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SOCA-CFAR Processor in A Homogeneous Erlang-Distributed Clutter: Exact Closed-Form Expression for the P_{fa}

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ABSTRACT

In this work, we have derived a closed-form expression for the probability of false alarm (P_{FA}) of the smallest-of cell-averaging constant false alarm rate (SOCA-CFAR) processor which operates in a homogeneous Erlang-distributed clutter environment. This expression indicates that the P_{FA} is independent of the rate parameter λ of the Erlang clutter. Moreover, as an intermediate step, we have derived accurate formulations for the probability density function (PDF) for the sum and the minimum sum of independent and identically distributed (i.i.d) Erlang random variates. The numerical simulations stipulate an enhancement in the SOCA-CFAR's P_{FA} as the shape parameter k of the Erlang clutter increases. The accuracy of the analytical outcomes presented in this work is corroborated through Monte Carlo simulations.

1. INTRODUCTION

In radar target detection, undesirable reflections from objects are termed clutter. In homogeneous clutter backgrounds, constant false alarm rate (CFAR) techniques are fundamental for maintaining a predetermined, consistent false alarm rate (FAR) [1-5].

A CFAR processor determines the optimal detection threshold by analyzing the clutter level. This analysis involves processing a set of strategically positioned reference cells surrounding the cell under investigation (CUI), also known as the test cell. By carefully examining these reference cells, the CFAR processor generates a robust estimate of the clutter magnitude, effectively differentiating between the signal of interest and unwanted clutter. This process is crucial for mitigating false alarms while preserving the sensitivity needed for effective target detection and tracking in radar systems.

The cell averaging (CA)-CFAR processor is a well-known CFAR variant [6, 7]. It methodically estimates the clutter's mean level by averaging the values from all cells surrounding the CUI. This comprehensive averaging approach provides a robust estimate of the average clutter magnitude, enabling effective signal-clutter discrimination. Numerous CFAR variants have been developed for specific operational contexts. These include the greatest-of (GO) CA-CFAR processor, introduced by some researchers [8, 9] to manage false alarm rates in clutter transition zones; the smallest-of (SO) CA-CFAR processor, described by Trunk (1978) and elaborated in later studies [10-13]; the ordered statistics OS-CFAR processor, proposed by Rohling (1983) and refined in subsequent research [14, 15]; and the Variability Index CFAR (VI-CFAR) introduced by Guida et al. [16]. Each of these variants offers distinct advantages and trade-offs. The OS-CFAR processor offers robustness against interfering targets through rank-based threshold estimation; however, it incurs computational overhead due to sorting operations. VI-CFAR adapts to clutter heterogeneity by exploiting statistical variability measures, achieving superior performance in mixed clutter environments at the cost of increased computational complexity (~10^4 operations per decision). Despite these advances, systematic comparative analysis of these processors under Erlang-distributed clutter—particularly relevant for maritime and urban radar scenarios—is lacking in existing literature. Most studies focus on Weibull or K-distributed clutter models, leaving a gap in understanding optimal CFAR selection for Erlang environments. This limitation motivates

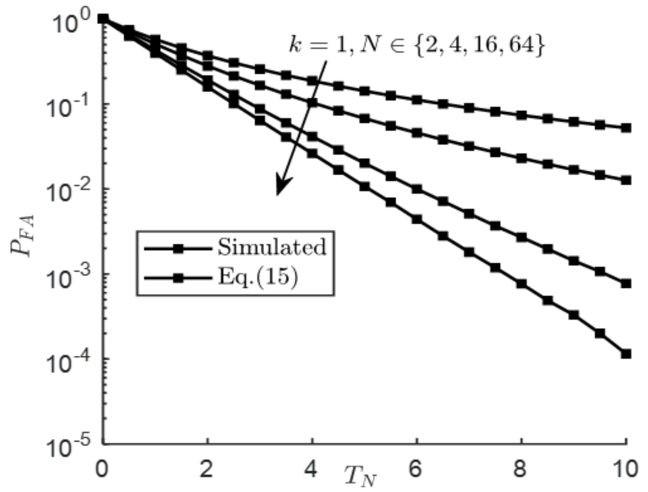

Figure 1. P_{FA} of SOCA-CFAR vs. T_N: (a) $k = 1$. The curves from top to bottom corresponds to N=2, 4, 16, and 64, respectively

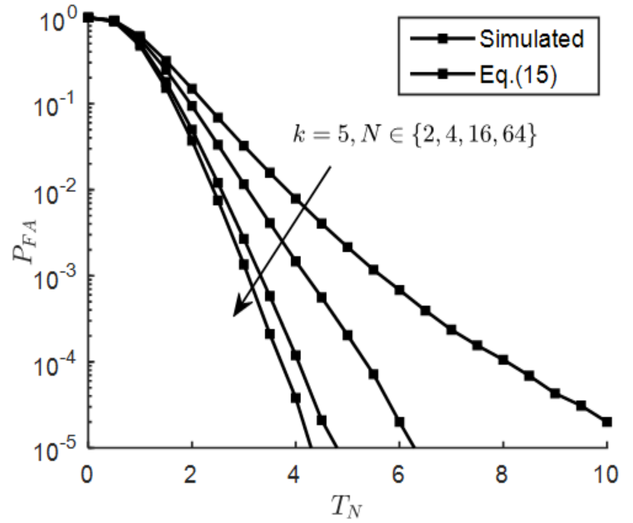

Figure 2. P_{FA} of SOCA-CFAR vs. T_N: (b) $k = 5$. The curves form top to bottom corresponds to $N = 2, 4, 16$, and 64, respectively

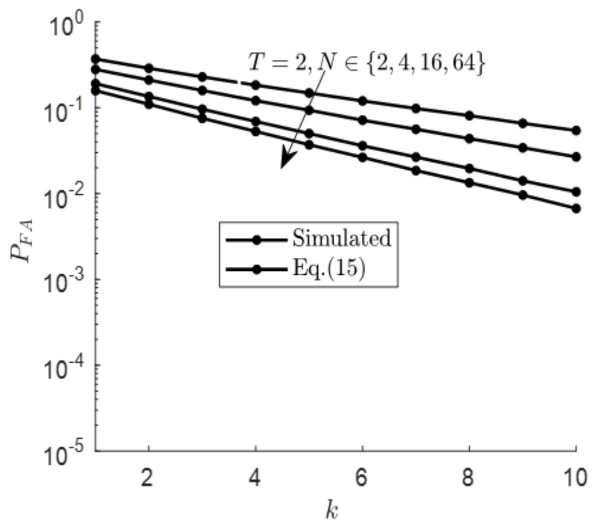

Figure 3. P_{FA} of SOCA-CFAR vs. k: (a) T_N=2. The curves from top to bottom corresponds to N=2, 4, 16, and 64, respectively

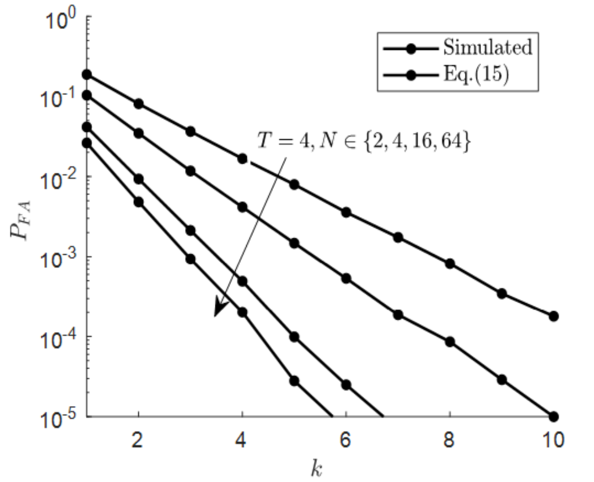

Figure 4. P_{FA} of SOCA-CFAR vs. k: (b) T_N=4. The curves from top to bottom corresponds to N=2, 4, 16, and 64, respectively

5.2 Computational analysis

For practical implementation, the infinite series can be truncated when successive terms fall below a prescribed tolerance ϵ. Typically, 20-50 terms suffice for engineering accuracy ($\epsilon = 10^{-6}$).

Remark: The expression reduces to known results for special cases:
- $N = 1$, $k = 1$: Classical CA-CFAR with $P_{\mathrm{FA}} = (1 + T_1)^{-1}$-Large N limit: Approaches Gaussian approximation.

Proposition 1 (Convergence Properties) *All infinite series in Theorem 1 converge absolutely for $T_N > 0$, $N \geq 1$, and $k > 0$.*

Proof. The convergence follows from:

1. For the series in p: $\left|\frac{(1)_p (Nk+k)_p}{(Nk+1)_p p! 2^p}\right| \sim \mathcal{O}(p^{-1})$ as $p \to \infty$.
2. For the series in q: geometric convergence with ratio $|T_N/(T_N + 1)| < 1$.
3. For the double series: dominated convergence via $\sum_{m,n} |a_{m,n}| \leq \sum_{m,n} \frac{C^{m+n}}{m!n!} < \infty$ for some constant C.

6. CONCLUSION

In this letter, we have assessed the performance of the SO-CFAR processor in the context of a clutter environment characterized by an Erlang distribution. To do so, we have derived the necessary statistics for the clutter's mean level in a form of the sum and the minimum sum of independent and identically distributed (i.i.d) Erlang random variates' PDFs. Based on that we have then derived a closed-form expression for the SO-CFAR processor' P_{FA}. Furthermore, aside from its ease of evaluation, this closed-form expression can also be instrumental in facilitating a realistic design and analysis of the SO-CFAR processor. Additionally, an examination of various parameters and their influence on the performance of the processor has been conducted. The numerical findings indicate that an augmentation in either the clutter's shape parameter λ, or the size of the reference window N, results in an enhancement of the processor's performance. Future work