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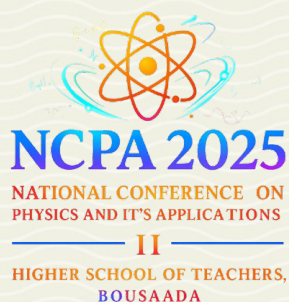
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The Organizing Committee of the Second National Conference on Physics and its Applications
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Mohammed Lakhdar NEBBAR

in recognition of his active participation in the conference with an **Poster** presentation
entitled: “ Critical depth calculation in triangular shaped-channel depends on flow vis-
cosity and surface roughness “

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THE 2nd NATIONAL CONFERENCE ON PHYSICS AND IT'S APPLICATIONS



NCPA 25
BOUSAADA
ON 20th NOVEMBER 2025

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Abstract submission deadline: October 28, 2025

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Influence of Cavity Size, Location, and Morphology on Partial Discharge in HV Cable Insulation.



Dr. Nadjib Baadji

University of M'sila

spintronics: Principles and perspectives.



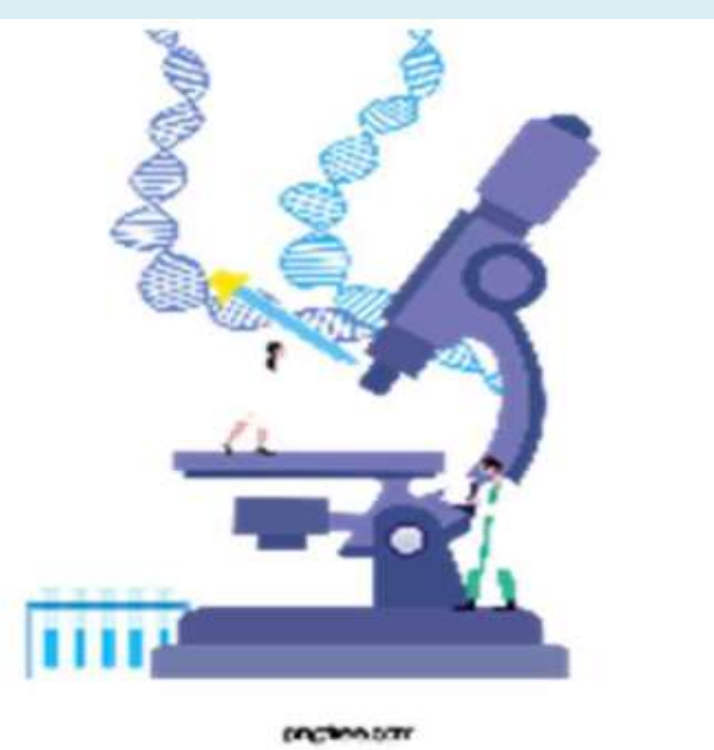
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Critical depth calculation in triangular shaped-channel depends on flow viscosity and surface roughness

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Abstract

In this paper, a procedure for the dimensioning of the triangular-shaped channel at critical flow, which is important in the practice of the hydraulic engineer. The proposed approach, which explores the potential offered by heuristic methods, for solving complex optimization problems, is based on the use of an analytical method, is presented and applied for the calculation of the critical depth y_c , which is governed by a cubic equation with no second order term. The resolution of this equation is essentially based on Cardan's theorem. This method takes into account, in particular, the effect of the absolute roughness ϵ , and the effect of the kinematic viscosity ν , through the Reynolds number Re . And also the effect of the channel bed slope, through the friction factor f . These parameters are easily measurable in practice. In this research, we relied mainly on the application of two universally accepted relations of Darcy-Weisbach and Colebrook formula, in a state of critical flow. Explicit relations are deduced, that governs the critical depth y_c , by a particular examination of two cases, one is a turbulent flow over a smooth surface, and the other is a flow over a rough surface. Using a detailed practical example, we show how to calculate the critical depth in a triangular channel from practical data. This calculation depends on the absolute roughness value of the channel walls and also the viscosity of the flowing fluid, which can be easily determined in practice.

Key words: Absolute roughness, Critical depth, Friction factor, Kinematic Viscosity, Smooth surface, Triangular-shaped channel, Turbulent flow.

1. Introduction

The study of critical flow is an important concept in the design and operation of open channel systems, as it affects the stability of the channel and the efficiency of flow control structures. Understanding the critical flow condition is essential for the safe and effective management of water resources [1].

Bakhmeteff [2] and Chow [3] stated that uniform flow in an open channel, is characterized by a specific depth known as the critical depth. This depth is defined as the depth of flow at which the specific energy of the flow is at a minimum, and it represents the transition point between subcritical and supercritical flow. Bakhmeteff [2] observed that a small variation in energy (specific energy) can result in a significant change in the depth of flow in an open channel. At critical depth, the Froude number is equal to unity, and the velocity of the flow is equal to the critical velocity.

The references showed what many authors have tried to express in order to calculate the critical depth, deduced from the criticality criterion, excluding the effect of turbulence, roughness and also the effect of viscosity. Swamee [4] developed explicit critical depth equation for some shapes of irrigation canals (as rectangular and triangular). Our study takes into consideration the following factors: absolute roughness ϵ , which characterizes the state of the channel wall, the kinematic viscosity ν , of the flowing liquid, the side slope m , and the critical slope S_c expressed in terms of the friction factor f_c . This can provide a more accurate estimate of critical depth y_c , through the functional relationship $\varphi(y_c, f_c, m, \epsilon, \nu, g) = 0$, which is well-defined in theory. This function is obtained by a theoretical analytical approach, based on the use of universal relations, such as the Darcy [5] and Colebrook [6] relations, in a critical flow regime. The mathematical solution of this functional relation, relies on the use of Cardan's theorem, due to its simplicity and accuracy [7].

Our choice fell on a triangular-shaped channel, because of its many uses in hydraulics engineer such as: providing efficient flow of fluids due to their shape, which reduces energy losses as compared to other shapes like rectangular or circular channels [8]. The triangular-shaped channel using in dissipate the energy by hydraulic jump [9]. In hydraulics engineering, the triangular channels can be easily designed to fit sloping terrain, and can be cost-effective to build and maintain, as they require less material and can be constructed quickly. Triangular channels can also be aesthetically pleasing in certain landscapes or urban environments. And for flow measurement in open channels, the triangular-shaped broad-crested weir is more useful for irrigation channels [10].

In this study, we will examine the equation that governs the critical depth, in two extreme cases of turbulent flow in a triangular-shaped channel, one is the turbulent flow over smooth surfaces, and the other is the turbulent flow over rough surfaces, the purpose of this is to know the effect of the roughness of the walls on the value of the critical depth. Also, To validate our findings, we will present practical examples that demonstrate the applicability of our results in real-world scenarios Karzan and Bahzad [11], as well as in field applications.

2. Methodology

2.1. Geometric characteristics of the channel

The channel considered in this study have a triangular-shaped, is symmetrical and discovered, having side slope m horizontal to 1 vertical, and its walls have an absolute roughness ϵ . At the free surface, the pressure is equal to the atmospheric pressure. The main elements that can be defined from the wet section are displayed in Figure 1 below, where the subscript "c" denotes the condition of the critical state of the flow.

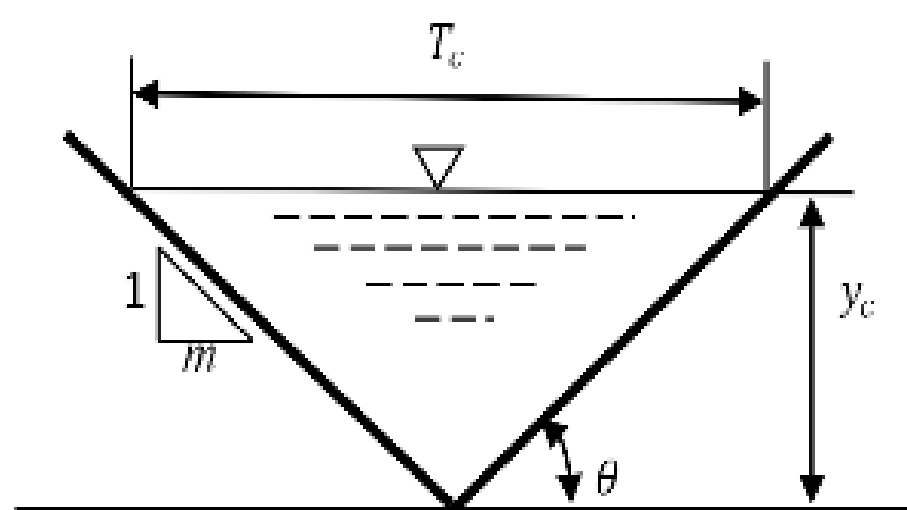


Figure 1: Schematic representation of the critical flow in triangular-shaped channel

The channel is characterized by the following critical geometric and hydraulic parameters: T_c : critical top width at the water surface, y_c : critical flow depth, A_c : is the critical wetted area, its expression is written :

$$A_c = m y_c^2 \quad (1)$$

P_c : the critical wet perimeter; his expression is written :

$$P_c = 2y_c\sqrt{1+m^2} \quad (2)$$

2.2. Sizing of the channel

The dimensioning of the studied channel essentially consists in determining the critical depth presented by the parameter y_c , for the given values of the parameters: f_c , m , ϵ and the viscosity kinematic ν .

2.2.1. Basic hydraulic parameters and formulas

In our calculation approach, we follow the following procedure:

- 1) The relationship that governs critical flow is used;
- 2) We introduce the *Darcy-Weisbach* equation;
- 3) *Colebrook* universal relationship is applied.

The criticality condition states that, regardless of the shape of all known channels, it is written:

$$Q_c^2/g = A_c^3/T_c \quad (3)$$

Q_c : the critical flow discharge (m³/s), A_c : is the area of the critical wet section (m²), and g is the acceleration of gravity (9.81 m/s²)

Also we have the critical hydraulic diameter $D_{h,c}$; is written under relations (1) and (2):

$$D_{h,c} = 4R_{h,c} = \frac{2m}{\sqrt{1+m^2}} y_c \quad (4)$$

The Reynolds number, characterizing the critical flow, is:

$$Re_{c} = \frac{\sqrt{2g}}{\nu} \frac{m}{\sqrt{1+m^2}} y_c^{3/2} \quad (5)$$

The relationship that will serve our study, is that of *Darcy-Weisbach* relationship developed for flows in circular pipes in loads. And its generalization for the open channels knows a great success [12], by replacing D by the hydraulic diameter D_h . Thus, Eq can be written in the following form:

$$S_c = \frac{f_c}{D_{h,c}} \frac{V_c^2}{2g} \quad (6)$$

The relation (6) expresses the proportionality between the critical bed slope S_c of the channel, the critical mean velocity V_c and the critical hydraulic diameter $D_{h,c}$; they will affect the friction factor f_c in turbulent flow.

According to Eq.(6) and Eq.(4), the friction factor f_c is expressed as $f_c = 8S_c \frac{m}{\sqrt{1+m^2}}$ (7)

On the other hand, the fact that the formula "Colebrook (1939)" was conducted in pipe, it appears to be of great interest when applied to open channels "(Falvey, 1987; Sinniger and Hager, 1989)". This formula expresses the friction factor f . And in critical parameters, it is written:

$$\frac{1}{\sqrt{f_c}} = -2 \log \left(\frac{\epsilon}{3.71 D_{h,c}} + \frac{2.51}{Re_{c} \sqrt{f_c}} \right) \quad (8)$$

The relationship (8) covers the entire Moody chart domain "Moody (1944)", and remains applicable throughout the range of relative roughness $0 < (\epsilon/D_{h,c}) < 5 \times 10^{-2}$, and for any critical Reynolds number value, $Re_{c,c} > 2300$. It is indeed exclusively intended for rough turbulent flow, transition and virtually smooth in critical regime. Given the implicit form of the relation (8), it is important to point out that the critical Reynolds number cannot be evaluated; since the critical depth y_c is the unknown parameter of the problem. This is the parameter we are trying to determine in the present study.

2.2.2. Explicit calculation of linear dimension

To calculate the critical linear dimension, represented by the depth y_c , we replace the expression of critical hydraulic diameter and the critical Reynolds number given respectively by relations (4) and (5) in relation (8) Colebrook's formula, it comes that:

$$\frac{1}{\sqrt{f_c}} = -2 \log \left(\left(\frac{\epsilon \sqrt{1+m^2}}{7.42m} \right) \frac{1}{y_c} + \left(\frac{2.51 \nu \sqrt{1+m^2}}{m \sqrt{2g f_c}} \right) \frac{1}{y_c^{3/2}} \right) \quad (9)$$

The relationship (9) is a decimal logarithm function. Then, Eq.(9) can be rewritten as:

$$\left(\frac{\epsilon \sqrt{1+m^2}}{7.42m} \right) \frac{10^{2\sqrt{f_c}}}{y_c} + \left(\frac{2.51 \nu \sqrt{1+m^2}}{m \sqrt{2g f_c}} \right) \frac{10^{2\sqrt{f_c}}}{y_c^{3/2}} - 1 = 0 \quad (10)$$

Multiplying the Eq.(10) by $y_c^{(3/2)}$, and we obtain the following equation:

$$y_c^{3/2} - 10^{2\sqrt{f_c}} \left(\frac{\epsilon \sqrt{1+m^2}}{7.42m} \right) y_c^{1/2} - 10^{2\sqrt{f_c}} \left(\frac{2.51 \nu \sqrt{1+m^2}}{m \sqrt{2g f_c}} \right) = 0 \quad (11)$$

Let's define the following parameters:

$$p = 10^{2\sqrt{f_c}} \left(\frac{\epsilon \sqrt{1+m^2}}{7.42m} \right) \quad (12)$$

$$q = 10^{2\sqrt{f_c}} \left(\frac{2.51 \nu \sqrt{1+m^2}}{m \sqrt{2g f_c}} \right) \quad (13)$$

Eq.(11) can be then expressed in the following form: $y_c^{3/2} - p y_c^{1/2} - q = 0$ (14)

The parameters p and q are well known, provided ϵ , m , ν and f_c are given. By adopting the following change of variables: $X = y_c^{(1/2)}$; Eq. (14) is reduced to: $X^3 - pX - q = 0$ (15)

Eq. (15) is a cubic equation without a second-order term. It can be easily solved for, and the resolution of this is based essentially on Cardan's theorem. Taking into account the change of variables made in the first step, the value of the critical depth y_c is therefore:

$$y_c = X^2 = \left(\frac{4p}{3} \cos(\beta/3) \right)^2 \quad (16)$$

Where the angle β is such that:

$$\cos \beta = \frac{3q}{2p} \sqrt{\frac{3}{p}} \quad (17)$$

3. Results and discussion

3.1. The design in extreme cases of the flow's regime

The relationship (3) applies to turbulent flow, provided the Reynolds number Re_c is greater than 2300 [13]. The turbulent flow is not only characterized by the value of the Reynolds number Re_c , but also by that of the relative roughness ϵ/D_h .

3.1.1. The case of a hydraulically smooth flow state

In order to simplify both the study and the calculation, consider the case of a smooth triangular-shaped channel ($\epsilon \rightarrow 0$), Eq.(11) becomes then:

$$y_c^{3/2} - 10^{2\sqrt{f_c}} \left(\frac{2.51 \nu \sqrt{1+m^2}}{m \sqrt{2g f_c}} \right) = 0 \quad (18)$$

Introducing Eq.(13) into Eq.(18) and simplifying results into:

$$y_c = \left(\frac{2.51 \times 10^{2\sqrt{f_c}}}{\sqrt{f_c}} \right)^{2/3} \left(\frac{\sqrt{1+m^2}}{m} \right)^{2/3} \left(\frac{\nu^2}{2g} \right)^{1/3} \quad (19)$$

From the relationship (19), we draw the following conclusions:

- 1) The quantity: $(\nu^2/g)^{1/3}$, have a dimension of length;
- 2) The critical depth y_c , there is inversely proportional to the inclination of the channel walls, i.e., when the side slope m , increases implies the significant decrease in critical depth y_c . This is because the exponent of the quantity $(\sqrt{1+m^2}/m)$, is strictly less than 1;
- 3) This equation shows that values of (y_c) are increasing with the decrease of (f_c) values for all channel side slopes.
- 4) Substituting expression y_c into equation (5) and simplify the results into:

$$Re_{c} = 2.51 \left(\frac{10^{2\sqrt{f_c}}}{\sqrt{f_c}} \right) \quad (20)$$

The relationship (20) shows that the critical Reynolds number Re_{c} , increases only with decreasing coefficient of friction f_c ;

5) The relationship (19), is explicit for the critical depth sought y_c , for hydraulically smooth regime, developed for the total absence of roughness, for Reynolds numbers greater than 2300.

3.1.2. The case of a fully rough turbulent flow state

Surface roughness can significantly influence the fluid dynamics, such as it can reduce the performances hydraulics [14]. So in this case, we are interested in the fully turbulent rough flow in a triangular-shaped channel.

We know that the rough turbulent flow state, corresponding to high values of the *Reynolds* number ($Re \rightarrow \infty$). This shows well, when we introducing Eq. (5) into Eq.(10) and simplifying results in:

$$\left(\frac{\epsilon \sqrt{1+m^2}}{7.42m} \right) \frac{10^{2\sqrt{f_c}}}{y_c} + \left(\frac{2.51}{\sqrt{f_c}} \right) \frac{10^{2\sqrt{f_c}}}{Re_{c}} - 1 = 0 \quad (21)$$

So, when ($Re_{c} \rightarrow \infty$), Eq.(21) can be rewritten as :

$$y_c = \frac{10^{2\sqrt{f_c}}}{7.42} \left(\frac{\sqrt{1+m^2}}{m} \right) \epsilon \quad (22)$$

Eq. (22) is explicit for the critical depth sought y_c , in the rough turbulent flow regime. From this equation, one can clearly observe that:

1. The roughness ϵ , has a direct effect on the value of the critical depth;
2. For the same roughness ϵ , the critical depth y_c increases as the side slope m increases, i.e. when θ decreases or when the apex angle increases;
3. At fully rough turbulent flow state, the friction factor f no longer depends on Re at all, and the curve of f becomes horizontal in Moody's diagram. So, for m and f_c are known parameters, the ratio of critical depth to absolute roughness is a constant, i.e. $(y_c/\epsilon) = \text{constant}$.

Generally, the relationship (22) shows that an increase in wall roughness can lead to an increase in friction losses, which can reduce the specific energy and increase the flow depth. Conversely, a decrease in wall roughness can lead to a decrease in frictional losses, which can lead to an increase in specific energy and a decrease in flow depth.

It is observed from relations (19) and (22), that the effect of the critical slope S_c , remains evident through the critical friction factor f_c .

Practical example

The Smooth triangular channel whose side slope of these walls is $m = 1.0$, longitudinal slope $S_{02} = 0.0$, experimental discharge $Q = 0.01357 \text{ m}^3/\text{s}$ and the critical depth $y_{c2} = 0.13028915 \text{ m}$ [12].

1. Calculate the critical depth y_c according Eq.(19);
2. Calculate the critical Reynolds number Re_{c} , and what is the regime flow?

Solution

1. The critical slope S_c is calculated using the Swamee formula "Swamee, 2012)":

$$S_c = \frac{\sqrt{1+m^2}}{6m} \left(\ln \left(\frac{\epsilon \sqrt{1+m^2}}{6.892} \left(\frac{g}{m^3 Q^2} \right) + 3.38 \left(\frac{\nu \sqrt{1+m^2}}{g m^2 Q^{3/2}} \right)^{0.9} \right) \right)^{-2} \quad (23)$$

The friction factor f_c expression is obtained by Inserting Eq.(23) into Eq.(7) and taking in consideration ($\epsilon \rightarrow 0$) yields:

$$f_c = \frac{4}{3} \left(\ln \left(3.38 \left(\frac{\nu \sqrt{1+m^2}}{g m^2 Q^{3/2}} \right)^{0.9} \right) \right)^{-2} = \frac{4}{3} \left(\ln \left(3.38 \left(\frac{10^{-6} \sqrt{2}}{(9.81 \times 0.01357)^{3/2}} \right)^{0.9} \right) \right)^{-2} \Rightarrow f_c = 0.016485671$$

According to Eq. (19), the critical depth y_c is:

$$y_c = \left(\frac{2.51 \times 10^{2\sqrt{0.016485671}}}{\sqrt{0.016485671}} \right)^{2/3} \left(\frac{(10^{-6})^2}{2 \times 9.81} \right)^{1/3} = 0.133756407 \text{ m}$$

We have: $y_c > y_{c2}$

The critical depth calculated by relation (19), is slightly higher than the value calculated in an experiment [12]. This is due to the effect of the viscosity of the flowing fluid. This in turn disrupts the flow, leading to an increase in depth.

2. According the relation (20) :

$$Re_{c} = 2.51 \left(\frac{10^{2\sqrt{f_c}}}{\sqrt{f_c}} \right) = 2.51 \left(\frac{10^{2\sqrt{0.016485671}}}{\sqrt{0.016485671}} \right) = 153216.8037 > 2300$$

By relating the values of ($\epsilon \rightarrow 0$) and $Re_{c} = 153216.8037$ in Moody's diagram, we have a point on the hydraulically smooth zone. So the turbulent flow regime in the channel is hydraulically smooth.

Conclusion

The concept of critical mode, is used in solving many hydraulic problems of calculation of water facilities. Here's why this study is essentially proposed for a more precise design, of a triangular-shaped channel in a critical flow state, taking into account all the geometrical parameters of the channel, such as surface roughness ϵ , and the bed slope of the channel through the friction factor f_c . Also the properties of the liquid in the flow state such as the kinematic viscosity ν . This study found the function $\varphi(y_c, f_c, m, \epsilon, \nu, g) = 0$, its defined by the Eq.(11), which calculates the critical depth y_c value, with more accurately. Which gives us the correct design of the channel, practically corresponds to the quality of using this channel. The equation (11) was inferred from using both universally accepted relationships of Darcy-Weisbach and Colebrook, which are valid for turbulent flow types.

This study also paid particular attention to the application of the relation (11) in two cases widely applied in water resources engineering. These are the turbulent flow over a smooth surface first, and then over a rough surface. This leads to two distinct relations, for the calculation of the critical depth, respectively Eq.(19) and Eq.(22). In the case of turbulent flow over a smooth surface, the critical Reynolds number Re_{c} , increases only with decreasing coefficient of friction f_c .

The approach adopted in this study allows us to confirm that the demonstrated formulas are characterized by high accuracy. They are useful to designers and engineers in determining critical depth, when designing suitable triangular-shaped channel that carry water. So, During the operation of free-flow hydraulic structures, different types of near-critical flows can occur and have negative effects on the condition of structures. Thus, the main parameters and particularities of these flows should be taken into account during the design, construction and operation of such structures.

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