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Closed-Form Solution for Energy Efficiency Maximization in Uplink IRS-Assisted Multi-User NOMA Network

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ABSTRACT

Given the growing concerns about energy consumption and its negative impact on the ecosystem, energy efficiency (EE) has become one of the most important key performance indicators in current and future wireless communication technologies. In this paper, we address the EE maximization problem in an uplink intelligent reflective surface (IRS)-assisted multi-user non-orthogonal multiple access (NOMA) network. This problem is formulated as a trade-off between the spectral efficiency (SE) and total power consumption, and it appears to be non-convex. To avoid the complexity associated with the traditional iteration-based Dinkelbach method, we opt for an alternative closed-form solution for the users' transmit power based on partial derivative analysis and Lambert function. Simulation results with a realistic power consumption models confirm the accuracy of our theoretical findings.

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1. INTRODUCTION

The evolution of information and communication technologies (ICTs), especially the mass deployment of 5G, is driving significant changes in the wireless communications landscape. Indeed, the rapid increase in the number, and diversity of connected devices [1][1], data traffic and supported applications is raising concerns about its impact on the environment, notably in terms of greenhouse gas (GHG) emissions. The trend shows that the ICTs sector contributes between 1.5% and 4% of worldwide GHG emissions[2, 3]. Furthermore, forecasts indicate that by 2040, GHG emissions generated by ICTs could account for over 16% of worldwide emissions, with a significant proportion coming from end-user devices [3]. In light of these facts, it is essential to investigate innovative approaches to develop green technological solutions aimed at mitigating environmental impacts and promoting sustainability. This involves integrating more eco-friendly energy sources and improving energy efficiency (EE).

One such innovative approaches aimed at improving EE leverages the integration of intelligent reflective surfaces (IRSs) [4]. IRS is a flat structure composed of a large number of passive, cost-effective and energy-efficient reflecting elements, which deviate from traditional reflection laws to adjust the phase and direction of electromagnetic waves, enabling precise and controlled reflection or refraction. In addition, the IRS has the ability to adapt dynamically and instantaneously to the variations in the wireless channel by modifying

[1]The 5G subscriptions keep growing fast, from 1.6 billion in 2023 to 2.27 billion in 2024, with a forecast of 6.35 billion by 2030.

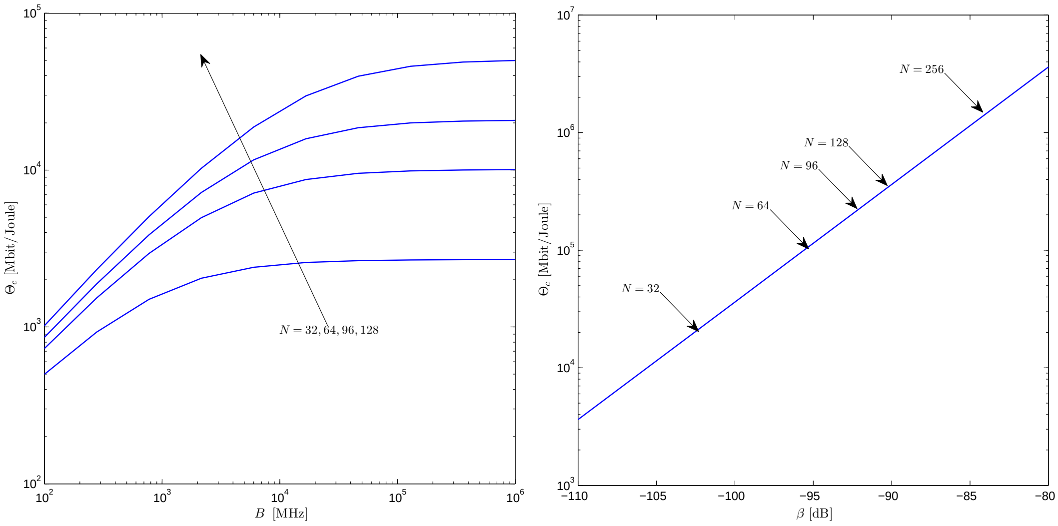
the phase shifts and/or amplitudes of its elements. As a result, the incident signals can be redirected according to the system's overall performance targets. Owing to their passive nature, IRSs require less energy consumption compared with conventional relays [5]. Thanks to this EE, IRSs are a crucial innovation for future green wireless communication systems [6].

Another promising technology for next-generation wireless networks is non-orthogonal multiple access (NOMA), meeting the requirements for massive connectivity, high spectral efficiency (SE), and improved EE [7–9]. Unlike traditional orthogonal multiple access (OMA) schemes, NOMA allows multiple users equipments (UEs) to share the same degrees of freedom, so that UEs are served on the same radio resources by superimposing their signals at the transmitter-end and decoding these signals at the receiver-end using successive interference cancellation (SIC) [10, 11]. In addition, the use of NOMA leads to a significant improvement in the system EE and SE compared to OMA. Nevertheless, this improvement is only achieved when the users' channel strengths exhibit significant differences, which is not necessarily the case in real communication situations. In light of the above, IRS can be effectively combined with NOMA to improve the overall system's performance, where IRS can artificially provide extra paths to boost a specific user's channel gain, thereby improving the SIC performance[12]. This synergy is often referred to as IRS-aided NOMA[13], IRS-assisted NOMA[14] or merely IRS-NOMA [15].

Several studies have been conducted on optimizing the EE in IRS-assisted NOMA downlink communication [14, 16–20]. However, those dealing with uplink scenario are relatively limited. G. Li et *al*. [21] proposed an iterative approach to solve the multivariate non-convex optimization problem to maximize system EE in IRS-empowered multiple input multiple-output (MIMO)-NOMA uplink systems. The approach involves the joint optimization of passive beamforming (BF) at the IRS, active BF at the base station (BS), and power allocation (PA). A comparison with baseline schemes shows a substantial gain in terms of system EE. In [22], the authors examined the joint optimization of users' transmit power, passive BF at the IRS and active BF at the BS, with a view to maximizing overall system EE in IRS-assisted multi-antenna NOMA uplink systems, while meeting users' minimum throughput constraints. To solve the highly challenging non-convex optimization problem, they developed an iterative solution using a block coordinate descent (BCD) approach. In [23], the authors sought to maximize EE for IRS-assisted millimeter-wave (mmWave) NOMA networks while considering constraints such as each device's minimum rate, maximum power and constant modulus (CM) of BF vectors. For this purpose, they presented two iterative algorithms: the first, based on majorization-minimization (MM), the concave-convex procedure (CCCP) and the BCD method, to obtain closed-form solution for the joint BF design problem; the second, based on successive convex approximation (SCA), BCD and Dinkelbach's methods, to achieve suboptimal closed-form PA for each iteration, given the designed passive and analog BFs. Recently, T. Qiao et *al* [24]. investigated EE maximization in a NOMA uplink system assisted by an active IRS, where reflecting elements amplified incident signals. Unlike conventional passive IRS architectures, their approach employed a cascaded channel-based user scheduling algorithm and jointly optimized transmit PA and active IRS's BF using Dinkelbach's method combined with SDR. Their results demonstrated substantial EE gains over passive IRS benchmarks, albeit at the expense of increased computational complexity and additional power consumption from active amplification.

Apart from the aforementioned studies, the EE maximization in the context of uplink IRS-assisted NOMA for mobile edge computing (MEC) and coordinated multipoint (CoMP) systems was investigated in [25] and [26], respectively. The authors in [25] aimed to minimize total energy consumption by jointly optimizing transmission time, offloading sharing and IRS phase shifts as well as transmit powers. They derived an optimal solution for transmit power in closed form and used an alternative optimization (AO) technique to solve the formulated problem. The authors in [26] investigated a transmit power minimization problem, which was further solved by leveraging an alternating method to iteratively optimize both transmit power and phase shifts.

In the previous studies, maximizing EE proved challenging due to the fractional form of its objective function. To overcome this, numerical fractional programming methods, such as the conventional Dinkelbach' method, were adopted[27]. This method aims to simplify the EE maximization problem by transforming it into a series of iterative non-linear problems. However, it frequently requires multiple unpredictable iterations, thereby increasing the overall computational complexity, especially for high-dimensional problems[28]. Furthermore, in the absence of a closed-form solution, the proposed methods are inadequate since they do not provide a sufficiently detailed analytical insight into the problem. To the best of our knowledge, none of these studies has provided a closed-form expression for the EE maximization and the EE-SE tradeoff. Such



(a) (b)

Figure 2. The system EE limit vs. (a) the bandwidth B, (b) the sum of UEs channel gains β.

Note that the result in (13) can be obtained when both $P_{\max}$ and B progress jointly to infinity, insofar as $P_{\max}$ has a slower convergence speed than B.

To evaluate the EE limit in (13), we plot it as a function of the bandwidth B in Fig. 2a, and the sum of all UE effective channel gains β in Fig. 2b. For clarity, we adopt the simulation parameters in Table 1. Fig. 2a illustrates the rate at which the UE reaches its limit when $B \to \infty$, for different values of N. For $N = 32$, this limit is reached at $B = 20\text{Ghz}$. However, it requires $5\times$ more bandwidth to reach the EE limit when doubling the size of the IRS unit, i.e., for $N = 64$. Furthermore, Fig. 2b shows that the EE limit increases monotonically with β, which is consistent with the finding in **Corollary** 1. The solid arrows in this figure indicate the values of N corresponding to fade levels. For example, when $N = 64$, we get $\beta = -95\text{dB}$.

Let us now focus on deriving the unique solution for determining the optimal transmit power $P^{\star}_{max}$ that maximizes Θ_c. To do so, Let us first consider the following **Lemma**.

Lemma 1. *Let a, b, c and $d \in \mathbb{R}^{+}$, and the function $f(x) = d\frac{\log_2(1+acx)}{ax+b}$. The global unique optimal solution that maximizes $f(x)$ is given by*

$$x^{*} = \frac{1}{ac}\left(\mathrm{e}^{\mathcal{W}\left(\frac{cb-1}{\mathrm{e}}\right)+1} - 1\right), \tag{14}$$

and its corresponding maxima is

$$f(x^{*}) = \frac{d}{\ln(2)}\frac{c}{\mathrm{e}^{\mathcal{W}\left(\frac{cb-1}{\mathrm{e}}\right)+1}}, \tag{15}$$

where $\mathcal{W}(\cdot)$ is the Lambert function defined as the inverse function of $x \mapsto xe^{x}$ [29], and e is the base of the natural logarithm function.

Proof. Please refer to Appendix A1.. □

Theorem 1. *The optimal transmit power that maximizes the system EE in* (11) *is given by*

$$P^{\star}_{max} = \frac{1}{\varepsilon\gamma}\left(\mathrm{e}^{\mathcal{W}\left(\frac{\gamma P_{sta}-1}{\mathrm{e}}\right)+1} - 1\right), \tag{16}$$

and the maximum system EE is

$$\Theta^{\star}_{c} = \frac{B}{\ln(2)}\frac{\gamma}{\mathrm{e}^{\mathcal{W}\left(\frac{\gamma P_{sta}-1}{\mathrm{e}}\right)+1}}. \tag{17}$$

Proof. This is a straightforward result of **Lemma** 1, for $a = \varepsilon$, $b = P_{sta}$, $c = \gamma$, and $d = B$. □

Remark 3. *Note that the Lambert function is increasing on* $\left[-\frac{1}{\mathrm{e}}, +\infty\right[$*, which holds true since,* $\frac{\gamma P_{sta}-1}{\mathrm{e}} \geq -\frac{1}{\mathrm{e}} \Rightarrow \gamma P_{sta} \geq 0$.

To deepen the analysis, we can relate the maximum system EE value (i.e., $\Theta_c^\star$) to its corresponding system SE (i.e., $\mathcal{R}_c^\star$). Plugging $P_{max}^\star$ in (10) yields

$$\mathcal{R}_c^\star = \frac{1}{\ln(2)} \left(\mathcal{W}\left(\frac{\gamma P_{sta} - 1}{\mathrm{e}} \right) + 1 \right). \tag{18}$$

After some mathematical manipulations in (17) , we get the following equality

$$\log_2 (\Theta_c^\star) + \mathcal{R}_c^\star = \log_2 \left(\frac{\gamma B}{\ln(2)} \right). \tag{19}$$

In (19), $\log_2 (\Theta_c^\star)$ and $\mathcal{R}_c^\star$ follow a linear dependence. We can therefore achieve an exponential gain in the system EE at the cost of a linear loss in the system SE.

Corollary 2. *As* γ *and/or* P_{sta} *get larger (i.e.,* $\gamma P_{sta} \gg 1$*), then*

$$\begin{aligned} P_{max}^\star &\approx \frac{1}{\varepsilon\gamma} \left(\frac{\gamma P_{sta} - 1}{\mathrm{e}} \right) \\ &\approx \frac{P_{sta}}{\varepsilon \mathrm{e}}, \end{aligned} \tag{20}$$

and

$$\begin{aligned} \Theta_c^\star &\approx \frac{B}{\ln(2)} \frac{\gamma}{\frac{\gamma P_{sta}-1}{\mathrm{e}}} \\ &\approx \frac{B\mathrm{e}}{\ln(2) P_{sta}}, \end{aligned} \tag{21}$$

and

$$\mathcal{R}_c^\star \approx \log_2 \left(\frac{\gamma P_{sta}}{\mathrm{e}} \right). \tag{22}$$

Proof. $\mathrm{e}^{\mathcal{W}(x)+1} \approx x$ for large values of x. □

Remark 4. *From* (20)*, and* (21)*, we can see that* $\Theta_c^\star$ *and* $P_{max}^\star$ *are linearly decreasing and increasing functions of* P_{sta}*, respectively. Both are also independent of the user channel conditions.*

3.2. Variable circuit power

In the previous subsection, P_{sta} is considered independent of system's bandwidth. This implies that circuit consumption remains constant with respect to the system's sampling rate, which in turn is directly related to B. In practice, however, the PD of the hardware components is strongly affected by the system's sampling frequency, as well as by the power consumed in backhaul transmission, digital signal processing, encoding/decoding, and so on. It is therefore much more practical to consider a model adapted to a VCP consumption scenario. Let us now present the adapted circuit power consumption model

$$\mathcal{P}_{\text{total}} = \varepsilon P_{\max} + \eta B \mathcal{R}_v + \underbrace{\vartheta B + K P_{\text{UE}} + N P_{\text{e}}}_{=P_{\text{sta}}}, \tag{23}$$

where $\eta, \vartheta \geq 0$ denote the hardware characteristic constants related to load-dependent power consumption and digital signal processing, respectively. Applying (6) in this scenario, we obtain the system EE as follows

$$\Theta_v = B \frac{\log_2 (1 + \varepsilon P_{max} \gamma)}{\varepsilon P_{\max} + \eta B \log_2 (1 + \varepsilon P_{max} \gamma) + P_{\text{sta}}}. \tag{24}$$

Similarly, to determine the optimal transmit power $P_{max}^\star$ that maximizes Θ_v, we rely on the following **Lemma**.