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Performance evaluation of CA-, GO- and SO-CFAR processors in a non-centered Lévy-distributed clutter

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Abstract:

Constant false alarm rate (CFAR) processors are critical for radar reliable target detection in radar systems. Traditional CFAR designs often assume Gaussian clutter, which may not reflect real-world conditions. Lévy distributions, with heavy tails and a location parameter (δ), provide a more accurate model for non-Gaussian and non-centered clutter in complex environments. This paper presents a comprehensive performance analysis of three widely used CFAR processors-cell-averaging (CA), greatest-of (GO), and smallest-of (SO) in homogeneous Lévy-distributed clutter with an arbitrary δ. We derive integral-form expressions for the probability of false alarm (PFA) for each processor, explicitly incorporating δ. Furthermore, we provide analytical formulations for the probability density function (PDF) of key statistics involving Lévy random variables, such as sums, minima, and maxima. Monte Carlo simulations validate the theoretical results, showing that the PFA performance improves with increasing δ, highlighting the critical impact of clutter location on CFAR detector performance. These findings offer valuable insights for designing robust CFAR detectors in non-Gaussian, non-centered clutter environments.

Keywords: Cell-averaging constant false alarm rate; Constant false alarm rate processors; Greatest-of constant false alarm rate; Lévy-distributed clutter; Probability of false alarm; Smallest-of constant false alarm rate

1. Introduction

In radar target detection, clutter refers to unwanted reflections from objects that can interfere with the detection of real targets. Under these conditions, Constant False Alarm Rate (CFAR) techniques are applied to maintain a fixed false alarm rate in homogeneous clutter environments. CFAR processors adaptively set detection thresholds by estimating the clutter level from reference cells surrounding the cell under investigation (CUI). Several CFAR variants have been developed to address different clutter scenarios [1–4]. Traditional Cell-Averaging CFAR (CA-CFAR) detectors estimate clutter level using the mean of surrounding cells [5], but their performance degrades in non-homogeneous clutter. To address these limitations, the Greatest Of CFAR (GO-CFAR), initially proposed by Hansen and Sawyers [6], employs the maximum of the sums from the leading and lagging reference windows for clutter level estimation, improving robustness in clutter transition regions. Weiss in [7] further analyzed GO-CFAR's performance in multi-target scenarios. In parallel, the SO-CFAR (Smallest Of CFAR) method, developed by Trunk [8] in the context of target resolution, mitigated the effects of target masking. Rickard and Dillard in [9] provided an early, comprehensive analysis of adaptive detectors, including SO-CFAR, laying the theoretical and practical foundations for their use in diverse environments. These foundational works paved the way for advanced applications of GO-CFAR and SO-CFAR in modern radar systems [10–14].
Modern high-resolution radars face challenges in statistical clutter modeling due to increased complexity. The radar's research community has proposed various non-Gaussian models to better represent high-resolution radar clutter, including Positive Alpha-Stable, Weibull, Log-Normal, Pareto, and K-distributions [15–19]. However, adopting these non-Gaussian models in CFAR schemes can make deriving closed-form expressions for key performance metrics, such as probability of detection and false alarm rate, challenging or even impossible. This complicates the evaluation and optimization of CFAR detectors in complex clutter environments.

3.2 Key results

Lemma 1 Let $\{X_i\}_{i=1}^{2N}$ be a sequence of i.i.d. RVs, where $X_i \sim \text{Lévy}(\delta,\gamma)$. The PDF and CDF of RV $Z=\sum_{i=1}^{2N} X_i$ are given, respectively, as follows:

$$f_z(z)=\sqrt{\frac{2N^2\gamma}{\pi}}\frac{e^{-\frac{2N^2\gamma}{z-2N\delta}}}{(z-2N\delta)^{\frac{3}{2}}} \quad z\geq 2N\delta,\gamma>0 \tag{5}$$

$$F_z(z)=\text{erfc}\left(\sqrt{\frac{2N^2\gamma}{z-2N\delta}}\right) \quad z\geq 2N\delta,\gamma>0 \tag{6}$$

Proof: The characteristic function $\varphi_Z(w)$ of Z is defined as:

$$\varphi_Z(w)=E[e^{jwZ}]=E[e^{jw\sum_{i=1}^{2N}X_i}]=(E[e^{jwX_1}])^{2N} \tag{7}$$

$$\begin{aligned}\varphi_Z(w)=&(E[e^{jwX_1}])^{2N}=\left(\int_0^{+\infty} e^{jwX_1} f_{X_1}(x_1)dx_1\right)^{2N}\\ =&\left(\sqrt{\frac{\gamma}{2\pi}}\underbrace{\int_0^{+\infty}\frac{e^{jwx}-\frac{\gamma}{2(x-\delta)}}{(x-\delta)^{\frac{3}{2}}}}_{=I}\right)^{2N}\end{aligned} \tag{8}$$

After evaluating the integral I, equation (8) can be inverted using the Fourier transform to yield the result in (5). The CDF in (6) can be easily derived by integrating the PDF obtained in (5). This completes the proof.

Lemma 2 Let $\{X_i\}_{i=1}^{2N}$ be a sequence of i.i.d. RVs, where $X_i \sim \text{Lévy}(\delta,\gamma)$. Define $Z_1=\sum_{i=1}^{N} X_i$ and $Z_2=\sum_{i=N+1}^{2N} X_i$. The PDF of RV $Z=\max(Z_1,Z_2)$ is given by:

$$f_z(z)=2\sqrt{\frac{N^2\gamma}{\pi}}\frac{e^{-\frac{N^2\gamma}{z-N\delta}}}{(z-N\delta)^{\frac{3}{2}}}\text{erfc}\left(\sqrt{\frac{N^2\gamma}{(z-N\delta)}}\right), z\geq N\delta,\gamma>0 \tag{9}$$

Proof: The PDF of $Z=\max(Z_1,Z_2)$ can be expressed as:

$$f_z(z)=2f_{Z_i}(z)F_{Z_i}(z),\ i\in\{1,2\} \tag{10}$$

From Lemma 1, the PDF and CDF of Z_i (for $i\in\{1,2\}$) are given, respectively, as follows:

$$f_{Z_i}(z)=\sqrt{\frac{N^2\gamma}{\pi}}\frac{e^{-\frac{N^2\gamma}{z-N\delta}}}{(z-N\delta)^{\frac{3}{2}}}, z\geq N\delta,\gamma>0 \tag{11}$$

$$F_{Z_i}(z)=\text{erfc}\left(\sqrt{\frac{N^2\gamma}{z-N\delta}}\right), z\geq N\delta,\gamma>0 \tag{12}$$

Substituting (11) and (12) into (14) yields the result in (9).

Lemma 3 Let $\{X_i\}_{i=1}^{2N}$ be a sequence of i.i.d. RVs, where $X_i \sim \text{Lévy}(\delta,\gamma)$. Define $Z_1=\sum_{i=1}^{N} X_i$ and $Z_2=\sum_{i=N+1}^{2N} X_i$. The PDF of RV $Z=\min(Z_1,Z_2)$ is given by:

$$f_z(z)=2\sqrt{\frac{N^2\gamma}{\pi}}\frac{e^{-\frac{N^2\gamma}{z-N\delta}}}{(z-N\delta)^{\frac{3}{2}}}\text{erfc}\left(\sqrt{\frac{N^2\gamma}{z-N\delta}}\right), z\geq N\delta,\gamma>0. \tag{13}$$

Proof: The PDF of $Z=\min(Z_1,Z_2)$ can be expressed as:

$$f_z(z)=2f_{Z_i}(z)(1-F_{Z_i}(z)),\ i\in\{1,2\} \tag{14}$$

Substituting (11) and (12) into (**??**) yields the result in (13).

4. Application to the P_{FA}'s evaluation of CFAR processors

Given the decision rule in (2) and the clutter level estimates defined in (1), the P_{FA} can be expressed as:

$$P_{FA}(T)=E_Z[\Pr(X_0>TZ\mid H_0)] \tag{15}$$

Since $X_0\sim\text{Lévy}(\delta,\gamma)$, $\Pr(X_0>TZ\mid H_0)$ can be written as:

$$\Pr(X_0>TZ\mid H_0)=1-F_{X_0}(TZ)=\text{erf}\left(\sqrt{\frac{\gamma}{2(TZ-\delta)}}\right). \tag{16}$$

Substituting (15) into (16) gives:

$$P_{FA}(T)=\int_{-\infty}^{+\infty}\text{erf}\left(\sqrt{\frac{\gamma}{2(TZ-\delta)}}\right)f_z(z)dz. \tag{17}$$

Now, substituting (5), (9), and (13) into (17) yields the P_{FA} expressions for the CA, GO, and SO-CFAR processors, given in (18), (19), and (20), respectively. Here, $T_{CA}=T/2N$, and $T_{GO}=T_{SO}=T/N$. The P_{FA} for the CA-CFAR processor is:

$$\begin{aligned}P_{FA,CA}(T_{CA})=\sqrt{\frac{2N^2\gamma}{\pi}}\int_{2N\delta}^{+\infty}\text{erf}\left(\sqrt{\frac{\gamma}{2(T_{CA}z-\delta)}}\right)\\ \frac{e^{-\frac{2N^2\gamma}{z-2N\delta}}}{(z-2N\delta)^{\frac{3}{2}}}dz.\end{aligned} \tag{18}$$

$$\begin{aligned}P_{FA,GO}(T_{GO})=2\sqrt{\frac{N^2\gamma}{\pi}}\int_{N\delta}^{+\infty}\text{erf}\left(\sqrt{\frac{\gamma}{2(T_{GO}z-\delta)}}\right)\\ \text{erfc}\left(\sqrt{\frac{N^2\gamma}{z-N\delta}}\right)\frac{e^{-\frac{N^2\gamma}{z-N\delta}}}{(z-N\delta)^{\frac{3}{2}}}dz.\end{aligned} \tag{19}$$

$$\begin{aligned}P_{FA,SO}(T_{SO})=2\sqrt{\frac{N^2\gamma}{\pi}}\int_{N\delta}^{+\infty}\text{erf}\left(\sqrt{\frac{\gamma}{2(T_{SO}z-\delta)}}\right)\\ \text{erf}\left(\sqrt{\frac{N^2\gamma}{z-N\delta}}\right)\frac{e^{-\frac{N^2\gamma}{z-N\delta}}}{(z-N\delta)^{\frac{3}{2}}}dz.\end{aligned} \tag{20}$$

Remark 3 Although the integrals presented in (18), (19), (20), defy straightforward analytical evaluation since they cannot be expressed in a simple closed form, they are ready to be evaluated numerically. Thereby, we can unravel the underlying mathematical intricacies and extract meaningful results.

5. Numerical results

This section presents the numerical evaluation of the CA-, GO-, and SO-CFAR processors' performance under homogeneous non-centered Lévy-distributed clutter. Unless otherwise specified, for all simulations, we set $\gamma=1/\sqrt{2}$, and the size of the reference window is fixed at $N=8$. The results are derived using the analytical expressions in equations (18), (19), and (20) and validated through Monte Carlo simulations.

Figs. 1 (a), 1 (b), and 1 (c) illustrate the PFA performance as a function of the scaling factor T for the CA-, GO-,