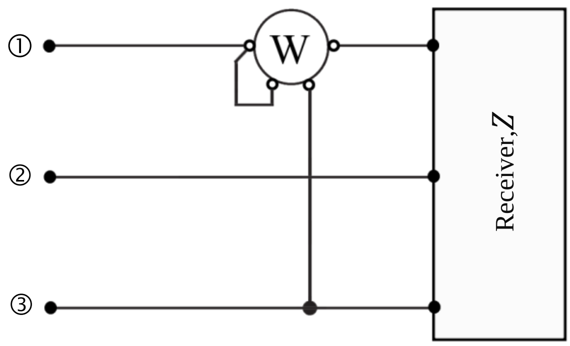


Figure 2-35 : Power Measurement P_{13-1}

Procedure: The rational use of wattmeters for the determination of active and reactive powers in three-phase systems involves certain preliminary measurements:

- The phase sequence of the distribution network must be known.
- Before use, the inputs and outputs of the wattmeter windings must be checked so that a positive power corresponds to a positive deflection. To do this, we measure a single-phase power which, by definition, is always active. We can measure, for example, P_{10-1} or P_{1N-1}, the wattmeter deflection must be positive: We deduce the input of the voltage winding, it is index 1; if the deflection is negative, the input will be index 0. The connection order thus obtained must be absolutely respected during subsequent measurements.

- To define a negative deflection, it is sufficient to temporarily cross two voltage wires, take the reading and assign it a minus sign.
- It is recommended to have an ammeter and a voltmeter in the circuit so as to check at any time whether the wattmeter range is suitable.
- A total active power is always positive. On the other hand, reactive power can be positive or negative.

Now, we will discuss the methods of measuring active power in three-phase systems.

❶Three-wattmeter method

- **4-Wire Distribution (3 Phases + Neutral): The Setup for This Configuration Is Shown in Figure 2.36.** The System Can Be Considered as a Set of Three Single-Phase Distributions, the Neutral Wire Being the Common Return.

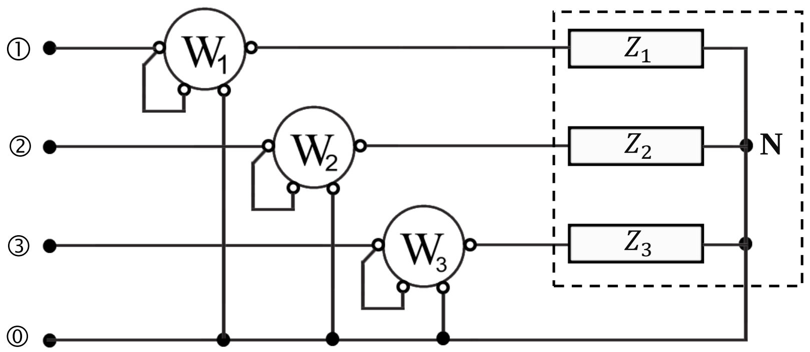


Figure 2-36: Setup of the three-wattmeter method for a 4-wire distribution

The total active power is equal to the sum of the active powers per phase.

$$P = \frac{1}{T}\int_0^T (v_{1\mathrm{N}} \cdot i_1 + v_{2\mathrm{N}} \cdot i_2 + v_{3\mathrm{N}} \cdot i_3)\, dt$$

The voltages $v_{1\mathrm{N}}$, $v_{2\mathrm{N}}$ and $v_{3\mathrm{N}}$ are the voltages between phase and neutral of the receiver.

The sum of the readings of the three wattmeters gives the total active power P.

$$P = P_{1\mathrm{N}-1} + P_{2\mathrm{N}-2} + P_{3\mathrm{N}-3}$$

It is an arithmetic sum because the active powers per phase are always positive.

The neutral N of the receiver being at potential 0, we deduce:

$$\left.\begin{array}{l} v_{1\mathrm{N}} = v_{10} \\ v_{2\mathrm{N}} = v_{20} \\ v_{3\mathrm{N}} = v_{30} \end{array}\right\} \Rightarrow P = \frac{1}{T}\int_0^T (v_{10} \cdot i_1 + v_{20} \cdot i_2 + v_{30} \cdot i_3)\, dt$$

Or even

$$P = P_{10-1} + P_{20-2} + P_{30-3}$$

If the system is current-balanced, a single wattmeter is sufficient since the powers per phase are equal. In this case,

$$P = 3 \cdot P_{10-1}$$

- **3-wire distribution without neutral:** The distribution of currents in the line wires is such that, at any time:

$$i_1 + i_2 + i_3 = 0$$

  - **Star-Connected Receiver: A Star-Connected Receiver Is Shown in Figure 2.37.** The Instantaneous Power Is as in 4-Wire

$$p = v_{1N} \cdot i_1 + v_{2N} \cdot i_2 + v_{3N} \cdot i_3$$

Since $\begin{cases} v_{1N} = v_{10} + v_{0N} \\ v_{2N} = v_{20} + v_{0N} \\ v_{3N} = v_{30} + v_{0N} \end{cases}$

$$p = (v_{10} + v_{0N}) \cdot i_1 + (v_{20} + v_{0N}) \cdot i_2 + (v_{30} + v_{0N}) \cdot i_3$$

$$p = v_{10} \cdot i_1 + v_{20} \cdot i_2 + v_{30} \cdot i_3 + v_{0N}(i_1 + i_2 + i_3)$$

and since $i_1 + i_2 + i_3 = 0$, then

$$p = v_{10} \cdot i_1 + v_{20} \cdot i_2 + v_{30} \cdot i_3$$

If the receiver's neutral is accessible, we measure: $P = P_{1N-1} + P_{2N-2} + P_{3N-3}$

If the 0 point of the network is accessible, we measure: $P = P_{10-1} + P_{20-2} + P_{30-3}$

If the receiver is balanced, the measurement of P_{1N-1} or P_{10-1} is sufficient.

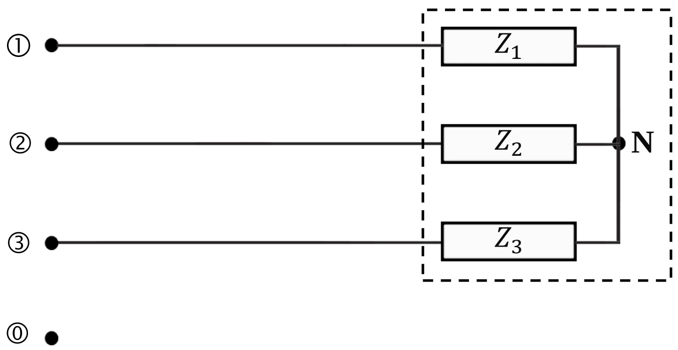


Figure 2-37: Star-connected receiver

  - **Delta-Connected Receiver: A Delta-Connected Receiver Is Shown in Figure 2.38.** The Total Power Absorbed Is the Sum of the Powers per Phase:

$$p = u_{12} \cdot i_{12} + u_{23} \cdot i_{23} + u_{31} \cdot i_{31}$$

By hypothesis:

$$u_{12} = v_{10} - v_{20}$$
$$u_{23} = v_{20} - v_{30}$$
$$u_{31} = v_{30} - v_{10}$$

from which

$$p = (v_{10} - v_{20}) \cdot i_{12} + (v_{20} - v_{30}) \cdot i_{23} + (v_{30} - v_{10}) \cdot i_{31}$$

$$p = v_{10}(i_{12} - i_{31}) + v_{20}(i_{23} - i_{12}) + v_{30}(i_{31} - i_{23})$$

$$p = v_{10} \cdot i_1 + v_{20} \cdot i_2 + v_{30} \cdot i_3$$

So

$$P = P_{10-1} + P_{20-2} + P_{30-3}$$

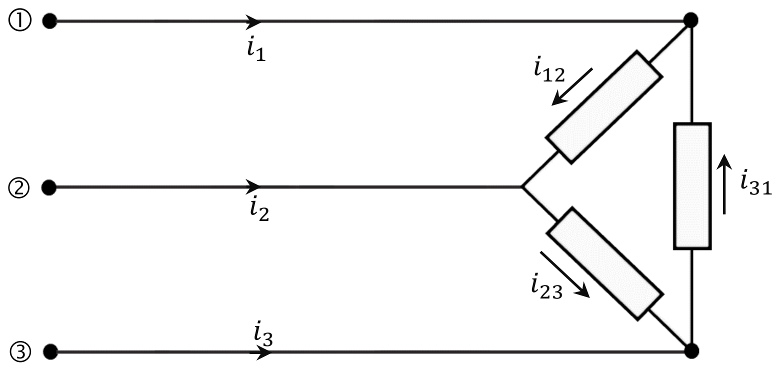


Figure 2-38: Delta-Connected Receiver

Conclusion: In a 3-wire distribution with a balanced or unbalanced receiver, connected in delta or star, the three-wattmeter method is always valid.

❷**Two-Wattmeter Method:** The setup for this configuration is shown in Figure 2.39.

The principle of this method is simple.

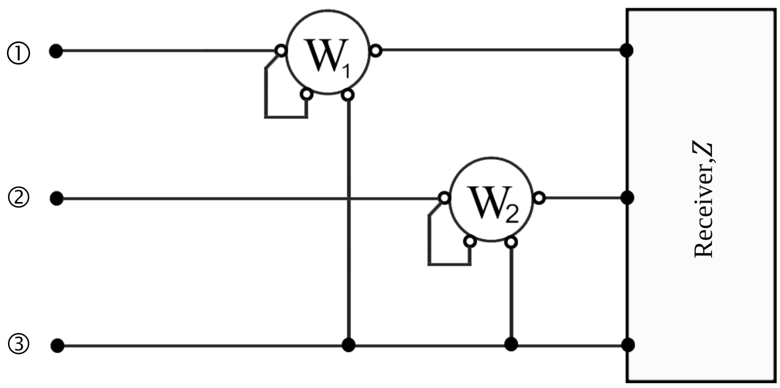


Figure 2-39: Two-Wattmeter Method Setup for a 3-Wire Distribution

Let's revisit the relationship

$$p = v_{10} \cdot i_1 + v_{20} \cdot i_2 + v_{30} \cdot i_3$$

and since

$$i_1 + i_2 + i_3 = 0 \Rightarrow i_3 = -i_1 - i_2$$

It follows that

$$p = v_{10} \cdot i_1 + v_{20} \cdot i_2 + v_{30} \cdot (-i_1 - i_2)$$

therefore

$$p = (v_{10} - v_{30}) \cdot i_1 + (v_{20} - v_{30}) \cdot i_2$$

$$p = u_{13} \cdot i_1 + u_{23} \cdot i_2$$

Using the RMS values of voltages and currents, the active power is given by:

$$P = U_{13} \cdot I_1 + U_{23} \cdot I_2$$

Which gives

$$P = P_{13-1} + P_{23-2}$$

Conclusion: The two-wattmeter method allows for the measurement, in a 3-phase 3-wire system, of the active power absorbed by a balanced or unbalanced receiver.

2.6.2.3 Reactive Power Measurement

There are generally two methods for measuring reactive power (whether in single-phase or three-phase): Direct measurement method and indirect measurement method.

The reactive power involved in a single-phase receiver is (see paragraph 2.6.2.1):

$$Q = U\, I \sin(\varphi)$$

➊**Direct Measurement Method:** Measurements are made using a varmeter, a device whose driving torque is of the form: $C_M = k\, U\, I \sin(\varphi)$. In this case, the measurement methods are identical to the active power measurements using a wattmeter.

Example: A wattmeter measures P_{10-1}, and a varmeter measures Q_{10-1}.

➋**Indirect Measurement Method:** These methods utilize wattmeters and are applied to polyphase networks. The connections are made such that the deflections no longer define an active power but rather a reactive power.

Their Principle Is as Follows. The Voltage Circuit of the Wattmeter Is Supplied by a Voltage, Phase-Shifted with Respect to the Initial Voltage by a Lagging Angle (See Figure 2.40). $U' U \pi/2$

In this case, the active power is given by

$$P = U'\, I \cos\left(\frac{\pi}{2} + \varphi\right)$$

If $|U| = |U'|$ then

$$P = Q = -U\, I \sin(\varphi)$$

Capacitive circuit: $\varphi > 0$ then $Q < 0$.

Inductive circuit: $\varphi < 0$ then $Q > 0$.

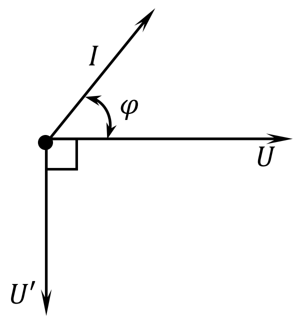


Figure 2-40: Single-Phase Voltage/Current Diagram

◆**Single-Phase Distribution:** The measurements of P, U, and I allow for the calculation of:

- **Apparent Power:** $S = UI$
- **Reactive Power:** $Q = \sqrt{S^2 - P^2}$

◆**4-wire three-phase distribution (3 phases + neutral): Th**e total reactive power is equal to the algebraic sum of the reactive powers involved in each phase.

Points **N** and **0** being at the same potential, it follows:

$$Q = V_{1N}\, I_1 \sin(\varphi_1) + V_{2N}\, I_2 \sin(\varphi_2) + V_{3N}\, I_3 \sin(\varphi_3)$$

$$Q = V_{10}\, I_1 \sin(\varphi_1) + V_{20}\, I_2 \sin(\varphi_2) + V_{30}\, I_3 \sin(\varphi_3)$$

And Since, the Diagram in Figure 2.41 Shows: $\sin(\varphi) = \cos\left(\frac{\pi}{2} - \varphi\right)$

$$V_{1N}\, I_1 \sin(\varphi_1) = U_{23}\, I_1 \sin(\varphi_1)/\sqrt{3}$$

$$V_{2N}\, I_2 \sin(\varphi_2) = U_{31}\, I_2 \sin(\varphi_2)/\sqrt{3}$$

$$V_{3N}\, I_3 \sin(\varphi_3) = U_{12}\, I_3 \sin(\varphi_3)/\sqrt{3}$$

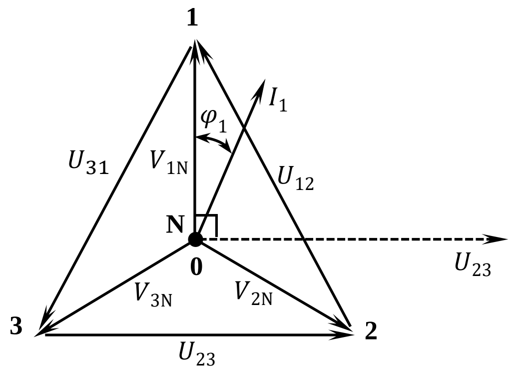


Figure 2-41: Three-Phase Voltage/Current Diagram

Finally,

$$Q = \frac{U_{23}\, I_1 \sin(\varphi_1) + U_{31}\, I_2 \sin(\varphi_2) + U_{12}\, I_3 \sin(\varphi_3)}{\sqrt{3}}$$

This Relationship Allows for the Establishment of the Three-Wattmeter Connection Diagram Shown in Figure 2.42.

The wattmeter W_1 indicates the power P_{23-1} which is the image of the reactive power Q_{1N-1}. We have

$$Q_{1N-1} = \frac{P_{23-1}}{\sqrt{3}}$$

The other two wattmeters W_2 and W_3 respectively indicate the powers P_{31-2} and P_{12-3} images of Q_{2N-2} and Q_{3N-3}, hence

$$Q_{2N-2} = \frac{P_{31-2}}{\sqrt{3}}$$

and

$$Q_{3N-3} = \frac{P_{12-3}}{\sqrt{3}}$$

So

$$Q = \frac{P_{23-1} + P_{31-2} + P_{12-3}}{\sqrt{3}}$$

If the receiver is balanced, the reading of a single wattmeter is sufficient.

$$Q = \sqrt{3}\, P_{23-1}$$

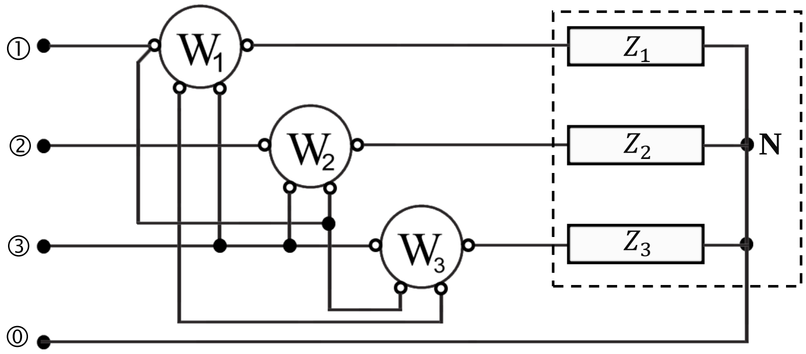


Figure 2-42: Setup of the indirect three-wattmeter method for measuring reactive power (Case of a 4-wire three-phase distribution)

2.7 Phase Shift Measurement

The phase shift existing between two sinusoidal quantities of the same frequency specifies the angular displacement of one of the quantities relative to the other. This phase shift angle is conventionally called φ, with the trigonometric convention allowing us to specify whether it is positive or negative.

2.7.1 Measurement of the Phase Shift of a Current Relative to a Voltage

➊**Phasemeter Method:** The phasemeter allows, through direct reading, the determination of the phase shift angle.φ Its connection in a circuit is shown in Figure 2.43.

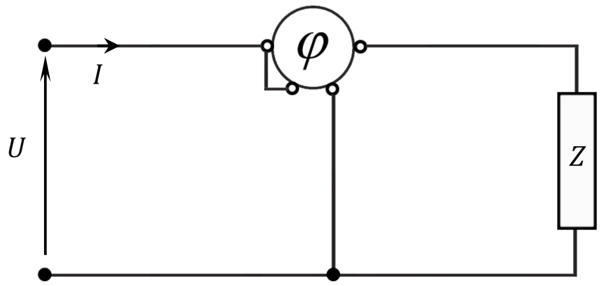


Figure 2-43: Setup of the phase meter method

➋**Wattmeter Method:** In this method, we use the relationship $P = U\ I\ \cos(\varphi)$. The measurement of , and allows the calculation of and of . $PUI\cos(\varphi)\varphi$

2.7.2 Measurement of the Phase Shift Between Two Voltages

➊**Voltmeter Method**

◆**The two voltages have a common point:** We successively measure the voltages: $U_1 = U_{13}$, $U_2 = U_{23}$, and $U_3 = U_{12}$ and a graphical construction allows us to deduce φ (See Figure 2.44).

Graphical construction: Using the chosen scale and starting from the two ends of U_1, arcs of length U_2 and U_3 should be drawn. The intersection point of the two arcs allows for the drawing of U_2 and U_3.

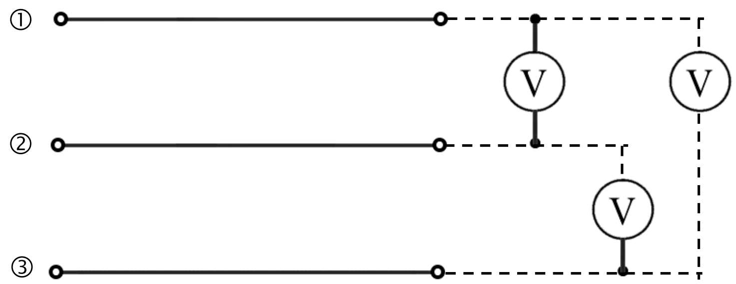


Figure 2-44: Phase shift measurement using the voltmeter method

The Diagram in Figure 2.45 Allows Us to Establish:

$$U_3^2 = U_1^2 + U_2^2 - 2\,U_1\,U_2\cos(\varphi)$$

It follows that

$$\cos(\varphi) = \frac{U_1^2 + U_2^2 - U_3^2}{2\,U_1\,U_2}$$

Note: the voltmeter method does not allow the determination of the sign of the phase shift.

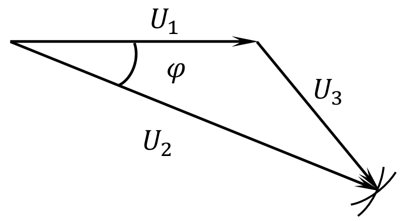


Figure 2-45: Diagram for calculating the phase shift between two voltages

◆**The two voltages have no common point:** In this case, two isolation transformers with a ratio of 1 are used (See Figure 2.46). The input and output voltages at the transformer terminals must remain in phase, which requires prior marking of the terminals. The graphical construction is identical to the previous case.

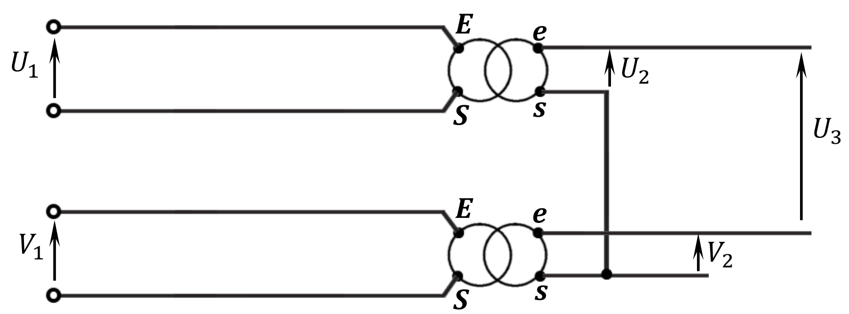


Figure 2-46: Diagram for calculating the phase shift between two voltages

❷**Oscilloscope Method**

◆**The two voltages have a common point:**The oscilloscope is in *Lissajous* mode. The input (horizontal deflection) is driven by the voltage whose instantaneous value is. The input (vertical deflection) is driven by the voltage whose instantaneous value is. $XVv = V_M\sin(\omega t)\,YUu = U_M\sin(\omega t + \varphi)$

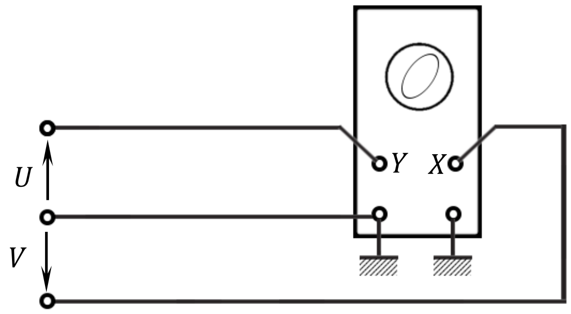


Figure 2-47: Phase shift measurement using an oscilloscope
(Case where the two voltages have a common point)

With proper gain adjustment, an ellipse appears on the screen.

If $v = 0$, $\sin(\omega t) = 0$ therefore$u = U_M \sin(\varphi)$

and then

$$\sin(\varphi) = \frac{u}{U_M}$$

$v = 0$, when the spot is on the axis of Y. We can therefore define and . uU_M

Graphical Determination: According to Figure 2.48, We Can Have

$$\sin(\varphi) = \frac{l}{L} = \frac{2\,u}{2\,U_M} = \frac{u}{U_M}$$

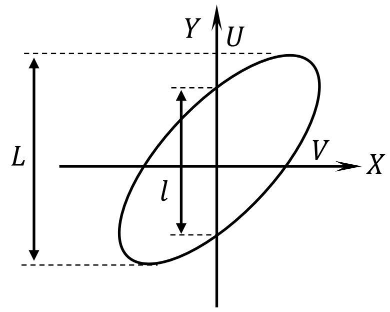
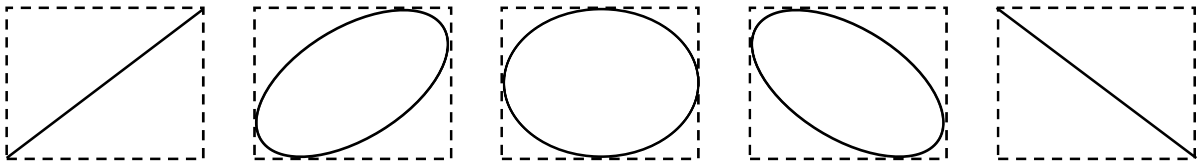


Figure 2-48: Graphical determination of phase shift using an oscilloscope

It is necessary that before the measurement, the spot is adjusted to the center of the screen.

Some Examples of Lissajous Curves Are Shown in Figure 2.49.



$\varphi = 0$ | $0 < \varphi < \frac{\pi}{2}$ | $\varphi = \frac{\pi}{2}$ | $\frac{\pi}{2} < \varphi < \pi$ | $\varphi = \pi$

Figure 2-49: Special cases of Lissajous curves

These correspond to the following particular cases:

- If the vertices of the ellipse are located in quadrants 1 and 3, then $0 < \varphi < \frac{\pi}{2}$.
- If the vertices of the ellipse are located in quadrants 2 and 4, then$\frac{\pi}{2} < \varphi < \pi$.
- The ellipse is reduced to an inclined line: $\varphi = 0$ or $\varphi = \pi$.

◆The ellipse has its axes aligned with those of the screen, $\varphi = \frac{\pi}{2}$. If the amplitudes of the traces are equal, the figure is then a circle.

Dual-trace oscilloscope:

A Dual-Trace Cathode-Ray Oscilloscope and or One Equipped with an Electronic Switch Allows the Comparison of Two Voltages by the Simultaneous Display and Superposition of Their Representative Curves (See Figure 2.50). $Y_1 Y_2 v_1(t) v_2(t)$

The two voltages with a phase difference of φ are applied to the terminals of Y_1 and Y_2. With their traces centered, their amplitudes are adjusted to be equal. The phase shift is given by the relationship:

$$\varphi = 360\frac{X}{a}$$

Or can be obtained by the following relationships:

$$\cos(\varphi) = \frac{B}{A}$$

or

$$\tan\left(\frac{\varphi}{2}\right) = \sqrt{\left(\frac{A}{B}\right)^2 - 1}$$

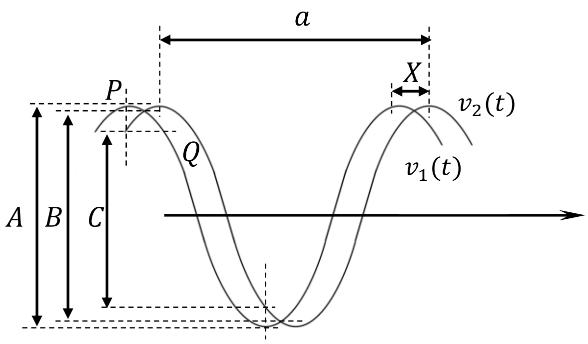


Figure 2-50: Phase shift measurement by simultaneous display of two voltages on an oscilloscope

◆**The two voltages have no common point:** As with the voltmeter method, two isolation transformers with a ratio of 1 are used.

2.8 Measurement of Frequencies and Periods

The frequency of a periodic electrical quantity with period T is defined by the following relationship:

$$f = \frac{1}{T}$$

The unit of frequency is the Hertz (Hz).

The angular frequency is given by

$$\omega = 2\pi f$$

Its unit is radians/second (rad/s).

Now, we will discuss the following methods of frequency measurement: Direct deflection method, Resonance method, and Lissajous curves method.

❶Direct deflection method: A frequency is commonly measured using a frequency meter. We distinguish vibrating blade, deflection, and digital display frequency meters.

❷Resonance Method: A fixed inductor L and a variable capacitor$C$ are used as shown in Figure 2.51. Adjusting the capacitance allows for obtaining, at resonance, a maximum deflection on the ammeter.

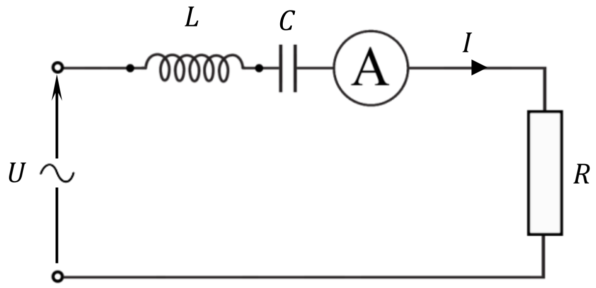


Figure 2-51: Frequency measurement by the resonance method

From the circuit above, we have

$$I = \frac{U}{R}$$

with

$$L\omega = \frac{1}{C\omega} \Rightarrow \omega^2 = \frac{1}{LC}$$

The frequency is then given by

$$f = \frac{1}{2\pi\sqrt{LC}}$$

❸Lissajous Curve Method: This is a comparison method that uses an oscilloscope. Its wiring diagram is shown in Figure 2.52.

The standard frequency f_E is delivered by the network, and f_x is the frequency to be measured. If $f_x = 2 \cdot f_E$, the rate of change of the frequency f_x is twice as fast as that of the frequency f_E : there will be two points of tangency with the Y axis and only one point of tangency with the X axis.

The above shows that the points of tangency are in the ratio of the frequencies.

To f_x, corresponds y points of tangency with the Y axis.

To f_E, corresponds x points of tangency with the X axis.

$$\frac{f_x}{f_E} = \frac{y}{x} \Rightarrow f_x = \frac{y}{x} f_E$$

The horizontal deflection plates are isolated from the time base and connected to the input X (*Lissajous* position).

Consider two signals: f_x connected to input X, f_E connected to input Y. Whenever the displayed curve is closed, we can assert that the frequencies are in a commensurable ratio.

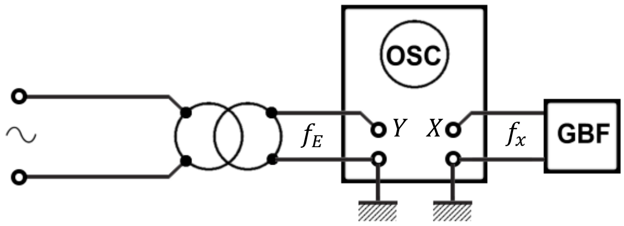


Figure 2-52: Frequency measurement by the comparison method

Examples: With a frequency $f_E = 50\text{Hz}$, we obtain:

- Figure 2.53 ① : $\begin{Bmatrix} y = 1 \\ x = 2 \end{Bmatrix} \Rightarrow f_x = 25\text{Hz}.$
- Figure 2.53 ② : $\begin{Bmatrix} y = 2 \\ x = 4 \end{Bmatrix} \Rightarrow f_x = 25\text{Hz}.$
- Figure 2.53 ③ : $\begin{Bmatrix} y = 2 \\ x = 3 \end{Bmatrix} \Rightarrow f_x = 33\text{Hz}.$
- Figure 2.53 ④ : $\begin{Bmatrix} y = 4 \\ x = 2 \end{Bmatrix} \Rightarrow f_x = 100\text{Hz}.$

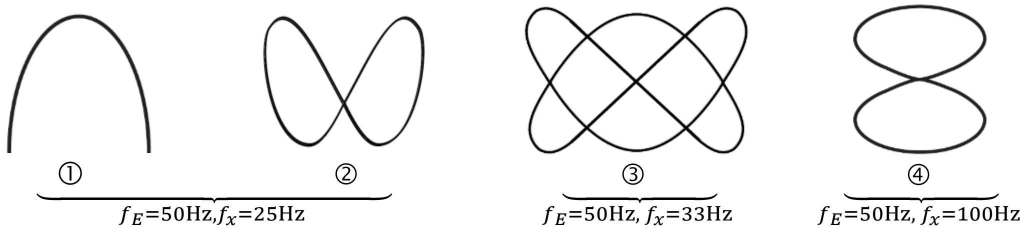


Figure 2-53: Lissajous curves

2.9 Application Exercises

Exercise 2.1 **Wheatstone Bridge**

We have a Wheatstone bridge whose ratio of proportion is equal to $\frac{R_3}{R_1}$ with $R_3 = 100\Omega$ and $R_1 = 1000\Omega$ on decades of 0,2%, the variable resistance R_2 is constituted by a combination of four decade boxes ($\times 1$, $\times 10$, $\times 100$, $\times 1000$) of precision 0,2%. The value of R_2 is 3528Ω.

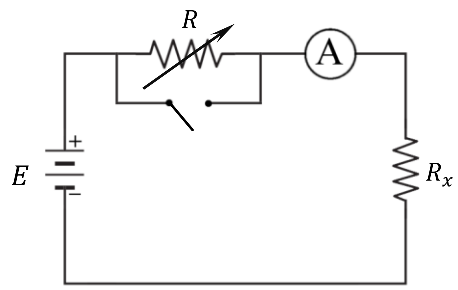
1. Represent the diagram illustrating this measurement method.
2. Give the expression and the value of the unknown resistance R_x.
3. Calculate ΔR_a, ΔR_b, ΔR_c and ΔR_d, then deduce $\frac{\Delta R_2}{R_2}$.
4. Determine the relative uncertainty $\frac{\Delta R_x}{R_x}$ then the absolute uncertainty ΔR_x.

Indication: $R_2 = R_d + R_c + R_b + R_a = r_{1000} + r_{100} + r_{10} + R_1$ (décades)

$$\Delta R_2 = \Delta R_d + \Delta R_c + \Delta R_b + \Delta R_a$$

Exercise 2.2

To measure the resistance of a motor winding, the following setup was used:



◆We close K and measure the current I_0.

◆iKs opened and iRs varied until a current f$I = \frac{I_0}{2}$lows in the circuit.

We are given $R = 529 \pm 21\Omega$.

1. Find the expression and the value of R_x.
2. Calculate the relative uncertainty $\frac{\Delta R_x}{R_x}$.

Chapter 3

Measurement Devices

Chapter objectives

At the end of Chapter 3, the student will be able to:

- Indicating instruments
- Moving coil instruments
- Electrodynamometer type
- Moving iron instruments
- Classification of instruments

3.1 Introduction

A measuring device is a system that translates a physical phenomenon that is not, or is difficult for our senses to access, into another phenomenon that can be visualized and estimated.

There are two main classes of devices:

- ***Deviation or analog devices:*** by their operating principle, theoretically give a value of the measured quantity exactly proportional to this quantity.
- ***Digital or logical measuring devices:*** they give a value representing the measured quantity to the nearest quantization step. This value is given in the form of a number (digital display).

3.2 Analog Measuring Devices

A measuring device generally includes one or more fixed inductors (permanent magnet or electromagnet) acting on a moving coil assembly around a fixed axis. The internal technology of these devices is based on three elements:

- The suspension of the moving assembly;

- The reading device, which can be a needle or a light spot;
- The damping device, which can be magnetic or by air.

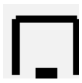
According to their terminology, there are several types of analog devices, namely:

- Ratiometers (electric balance);
- Integrating devices (meter, fluxmeter);
- Electronic devices;
- Analog devices with digital display;
- Deviation devices.

3.2.1 Deviation Devices

The usual classification of deviation devices uses the nature of the physical phenomenon that governs the operation of the device. There are several types of devices, the main types being:

- ***Magneto-electric Devices:*** The fixed inductor is a fixed *U*-shaped magnet, the deflection of the needle is proportional to the average current flowing through a coil placed inside the magnetic field created by the fixed magnet.

  Symbol: 

- ***Ferromagnetic Devices:*** The operating principle of a ferromagnetic device is based on the action of a field created by a circuit carrying a current on one or more soft iron parts, some of which are mobile. There are two types of devices: attraction or repulsion type.

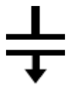
  For the attraction type device, the principle used is the magnetic action produced by a fixed coil carrying a current on a soft iron vane (moving element) mounted on two pivots. This moving assembly is equipped with a needle and a damping device. For the repulsion type device, the magnetic field created by the fixed coil acts on two vanes placed in this field which undergo magnetization in the same direction. The repulsion of the two vanes deflects the needle. A ferromagnetic device is very simple to construct, robust, usable in direct and alternating current. The graduation of its scale is non-linear.

  Symbol: 

- ***Electrodynamic Devices:*** An electrodynamic device is formed mainly by a fixed circuit (generally two half-coils) creating a magnetic field inside which a moving frame of low inertia mounted on two pivots and driving a needle moves. Electrodynamic devices are non-polarized. They are usable in direct and alternating current. They are generally used for the manufacture of wattmeters.

  Symbol: 

- ***Electrostatic Devices:*** This type of device is characterized by a force exerted by the fixed armature of a capacitor on its moving armature. This type of device is always used as a voltmeter. When a voltage is applied between the two plates of this device, one becomes positively charged and the other negatively charged, which produces an attractive force that tends to rotate the moving plate which is integral with a needle. They are usable in direct and alternating current and have a non-linear scale.

  Symbol: 

- ***Thermal Devices:*** The operating principle of this type of device is based on the expansion of a conductive wire that heats up when an electric current of intensity passes through it. This effect is the direct consequence of the power dissipated by the Joule effect in the expansion wire. The expansion wire used is generally made of bronze or a platinum and silver alloy. Thermal devices are non-polarized, usable in direct current and alternating current.

  Symbol: 

3.2.2 Moving-Coil Galvanometers

They consists of a rectangular frame abed on which N turns of a fine copper wire are wound. The ends of this winding are soldered to two very fine wires f and f') which serve as:

- Suspension wires for the frame;
- Connection wires;
- Torsion wires, exerting a restoring torque on the frame, which tends to bring the frame back to the equilibrium position it occupies when no current flows through it.

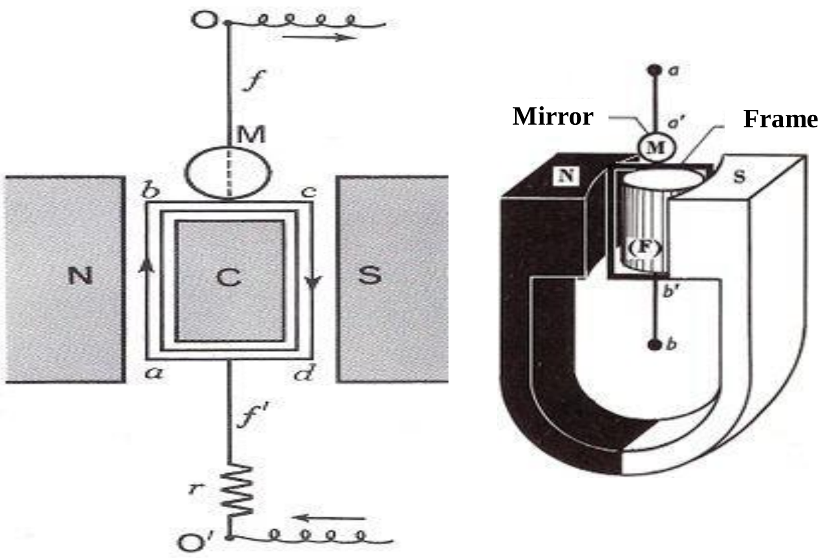


Figure 3-1 : Magneto-Electric Galvanometer.

The frame can move in the air gap of a *U*-shaped magnet equipped with two hollowed pole pieces so as to leave a cylindrical cavity between them; in this cavity, a soft iron cylinder is placed which channels the lines of induction. Between the pole pieces and the cylinder, the lines of induction are directed along the radii of the two coaxial cylinders: *the field is radial.*

Therefore, whatever the position of the frame in the air gap:

- The magnetic induction vectors $\vec{B}$ at all points of the vertical sides ab and ed of the frame are always perpendicular to these sides and contained in the plane of the coils;
- The intensity B of these magnetic induction vectors is practically constant.

Finally, a mirror M, attached to the frame, allows small rotations of the moving part to be observed by the optical method known as the *Poggendorff method*.

A. Frame Equilibrium Equation: If no current flows through the frame, it takes an equilibrium position corresponding to zero torsion of the suspension wires. When a current flows through the frame, it rotates and stops in an equilibrium position defined by the equality of two moments:

- C_M: The moment of the electromagnetic torque (motor torque);
- C_r: The moment of the torsion torque (resisting torque = restoring torque).

▪ ***Electromagnetic torque***

- The horizontal sides ad and bc, of length l', are subjected to a substantially horizontal induction, therefore to Laplace forces $\vec{F}_1$ and $\vec{F}_2$ which are almost vertical which have no rotational effect on the frame.
- The vertical sides ba and cd, of length l, being perpendicular to the lines of induction are subjected to Laplace forces $\vec{F}_3$ and $\vec{F}_4$ normal to the plane of the coils, in opposite directions and having a common intensity:

$$F = F_3 = F_4 = I \times B \times l \tag{3.1}$$

where I is the current intensity.

These two forces therefore form a couple of moment:

$$C_M = F \times l' = I \times B \times l \times l' = I \times B \times S \tag{3.2}$$

where $S = l \times l'$: denotes the cross-section of the moving frame.

Since the frame comprises N identical turns, the moment of the resulting electromagnetic torque is written:

$$C_M = N \times I \times B \times S \triangleq N \times I \times \phi_0 \tag{3.3}$$

where ϕ_0 is the flux that crosses the moving frame, C_M in meter-newton; I in amperes; S in square meters and B in tesla.

- ***Torsion Torque***

The electromagnetic torque rotates the frame in the direction of the forces $\vec{F}_3$ and $\vec{F}_4$. During this rotation, the suspension wires are twisted by an angle α and exert *a restoring torque* (torsion torque) proportional to this angle. The moment of this opposite torque is:

$$C_r = K \times a \tag{3.4}$$

where C_r is in meter-newton; K is torsion constant in meter-newton per radian and α in radian.

- ***Equilibrium***

At equilibrium, we have

$$C_M = C_r \tag{3.5}$$

From this equality, we deduce the expression for the deflection α of the coil, which corresponds to the current of intensity I:

$$\alpha = \frac{N\phi_0}{K} I \tag{3.6}$$

where K represents the specific resisting torque of the spring, it is expressed in J/rad.

We notice that the coil rotates by an angle proportional to the current intensity.

- ***Angle Reading***

The rotation of the coil is measured by the method of Poggendorff. The coil is surmounted by a small mirror M which gives, from a thin illuminated slit, a light beam, the spot, on a translucent graduated ruler. If the coil rotates by an angle α, the reflected ray rotates by an angle 2α and the spot then moves from *F'* to *F''*. We therefore have, if the angle α is small, tan $\alpha \approx 2\alpha$ radians and, consequently:

$$\alpha \approx \frac{d}{2D} \tag{3.7}$$

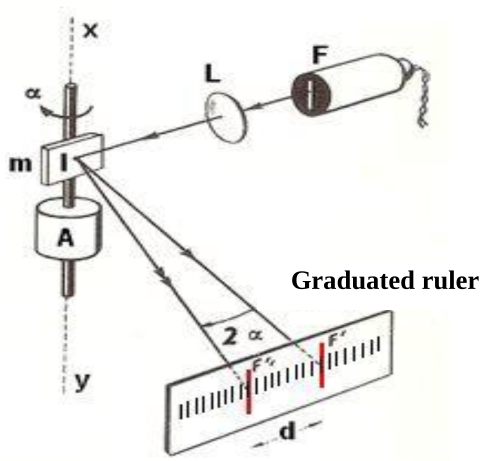


Figure 3-2: The Reading Device of a Magneto-Electric Galvanometer.

Now, equation (3.6) becomes:

$$I = \frac{K}{N\phi_0}\frac{d}{2D} \tag{3.8}$$

B. Sensitivity

The galvanometer is all the more sensitive as, for the same intensity I, the angle of rotation a is larger:

$$S_i = \frac{\alpha}{I} \tag{3.9}$$

where S_i is the current sensitivity of the moving element. From the relationship giving α, we deduce that

$$S_i = \frac{N\phi_0}{K} \tag{3.10}$$

To improve sensitivity, it will be necessary to increase B, N, S and decrease K.

3.2.3 Multi-range meters

A multi-range meter consists of the moving element, resistors, and a battery. For resistance measurement, it will therefore have several resistors depending on the desired ranges. For a moving element of the moving coil to measure a voltage, it is sufficient to mount a multiplier resistor with it (additional resistor). When measuring a current, a shunt must be mounted across its terminals (resistor placed in parallel). In both cases, the resistors serve to limit the current passing through the moving element to the maximum it can withstand.

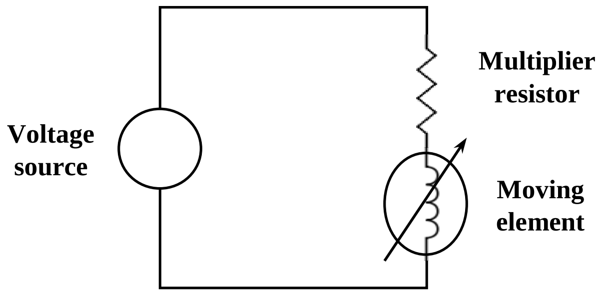


Figure 3-3: Passive multimeter for voltage measurement

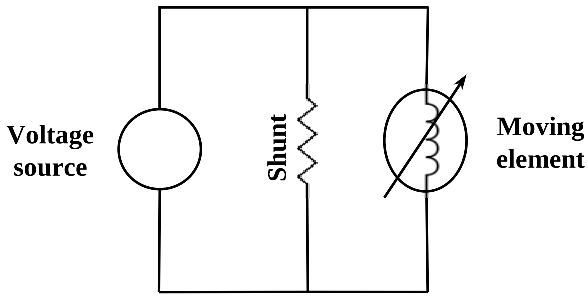


Figure 3-4: Passive multimeter for current measurement

A. Multi-range Ammeter

The current range of a d.c. ammeter may be further extended by a number of shunts, selected by a range switch. Such meter is called a multi-range ammeter. Figure 3.5 shows a schematic diagram of Multi-range ammeter. The circuit has four shunts R_{Sh_1}, R_{Sh_2}, R_{Sh_3} and R_{Sh_4}, which can be put in parallel with the meter movement to give four different current ranges I_1, I_2, I_3 and I_4.

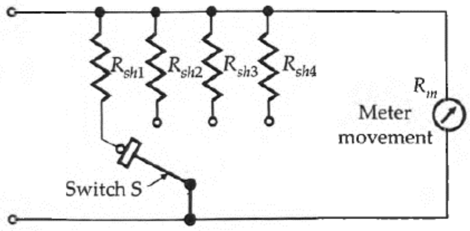


Figure 3-5: Multi-range ammeter.

Let m_1, m_2, m_3 and m_4, be the shunt multiplying powers for currents I_1, I_2, I_3 and I_4, respectively, so

$$R_{Sh_1} = \frac{R_m}{m_1 - 1}$$

$$R_{Sh_2} = \frac{R_m}{m_2 - 1}$$

$$R_{Sh_3} = \frac{R_m}{m_3 - 1}$$

$$R_{Sh_3} = \frac{R_m}{m_3 - 1}$$

Low range ammeters use a multi-position make-before-break switch provided on the case of the instrument. This type of switch is essential in order that the meter movement is not damaged when changing from the current range one to another. If we provide an ordinary switch, the meter remains without a shunt and as such is unprotected and therefore can be damaged when the range is changed. Multi-range ammeters are used for ranges from 1mA to 50A. When using a multi-range ammeter, first use the highest current range, then decrease the current range until good upscale reading is obtained.

Remark: *For an ammeter, we note that the higher the range, the lower the internal resistance value. An ammeter must therefore have the lowest possible resistance.*

B. Multi-range Voltmeter

In a multirange voltmeter, different full scale voltage ranges may be obtained by the use of individual multiplier resistors or by a potential divider arrangement.

We can obtain different voltage ranges by connecting different values of multiplier resistors in series with the meter. The number of these resistors is equal to the number of ranges required. Figure 3.6

shows multiplier resistors R_{S_1}, R_{S_2}, R_{S_3} and R_{S_4}, which can be connected in series with the meter by a range selector switch. Consider that the ranges desired are V_1, V_2, V_3 and V_4, then the corresponding multiplier resistances can be obtained as follows:

$$R_{S_1} = (m_1 - 1)R_m$$
$$R_{S_2} = (m_2 - 1)R_m$$
$$R_{S_3} = (m_3 - 1)R_m$$
$$R_{S_3} = (m_4 - 1)R_m$$

where $m_1 = V_1/v$, $m_2 = V_2/v$, $m_3 = V_3/v$, and $m_4 = V_4/v$.

3.3 Digital Measuring Devices

Digital measuring devices are increasingly used thanks to their reliability, precision, and ease of reading. It is necessary for users of digital devices to understand the language adopted by the manufacturers of these devices.

The principle is to convert an analog quantity into a numerical value that can be displayed. For this, it is necessary to use electronic circuits, the main ones being: analog-to-digital converters, the oscillator, the counter, and the display.

3.3.1 Analog-to-Digital Converters (ADC)

There are two types of analog-to-digital converters:

- *Single-slope ADCs.*
- *Dual-slope ADCs*.

The former are used (reserved) for applications where high precision is not required. The operating principle of ADCs is based on the charging and discharging of a capacitor C with a constant current.

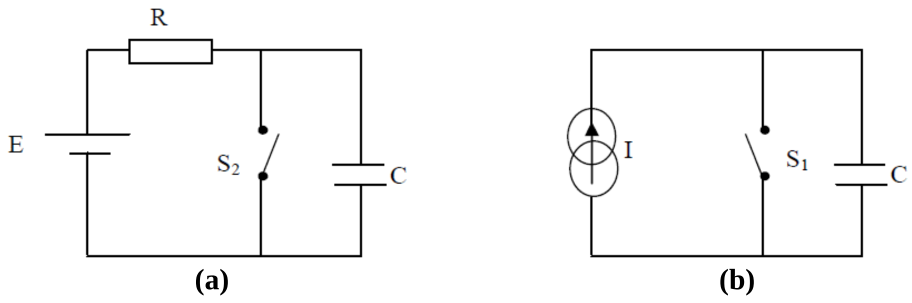


Figure 3-6: Capacitor charging circuits.

In the first circuit shown on Figure 3.6 (a), the capacitor charges exponentially according to the following equation.

$$V_C = E\left(1 - e^{-t/\tau}\right) \tag{3.13}$$

where $\tau = RC$.

In the second circuit shown on Figure 3.6 (b), the capacitor has a linear charging (See equation below).

$$V_C = \frac{i \times t}{C} \tag{3.14}$$

where $i = dq/dt$.

We note that the charging time is directly proportional to the current or the voltage. This means that these circuits will be used to transform a current or a voltage into a time. This important property will help us understand the operation of the ADC.

A. Single-Slope Analog-to-Digital Converter

The ADC converter includes a comparator to which the capacitor charging voltage is applied to the inverting input, and the unknown voltage to be measured is applied to the non-inverting input, an AND logic gate to which the oscillator signal is applied, the frequency of which can be changed according to the chosen range, and the comparator output signal, and finally a decade counter and displays.

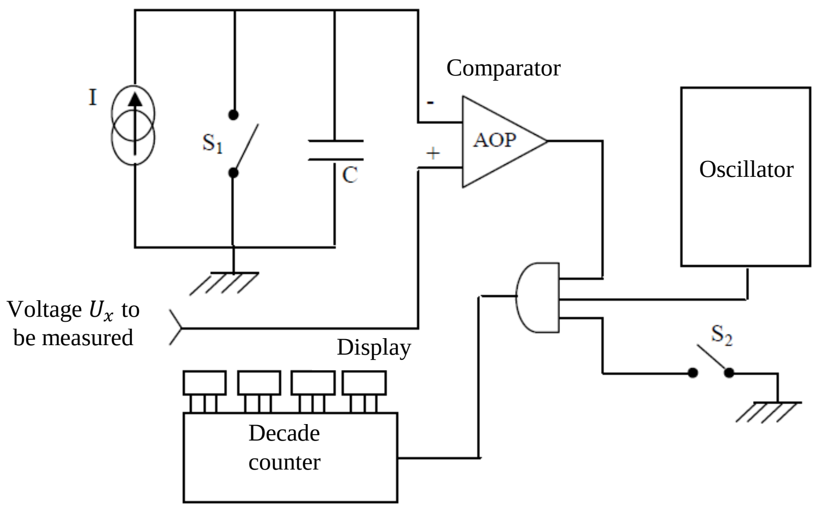


Figure 3-7: Single-Slope Analog-to-Digital Converter Circuit.

The capacitor charging voltage is compared to a voltage U_x to be measured. The AOP compares the voltage V^- at the inverting terminal and the voltage V^+ at the non-inverting terminal, therefore:

- If $V^+ > V^- \Rightarrow Vs > 0$Volt $\Rightarrow$ 1 Logic.
- If $V^+ < V^- \Rightarrow Vs < 0$Volt $\Rightarrow$ 0 Logic.

At the beginning, we assume that the comparator is at the high level (1 logic), both switches S_1 and S_2 are open. In this case, the capacitor is fully charged when $U_C > U_x \Rightarrow V^- > V^+$, a logical "0" is introduced, therefore the "AND" gate closes $\Rightarrow$ the measurement is finished.

The role of switch S_2 is to activate the timer including the oscillator, the counter, the displays and the AND gate.

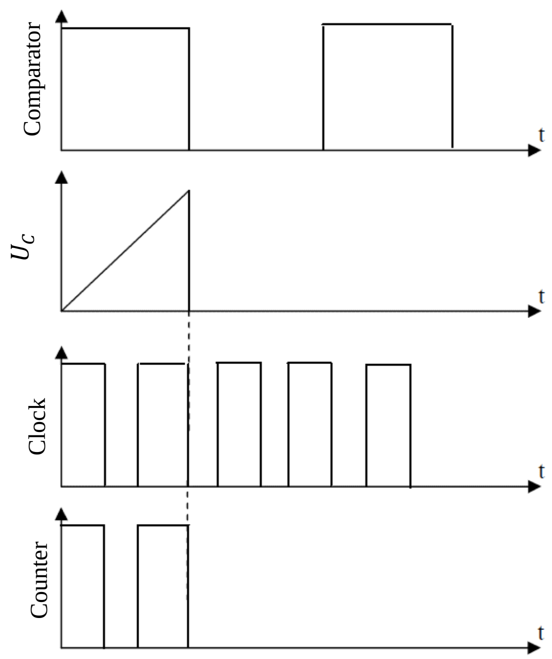


Figure 3-8: Oscillogram of the Single-Slope Analog-to-Digital Converter.

B. Dual-slope analog-to-digital circuit

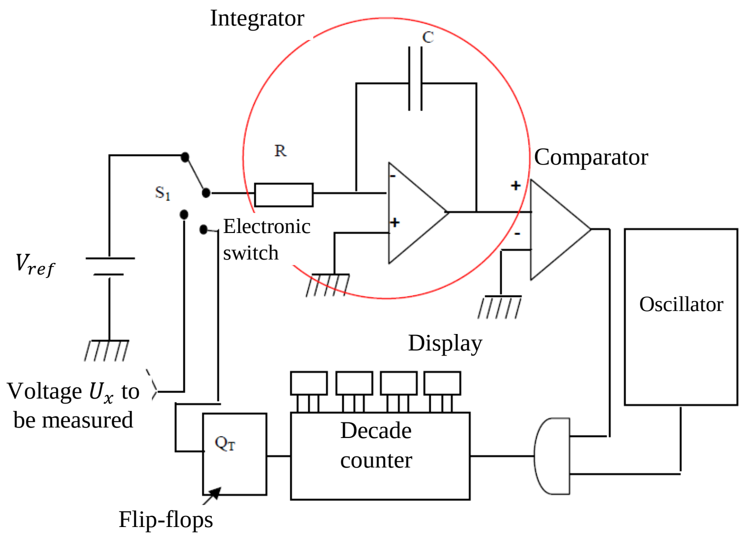


Figure 3-9: Dual-Slope Analog-to-Digital Converter Circuit.

The role of the integrator is to charge the capacitor linearly. Compared to the single-slope ADC, we observe that it has an electronic switch and a flip-flop to control this switch. We also note the presence of an integrator which serves to charge the capacitor linearly. The input of the integrator is alternately subjected to two continuous voltages, one a fixed reference, and the other, the unknown voltage to be measured, is of negative sign.

The charging (or discharging) is done through the resistor R with a constant current of value:

$$I = \frac{V}{R} \quad \text{or} \quad V = \begin{cases} -U_x \\ V_{ref} \end{cases}$$

At the beginning of the measurement, the voltage $-U_x$ will charge the capacitor for a very specific time T_1. The voltage at the comparator input will exceed $0V$, and consequently, the comparator output will switch to the high state. This comparator signal and the signal delivered by the clock are applied to an AND gate, whose output drives a counter. During the time T_1, n pulses would have passed, the $(n + 1)th$ pulse, which is an overflow signal, is received that will switch the position of the switch to V_{ref} and start the counting.

At this moment, the second slope of the measurement is initiated, meaning that the current produced by V_{ref} will discharge the capacitor during the time T_2 until the voltage across C is zero due to the change of the comparator state to low, consequently the counting is stopped, and the result is displayed.

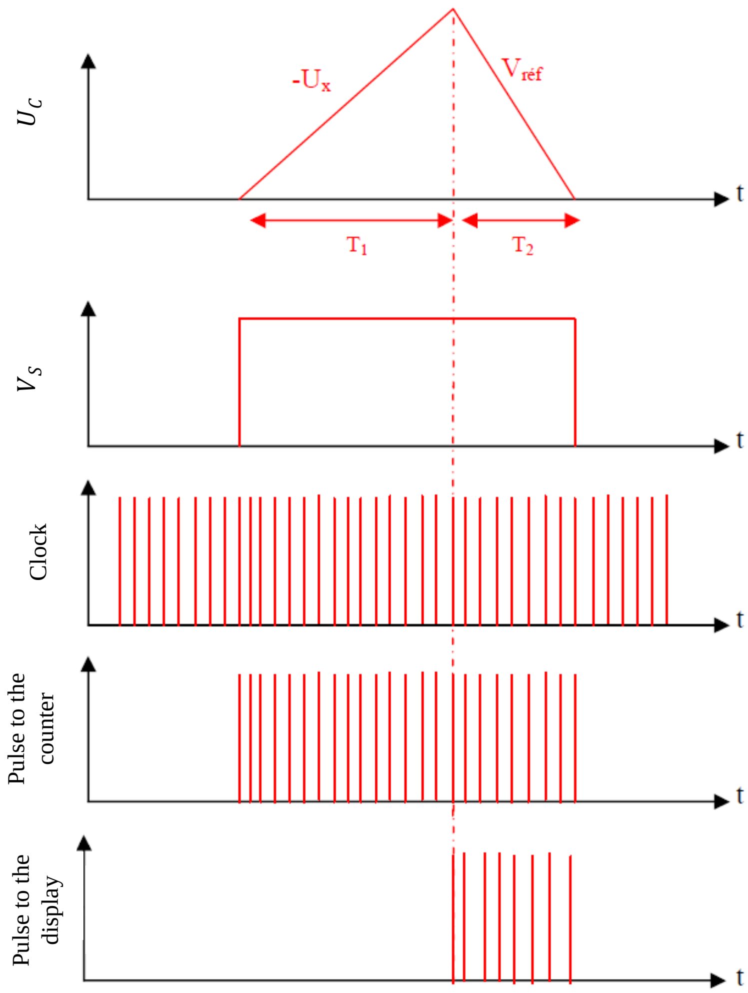


Figure 3-10: Oscillogram of the Dual-Slope Analog-to-Digital Converter.

$$U_x = V_{ref}\frac{T_2}{T_1} \tag{3.35}$$

We note that the voltage to be measured depends only on V_{ref} and of the report T_2/T_1. According to the relationship found, we can say that the calibration can be done by changing the reference voltage because when $V_{ref} \nearrow\Longrightarrow \frac{U_x}{V_{ref}} \searrow, \frac{T_2}{T_1} \searrow \Longrightarrow T_2$ decreases (T_1 remains unchanged).

It depends on the oscillation frequency and the counter, therefore the number of pulses will decrease but this will be remedied by setting the appropriate (necessary) scale.

- ***Accuracy:*** According to the given relationship, we note that the voltage to be measured depends on the voltage V_{ref} and the ratio T_2/T_1, therefore, neither the oscillator frequency nor the value of the capacitance, nor the offset voltage, and we can conclude that by this method, the main cause of error in the single-slope ADC has been eliminated.

3.3.2 Examples of digital measuring devices

A. The multimeter

The Digital Multimeter, shown in Figure 3.11, is built around a digital voltmeter and includes at least a current-to-voltage converter allowing it to function as an ammeter and a constant current generator to function as an ohmmeter.

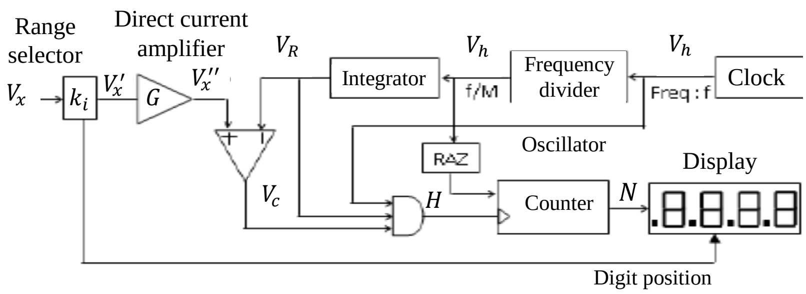


Figure 3-11: Digital Multimeter.

The choice of measurement type (of the instrument), the range or measurement scale is generally made using a rotary switch; push buttons can control additional functions. The most recent multimeters, often the easiest to use, automatically choose the correct mode and range. Other measurement functions may be available depending on the sophistication of the multimeter:

- Continuity test with or without beeper;
- Amplification to measure very low voltages and high resistances;
- Measurement of the capacitance of a capacitor or a capacitive circuit;
- Measurement of the inductance of a coil or an inductive circuit (self);

- Temperature measurement, with the help of an external probe;
- Discrete semiconductor tests: diodes, transistor gain (*h*fe);
- Measurement of electrical signal frequency;
- Measurement of voltage peaks (high and low) (peak *h*old).

A. The oscilloscope

Unlike analog models, the signal to be displayed is first digitized by an analog-to-digital converter. The ability of the device to display a high-frequency signal without distortion depends on the quality of this interface. The main characteristics to consider are:

- The resolution of the analog-to-digital converter.
- The sampling frequency in MS/s (mega samples per second) or GS/s (giga samples per second).
- The memory depth.

The device is coupled to memories allowing these signals to be stored and to a number of analysis and processing units that allow many characteristics of the observed signal to be obtained:

- Measurement of signal characteristics: peak value, RMS value, period, frequency, etc.
- Fast Fourier transform which allows the signal spectrum to be obtained.
- Sophisticated filters which, applied to this digital signal, allow the visibility of details to be increased.

The result is increasingly displayed on a liquid crystal screen, which makes these devices easy to move and much less energy-consuming. Digital oscilloscopes have now completely supplanted their analog predecessors, thanks to their greater portability, greater ease of use and, above all, their reduced cost.

3.4 Application Exercises

Exercise 3.1

Consider the magneto-electric motor element.

1. Find the expression of the force applied to the sides of the moving frame.
2. Give the expression of the motor torque exerted on the frame.

Exercise 3.2

An AC voltmeter is equipped with the magneto-electric motor element with a full-wave rectifier and is graduated in RMS values with the following values from 0 to 100 with a step of 10.

0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100.

- Re-graduate this voltmeter in average value.

Exercise 3.3

A milliammeter with a range of 0.1mA and an internal resistance of 1kΩ is available.

1. Calculate the voltage across its terminals when the needle deflection is maximum.
2. With the previous milliammeter, we can create an ammeter with a caliber of 0.1A and 1A, what is the necessary shunt resistance in this case?

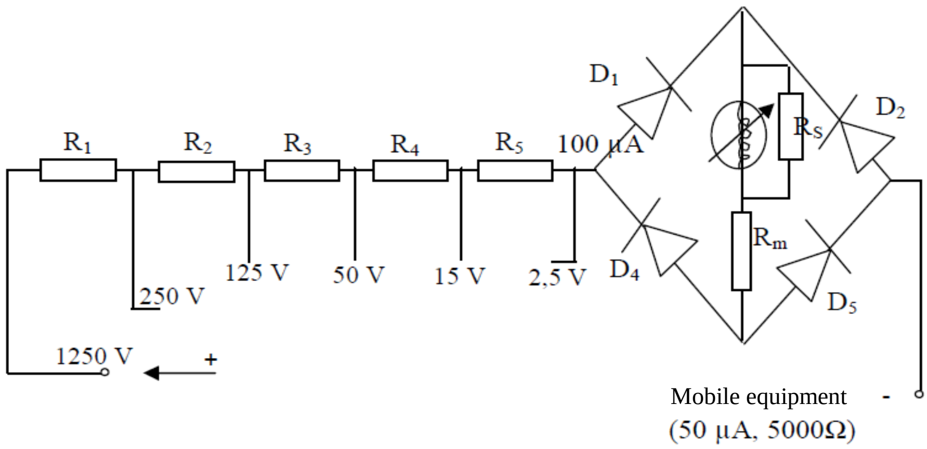
Exercise 3.4

Using the milliammeter of the previous exercise, we want to create a voltmeter with the following calibers: 15V, 30V, 150V. Calculate:

1. The values of the additional resistances r1, r2, r3 corresponding to the different calibers.
2. The internal resistances of a voltmeter in each case.

Exercise 3.5

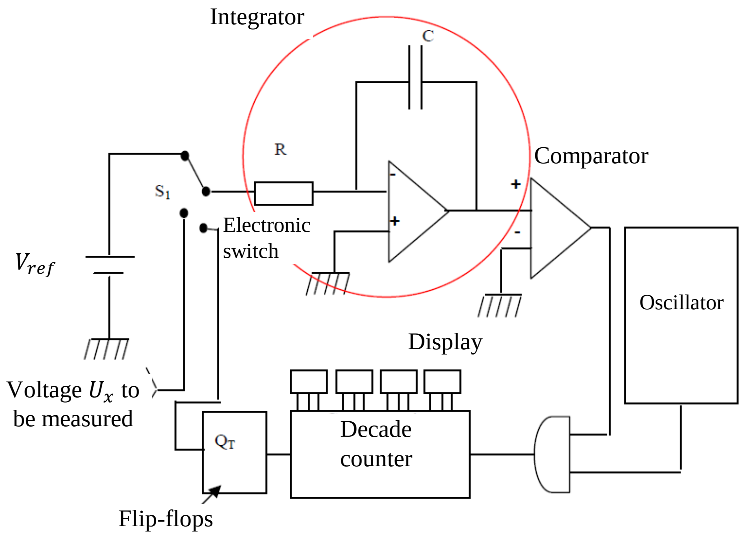
Consider the AC circuit equipped with a full-wave rectifier bridge and a magneto-electric motor element whose calibration is given in average value.



1. Calculate r_m and r_s of the moving coil as well as the resistances of the different calibers.
2. Assuming that the diodes are ideal and must have a full-scale current of $100\mu A$. Recalibrate this voltmeter in peak values.

Exercise 3.6

Consider the analog-to-digital converter given by the following figure:



1. Explain the operating principle of this converter.
2. If $f_0 = 10\ kHz$, the charging time $T_1 = 2s$, the $V_{ref} = 10V$, we want to measure the unknown voltage U_x which causes a discharge time of the capacitor C, $T_2 = 1s$. Calculate the unknown voltage U_x and the number of counted pulses corresponding to the measurement.
3. Assuming that the frequency of the oscillator drifts due to the rapid rise in ambient temperature $f_0 = 12.5\ kHz$. Calculate the new voltage to be measured U_x.

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